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ADAPTIVE PREDICTIVE CONTROL OF A DISTILLATION COLUMN

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Distillation processes reveal complicated multivariable nonlinear dynamics for which it is difficult to design a high-performance control system. This paper proposes an adaptive control scheme for a distillation column. The proposed adaptive system consists of a multivariable receding-horizon predictive controller using a transfer function model and a recursive least-squares (RLS) based estimator. Simulations show a consistent closed-loop performance despite the uncertain nonlinear characteristics of the distillation column.

Keywords: adaptive control, multivariable predictive controller, distillation column.

1. Introduction

Distillation is the most important separation process in chemical and petroleum industry, and is also very energy-intensive (Yang and Lee, 1997). Designing a high-performance control system for a distillation column is therefore important for improved product quality and energy saving. However, distillation processes feature quite complicated multivariable nonlinear dynamics, for which it is difficult to find simplified models that can be used for controller design. In this paper, we present an adaptive control strategy for a distillation column using a transfer-function model.

It is widely recognized that Generalized Predictive Control (GPC) is one of the most effective control algorithms for adaptive systems. However, the problem with GPC is that there is no clear theory guaranteeing closed-loop stability in terms of GPC tuning knobs. For this problem, solutions were later presented independently by Clarke and Scattolini (1991), Mosca and Zhang (1992) and Kouvaritakis *et al.* (1992). In their methods, an appropriate number of terminal equality constraints are imposed on the output of a plant such that the receding-horizon cost is monotonically non-increasing, thereby guaranteeing closed-loop stability. These schemes can be regarded

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as input/output realizations of the state space controller of Kwon and Pearson (1978), where the state variables are constrained to zero at the end of the costing horizon. The effectiveness of such a scheme for adaptive control is demonstrated in Yoon and Clarke (1994). Recently, a multivariable extension of the stable predictive scheme of Clarke and Scattoloni (1991) is proposed (Yoon and Chow, 1995; Yoon and Kwon, 1998). This type of predictive control method is employed here for adaptive control of a distillation column.

2. Distillation Dynamics

Consider a binary distillation column with multiple trays, a total condenser and a reboiler, which is depicted in Fig. 1.

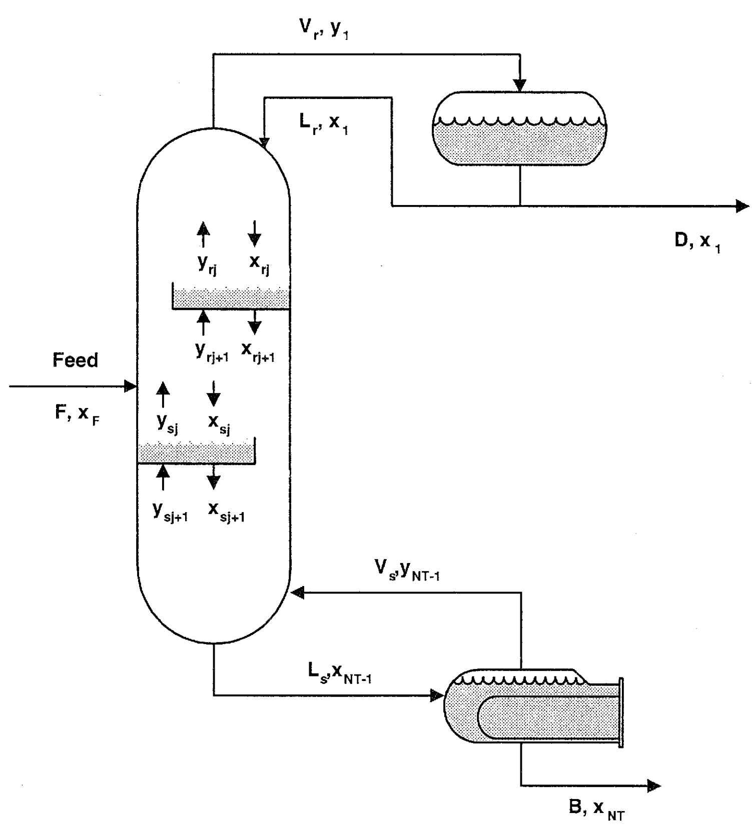

Fig. 1. Binary distillation column.

For simplicity, the following assumptions are made:

- Molar overflow is constant;
- Liquid holdup in each tray is constant;
- Vapor holdup is negligible;
- Vapor leaves a tray at equilibrium with liquid;
- The holdups in the accumulators are constant.

