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DISTRIBUTED FAULT ESTIMATION OF MULTI–AGENT SYSTEMS USING A PROPORTIONAL–INTEGRAL OBSERVER: A LEADER–FOLLOWING APPLICATION

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This paper proposes a methodology for observer-based fault estimation of leader-following linear multi-agent systems subject to actuator faults. First, a proportional-integral distributed fault estimation observer is developed to estimate both actuator faults and states of each follower agent by considering directed and undirected graph topologies. Second, based on the proposed quadratic Lyapunov equation, sufficient conditions for the asymptotic convergence of the observer are obtained as a set of linear matrix inequalities. Finally, a numerical example is provided to illustrate the proposed approach.

Keywords: multiagent systems, fault estimation, state and fault observers, linear matrix inequalities.

1. Introduction

In recent years, cooperative control for multi-agents systems (MASs) has been widely studied as a solution for problems where multiple systems have to collaborate to reach a common goal. In this scenario, an individual control law for each of the agents cannot provide a satisfactory performance of the global control task. Particularly, in cooperative control on graphs, the control protocols must be distributed since they need an information exchange between the agents (Lewis *et al.*, 2013), e.g., distributed cooperative control of microgrids (Nasirian *et al.*, 2014), trajectory tracking and decentralized navigation (Prodan *et al.*, 2013), cooperative formation control of autonomous underwater vehicles (Das *et al.*, 2016), formation control of unmanned aerial vehicles (UAVs) (Kuriki and Namerikawa, 2014), cooperative control of manipulators (Li *et al.*, 2016), to mention a few.

Among different topics of MASs, the leader-following, also called cooperative tracking control, has become the most popular consensus problem. In this case, the leader sends information to the agents; then the controller tries to reduce the error so all follower agents can track the desired trajectory generated by the leader (Lewis *et al.*, 2013; Zhai, 2015). In the literature, there is extensive research about this problem; for example, in the work of Wang and Wu (2012) a leader-following formation control for a second-order nonlinear multi-agent system under fixed and switching topologies is exposed. Cai and Huang (2014) present a

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leader-following control for multiple spacecraft systems. Zhang *et al.* (2014) discuss an adaptive technique for fault estimation. Ma *et al.* (2015) and Zhao *et al.* (2017) outline two different approaches for second-order multi-agent systems, one being an optimal strategy and the other an event-triggered strategy for the communication graph, respectively. However, few works are dedicated to detect and isolate faults for MASs.

The purpose of a fault diagnosis system is to generate an alarm when a fault occurs as well as to detect, locate, and estimate the magnitude of the faulty element (López-Estrada *et al.*, 2019; Bermúdez *et al.*, 2018). Specifically, fault estimation has been well discussed for single-agent systems by Yang and Yin (2018; 2019b), who addressed the state and fault estimation for Markovian jump systems in the presence of simultaneous sensor and actuator faults and recently for applications as wheeled mobile manipulators (Yang and Yin, 2019a). Nevertheless, few works have been proposed for fault diagnosis in MASs. In particular, for leader-following cooperative control problems, Shi *et al.* (2014) use a bank of optimal robust observers to detect and isolate actuator faults. Chen and Song (2015) developed an actuator fault detection module for directed graphs, while Li (2015) proposed a controller for multi-agent systems subject to a loss of actuator effectiveness with an adaptive observer.

Furthermore, few works related to fault estimation have been reported, e.g., Zhou *et al.* (2014) achieve fault tolerant cooperative control with a sliding mode observer to estimate faults. Ye *et al.* (2017) presented an adaptive observer to estimate the states and bias faults with multiple leaders. An unknown input observer (UIO) is designed by Wu *et al.* (2018) to estimate states and faults for directed graphs in the presence of exosystem disturbances. Yang *et al.* (2018) present a distributed adaptive fault estimation algorithm for undirected graphs. Nevertheless, to the best of the authors' knowledge, proportional-integrative (PI) distributed fault estimation observers (DFEOs) (Zhang *et al.*, 2015) have not been reported in the literature for fault estimation in leader-following applications.

Therefore, the main contribution is to propose a proportional-integral distributed fault estimation observer (PI-DFEO) for MASs with a distributed approach. Moreover, the PI-DFEO estimates both the system states and actuator faults without requiring a bank of observers. Furthermore, to guarantee robustness against measurement noise and disturbances, an H_∞ criterion was considered. As a result, sufficient conditions to compute the observer gains are given by a set of feasible linear matrix inequalities. In order to reach the main goal, it is assumed that the control law of each agent depends on its own information and the information provided by its neighbors, and a graph topology is considered in order to show the connection between the agents. The proposed approach can be applied to directed and undirected graphs, according to graph theory. Finally, the performance of the proposed method is tested through numerical examples of formation control.

