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FAULT DETECTION IN LINEAR DISCRETE DYNAMIC SYSTEMS USING GENERALIZED–LIKELIHOOD–RATIO TECHNIQUE

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Generalized–likelihood–ratio (GLR) technique is used for fault detection in linear discrete dynamic systems. Based on fault detectability, three detection methods are first introduced; a reduced order step–hypothesized GLR (SH-GLR) method, a reduced order tracking functional subspace method(TFSM), which makes use of the system information about input and observation, and a pattern recognition based method (PRBM), which recognizes the pattern of the curve of the reduced order SHGLR to detect the fault. Finally, the robustness on uncertainties of the system is considered.

1. Introduction

Fault detection and/or diagnosis are important from the viewpoints of improving system availability and protecting against disasters. From this perspective, many fault detection and diagnosis methods have been developed (for example, see Willsky, 1976; Isermann, 1984; Basserville and Benvensite, 1986). Specifically, GLR technique is well–known for its rapid fault detection for the dynamic systems whose mathematical models are known (Willsky, 1976). But, in its application, adequate hypotheses must be adopted. The step hypothesis, which models the anomaly vector function appearing in the system by a step function, has been often used for its convenience, because it only necessitates the anomaly function to have a meaningful bias over an interval (Tylee, 1982; Ono and Kumamaru, 1984). This hypothesis is sufficiently robust in the sense that it does not require any exact mode of the fault.

This paper first introduces, based on fault detectability, a reduced order step–hypothesized GLR (SHGLR) method which considerably improves the detection performance. Using only the SHGLR method, however, good detection result cannot always be expected because the actual anomaly vector function cannot necessarily be well modelled by a step function. Thus, the paper furthermore presents other two more general detection methods; one of which, named here TFSM, makes use of the system information about input and observation effectively to track and estimate the unknown anomaly vector function, whereas the other, called here PRBM, recognizes the pattern of the curve of the reduced order SHGLR to detect the fault.

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2. System Description

The system to be considered is :

$$x(k+1) = Ax(k) + Bu(k) + \Gamma w(k) \tag{1}$$

$$y(k) = Hx(k) + v(k) \tag{2}$$

where $x(k) \in R^n$: the state, $u(k) \in R^r$: the input, $y(k) \in R^m$: the observation, and $w(k) \in R^p$ and $v(k) \in R^m$ are mutually independent white Gaussian noises with zero means and covariances Q and R (positive definite). The initial state $x(0)$ is assumed to be a Gaussian random variable with known mean and covariance independent of the noises.

When an anomaly/fault occurs to the plant or the sensors, it generally causes changes ΔA and ΔB or ΔH in the system matrices A, B and H. This means that an unexpected time varying function such as $\Delta Ax(k) + \Delta Bu(k)$ or $\Delta Hx(k)$ appears to the dynamics or to the sensor equation after the anomaly. For dynamics fault, for instance, we have

$$x(k+1) = Ax(k) + Bu(k) + \Gamma w(k) + \sigma(k+1, \theta) f(k) \tag{3}$$

where θ and $f(k)$ are respectively an unknown onset time of the fault and an unknown anomaly vector function of dimension n, and $\sigma(k, \theta)$ is the unit step function which takes the value 1 for $k \geq \theta$.

A similar discussion applies to sensor fault, but for clarification we are concerned here only with the dynamics fault.

3. Detectability by the Conventional SHGLR Method

We use a Kalman filter to estimate the state. The filter equations are given by:

$$\hat{x}(k|k-1) = A\hat{x}(k-1|k-1) + Bu(k-1) \tag{4}$$

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)\gamma(k) \tag{5}$$

$$\gamma(k) = y(k) - H\hat{x}(k|k-1) \tag{6}$$

$$P(k|k-1) = AP(k-1|k-1)A^T + \Gamma Q \Gamma^T \tag{7}$$

$$P(k|k) = P(k|k-1) - K(k)HP(k|k-1) \tag{8}$$

where

$$K(k) = P(k|k-1)H^T V^{-1}(k) \tag{9}$$

$$V(k) = HP(k|k-1)H^T + R \tag{10}$$

In normal operation, the innovation sequence $\gamma(k)$ in the filter behaves as a white Gaussian sequence with mean zero and a known covariance $V(k)$. We denote it by $\gamma^0(k)$. But, in fault condition, the anomaly function $f(k)$ affects the innovation sequence as follows:

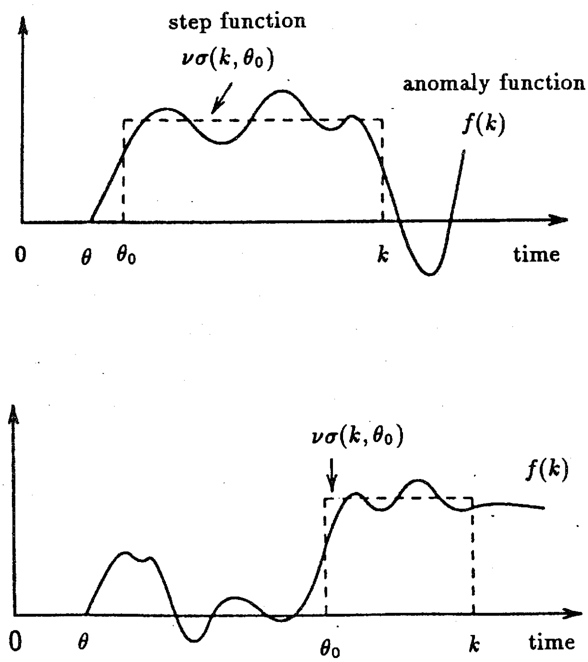
$$\gamma(k) = \gamma^0(k) + \Delta\gamma(k;\theta) \tag{11}$$

where

$$\Delta\gamma(k;\theta) = G^0(k;\theta) * f(k) \triangleq \sum_{j=\theta}^{k} G^0(k;j) f(j-1) \tag{12}$$