Let the total number of trays be N_T and each tray be numbered from the top (condenser). We then derive a model, which consists of the following equations:

- Total condenser

$$H_c \frac{\mathrm{d}x_1}{\mathrm{d}t} = V_r(y_1 - x_1) = V_r y_1 - (L_r + D)x_1$$

- Rectifying section

$$H_r \frac{\mathrm{d}x_{rj}}{\mathrm{d}t} = V_r(y_{rj} - y_{rj-1}) + L_r(x_{rj-1} - x_{rj}) + (1-q)F_j x_F$$

where $F_j = F$ if $j = N_F - 1$, otherwise $F_j = 0$.

- Stripping section

$$H_s \frac{\mathrm{d}x_{sj}}{\mathrm{d}t} = V_s(y_{sj} - y_{sj-1}) + L_s(x_{sj-1} - x_{sj}) + qF_j x_F$$

where $F_j = F$ if $j = N_F$, otherwise $F_j = 0$.

- Reboiler

$$H_b \frac{\mathrm{d}x_{N_T}}{\mathrm{d}t} = L_s x_{N_T-1} - V_s y_{N_T-1} - B x_{N_T}$$

- Other relations

$$\begin{aligned} L_s &= L_r + qF \\ V_r &= V_s + (1-q)F \\ y_j &= K_j x_{j+1} \end{aligned}$$

The symbols introduced in the above equations are explained in Table 1, and subscripts c, r, s and b denote the condenser, rectifying, stripping and reboiler sections, respectively. The subscript i denotes the i-th tray. The control objective for this distillation column is to regulate the overhead and bottom compositions x_1 and x_{N_T} using the reflux and steam flow rates L_r and V_s.

Table 1. Symbols used to describe the distillation column.

x, y	Liquid and vapor compositions
D, B	Overhead and bottom products
L, V	Liquid and vapor flow rates
F, x_F	Feed and feed composition
q	Feed quality
H	Liquid hold up
K	Vapor-liquid equilibrium constant

3. Predictive Control

In this section, we employ the multivariable predictive control algorithm proposed by Yoon and Chow (1995). The stability properties are obtained in (Yoon and Kwon, 1999).

3.1. Model and Prediction

Consider a MIMO process described by the following CARIMA model:

$$\boldsymbol{A}(q^{-1})\boldsymbol{y}(t) = \boldsymbol{B}(q^{-1})\boldsymbol{u}(t-1) + \frac{\boldsymbol{T}(q^{-1})}{\Delta}\boldsymbol{\xi}(t) \tag{1}$$

where $\boldsymbol{u}(t)$ and $\boldsymbol{y}(t)$ are the input and output vectors, $\boldsymbol{\xi}(t)$ is a vector of uncorrelated random noise sequences, $\boldsymbol{A}(q^{-1})$, $\boldsymbol{B}(q^{-1})$ and $\boldsymbol{T}$ are polynomial matrices, and Δ is the difference operator (i.e. $1-q^{-1}$). It is assumed without loss of generality that $\boldsymbol{A}$ and $\boldsymbol{T}$ are diagonal. It is also supposed for simplicity here that the vectors $\boldsymbol{y}$, $\boldsymbol{u}$ and $\boldsymbol{\xi}$ have the same dimension n (i.e. $\boldsymbol{y}, \boldsymbol{u}, \boldsymbol{\xi} \in \mathbb{R}^n$). For the purpose of formulation, we write the relation between the i-th output and the inputs as

$$A_i(q^{-1})y_i(t) = \boldsymbol{B}_i(q^{-1})\boldsymbol{u}(t-1) + \frac{T_i(q^{-1})}{\Delta}\xi_i(t) \tag{2}$$

where A_i and T_i are the i-th diagonal elements of $\boldsymbol{A}$ and $\boldsymbol{T}$, $\boldsymbol{B}_i$ is the i-th row of $\boldsymbol{B}$, and ξ_i is the i-th element of $\boldsymbol{\xi}$. The optimal prediction $\hat{y}_i(t+k)$ for $y_i(t+k)$ is then obtained by

$$\begin{aligned} \hat{y}_i(t+k) &= \boldsymbol{G}_i^k \Delta \boldsymbol{u}(t+k-1) + f_i(t+k) \\ f_i(t+k) &= \frac{F_i^k}{T_i} y_i(t) + \frac{\boldsymbol{H}_i^k}{T_i}\Delta \boldsymbol{u}(t-1) \end{aligned} \tag{3}$$

where the polynomials F_i^k, $\boldsymbol{G}_i^k$ and $\boldsymbol{H}_i^k$ satisfy the Diophantine identities

$$\begin{aligned} T_i &= A_i \Delta E_i^k + q^{-k} F_i^k \\ E_i^k \boldsymbol{B}_i &= \boldsymbol{G}_i^k T_i + q^{-k}\boldsymbol{H}_i^k \end{aligned} \tag{4}$$

Note that G_i^k is a polynomial (row) vector of order $k-1$ whose coefficients are equivalent to the first k step responses of the system B^i/A^i, i.e.

$$\frac{B_i}{A_i\Delta} = \sum_{l=1}^{\infty} g_i^l q^{-l+1}, \qquad G_i^k = \sum_{l=1}^{k} g_i^l q^{-l+1} \tag{5}$$

On the basis of the predictions given in eqn. (3), the MIMO receding-horizon control law is derived below.