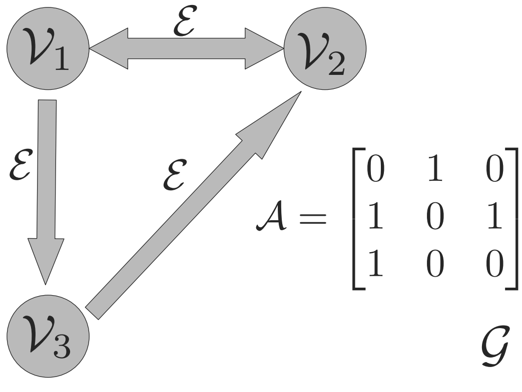

Fig. 1. Graph example.

The paper is organized as follows: Section 2 introduces the mathematical principles related to MASs and the problem statement Section 3 presents the main result, where an LMI is developed based on the appropriate Lyapunov function. Section 4 provides a numerical example in order to validate the theoretical result. Finally, Section 5 presents the conclusions and future work.

2. Background

2.1. Mathematical preliminaries. Consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a set of N nodes $\mathcal{V} = (v_1, v_2, \dots, v_N)$; a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ that interconnects the nodes in the graph and its associated adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ whose entries depend on the communication between agents in the graph as it is explained later.

Elements of $\mathcal{E}$ rooted at node j and ended at node i are denoted by (v_j, v_i), which means that information can flow from node j to node i. Elements a_{ij} are the weights of the edge (v_j, v_i), $a_{ij} = 1$ if edge $(v_j, v_i) \in \mathcal{E}$, i.e., exist a connection between agent v_j and agent v_i, otherwise $a_{ij} = 0$. For example, for a system of three agents, the corresponding graph is as in Fig. 1. For undirected graphs we have $(v_j, v_i) = (v_i, v_j)$, i.e., communication between agents is bidirectional and therefore $\mathcal{A} = \mathcal{A}^T$. Otherwise, it is considered a directed graph or a digraph. In this paper, only graphs with $a_{ii} = 0$ are considered. Another important definition is the weighted in-degree of node v_i which expresses the i-th row sum of $\mathcal{A}$:

$$d_i = \sum_{j=1}^{N} a_{ij}, \tag{1}$$

with which $D = \operatorname{diag}(d_i)$ is the diagonal in-degree matrix. Finally, let $L = D - \mathcal{A}$ be the Laplacian matrix that includes all the communication information between the agents.

A graph that contains a node that acts like a command generator (leader node) is modeled with an augmented graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ with a set of $N+1$ nodes $\bar{\mathcal{V}} = (v_0, v_1, \dots, v_N)$ and a set of edges $\bar{\mathcal{E}} \subset \bar{V} \times \bar{V}$. With no loss of generality, the leader node is labeled as v_0. Then, if there exists a connection between the leader node and the nodes of the i-th follower, an edge (v_0, v_i) is said to exist with $g_i = 1$ as the weight. These weights are called pinning gains and the diagonal matrix of pinning gains is defined as $G = \mathrm{diag}(g_i)$; it represents the connections between the leader node and the i-th agents (Lewis *et al.*, 2013). Finally, to express a complete space-state model of all agents the Kronecker product $\otimes$ is used which is defined, given two matrices $A = [a_{ij}]$ and B, as $A \otimes B = [a_{ij}B]$.

2.2. Problem statement and system description. Consider a collection of $N+1$ identical agents where the follower nodes are represented with faults through the following space-state model:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + Hf_i(t), \\ y_i(t) &= Cx_i(t), \end{aligned} \tag{2}$$

where $i = 1, \dots, N$ refers to the i-th agent in the multi-agent system, $x_i(t) \in \mathbb{R}^n$ is the state vector, $u_i(t) \in \mathbb{R}^m$ is the control input vector, $f_i(t) \in \mathbb{R}^r$ represents the system component or actuator fault vector, and $y_i(t) \in \mathbb{R}^p$ is the output vector. The pair (A, C) is assumed to be observable and matrix H is constant with appropriate dimensions.

Without loss of generality, the leader agent labeled with the subscript 0 is modeled as follows:

$$\dot{x}_0(t) = Ax_0(t). \tag{3}$$

Remark 1. Note that the leader agent does not have any input. This holds for the standard multi-agent systems theory and for the purpose of this work. However, it is worth mentioning that a controller for the leader agent can be developed independently applying any other single agent control theory (Lewis *et al.*, 2013).

For the development of this work, the following assumptions and lemma are needed:

Assumption 1. (*Lewis* et al.*, 2013*) In order for all follower agents to track the state of the leader, the graph must have a spanning tree with the leader node as the root, i.e., the leader node can send information directly or indirectly to all follower agents.