Here, $G^0(k;j)$ and $*$ represent respectively a fault signature matrix for a dynamics jump and the symbol of a convolution (Willsky and Jones, 1976; Tanaka *et al.*, 1987). From (11), (12) we find that the fault detection can be made by noticing the change in the innovation sequence $\gamma(k)$.

The conventional SHGLR method is the method which detects the fault by modelling the anomaly vector function $f(k)$ by a step function $\nu\sigma(k,\theta_0)$ $(\nu \in R^n)$ and estimating it. The situation of the modelling is graphically shown in Figure 1 for two cases.



θ : onset time of the fault

k : current time

Fig. 1. Modelling of anomaly function by a step function.

More accurately, the maximum–likelihood–estimate (MLE) $\hat{\boldsymbol{\nu}}(k;\theta_0)$ of the step vector $\boldsymbol{\nu}$ is obtained under the step hypothesis that a step function $\boldsymbol{\nu}\sigma(k,\theta_0)$ has added from the time θ_0 and the step–hypothesized GLR(SHGLR) defined as

$$\ell(k;\theta_0) = \ell n \left[\frac{P(\gamma^k/H_1, \theta_0, \hat{\boldsymbol{\nu}}(k;\theta_0))}{P(\gamma^k/H_0)} \right] \tag{13}$$

is computed, and whether the system is faulty or not is judged with the SHGLR. Note that in (13) γ^k means $\gamma(j)(\theta_0 \leq j \leq k)$, and H_0, H_1 represent respectively the hypotheses of no fault and fault.

In implementation, however, the maximum SHGLR over a window $\Theta = [k-M, k-N]$ is adopted to raise the detection performance. Anyway, with the SHGLR method, the value of the SHGLR serves as the index for judging the occurrence of an anomaly.

Calculating the SHGLR, we have

$$\ell(k;\theta_0) = (1/2)\hat{\boldsymbol{\nu}}(k;\theta_0)^T \boldsymbol{C}(k;\theta_0)\hat{\boldsymbol{\nu}}(k;\theta_0) \tag{14}$$

where

$$\hat{\boldsymbol{\nu}}(k;\theta_0) = \boldsymbol{C}^{-1}(k;\theta_0) \sum_{j=\theta_0}^{k} \boldsymbol{G}^T(j;\theta_0)\boldsymbol{V}^{-1}(j)\gamma(j) \tag{15}$$

$$\boldsymbol{C}(k;\theta_0) \triangleq \sum_{j=\theta_0}^{k} \boldsymbol{G}^T(j;\theta_0)\boldsymbol{V}^{-1}(j)\boldsymbol{G}(j;\theta_0) \tag{16}$$

$$\boldsymbol{G}(j;\theta_0) \triangleq \sum_{i=\theta_0}^{j} \boldsymbol{G}^0(j;i) \tag{17}$$

We introduce now the inner product and the norm defined by

$$\langle \boldsymbol{a}(k), \boldsymbol{b}(k)\rangle_F \triangleq \sum_{j=\theta_0}^{k} \boldsymbol{a}^T(j)\boldsymbol{V}^{-1}(j)\boldsymbol{b}(j) \tag{18}$$

$$||\boldsymbol{a}(k)||_F^2 \triangleq \langle \boldsymbol{a}(k), \boldsymbol{a}(k)\rangle_F \tag{19}$$

and consider the vector $\boldsymbol{\nu}_0$ such that

$$J(\boldsymbol{\nu}_0) \leq J(\boldsymbol{\nu}) \text{ for all } \boldsymbol{\nu} \in R^n \tag{20}$$

where

$$J(\boldsymbol{\nu}) \triangleq ||\Delta\gamma(k;\theta) - \boldsymbol{G}^0(k;\theta_0) * \boldsymbol{\nu}\sigma(k,\theta_0)\ ||_F^2$$

$$= ||\boldsymbol{G}^0(k;\theta) * \boldsymbol{f}(k) - \boldsymbol{G}(k;\theta_0)\boldsymbol{\nu}||_F^2 \tag{21}$$