3.2. Control Law

As is standard, we consider the quadratic cost function of the form

$$J = \sum_{i=1}^{n} J_i \tag{6}$$

where

$$\begin{aligned} J_i \; = \; & \sum_{k=N_1}^{N_y-1} \mu_i(k)\big[w_i(t+k) - \hat{y}_i(t+k)\big]^2 \\ & + \sum_{k=N_y}^{N_{2i}} \frac{\mu_i(N_y)}{\gamma}\big[w_i(t+N_y) - \hat{y}_i(t+k)\big]^2 + \sum_{k=0}^{N_u-1} \rho_i(k)\,\Delta u_i(t+k)^2 \end{aligned} \tag{7}$$

This is a MIMO extension of the performance index considered in (Yoon and Clarke, 1995a): $\mu_i(k)$ and $\rho_i(k)$ are positive weighting sequences for the i-th tracking errors and control increments, N_1 and N_{2i} are the lower and upper prediction horizons, N_u is the control horizon, and γ is a non-negative number (≤ 1) introduced to place heavier weighting on the errors further ahead than N_y. Note that if $\gamma = 0$, then we have the following equality constraints:

$$w_i(t+N_y) = \hat{y}_i(t+N_y+k) \quad \text{for} \quad k \in [0, m_i - 1] \tag{8}$$

where m_i is the number of constraints, i.e. $m_i = N_{2i} - N_y + 1$. It is also assumed that the control $\boldsymbol{u}(t)$ does not move after the interval N_u (i.e. $\Delta\boldsymbol{u}(t+k) = \boldsymbol{0}$ if $k \geq N_u$), and that the first term of the cost (7) is ignored when N_1 equals N_y. The weighting sequences $\mu_i(k)$ and $\rho_i(k)$ are normally set to be time-invariant. However, time-varying weighting can be specified to enhance the performance: as in (Yoon and Clarke, 1993), it is suggested that

$$\mu_i(k) \; = \; \alpha^{-2k}, \quad \rho_i(k) \; = \; \alpha^{-2k}\bar{\rho}_i \tag{9}$$

with $\bar{\rho}_i > 0$. This exponential weighting places the closed-loop poles within a circle of radius α (if the control law is stabilizing). The cost function (7) is general in that it can lead to a wide range of existing predictive control methods including GPC ($\gamma = 1$). The use of a nonzero number (< 1) for γ is expected to have effects similar to those observed in receding-horizon control with finite end-point weighting as discussed by Kwon and Byun (1989) and Demircioglu and Clarke (1993).

In order to rewrite the cost (7) in a simple vector form, we define

$$\begin{aligned}
\Delta \boldsymbol{U} &= \left[\Delta \boldsymbol{u}(t)^T \quad \Delta \boldsymbol{u}(t+1)^T \quad \cdots \ \Delta \boldsymbol{u}(t+N_u-1)^T\right]^T \\
\boldsymbol{w}_i &= \left[w_i(t+N_1)\ w_i(t+N_1+1) \cdots \ w_i(t+N_y-1)\right]^T \\
\boldsymbol{y}_i &= \left[\hat{y}_i(t+N_1)\ \hat{y}_i(t+N_1+1) \cdots \ \hat{y}_i(t+N_y-1)\right]^T \\
\boldsymbol{f}_i &= \left[f_i(t+N_1)\ f_i(t+N_1+1) \cdots \ f_i(t+N_y-1)\right]^T \\
\bar{\boldsymbol{w}}_i &= \left[w_i(t+N_y)\ w_i(t+N_y) \quad \cdots \ w_i(t+N_y)\right]^T \\
\bar{\boldsymbol{y}}_i &= \left[\hat{y}_i(t+N_y)\ \hat{y}_i(t+N_y+1) \cdots \ \hat{y}_i(t+N_{2i})\right]^T \\
\bar{\boldsymbol{f}}_i &= \left[f_i(t+N_y)\ f_i(t+N_y+1) \cdots \ f_i(t+N_{2i})\right]^T
\end{aligned} \tag{10}$$