Assumption 2. In order to eliminate the derivative of the fault in the dynamic estimation error, it is assumed that $\dot{f}(t) \approx 0$, which is also known as the slow variation condition. Note that, from a practical point of view, this condition can be relaxed as discussed by Chadli *et al.* (2013) and Rotondo *et al.* (2016).

Lemma 1. (Lewis *et al.*, 2013) *Under Assumption 1, the matrix $(L+G)$ is nonsingular. In addition, this matrix is positive definite.*

In order to estimate states, the following distributed observer is proposed:

$$\begin{aligned} \dot{\hat{x}}_i(t) &= A\hat{x}_i(t) + Bu_i(t) - R\zeta_i(t) + H\hat{f}_i(t), \\ \hat{y}_i(t) &= C\hat{x}_i(t), \\ \hat{f}_i(t) &= -\Gamma F\Big(\zeta_i(t) + \int_{t_f}^{t} \zeta_i(t)\,\mathrm{d}t\Big), \end{aligned} \tag{4}$$

where $i = 1, \dots, N$, $\hat{x}_i(t) \in \mathbb{R}^n$ is the estimated state, $\hat{y}_i(t) \in \mathbb{R}^p$ is the estimated output, $\zeta_i(t) \in \mathbb{R}^q$ is the relative output estimation error of the i-th agent in the communication graph defined later, $R \in \mathbb{R}^{n\times q}$ is the observer gain matrix to be designed, and $\hat{f}_i(t)$ is the estimated fault. To deal with fault estimation, a distributed PI fault estimator $\hat{f}_i(t)$ is proposed where the relative output estimation error ζ_i is used. Additionally, an integral term of ζ_i is added that allows the observer to have a faster convergence to the fault, $F \in \mathbb{R}^{r\times p}$ is the fault estimator gain matrix to be designed, and matrix $\Gamma = \Gamma^T > 0$ is the learning rate. Note that t_f indicates the time when the fault occurs.

Then, the problem is reformulated to find the gain matrices R and F, such that the estimation error between system (2) and observer (4) tends asymptotically to zero for all follower agents. Communication between follower agents and the leader agent is established through the pinning gain matrix G defined in Section 2.1.

3. Main contribution

The main idea of this paper is to estimate states and faults of the follower agents in a MAS using the observer proposed in (4). Then, according to MAS theory, the relative output estimation error ζ_i, which expresses the information exchanged between the agents, is defined as (Lewis *et al.*, 2013)

$$\begin{aligned} \zeta_i(t) = &\sum_{j=1}^{N} a_{ij}\Big(\big(\hat{y}_i(t) - y_i(t)\big) - \big(\hat{y}_j(t) - y_j(t)\big)\Big) \\ &+ g_i\Big(\big(\hat{y}_i(t) - y_i(t)\big) - \big(\hat{y}_0(t) - y_0(t)\big)\Big), \\ &\qquad i = 1, \dots, N, \end{aligned} \tag{5}$$

where $\hat{y}_i(t)$ is the observer output of the i-th agent, $\hat{y}_j(t)$ is the observer output of the j-th agent that constitutes a neighbors of agent i in the collection of systems, g_i represents the nodes pinned to the leader node.

From Remark 1, since the leader agent can be treated independently and an observer for the leader agent is not needed, it is reasonable to assume for the MAS model that the output estimation error of the leader agent is equal to zero, i.e., $\hat{y}_0(t) - y_0(t) = 0$. Then, the relative output estimation error is reduced to the following expression:

$$\zeta_i(t) = \sum_{j=1}^{N} a_{ij}\Big(\big(\hat{y}_i(t) - y_i(t)\big) - \big(\hat{y}_j(t) - y_j(t)\big)\Big) + g_i\big(\hat{y}_i(t) - y_i(t)\big), \quad i = 1,\dots,N. \tag{6}$$

In order to develop the DFEO, it is necessary to define the dynamic error for both state estimation and fault estimation. The state estimation error vector for the i-th agent is defined as

$$e_{x_i}(t) = \hat{x}_i(t) - x_i(t). \tag{7}$$

Then the dynamic state estimation error is given as

$$\begin{aligned}\dot{e}_{x_i}(t) &= \dot{\hat{x}}_i(t) - \dot{x}_i(t)\\ &= A\hat{x}_i(t) + Bu_i(t) + H\hat{f_i}(t) - R\zeta_i(t)\\ &\quad - Ax_i(t) - Bu_i(t) - Hf_i(t)\\ &= A(\hat{x}_i(t) - x_i(t)) + H(\hat{f}_i(t) - f_i(t))\\ &\quad - R\zeta_i(t),\end{aligned} \tag{8}$$

where the relative output estimation error ζ_i can be expressed as

$$\begin{aligned}\zeta_i(t) &= \sum_{j=1}^{N} a_{ij}(C\hat{x}_i(t)) - \sum_{j=1}^{N} a_{ij}(Cx_i(t))\\ &\quad - \sum_{j=1}^{N} a_{ij}(C\hat{x}_j(t)) + \sum_{j=1}^{N} a_{ij}(Cx_j(t)) + g_i(C\hat{x}_i(t))\\ &\quad - g_i(Cx_i(t)).\end{aligned} \tag{9}$$