(21) is nothing but the approximation error in the anomaly effect $\Delta\gamma(k;\theta) = G^0(k;\theta) * f(k)$ in the interval $[\theta_0, k]$ by the step hypothesis. We find that ν_0 is the step vector which optimally models the anomaly function $f(k)$ under the step hypothesis in the interval $[\theta_0, k]$ in the sense that the difference in the innovation sequence is minimum. Note that ν_0 is not necessarily an arithmetic mean of $f(k)$ on the interval $[\theta_0, k]$, but a vector of such a kind. At any rate, expressing the effect $\Delta\gamma(k;\theta)$ by:

$$\Delta\gamma(k;\theta) = G(k;\theta_0)\nu_0 + \Delta\gamma^{\#}(k;\theta) \tag{22}$$

we obtain a condition for $\Delta\gamma^{\#}(k;\theta)$ as follows:

$$\langle\Delta\gamma^{\#}(k;\theta),\ G(k;\theta_0)\nu\rangle_F = 0 \quad \text{for any } \nu \in R^n \tag{23}$$

Substituting $\gamma(j) = \Delta\gamma(j;\theta) + \gamma^0(j)$ (see (11)) into (15) and considering the relations (22) and (23), we immediately find that the MLE $\hat{\nu}(k;\theta_0)$ obeys Gaussian distribution with mean ν_0 and covariance $C^{-1}(k;\theta_0)$ and that the probability distribution of the SHGLR $\ell(k;\theta_0)$ becomes from (14) the non-central Chi-squared distribution with n degrees-of-freedom and the non-centrality parameter

$$\delta(k;\theta_0)^2 = \nu_0^T C(k;\theta_0)\nu_0 \tag{24}$$

We find from (16) and (19) that $\delta(k;\theta_0)^2$ is the squared norm (in the sense of (19)) of the function $G(j;\theta_0)\nu_0$. This function approximates the anomaly effect $\Delta\gamma(j;\theta)$ in the interval $[\theta_0, k]$ optimally under the step hypothesis, and ν_0 represents the step vector of the step function. This distribution is denoted here by $\chi^2(n,\delta^2)$. On the other hand, letting $\delta^2 = 0$, we get Chi-squared distribution with n degrees-of-freedom as the distribution of the SHGLR under no fault. We denote it by $\chi^2(n)$. Thus, whether fault detection can be easily accomplished or not depends on the difference between the two distributions $\chi^2(n,\delta^2)$ and $\chi^2(n)$.

Introducing the discriminating measure *divergence* (Kullback, 1959) to evaluate the distance, we can see that the divergence is characterized by the non-centrality parameter $\delta(k;\theta_0)^2$ and that it is a monotonically increasing function of the parameter $\delta(k;\theta_0)^2$ (Tanaka, 1988). From these facts, we see that we can define the fault detectability by the conventional step-hypothesized GLR method as the non-centrality parameter $\delta(k;\theta_0)^2$. We thus see that the conventional SHGLR method is effective when the anomaly vector function $f(k)$ has a comparatively large non-centrality parameter δ^2 somewhere after the fault. This is satisfied if $f(k)$ has a meaningful bias on some interval.

4. Order Reduction in the Step Hypothesis

It is noteworthy that there is a weakly-diagnosable-space(WDS) whose constituent vector ν has a particularly weak influence on the non-centrality parameter δ^2.

Below, we give only the result for the WDS for the dynamics fault for simplicity. We assume, however, that the fault occurs in the stationary state of the

Kalman filter, because it is quite rare for fault to occur. Of course, the result is valid even in quasi–stationary state of the filter.

Theorem 1. *We assume that the fault modelled well by a step function occurs in the stationary state of the Kalman filter. Then, the WDS for the dynamics step vector is given by the range space of* $(I_n - A)$ *of* $N(H)$, *i.e.,* $(I_n - A)N(H)$, *where* $N(H)$ *denotes the null space of the observation matrix* H *(Tanaka, 1988; Tanaka, 1989).*

Proof. Using the recursive equations on $G^0(k;\theta_0)$ (Willsky and Jones, 1976) and equation (17), the anomaly effect by the dynamics step $\nu\sigma(k,\theta_0)$ on the innovation sequence can be expressed as follows:

$$G(k;\theta_0)\nu = G^0(k;\theta_0) * \nu\sigma(k,\theta_0) = H[I_n + \sum_{j=\theta_0}^{k-1}(\Pi_{k-1}\Pi_{k-2}\cdots\Pi_j)]\nu \tag{25}$$

where $\Pi_j = A(I_n - K(j)H)$ and I_n is the $n \times n$ identity matrix. Thus, in the stationary state, we have

$$G(k;\theta_0)\nu = H\sum_{j=0}^{k-\theta_0}\Pi^j\nu \tag{26}$$

where $\Pi = A(I_n - K^*H)$. Here, K^* denotes a Kalman gain in the stationary state. Since we are now assuming the stationary state, Π is a stable matrix. This is easily verified if we consider that the recursive equation for the estimation error $e(k|k) = x(k) - \hat{x}(k|k)$ is given by

$$e(k|k) = (I_n - K(k)H)Ae(k-1|k-1)$$
$$+(I_n - K(k)H)\Gamma w(k-1) - K(k)v(k) \tag{27}$$

and that the matrices $(I_n - K(k)H)A$ and $A(I_n - K(k)H)$ are similar to each other. The similarity of the two matrices comes from the non–singularity of the matrix $(I_n - K(k)H)$ which is the result of the positive definiteness of the estimation error covariance of $x(k)$ (see Appendix).