Using these vectors, we re-express eqn. (7) as

$$\begin{aligned}
J &= \sum_{i=1}^{n}\left([\boldsymbol{y}_i-\boldsymbol{w}_i]^T \boldsymbol{Q}\,[\boldsymbol{y}_i-\boldsymbol{w}_i] + \frac{\alpha^{-2N_y}}{\gamma}\,[\bar{\boldsymbol{y}}_i-\bar{\boldsymbol{w}}_i]^T[\bar{\boldsymbol{y}}_i-\bar{\boldsymbol{w}}_i]\right) \\
&\quad + \Delta \boldsymbol{U}^T \boldsymbol{\Lambda}\, \Delta \boldsymbol{U},
\end{aligned} \tag{11}$$

with

$$\begin{aligned}
\boldsymbol{Q} &= \text{diag}\left[\alpha^{-2N_1},\ \alpha^{-2(N_1+1)},\ \ldots,\ \alpha^{-2(N_y-1)}\right] \\
\boldsymbol{\Lambda} &= \text{diag}\left[\boldsymbol{\Lambda}_o,\ \alpha^{-2}\boldsymbol{\Lambda}_o,\ \ldots,\ \alpha^{-2(N_u-1)}\boldsymbol{\Lambda}_o\right] \\
\boldsymbol{\Lambda}_o &= \text{diag}\left[\bar{\rho}_1,\ \bar{\rho}_2,\ \ldots,\ \bar{\rho}_n\right]
\end{aligned} \tag{12}$$

This cost function is subject to the predictions:

$$\boldsymbol{y}_i = \boldsymbol{G}_i \Delta \boldsymbol{U} + \boldsymbol{f}_i, \quad \bar{\boldsymbol{y}}_i = \bar{\boldsymbol{G}}_i \Delta \boldsymbol{U} + \bar{\boldsymbol{f}}_i \tag{13}$$

where the matrices $\boldsymbol{G}_i$ and $\bar{\boldsymbol{G}}_i$ are given by:

$$\begin{aligned}
\boldsymbol{G}_i &= \begin{bmatrix} \boldsymbol{g}_i^{N_1} & \boldsymbol{g}_i^{N_1-1} & \cdots & \boldsymbol{0} \\ \boldsymbol{g}_i^{N_1+1} & \boldsymbol{g}_i^{N_1} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{g}_i^{N_y-1} & \boldsymbol{g}_i^{N_y-2} & \cdots & \boldsymbol{g}_i^{N_y-N_u} \end{bmatrix} \\
\bar{\boldsymbol{G}}_i &= \begin{bmatrix} \boldsymbol{g}_i^{N_y} & \boldsymbol{g}_i^{N_y-1} & \cdots & \boldsymbol{g}_i^{N_y-N_u+1} \\ \boldsymbol{g}_i^{N_y+1} & \boldsymbol{g}_i^{N_y} & \cdots & \boldsymbol{g}_i^{N_y-N_u+2} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{g}_i^{N_{2i}} & \boldsymbol{g}_i^{N_{2i}-1} & \cdots & \boldsymbol{g}_i^{N_y-N_u+m_i} \end{bmatrix}
\end{aligned} \tag{14}$$

Minimizing the cost (11) results in the optimal $\Delta \boldsymbol{U}$:

$$\Delta \boldsymbol{U} = \left(\boldsymbol{\Lambda} + \sum_{i=1}^{n}\left[\boldsymbol{G}_i^T \boldsymbol{Q}\, \boldsymbol{G}_i + \frac{\alpha^{-2N_y}}{\gamma}\bar{\boldsymbol{G}}_i^T \bar{\boldsymbol{G}}_i\right]\right)^{-1} \times \left(\sum_{i=1}^{n}\left[\boldsymbol{G}_i^T \boldsymbol{Q}\,(\boldsymbol{w}_i - \boldsymbol{f}_i) + \frac{\alpha^{-2N_y}}{\gamma}\bar{\boldsymbol{G}}_i^T(\bar{\boldsymbol{w}}_i - \bar{\boldsymbol{f}}_i)\right]\right) \quad (15)$$

The control law (15) may, however, produce numerical difficulties as γ approaches zero, and it is obvious that γ cannot be made zero. To deal with such a situation, we introduce

$$\boldsymbol{p}_i = \frac{\alpha^{-2N_y}}{\gamma}\left[\bar{\boldsymbol{G}}_i \Delta \boldsymbol{U} - (\bar{\boldsymbol{w}}_i - \bar{\boldsymbol{f}}_i)\right] \quad (16)$$