Then, from (1),

$$\begin{aligned}\zeta_i(t) &= d_iC\hat{x}_i(t) - d_iCx_i(t) - \begin{bmatrix}a_{i1} \dots a_{iN}\end{bmatrix}\begin{bmatrix}C\hat{x}_1(t)\\ \vdots\\ C\hat{x}_N(t)\end{bmatrix}\\ &\quad + \begin{bmatrix}a_{i1} & \dots & a_{iN}\end{bmatrix}\begin{bmatrix}Cx_1(t)\\ \vdots\\ Cx_N(t)\end{bmatrix}\\ &\quad + g_iC\hat{x}_i(t) - g_iCx_i(t).\end{aligned} \tag{10}$$

Define the global vectors

$$\begin{aligned}\zeta(t) &= \begin{bmatrix}\zeta_1^T(t), \dots, \zeta_N^T(t)\end{bmatrix}^T \in \mathbb{R}^{pN},\\ x(t) &= \begin{bmatrix}x_1^T(t), \dots, x_N^T(t)\end{bmatrix}^T \in \mathbb{R}^{nN},\\ e_x(t) &= \begin{bmatrix}e_{x_1}^T(t), \dots, e_{x_N}^T(t)\end{bmatrix}^T \in \mathbb{R}^{nN}.\end{aligned}$$

Given the adjacency matrix $\mathcal{A} = [a_{ij}]$, the diagonal in-degree matrix $D = \mathrm{diag}\{d_i\}$, the diagonal pinning gain matrix $G = \mathrm{diag}\{g_i\}$, and the Laplacian matrix defined as $L = D - \mathcal{A}$, we get

$$\begin{aligned}\zeta(t) &= (D \otimes C)\hat{x}(t) - (D \otimes C)x(t) - (\mathcal{A} \otimes C)\hat{x}(t)\\ &\quad + (\mathcal{A} \otimes C)x(t) + (G \otimes C)\hat{x}(t)\\ &\quad - (G \otimes C)x(t)\\ &= ((D - \mathcal{A} + G) \otimes C)(\hat{x}(t) - x(t))\\ &= ((L + G) \otimes C)e_x(t).\end{aligned} \tag{11}$$

Then, the global distributed state estimation dynamic error for the whole MAS is given by

$$\begin{aligned}\dot{e}_x(t) &= (I_N \otimes A)e_x(t) + (I_N \otimes H)e_f(t)\\ &\quad - (I_N \otimes R)[((L + G) \otimes C)e_x(t)]\\ &= (I_N \otimes A - (L + G) \otimes RC)e_x(t)\\ &\quad + (I_N \otimes H)e_f(t).\end{aligned} \tag{12}$$

The fault estimation error vector for the i-th agent is defined as

$$e_{f_i}(t) = \hat{f}_i(t) - f_i(t), \tag{13}$$

where the fault estimation dynamic error can be obtained with

$$\begin{aligned}\dot{e}_{f_i(t)} &= \dot{\hat{f}}_i(t) - \dot{f}_i(t)\\ &= -\Gamma F(\zeta_i(t) + \dot{\zeta}_i(t)) - \dot{f}_i(t).\end{aligned} \tag{14}$$

Let $\dot{f}(t) = \begin{bmatrix}\dot{f}_1^T(t), \dots, \dot{f}_N^T(t)\end{bmatrix}^T \in \mathbb{R}^{nN}$ be the global fault vector. Then, the global distributed fault estimation dynamic error for the whole MAS is given by

$$\dot{e}_f(t) = -((L+G)\otimes \Gamma FC)(e_x(t)+\dot{e}_x(t)) - \dot{f}(t). \tag{15}$$