Combining the stability of the matrix Π with (26), for an appropriately large $(k - \theta_0)$, we have:

$$G(k;\theta_0)\nu = H(I_n - \Pi)^{-1}\nu \tag{28}$$

This means that the non–centrality parameter $\delta(k;\theta_0)^2$ by the step vector ν which satisfies $H(I_n - \Pi)^{-1}\nu = 0$ is considerably quickly saturated with time k (see (16) and (24)). We call the space constructed by such a vector ν a weakly–diagnosable–space (WDS). This space is obviously the null space of $H(I_n - \Pi)^{-1}$. Using the definition of Π, we can rewrite the space as follows:

$$N(H(I_n - \Pi)^{-1}) = (I_n - \Pi)N(H)$$

$$= (I_n - A + AK^*H)N(H) = (I_n - A)N(H) \quad (29)$$

This completes the proof (Q.E.D.).

From Theorem 1, we see that the WDS for the dynamics step has the dimension of $(n-m)$, where m is the rank of H. Here, H is assumed to be of full rank and so $\text{rank}[H] = m$.

This WDS property obviously comes from the saturation of the $(n-m)$ smallest eigenvalues of the matrix $C(k;\theta_0)$ with the time k. Theoretically, the saturation occurs when $(k-\theta_0)$ becomes appropriately large. But, even at the time k when the saturation does not occur, the subspace spanned by the eigenvectors corresponding to the $(n-m)$ smallest eigenvalues is very close to the WDS. From this fact, it seems quite reasonable for the step hypothesis to assume only the step function whose step vector is included in the orthogonal complementary space of the WDS (hereafter the space will be called SDS (strongly–diagnosable–space)). Because, by this procedure, the number of unknown parameters to be estimated can be reduced from n to m with the non–centrality parameter $\delta(k;\theta_0)^2$ almost unchanged. This means that the degrees of freedom of the central and non–central Chi–squared distributions, by which the SHGLR's under no fault and fault conditions are respectively governed, are decreased from n to m with the non–centrality parameter almost unchanged, and thus a much larger divergence is obtained between the two distributions than before.

Summarizing, modelling the anomaly vector function $f(k)$ by the step function $\nu_s\sigma(k,\theta_0)$ ($\nu_s \in$SDS) can offer a much higher detection performance than before.

The maximum–likelihood–estimate of the reduced order step vector ν_s and the reduced order SHGLR are obtained as:

$$\hat{\nu}_s(k;\theta_0) = S[S^T C(k;\theta_0)S]^{-1} \times [S^T \sum_{j=\theta_0}^{k} G^T(j;\theta_0)V^{-1}(j)\gamma(j)] \quad (30)$$

$$\ell_s(k;\theta_0) = \frac{1}{2}\hat{\nu}_s^T(k;\theta_0)C(k;\theta_0)\hat{\nu}_s(k;\theta_0) \quad (31)$$

where S is the matrix composed of the orthonormal basis vectors of the subspace SDS.

$$S \triangleq [s_1, s_2, \cdots, s_m] \quad (32)$$

5. Reduced Order Tracking Functional Subspace Method

For the system where the step hypothesis does not work well, we can take a direct approach of estimating the anomaly function $f(k)$ as a linear combination of appropriate basis functions.

Letting the basis functions (n–dimensional) be

$$\phi_i(k) = \psi_i(k, \theta_0)\sigma(k, \theta_0) \quad (1 \le i \le q) \tag{33}$$

the MLE of the expansion coefficients $\{c_i\}$ of the basis functions are obtained by

$$\underset{\{c_i\}}{Min} \|\gamma(k) - G^0(k;\theta_0) * \sum_{i=1}^{q} c_i\phi_i(k)\|_F^2 \tag{34}$$

and the Generalized–Likelihood–Ratio (GLR) for this case is expressed as

$$\ell(k;\theta_0) = \frac{1}{2}\{\|\gamma(k)\|_F^2 - \underset{\{c_i\}}{Min} \|\gamma(k) - \sum_{i=1}^{q} c_i G^0(k;\theta_0) * \phi_i(k)\|_F^2\} \tag{35}$$

Introducing now the new basis functions $\epsilon_i = G^0(k;\theta_0) * \phi_i(k)$ $(1 \le i \le q)$ and defining the functional subspace S spanned by the basis functions, we can see that the GLR (35) becomes the squared norm of $\gamma^*(k)$, i.e.,

$$\ell(k;\theta_0) = \frac{1}{2}\|\gamma^*(k)\|_F^2 \tag{36}$$

where $\gamma^*(k)$ is the orthogonal projection of the $\gamma(k)$ onto the space S (see Fig. 2). The conventional and reduced order step hypotheses mentioned before are the special cases where $\phi_i(k) = e_i\sigma(k,\theta_0)(1 \le i \le n)$ and $\phi_i(k) = s_i\sigma(k,\theta_0)$ $(1 \le i \le m)$, respectively (where e_i is the i–th natural basis vector of R^n and s_i is the vector defined in (32)).