From (15) and (16), it follows that $\Delta \boldsymbol{U}$ can also be found by forming the augmented linear equation:

$$\begin{bmatrix} \boldsymbol{\Lambda} + \sum_{i=1}^{n}\boldsymbol{G}_i^T \boldsymbol{Q}\, \boldsymbol{G}_i & \bar{\boldsymbol{G}}_1^T & \cdots & \bar{\boldsymbol{G}}_n^T \\ \bar{\boldsymbol{G}}_1 & -\gamma\alpha^{2N_y}\boldsymbol{I}_{m_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\boldsymbol{G}}_n & 0 & \cdots & -\gamma\alpha^{2N_y}\boldsymbol{I}_{m_n} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{U} \\ \boldsymbol{p}_1 \\ \vdots \\ \boldsymbol{p}_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}\boldsymbol{G}_i^T \boldsymbol{Q}\,(\boldsymbol{w}_i - \boldsymbol{f}_i) \\ \bar{\boldsymbol{w}}_1 - \bar{\boldsymbol{f}}_1 \\ \vdots \\ \bar{\boldsymbol{w}}_n - \bar{\boldsymbol{f}}_n \end{bmatrix} \quad (17)$$

where $\boldsymbol{I}_{m_i}$ is the identity matrix of dimension m_i. This description of the control law now allows γ to be zero, and thus is numerically superior to eqn. (15) for $\gamma \ll 1$. In order to simplify further eqn. (17), we define

$$\begin{aligned} \boldsymbol{G} &= \left[\boldsymbol{G}_1^T\ \boldsymbol{G}_2^T\ \cdots\ \boldsymbol{G}_n^T\right]^T, & \bar{\boldsymbol{G}} &= \left[\bar{\boldsymbol{G}}_1^T\ \bar{\boldsymbol{G}}_2^T\ \cdots\ \bar{\boldsymbol{G}}_n^T\right]^T \\ \boldsymbol{p} &= \left[\boldsymbol{p}_1^T\ \boldsymbol{p}_2^T\ \cdots\ \boldsymbol{p}_n^T\right]^T, & \boldsymbol{w} &= \left[\boldsymbol{w}_1^T\ \boldsymbol{w}_2^T\ \cdots\ \boldsymbol{w}_n^T\right]^T \\ \bar{\boldsymbol{w}} &= \left[\bar{\boldsymbol{w}}_1^T\ \bar{\boldsymbol{w}}_2^T\ \cdots\ \bar{\boldsymbol{w}}_n^T\right]^T, & \boldsymbol{f} &= \left[\boldsymbol{f}_1^T\ \boldsymbol{f}_2^T\ \cdots\ \boldsymbol{f}_n^T\right]^T \\ \bar{\boldsymbol{f}} &= \left[\bar{\boldsymbol{f}}_1^T\ \bar{\boldsymbol{f}}_2^T\ \cdots\ \bar{\boldsymbol{f}}_n^T\right]^T, & \boldsymbol{M} &= \text{diag}\left[\boldsymbol{Q},\, \boldsymbol{Q},\, \cdots,\, \boldsymbol{Q}\right] \end{aligned} \quad (18)$$

Then the control law is rewritten as

$$\begin{bmatrix} \boldsymbol{\Lambda} + \boldsymbol{G}^T \boldsymbol{M}\, \boldsymbol{G} & \bar{\boldsymbol{G}}^T \\ \bar{\boldsymbol{G}} & -\gamma\alpha^{2N_y}\boldsymbol{I} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{U} \\ \boldsymbol{p} \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}^T \boldsymbol{M}\,(\boldsymbol{w} - \boldsymbol{f}) \\ \bar{\boldsymbol{w}} - \bar{\boldsymbol{f}} \end{bmatrix} \quad (19)$$

which is in exactly the same form as in (Yoon and Clarke, 1995a). When $\gamma = 0$, the vector $\boldsymbol{p}$ ($\boldsymbol{p}_i$) turns out to be the Lagrange multiplier for the optimization problem subject to $\bar{\boldsymbol{G}}\Delta\boldsymbol{U} = \bar{\boldsymbol{w}} - \bar{\boldsymbol{f}}$ ($\bar{\boldsymbol{G}}_i\Delta\boldsymbol{U} = \bar{\boldsymbol{w}}_i - \bar{\boldsymbol{f}}_i$).

Having computed $\Delta\boldsymbol{U}$ using eqn. (17) or (19), only $\Delta\boldsymbol{u}(t)$ (the first n elements of $\Delta\boldsymbol{U}$) is actually applied at time t, and the whole procedure is repeated at the next sample instant: hence the name 'receding-horizon.' The resulting control law is simple to implement, and is independent of the input-output ordering in the system description. Also, this predictive scheme has stability guarantees as follows:

Theorem 1. *For the MIMO process described by* (1) *or* (2) *and the receding-horizon predictive control law* (17) *or* (19), *the closed-loop is guaranteed to be stable if*

$$\begin{aligned} \gamma &= 0, \quad \alpha \leq 1 \\ m_i &= \deg(A_i) + 1 \\ N_u &\geq \deg\left(\mathrm{lcm}(A_1, A_2, \cdots, A_n)\right) + 1 \\ N_y &= N_u + \max_{1 \leq i,j \leq n}\left(\deg(B_{ij}) - \deg(A_i)\right) \end{aligned} \tag{20}$$

where m_i is the number of equality constraints (i.e. $N_{2i} - N_y + 1$), B_{ij} is the j-th element of $\boldsymbol{B}_i$, and lcm denotes the least common multiple.