If only faults with small variations are considered, we assume that $\dot{f} \approx 0$ (Estrada *et al.*, 2015), which leads to

$$\dot{e}_f(t) = -((L + G) \otimes \Gamma FC)(e_x(t) + \dot{e}_x(t)). \tag{16}$$

Now, the H_∞ criterion (Hu *et al.*, 2016) that is included in the developed DFEO in order to provide robustness to the observer is formed as

$$J_{rd} := \dot{V}_{e(t)} + J_1 < 0, \tag{17}$$

where $V_{e(t)}$ is the candidate Lyapunov function, $J_1 = e_x^T(t)e_x(t) - \gamma^2 e_f^T(t)e_f(t)$, $\gamma > 0$ is a scalar value. J_1 can be expressed in matrix form:

$$J_1 = \begin{bmatrix}e_x^T(t) & e_f^T(t)\end{bmatrix}\begin{bmatrix}I & 0\\ 0 & -\gamma^2 I\end{bmatrix}\begin{bmatrix}e_x(t)\\ e_f(t)\end{bmatrix}. \tag{18}$$

Theorem 1. *Assume that there exist a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$, and matrices $Y \in \mathbb{R}^{n\times p}$ and $F \in \mathbb{R}^{r\times p}$ that satisfy*

$$\begin{bmatrix} I_N \otimes (A^T P + PA) - \Lambda & \Phi \\ * & \phi \end{bmatrix} < 0, \tag{19}$$

$$H^T P = FC, \tag{20}$$

where $\Lambda = (L+G)\otimes(YC) + (L+G)^T \otimes C^T Y^T) - I$, $\Phi = I_N \otimes PH - (L+G)^T \otimes (A^T PH + PH) + (L+G)^{2T} \otimes (C^T Y^T H)$, $\phi = -(L+G)^T \otimes (H^T PH) - (L+G)\otimes(H^T PH) - \gamma^2 I$. Then the observer gain matrix R can be calculated by $R = P^{-1}Y$.

Proof. Consider the following Lyapunov function:

$$V_{e(t)} = e_x^T(t)(I_N \otimes P)e_x(t) + e_f(t)^T(I_N \otimes \Gamma^{-1})e_f(t). \tag{21}$$

Calculating its derivative, we get

$$\dot V_{e(t)} = \dot e_x^T(t)(I_N \otimes P)e_x(t) + e_x^T(t)(I_N \otimes P)\dot e_x(t) + \dot e_f^T(t)(I_N \otimes \Gamma^{-1})e_f(t) + e_f^T(t)(I_N \otimes \Gamma^{-1})\dot e_f(t). \tag{22}$$

Substituting (12) and (16) in (22), we obtain

$$\begin{aligned}\dot V_{e(t)} &= \Big(e_x^T(t)(I_N \otimes A - (L+G)\otimes RC)^T + e_f^T(t)(I_N\otimes H)^T\Big)(I_N\otimes P)e_x(t) \\ &\quad + e_x^T(t)(I_N\otimes P)\Big((I_N\otimes A - (L+G)\otimes RC) e_x(t) + (I_N\otimes H)e_f(t)\Big) - \Big((e_x(t)+\dot e_x(t))^T ((L+G)^T \otimes \Gamma FC)^T\Big)(I_N\otimes\Gamma^{-1})e_f(t) \\ &\quad - e_f(t)^T(I_N\otimes\Gamma^{-1})\Big(((L+G)\otimes\Gamma FC)(e_x(t) + \dot e_x(t))\Big) \\ &= e_x^T(t)(I_N\otimes A^T P - (L+G)^T\otimes C^T R^T P)e_x(t) + e_f^T(t)(I_N\otimes H^T P)e_x(t) \\ &\quad + e_x^T(t)(I_N\otimes PA - (L+G)\otimes PRC)e_x(t) + e_x^T(t)(I_N\otimes PH)e_f(t) \\ &\quad - (e_x(t)+\dot e_x(t))^T((L+G)^T\otimes(C^T F^T))e_f(t) - e_f^T(t)((L+G)\otimes(FC))(e_x(t)+\dot e_x(t))\end{aligned} \tag{23}$$

with $Y = PR$ and $H^T P = FC$. From Theorem 1, (23) can be transformed to

$$\begin{aligned}\dot V_{e(t)} &= e_x^T(t)(I_N\otimes(A^T P + PA) - (L+G)^T\otimes C^T Y^T - (L+G)\otimes YC)e_x(t) \\ &\quad + 2e_x^T(t)(I_N\otimes PH)e_f(t) - 2e_x^T(t)\Big((L+G)^T\otimes PH\Big)e_f(t) \\ &\quad \underbrace{-2\dot e_x^T(t)\Big((L+G)^T\otimes PH\Big)e_f(t)}_{\mathsf{e_x}},\end{aligned} \tag{24}$$

where

$$\begin{aligned}\mathsf{e_x} &= -2\Big(e_x^T(t)(I_N\otimes A - (L+G)\otimes RC)^T + e_f^T(t)(I_N\otimes H)^T\Big)\Big((L+G)^T\otimes PH\Big)e_f(t) \\ &= -2e_x^T(t)\Big((L+G)^T\otimes A^T PH - (L+G)^{2T}\otimes C^T Y^T H\Big)e_f(t) \\ &\quad - 2e_f^T(t)\Big((L+G)^T\otimes H^T PH\Big)e_f(t).\end{aligned} \tag{25}$$