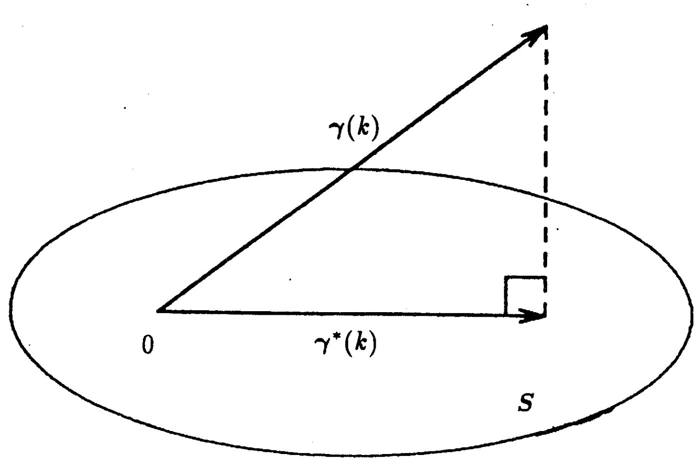


Fig. 2. Orthogonal projection of the innovation sequence $\gamma(k)$ onto the subspace S.

Anyway, with this method, the GLR obeys the non–central Chi– squared distribution $\chi^2(q,\tilde{\delta}^2)$ with q degrees of freedom and the non–centrality parameter:

$$\tilde{\delta}^2 = \| \text{ orthogonal projection of } G^0(k;\theta) * f(k) \text{ onto the space } S \|_F^2 \tag{37}$$

Under no fault, the GLR obeys of course the central Chi-squared distribution $\chi^2(q)$ with the same degrees of freedom q.

From the fact, we find that we should not use extra basis functions when adopting the method. This is because when we use extra basis functions, the degrees of the two distributions increase whereas the non-centrality parameter $\tilde{\delta}^2$ saturates and thus the *divergence* between the two distributions becomes small and fault detectability considerably degrades. From this viewpoint, making use of system information is recommended in the construction of the basis functions, instead of adopting orthogonal series. As mentioned before, anomaly effect of the anomaly function $f(k)$ appearing in the innovation sequence is $G^0(k;\theta) * f(k)$, where $f(k) = \Delta A x(k) + \Delta B u(k)$ (we are concerned here with the anomaly of parameter change). We thus see that an effective approach is to adopt the basis functions $G^0(k;\theta_0) * e_i u_j(k)$ $(1 \leq i \leq n,\ 1 \leq j \leq r)$ and $G^0(k;\theta_0) * e_i y_j(k)$ $(1 \leq i \leq n,\ 1 \leq j \leq m)$. Of course, these basis functions are defined in the innovation space. The reason why the observation signal $y(k)$ is adopted instead of $x(k)$ is that $x(k)$ is not directly available.

According to our previous discussion, the smaller the number of basis functions, the higher the detectability. From this viewpoint, we had better adopt the following basis functions:

$$\{\eta_j(k)\} \triangleq \{G^0(k;\theta_0) * s_i u_j(k) \ \ (1 \leq i \leq m,\ \ 1 \leq j \leq r),$$
$$G^0(k;\theta_0) * s_i y_j(k) \ \ (1 \leq i \leq m,\ \ 1 \leq j \leq m)\}$$

where $s_i (1 \leq i \leq m)$ denote the basis vectors of the strongly-diagnosable-space (SDS) (Tanaka *et al.*, 1987; Tanaka, 1989). Taking into account, however, the magnitude and the linear independence of the functions, further order reduction is possible. That is, by considering the Gram matrix G whose (i,j) element is defined by

$$g(i,j) \triangleq \langle \eta_i(k), \eta_j(k) \rangle_F \tag{38}$$

and considering the p eigenvectors $\xi_i (1 \leq i \leq p)$ corresponding to the largest (i.e., dominant) p eigenvalues of the matrix, desirable basis functions to be used are given by

$$\sum_j \xi_{ij} \eta_j(k) \quad (1 \leq i \leq p)$$

where ξ_{ij} is the j-th element of the i-th eigenvector ξ_i.

According to our recent research, we found that the vectors $\{s_i\}$ should be changed according to the signal to be used. For example, for the signal $u_j(k)$ the optimal vectors $\{s_i\}$ are obtained as the dominant eigenvectors of the $n \times n$ matrix which is symbolically defined by

$$\langle G^0(k;\theta_0) * u_j(k),\ G^0(k;\theta_0) * u_j(k)\rangle_F$$

The (i_1, i_2) element of the matrix is defined by

$$\langle g^0_{i_1}(k;\theta_0) * u_j(k),\ g^0_{i_2}(k;\theta_0) * u_j(k)\rangle_F$$

where $g^0_i(k;\theta_0)$ is the i–th column vector of the matrix $G^0(k;\theta_0)$. For each signal of $\{u_j(k)\}$ and $\{y_j(k)\}$ the vectors $\{s_i\}$ which define the elements in the set $\{\eta_j(k)\}$ are different. This consideration comes from the sensitivity analysis that desirable basis functions must affect the innovation sequence very largely. Evaluating this effect by the norm defined by (18), (19), we get the result mentioned above.