Outline of proof. As in the proof of the corresponding theorem for SISO systems given in (Yoon and Clarke, 1995a), it can be shown using $N_u \geq \deg(\mathrm{lcm}(A_1, A_2, \ldots, A_n)) + 1$ that there exist control inputs such that the outputs settle in a finite time. It then follows from $N_y = N_u + \max_{1 \leq i,j \leq n}(\deg(B_{ij}) - \deg(A_i))$ that each set of m_i terminal equality constraints can be satisfied and the resulting receding-horizon cost can be shown to be monotonically non-increasing. This completes the proof. ∎

Remark 1. For a detailed proof, see (Yoon and Kwon, 1999).

Remark 2. When N_1 is set to N_y, the control law is shown to be independent of the weighting parameter $\bar{\rho}_i$. Therefore, if we require the others to satisfy the conditions in (20), then only two parameters remain to be determined: N_u and α. This is very useful as N_u and α can easily be chosen by considering the rise-time of the process and the desired settling time of the closed-loop, respectively. We also take this approach here.

3.3. Estimation Law

Having stated the control law, the adaptive control design can be completed by describing the estimator. To this end, we employ the following set of estimators:

$$\hat{\boldsymbol{\theta}}_i(t) = \hat{\boldsymbol{\theta}}_i(t-1) + \frac{\boldsymbol{P}_i(t-1)\boldsymbol{\phi}_i(t-1)}{1 + \boldsymbol{\phi}_i(t-1)^T\boldsymbol{P}_i(t-1)\boldsymbol{\phi}_i(t-1)} e_i(t)$$

$$\bar{\boldsymbol{P}}_i(t) = \boldsymbol{P}_i(t-1) - \frac{\boldsymbol{P}_i(t-1)\boldsymbol{\phi}_i(t-1)\boldsymbol{\phi}_i(t-1)^T\boldsymbol{P}_i(t-1)}{1+\boldsymbol{\phi}_i(t-1)^T\boldsymbol{P}_i(t-1)\boldsymbol{\phi}_i(t-1)}$$

$$\boldsymbol{P}_i(t) = \left(1-\frac{\lambda_0}{\lambda_1}\right)\bar{\boldsymbol{P}}_i(t) + \lambda_0\boldsymbol{I}, \quad \boldsymbol{P}_i(0) = \lambda_1\boldsymbol{I} \tag{21}$$

where $i \in [1, n]$. The vector $\hat{\boldsymbol{\theta}}_i$ contains the estimates of the parameters of the polynomials $1 - A_i(q^{-1})$ and $\boldsymbol{B}_i(q^{-1})$ in (2), the regressor vector $\boldsymbol{\phi}_i$ consists of filtered input and outputs ($\boldsymbol{u}^f$, y_i^f), $\boldsymbol{P}$ is the covariance matrix, and $e_i(t)$ is the estimation error given by

$$e_i(t) = y_i^f(t) - \boldsymbol{\phi}_i(t-1)^T\hat{\boldsymbol{\theta}}_i(t-1). \tag{22}$$

Note also that T_i's in (2) are not estimated since they are considered important design polynomials used to enhance robustness, rather than representing noise characteritics. Regarding the robustness effects of the so-called T polynomial for SISO predictive control, see (Yoon and Clarke, 1995b).

The covariance modification in (21) is employed to impose lower and upper bounds (λ_0, λ_1) on the eigenvalues of the covariance matrix $\boldsymbol{P}$. It is necessary to keep the estimator alert ($\lambda_0 > 0$) due to the nonlinear and time-varying nature of distillation dynamics.

4. Adaptive Control of the Distillation Column

The overall adaptive control algorithm is applied to a binary distillation column with 15 trays. The detailed parameters of the column used in simulations are given in the Appendix.

4.1. Design Procedure

The design procedure involves selecting the sampling time, specifying the model structure, and setting up the control and estimation parameters.

The open-loop system is seen to have a rise-time of around 1 hour, and the desired rise-time is 20 minutes. We thus set the sampling time to 2 min (= 20/10).

Having selected the sampling time, we now specify the plant model (2). By observing the responses obtained through some open-loop simulations, it is assumed that

$$A_i(q^{-1}) = 1 - a_i q^{-1}$$

$$B_{ij}(q^{-1}) = b_{ij1} + b_{ij2}q^{-1} + b_{ij3}q^{-2} + b_{ij4}q^{-3} \tag{23}$$

We then perform initial estimation by applying PRBS inputs to the process. As shown in Fig. 2, the estimated outputs (dotted lines) tend to the actual outputs (solid lines) as the number of observations increases.