Then, substituting (25) in (24), we get

$$\begin{aligned}\dot V_{e(t)} &= e_x^T(t)(I_N\otimes(A^T P+PA) - (L+G)^T\otimes C^T Y^T - (L+G)\otimes YC)e_x(t) + 2e_x^T(t)(I_N\otimes PH \\ &\quad - (L+G)^T\otimes(PH + A^T PH) + (L+G)^{2T}\otimes C^T Y^T H)e_f(t) \\ &\quad - 2e_f^T(t)((L+G)^T\otimes H^T PH)e_f(t).\end{aligned} \tag{26}$$

Then, the term J_1 from (17) is added to (26) to apply the H_∞ criterion

$$\begin{aligned}J_{rd} &= e_x^T(t)(I_N\otimes(A^T P+PA) - (L+G)^T\otimes C^T Y^T - (L+G)\otimes YC)e_x(t) + 2e_x^T(t)(I_N\otimes PH \\ &\quad - (L+G)^T\otimes(PH + A^T PH) + (L+G)^{2T}\otimes C^T Y^T H)e_f(t) \\ &\quad - 2e_f^T(t)((L+G)^T\otimes H^T PH)e_f(t) + e_x^T(t)e_x(t) - \gamma^2 e_f^T(t)e_f(t),\end{aligned} \tag{27}$$

which leads to the matrix representation provided in Theorem 1. This completes the proof. ■

Remark 2. From Section 2.1, undirected graphs imply $(L+G) = (L+G)^T$ that is a special case of directed graphs. Theorem 1 can account for both $(L+G) = (L+G)^T$ and $(L+G) \neq (L+G)^T$. Thus it can be used for directed and undirected graphs topologies.

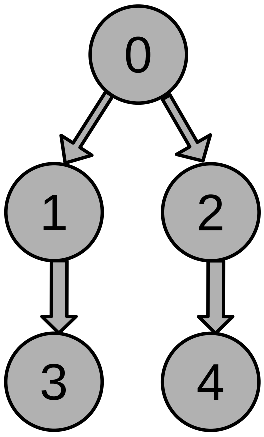

Fig. 2. Communication graph for the numerical example.

Remark 3. It is important to remark here that the proposed PI observer estimates simultaneously the states and the faults of the MAS. This cannot be addressed with a proportional observer, for which a bank of observers is required in order to isolate the fault. Also, the PI-DFEO proposed here has not been reported in the literature in the MAS context.

4. Results

In this section, a numerical example is given to illustrate the effectiveness of the theoretical results. To this end, the system investigated by Zhang *et al.* (2015) is considered. The problem regards a collection of five identical aircraft. The leader agent and four follower agents, with the directed communication topology, are depicted in Fig. 2 (the formation graph).

Each aircraft is modeled in a state-space representation as

$$A = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.7070 & 1.4200 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

where the state vector is defined as

$$x_i(t) = \begin{bmatrix} v_h(t) & v_v(t) & q(t) & \theta(t) \end{bmatrix}^T$$

and whose elements are the horizontal velocity, the vertical velocity, the pitch rate, and the pitch angle, respectively. The input vector includes collective pitch control and longitudinal cyclic pitch control.

The adjacency matrix $\mathcal{A}$, the in-degree matrix D, the Laplacian matrix L and the pinning gain matrix G are

Table 1. Magnitudes of faults for Agent 2.

Actuator 1	Time	Actuator 2
0	$0\,\mathrm{s} \le t < 2\,\mathrm{s}$	0
$0.2u_{2,1}$	$2\,\mathrm{s} \le t < 4\,\mathrm{s}$	$0.15u_{2,2}$
0	$4\,\mathrm{s} \le t < 6\,\mathrm{s}$	0
$-0.2u_{2,1}$	$6\,\mathrm{s} \le t < 8\,\mathrm{s}$	$-0.15u_{2,2}$
0	$8\,\mathrm{s} \le t < 12\,\mathrm{s}$	0

Table 2. Magnitudes of faults for Agent 4.