6. Pattern Recognition Based Detection Method

We introduce here another detection method. This is the pattern recognition based detection method (PRBM) which monitors the pattern of the curve of the maximum reduced order SHGLR and detects the fault. This method is quite easy to implement and also requires no computational burden compared to the conventional SHGLR method. The validity of this approach can be easily understood by our intuition. That is, many experiences show that even when after the anomaly the maximum SHGLR does not take as large values as we might expect, the time in which it takes comparatively large values is long and thus the curve of the SHGLR suggests to us that an anomaly has occurred. By adopting this approach we can expect not only the effective detection of the faults which are difficult to detect with the reduced order SHGLR method and the reduced order tracking functional subspace method, but also a decrease in false alarms caused by occasional large maximum GLR's taken before the anomaly.

With this method, furthermore, the fault, where some parameters change sequentially as time passes, can be detected much faster than with the reduced order tracking functional subspace method. This is because, to detect the fault effectively, the latter method must wait until the time when the changes ΔA and ΔB of the matrices A and B settle down to their final values.

Before handling the maximum reduced order SHGLR's and describing the idea of the method, we first consider the behavior of the M reduced order SHGLR's which are respectively computed for the pairs (k_i, θ_{i0}) $(1 \le i \le M)$ of the M successive windows Θ_i $(1 \le i \le M)$ (with no intersection, see Fig. 3). That is, we consider the M reduced order SHGLR's defined by (Tanaka and Müller, 1990)

$$\ell_{Fi} \triangleq \ell(k_i;\theta_{i0}) \ \ (1 \le i \le M) \tag{39}$$

where k_i and θ_{i0} are respectively the upper and lower edges of the i–th window Θ_i. k_1 of course implies the current time k.

We call the SHGLR's of (39) fixed–SHGLR's, because the times at which step functions are assumed to have added are respectively fixed at the initial times $\{\theta_{i0}\}$ of the M windows $\{\Theta_i\}$.

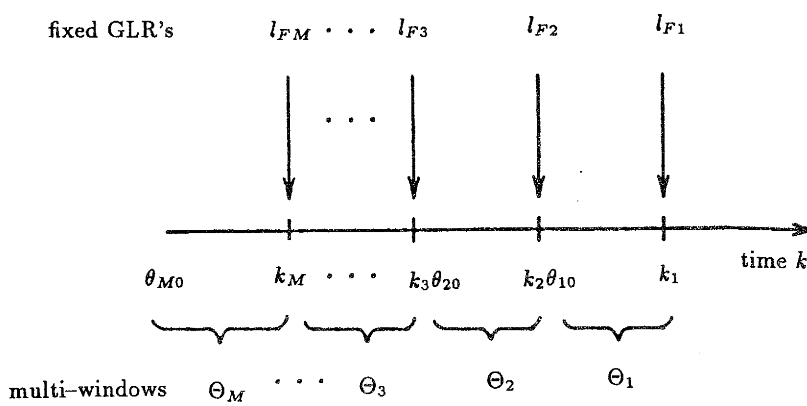


Fig. 3. Introduction of multi–windows for effective tracking of the anomaly function.

Under no fault condition, each fixed–SHGLR ℓ_{Fi} obeys the Chi–squared distribution with m degrees of freedom $\chi^2(m)$. But, once an anomaly has occurred to the system, ℓ_{Fi} obeys the non–central Chi–squared distribution $\chi^2(m, \delta_i^2)$ with m degrees of freedom and the non–centrality parameter:

$$\delta_i^2 = \boldsymbol{\nu}_i^T C(k_i; \theta_{i0}) \boldsymbol{\nu}_i \tag{40}$$

where $\boldsymbol{\nu}_i$ is a reduced order step vector which models the anomaly vector function in the i–th window Θ_i. Since the M successive windows are separated from each other and thus the M fixed–SHGLR's $\ell_{Fi}(1 \le i \le M)$ are mutually independent, the divergence between the two random vectors $\boldsymbol{L} = (\ell_{F1}, \ell_{F2}, \dots, \ell_{FM})^T$ arising from no fault and fault conditions can be calculated as:

$$D(\boldsymbol{L}/H_0; H_1) = \sum_{i=1}^{M} D(\ell_{Fi}/H_0; H_1) \tag{41}$$

If we assume that $\delta_i^2(1 \le i \le M)$ take nearly equal values for $\theta \le \theta_{M0}$, we can get an almost M times larger divergence than before by considering such M fixed–SHGLR's $\ell_{Fi}(1 \le i \le M)$. This means that fault detection becomes easier as we use many windows in the method.

This fault detection can be regarded as a classification problem between fault and no fault patterns. Applying the Bayesian decision rule, which minimizes the probability of errors, to the classification problem, we get the following suboptimal decision boundary (Tanaka and Müller, 1990; Fukunaga, 1972):