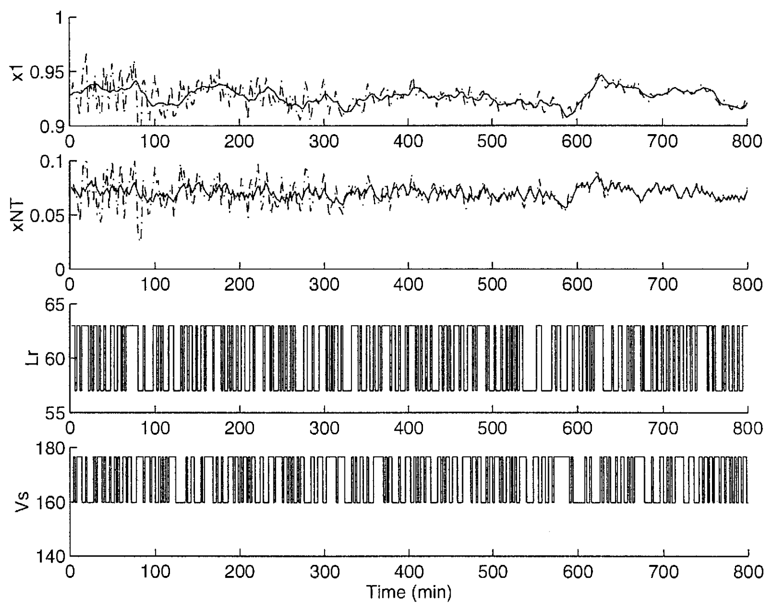

Fig. 2. PRBS test for initial estimation.

Table 2. Design objective and parameters.

Objective	a rise-time of 20 min
Sampling time	2 min
λ_0, λ_1	0.1, 100
N_u, α	20, 0.8
Estimator filter	$0.1/(1-0.9q^{-1})$

After the model structure and initial parameters have been decided, we select the adaptive control parameters as summarized in Table 2.

The choice $\alpha = 0.8$ is reasonable as the desired rise time is 10 samples and $1 - 0.8^{10} \approx 0.89$. Note also that the estimator filter is set to $1/(1-0.9q^{-1})$ as it is observed that the open-loop poles (a_1, a_2) lie in the interval $(0.9, 1)$.

4.2. Simulations

Three simulation results are reported here. Firstly, a set-point following performance is investigated. As illustrated in Fig. 3, consistent transient behavior is observed, and the objective of achieving a rise-time of 20 min is seen to be reached (dotted lines: set-points, solid lines: outputs). Also the interaction between the two channels seems to be handled quite well.

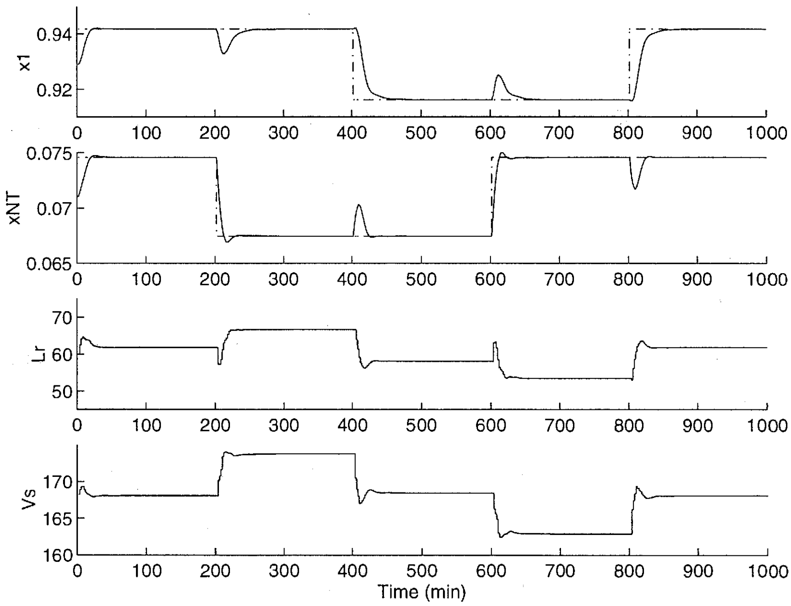

Fig. 3. Responses to varying set-points.

Secondly, we look at the regulation performance in the presence of a feed change. As shown in Fig. 4, the effect of feed change is rapidly eliminated by the adaptive controller. However, it can be argued that this satisfactory performance may be due to successful implementation of linear predictive control, rather than to adaptation. We thus consider the case where the feed quality q is varying, in order to demonstrate the enhanced performance resulting from the use of adaptation. The corresponding simulation results are shown in Fig. 5, where it is clear that adaptation leads to an improved performance (cf. Figs. 2 and 3 with Figs. 4 and 5) in the presence of the feed quality change.