Actuator 1	Time	Actuator 2
0	$0\,\mathrm{s} \le t < 2\,\mathrm{s}$	0
$0.05u_{4,1}$	$2\,\mathrm{s} \le t < 4\,\mathrm{s}$	0
0	$4\,\mathrm{s} \le t < 6\,\mathrm{s}$	0
$-0.05u_{4,1}$	$6\,\mathrm{s} \le t < 8\,\mathrm{s}$	0
0	$8\,\mathrm{s} \le t < 12\,\mathrm{s}$	0
0	$t \ge 4\,s$	$0.2\sin(0.5t)u_{4,2}$

obtained from Fig. 2,

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \qquad G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

As can be seen from Fig. 2, a spanning tree in the proposed graph exists and all eigenvalues of $(L + G)$ are 1, i.e., $(L + G) > 0$, which fulfills Lemma 1.

For this numerical experiment, the following notation is used to define all vectors:

$$q_a(t) = \begin{bmatrix} q_{a,b}(t) & q_{a,b}(t) \end{bmatrix}^T,$$

where a is the agent label and b is the vector element. Then, as each agent has only two actuators, faults in Agents 2 and 4 can be represented as $f_2(t) = \begin{bmatrix} f_{2,1}(t) & f_{2,2}(t) \end{bmatrix}^T$ and $f_4(t) = \begin{bmatrix} f_{4,1}(t) & f_{4,2}(t) \end{bmatrix}^T$, respectively. Faults occur simultaneously in Agents 2 and 4, and their evolution are described in Tables 1 and 2, respectively.

The magnitude of 0.2 represents a degradation of 20% on the actuator $u_{2,1}$. Therefore, $0.15u_{2,2}$ represents a 15% of degradation of Actuator 2. A similar analysis can be done for Agent 4 based on Table 2. It is worth noting that faults are co-occurring in both agents. Note that this degradation affects the thrust given by the rotors, and this may have several consequences on the system.

The initial conditions of the agents were randomly chosen in the interval $[0, 3]$ in order to be nonzero. Thus, the initial conditions are

$$agent_1 = \begin{bmatrix}1.6436 & 0.9044 & 2.0943 & 2.9972\end{bmatrix}^T,$$

$$agent_2 = \begin{bmatrix}2.8282 & 2.1033 & 1.9996 & 0.5134\end{bmatrix}^T,$$

$$agent_3 = \begin{bmatrix}1.2532 & 1.9990 & 0.5344 & 0.0978\end{bmatrix}^T,$$

$$agent_4 = \begin{bmatrix}2.9492 & 1.6174 & 0.3840 & 1.6836\end{bmatrix}^T.$$

The initial conditions of the agent observers were randomly chosen at different intervals:

$$observer_1 = \begin{bmatrix}1.6456 & 0.3822 & 0.9343 & 0.4461\end{bmatrix}^T,$$

$$observer_2 = \begin{bmatrix}1.0151 & 2.8898 & -0.7424 & -2.2763\end{bmatrix}^T,$$

$$observer_3 = \begin{bmatrix}2.5713 & 2.4692 & 2.5728 & 3.7685\end{bmatrix}^T,$$

$$observer_4 = \begin{bmatrix}0.7378 & 1.7110 & 0.8565 & 0.4524\end{bmatrix}^T.$$

In order to test robustness, white Gaussian noise with variance 0.01 and zero mean is assumed for the sensors. Defining $H = B$, and applying Theorem 1, the following constant matrices P, Y, R and F are obtained with the YALMIP toolbox and the SEDUMI solver:

$$P = \begin{bmatrix}1.2439 & 0.0126 & 0.1161 & 0.3070\\ 0.0126 & 0.0378 & 0.0461 & 0.0349\\ 0.1161 & 0.0461 & 0.0862 & 0.0470\\ 0.3070 & 0.0349 & 0.0470 & 0.1608\end{bmatrix},$$

$$Y = \begin{bmatrix}0.8754 & 0.1567 & 0.3070\\ 0.0504 & 0.0614 & 0.0782\\ 0.1261 & 0.0765 & 0.1178\\ -0.3325 & -0.0399 & 0.0381\end{bmatrix},$$

$$R = \begin{bmatrix}5.2205 & 1.1986 & 1.0049\\ 29.1051 & 8.5784 & 6.7898\\ -13.2428 & -3.4711 & -2.2591\\ -14.4876 & -3.3858 & -2.4964\end{bmatrix},$$

$$F = \begin{bmatrix}-0.0460 & -0.1149 & -0.2609\\ 0.6444 & -0.0780 & 0.0572\end{bmatrix},$$

$$\Gamma = \begin{bmatrix}13 & 3\\ 3 & 13\end{bmatrix}.$$

Figure 3 presents the fault estimate $\hat{f}_{2,1}$ for Actuator 1 in Agent 2 and Fig. 4 shows the fault estimate $\hat{f}_{4,1}$ for Actuator 1 in Agent 4. In both figures, it is possible to see that the estimates converge asymptotically to the fault quickly even when the dynamics of the fault changes (observe the behavior of the fault estimator after 2 s, 4 s, 6 s and 8 s). Finally, Figs. 5 and 6 show the state estimation error for Agents 1–4. They all converge asymptotically to zero. Note that state estimation error increase in the presence of faults at times 2 s, 4 s, 6 s, and 8 s for Agents 2 and 4 that are the only agents with faults.