$$\sum_{i=1}^{M} \ell_{Fi} = \epsilon_0 \tag{42}$$

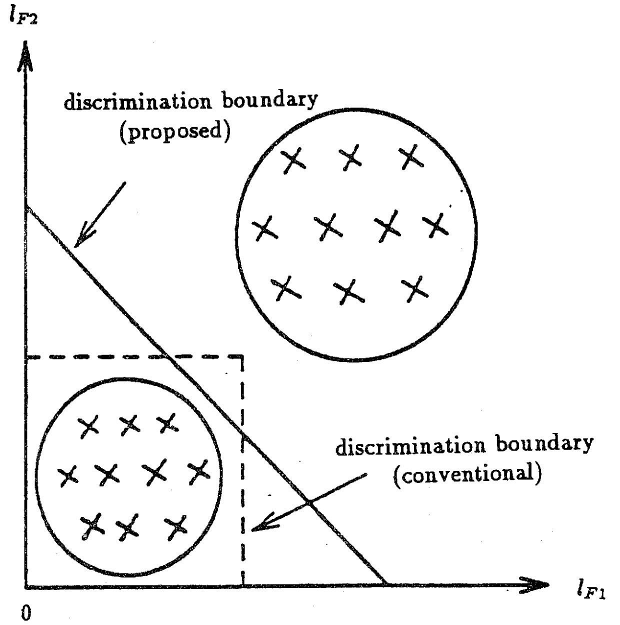


Fig. 4. Discrimination boundaries in the space of the fixed–GLR's.

Thus, we get a detection rule such that

$$\sum_{i=1}^{M} \ell_{Fi} = \begin{cases} \leq \epsilon_0 \rightarrow & \text{No Fault} \\ > \epsilon_0 \rightarrow & \text{Fault} \end{cases} \tag{43}$$

The parameter ϵ_0 is a threshold in detecting the fault. Using the terminologies in pattern recognition theory, the proposed method corresponds to introducing a linear discriminant function into the L–space, where $L = (\ell_{F1}, \ell_{F2}, \ldots, \ell_{FM})^T$, in order to detect the fault, whereas the conventional SHGLR method judging the fault occurrence by the value of $\{\ell_{Fi}\}$ corresponds to adopting a piece–wise linear discriminant function (see Fig. 4).

We next describe some modifications in applying the method. As previously mentioned, the larger the non–centrality parameter $\{\delta_i^2\}$, the more easily the fault can be detected. Thus, if the maximization of δ_i^2 is attempted with respect to the parameter θ_i in each window Θ_i, instead of fixing θ_i at the initial time θ_{i0} of the window, we can get much larger non–centrality parameters $\delta_i^2 (1 \leq i \leq M)$. Of course, we cannot *a priori* guess where these optimal $\theta_i (1 \leq i \leq M)$ are located in the windows. However,we can naturally imagine that for the optimal location we would be able to have larger GLR's than those which are computed for the fixed θ_i's, i.e., $\{\theta_{i0}\}$. We thus recommend, as a practical approach along this thought, to use the maximum reduced order SHGLR's computed in the windows $\{\Theta_i\}$, i.e.,

$$\ell_i \triangleq \max_{\theta_i \in \Theta_i} \ell(k_i; \theta_i) = \ell(k_i; \hat{\theta}_i) \quad (1 \leq i \leq M) \tag{44}$$

instead of the fixed SHGLR's $\ell_{Fi} = \ell(k_i; \theta_{i0})$ $(1 \leq i \leq M)$. By this approach, however, the actual interval from which fault information is extracted becomes shorter than before, from $[\theta_{i0}, k_i]$ to $[\hat{\theta}_i, k_i]$ in each window. This means that all the observations from θ_{M0} to the current time k are not utilized, which generally yields some detection delay. In order to avoid the detection delay, it is necessary to close the times $k_i (1 \leq i \leq M)$, at which each maximum reduced order SHGLR is computed, to each other. Regretfully, however, we do not have *a priori* any knowledge of how to close them to each other.

Thus, we propose to adopt the statistic defined by the sum of the all maximum reduced order SHGLR's calculated in a time interval $[k - M', k]$ and to monitor the value to see when it exceeds a threshold. This time interval must be comparatively long to raise the reliability of detecting the fault. We call this method a LIPRBM (long interval pattern recognition based method). However, this type of detection method inherently yields a detection delay for the fault, for which a single-step hypothesis can be effectively applied. We therefore finally recommend implementing the reduced order SHGLR method or a SIPRBM (short interval PRBM) together with the originally proposed LIPRBM. That is, we finally have the following detection rule:

$$\begin{cases} \sum_{i=1}^{N'} \ell_i \leq \epsilon_1 \text{ and } \sum_{i=1}^{M'} \ell_i \leq \epsilon_2 & \rightarrow \text{ No Fault} \\ \text{otherwise} & \rightarrow \text{ Fault} \end{cases} \tag{45}$$

where N' and M' are, respectively, the lengths of the short and long intervals on which the newest N' and M' maximum reduced order SHGLR's are summed up and $\ell_i (i = 1, 2, \ldots, N', \ldots, M')$ represent, the maximum reduced order SHGLR's computed at the last M' times including the current time k, i.e.,

$$\ell_i \triangleq \max_{\theta_i} \ell(k_i; \theta_i); \qquad k_i = k - (i - 1)$$

7. On Robustness

The fault detection methods described above all assume the exact knowledge of the system. But, practically, the assumption does not hold. Therefore, it yields a little larger GLR even under normal operation of the system. One approach which prevents the detection methods from causing false alarms is making the threshold a little higher. Here, we give some remarks on robustness of the proposed detection methods and also some ideas to overcome uncertainties of the system.