5. Conclusion

We have presented an adaptive control scheme for a binary distillation column. The proposed scheme is based on a recently developed multivariable receding-horizon predictive control method and an RLS algorithm with covariance regularization. An important feature of the overall adaptive controller is the ease of tuning; only two parameters need setting for control. Furthermore, these have close relationships with time-domain terms such as the rise-time. As discussed in Section 4.1, N_u and α are determined simply by considering the open-loop and desired closed-loop rise times. Despite its simplicity, the resulting adaptive control strategy is seen to result in a satisfactory performance through simulations. It is also demonstrated that adaptation leads to an enhanced performance in the presence of a feed quality change.

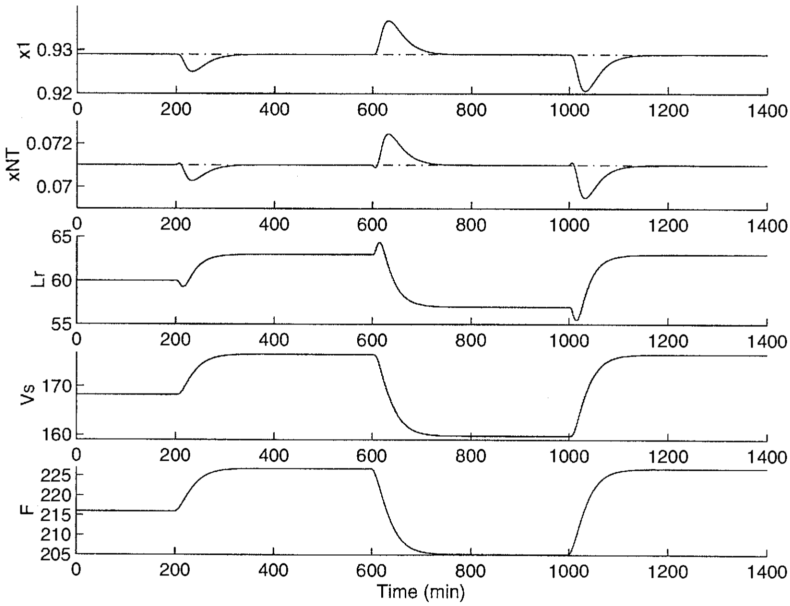

Fig. 4. Responses to a feed change.

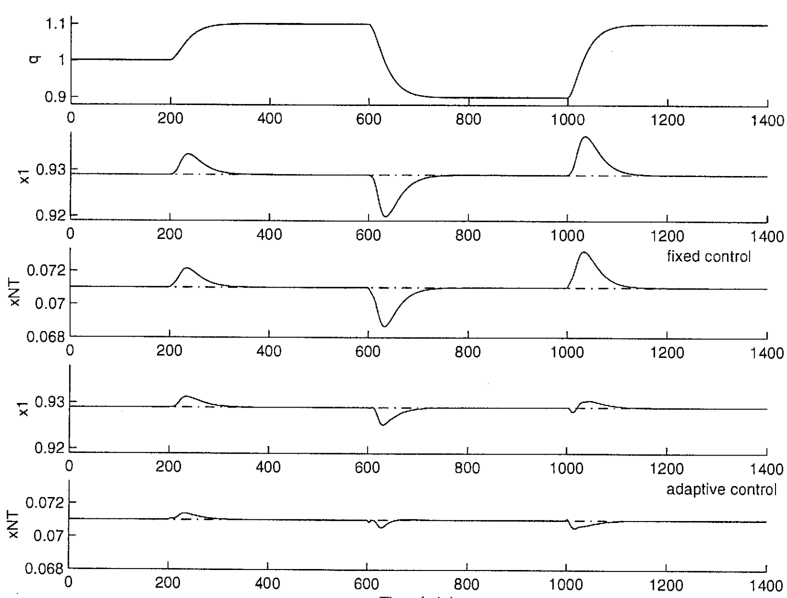

Fig. 5. Responses to a feed quality change.

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Appendix

Steady-State Conditions for the Distillation Column

$$K = [1.1242\ 1.2118\ 1.3185\ 1.3990\ 1.4854\ 1.5588\ 1.5588\ 1.5599\ 1.5661\ 1.5788\ 1.6305\ 1.8240\ 2.5984\ 4.7013]$$

$$x_F = 0.5, \quad q = 1.001$$

$$F = 216, \quad D = 108, \quad B = 108\,[\text{mole/hr}]$$

$$H_c = 15, \quad H_r = 2.5, \quad H_s = 3.5, \quad H_b = 24\,[\text{mole}]$$

$$N_T = 15 \text{ (total number of trays)}, \quad N_F = 7 \text{ (feed stage)}$$

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