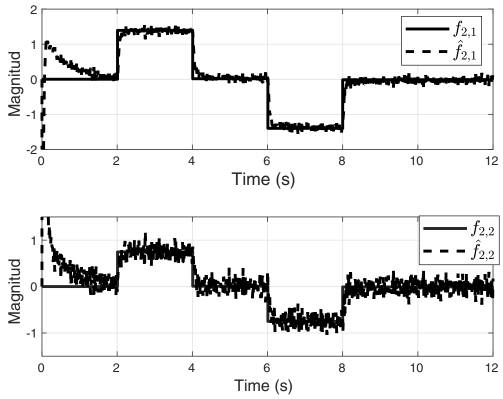

Fig. 3. Fault estimation for Agent 2.

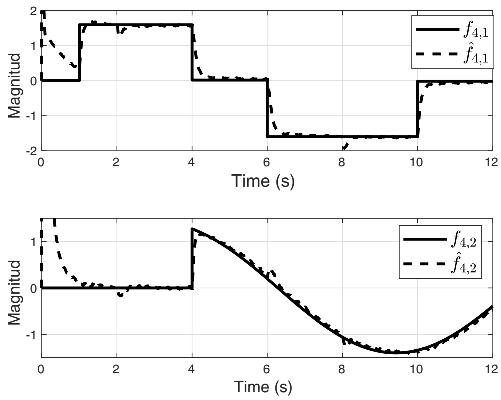

Fig. 4. Fault estimation for Agent 4.

Nevertheless, the observer is robust enough to estimate states even with multiple and simultaneous faults, as can be seen in Figs. 5 and 6.

Other tests were performed with different initial conditions of the observer to validate the convergence. These initial conditions were chosen randomly to illustrate the observer performance. Nonetheless, due to space limitations, only one test is reported here. As expected, the observers adequately converge to the values of the states and faults. Note that all observers converge regardless of the initial conditions. However, the convergence time varies depending on the difference between the initial state of the observer and the systems. From a practical point of view, the tolerance of the convergence time depends on the precision required in the application. In any case, this convergence time could be improved by choosing the initial conditions as close as possible to the real states.

5. Conclusions

According to the results presented, the proposed proportional-integral observer for leader-following

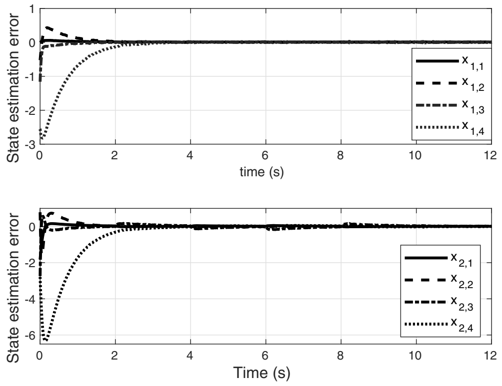

Fig. 5. Estimation error for States 1 and 2.

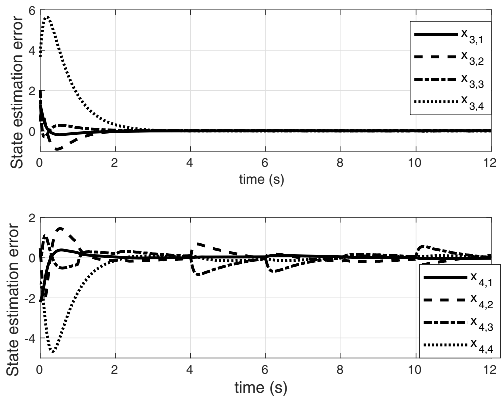

Fig. 6. Estimation error for States 3 and 4.

applications estimates fast and accurately the actuator faults in multiple follower agents in the multi-agent system. In this case, the learning rate in the fault estimator Γ was chosen heuristically since there is no established methodology to select an optimal value. Furthermore, the graphs can be indistinctly directed or undirected because the Laplacian matrix does not need to be symmetric in Theorem 1, which means that bidirectional or directed communication between agents is supported.

This research is focused to the observer design and does not integrate a control algorithm for the multi-agent system. Future work will be focused on extending the proposed method to nonlinear multi-agent systems. Another path to follow will be to provide a method to select a suitable learning rate for proportional-integral observers. Note also that the information given by the proportional-integral observer can be integrated into an active fault-tolerant control scheme. This problem will be addressed in a future contribution.

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