We first assume that the dynamic equation (1) has the following modelling errors: $\sum_i \alpha_i \Delta \boldsymbol{A}_i^0$, $\sum_j \beta_j \Delta \boldsymbol{B}_j^0$ respectively in the system matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, where $\{\alpha_i\}$, $\{\beta_j\}$ are unknown, whereas $\{\Delta \boldsymbol{A}_i^0\}$ and $\{\Delta \boldsymbol{B}_j^0\}$ are known. This assumption corresponds to considering the case where the system has some uncertainty on the values of its physical parameters. The parameters $\{\alpha_i\}$ and $\{\beta_j\}$ may be constant or time-varying. This case, with the reduced order SHGLR and PRBM methods, the reduced order SHGLR takes large values because of the anomaly function

$$\sum_i \alpha_i \Delta A_i^0 x(k) + \sum_j \beta_j \Delta B_j^0 \; u(k)$$

due to the modelling errors even when the system is operating normally. It is thus required to remove the effect of the modelling errors when using the reduced order SHGLR and PRBM methods.

One approach is minimizing the sum of the maximum reduced order SHGLR's computed over an interval: $\sum_{i=1}^{M'} \ell_i$ with respect to the unknown parameters $\{\alpha_i\}$ and $\{\beta_j\}$ and monitoring the resulting reduced order SHGLR. The reason why we minimize the sum is to raise the reliability of identifying the unknown parameters. After the minimization procedure, the reduced order SHGLR itself is monitored for the reduced order SHGLR method, and the sum of the reduced order SHGLR's for the PRBM method. Powell method, which is known as an optimization method, is available for the minimization. The minimization must be of course achieved on the admissible regions of the unknown parameters $\{\alpha_i\}$ and $\{\beta_j\}$. In order to decrease the computational burden for realizing an on-line use, we can execute the minimization only at appropriately-spaced times and renew the initial values using the data obtained in the preceding minimization procedure.

On the other hand, with the TFS method, too, the GLR under normal operation takes a little larger value because of parameter's misunderstanding. So, it will be desirable not only to make the threshold a little higher, but also to monitor the change of the estimated expansion coefficients, in order to raise the detection performance.

We next consider the case where the noise is correlated against our expectation that the noise is white. If the characteristics of the correlated noise is accurately known, the discussion is still valid through introduction of an augmented state vector, and the detection methods work perfectly. If not, the detection performance by the detection methods will degrade. But, TFSM is expected to have a higher robustness compared to the other two methods, because the detection method utilizes the system information on input and observation over a long interval and enables the anomaly function due to the fault to be discriminated from the correlated noise. If we monitor the change of the expansion coefficients simultaneously, the fault detection can be made more easily, because in normal operation the change is large depending on the noise whereas not large in anomalous operation of the system. To make the other two methods (i.e., SHGLR and PRBM methods) be robust against the noise characteristics, too, we may have to introduce the space of the pattern which are composed of not only the sequential reduced order SHGLR's, but also the MLE's $\{\hat{\nu}_s\}$ and the operating condition of the system. This extension is now under consideration.

8. Numerical Examples

By an example of a 2nd-order servo-system, the superiority of the non-reduced-order PRBM in comparison to the conventional SHGLR method and a Chi-squared test was demonstrated in (Tanaka and Müller, 1990). Also, the reduced order SHGLR and TFS methods were respectively shown to be superior to the non-reduced order SHGLR and TFS methods with a three dimensional system and a linear motor car system (Tanaka and Müller, 1993). Obviously, PRBM recognizing the pattern of the reduced order SHGLR offers a much more higher detection performance than that recognizing the curve of the conventional SHGLR. The merit of the PRBM is that it can, in principle, detect any type of complicated fault. The fault detection methods introduced here should be used according to the fault mode and the system.

9. Conclusions

In the framework of the generalized-likelihood-ratio (GLR) technique, a reduced order SHGLR method, a reduced order tracking functional subspace method (TFSM), and a pattern recognition based method (PRBM) were respectively proposed based on fault detectability. Finally, the robustness on the uncertainties of physical parameters and noise characteristics was discussed.

Although the paper dealt with only fault detection methods, fault diagnosis can be, for example, achieved with a multi-hypotheses test. In addition to the importance of fault detection and diagnosis methods, optimal location of sensors is also important from the viewpoint of raising the detectability and also the separability between dynamics and sensor faults. These considerations are possible within the framework of GLR technique, though omitted here (see (Tanaka, 1989) for the details).

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Appendix

We show here that the matrix $(\boldsymbol{I}_n - \boldsymbol{K}(k)\boldsymbol{H})$ in Theorem 1 is non–singular.

Letting $\boldsymbol{P}(k|k-1)$ and $\boldsymbol{P}(k|k)$ be respectively prediction and estimation error covariances of the state $\boldsymbol{x}(k)$ in the filter, then we have:

$$\boldsymbol{P}(k|k) = (\boldsymbol{I}_n - \boldsymbol{K}(k)\boldsymbol{H})\boldsymbol{P}(k|k-1)$$

From the assumption that the error covariances are positive definite, we can conclude that the matrix $(\boldsymbol{I}_n - \boldsymbol{K}(k)\boldsymbol{H})$ is non–singular.

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