

SUB-OPTIMAL NONLINEAR PREDICTIVE AND ADAPTIVE CONTROL BASED ON THE PARAMETRIC VOLTERRA MODEL

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Predictive control algorithms have been worked out mainly to control linear plants. There is a great demand to apply different control ideas to nonlinear systems. Using predictive control algorithms for nonlinear systems is a promising technique. Extended horizon one-step-ahead and long-range optimal predictive control algorithms are given here for the parametric Volterra model (which includes also the generalized Hammerstein model). A quadratic cost function is minimized which considers the quadratic deviations of the reference signal and the output signal at a future point (or points) beyond the dead time and also penalizes large control signal increments. For prediction of the output signal, a predictive model is applied which uses information about the input and output signals up to the current time. A predictive transformation of the nonlinear dynamic model is given. The incremental model is advantageous since the cost function contains the control increment and not the control signal itself. An incremental transformation of the predictive forms is also described. Sub-optimal solutions to the optimal control algorithms are discussed with different assumptions for the control signal during the control horizon. The effect of the different strategies and the effect of the tuning parameters is investigated through simulation examples.

Keywords: predictive control, nonlinear control, optimal control, nonlinear systems, adaptive control.

1. Introduction

Predictive control algorithms were suggested first in the mid 1970s and originated from industrial applications. The main idea is to calculate the series of control signal values which minimizes the quadratic deviation of the reference signal and the output signal predicted in a future horizon and also penalizes the squares of the control inputs or control increments. With the so-called receding horizon strategy only the first control signal is applied and at the next sampling point the procedure is repeated.

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A detailed theoretical analysis only followed the reports on successful industrial applications. Nowadays predictive control algorithms are declared as the second most accepted algorithms (after the PID algorithms) in industrial process control. The technique can be considered well-studied for control of linear plants. There are a variety of different versions of the algorithms (long-range or extended-horizon one-step-ahead optimal control based on non-parametric or parametric system models, etc.). The appropriate choice of different tuning parameters (prediction horizon, control horizon, control increment penalizing factors, etc.) could ensure good results for different circumstances and requirements.

There are several ways to model nonlinear processes. Non-parametric models are extensions of the linear weighting function series. With the Hammerstein series model square terms of the shifted input signals are considered and with the Volterra series model also cross-product terms of differently shifted input terms are included. The drawback of such models lies in the great number of parameters which have to be estimated. With a memory length of $m = 10$, the number of components in the Hammerstein series model is 21 and in the Volterra series model is 64. Parametric models are extensions of the linear pulse-transfer function model. With the simple Hammerstein cascade model the linear dynamic term is preceded by a nonlinear static characteristics. Its extension, the generalized Hammerstein model, is linear in the parameters, which makes adaptive control possible. The generalized Hammerstein model takes only equally shifted quadratic input terms into account. The parametric Volterra model includes the shifted input and output terms in linear form and the product terms of the equally and differently shifted input signals. Therefore, the parametric Volterra model approximates the whole Volterra series with only few parameters. Assuming a second-order memory the number of parameters becomes 7 and an adaptive tuning is easily performable.

Further on, the control based on the single-input, single-output parametric Volterra model is investigated. Control based on the generalized Hammerstein model remains a special case.

Non-parametric models are predictive ones, which means that the future output signal does not depend on the output signal terms not known at present. Parametric nonlinear dynamic models are non-predictive but can be transformed to such a form. If control increments are penalized in the cost function, an incremental predictive form of the system model is expedient where control increments are used instead of the control values.

One-dimensional analytical optimization (without any constraints) assumes a predictive form of the nonlinear dynamical model under consideration. Cascade models can be partitioned into a static nonlinear part and a linear dynamic part. The predictive control algorithm can be applied then for the linear part if the steady-state characteristic is known (Zhu *et al.*, 1991). With the generalized Hammerstein model this separation cannot be done and a nonlinear dynamical control algorithm is needed (Haber *et al.*, 1997; 1998). The parametric Volterra model is a generalization of the generalized Hammerstein model and approximates different nonlinear structures better than the Hammerstein model. The parametric Volterra model can be described by few parameters. This model is advantageous over the non-parametric Volterra

series. Predictive control based on the Volterra series was proposed e.g. by Bars and Haber (1988).

There are two ways to compute (predict) the output signal in the prediction horizon: sequential simulation of the non-predictive model equations till the end of the prediction horizon or using the corresponding prediction model for each point of the prediction horizon. The second method is especially advantageous when the change of the input signal during the control horizon is restricted, which means that the future input increments are any functions of the current input signal (e.g. they are equal to one another), then all predicted output signals in the prediction horizon can be expressed analytically and the cost function becomes an analytical function of the current control increment. In the case of the parametric Volterra model with polynomial steady-state characteristics the cost function is a polynomial of the current control increment. The optimal input increment can be computed by any constrained minimization algorithm. If hard input constraints are not taken into account, then the current control increment can be searched by a root finding algorithm and by a straightforward selection of the roots. Regardless of the degree of the nonlinear characteristics, a one-dimensional optimization can be used instead of a multi-dimensional one. (Another sub-optimal solution was recommended by Zheng (1998): the future control increments in the control horizon are approximated by linear controllers which can be computed analytically off-line based on linearized models.)

Long-range optimal and extended horizon one-step-ahead predictive optimal and sub-optimal control algorithms are given here for the parametric Volterra model (including the generalized Hammerstein model). Some properties of these algorithms are shown through simulation examples. Also adaptive control is demonstrated.

2. Control Aim and Control Strategy

The control signal is calculated by minimizing the following cost function:

$$J = \sum_{n_e=n_{e1}}^{n_{e2}} \gamma_{yn_e} \left[w(k+d+n_e) - \hat{y}(k+d+n_e | k) \right]^2 + \sum_{j=1}^{n_u} \gamma_{u,j-1} \Delta u^2(k+j-1) \Rightarrow \underset{\Delta u}{\text{MIN}} \quad (1)$$

Here w denotes the reference signal, $\hat{y}(k+d+n_e | k)$ is the n_e steps over the dead time d ahead predicted value of the output signal on the basis of the system model using information available up to the current time k , while d denotes the discrete dead time relative to the sampling time.

The tuning parameters of the control algorithm are: the prediction horizon $n_{e2} - n_{e1}$ (if $n_{e2} = n_{e1} \equiv n_e$, extended horizon one-step-ahead predictive control is applied), the control horizon n_u (the number of the supposed consecutive changes in the control signal), the weighting factors of the control error $\gamma_{yn_{e1}}, \dots, \gamma_{yn_{e2}}$, usually assumed to be equal to 1 ($\gamma_y = 1$), the weighting factors of the control increments $\gamma_{u0}, \dots, \gamma_{u,n_u-1}$, usually assumed to be equal to each other (and denoted then by γ_u).

The control increments $\Delta u(k), \dots, \Delta u(k+n_u-1)$ have to be calculated. Only the first one is applied as an input signal, and at the next time point the procedure

is repeated (a receding horizon strategy). The optimization problem formulation is similar to the LQ problem. The main difference is in the receding horizon strategy. A term penalizing the deviation of the output from its required value at the final point was not used. A new feature here is the control horizon which could be less than the prediction horizon. With an appropriate choice of the control horizon a good control performance could be achieved e.g. for non-minimum-phase plants. With $n_{e1} > 0$ an extension beyond the dead time is realized resulting in more moderated control signals. Penalizing control increments instead of the control values themselves introduces an integrating effect in the control system, which is advantageous when considering the static accuracy.

Depending on the prediction horizon lengths, two different control strategies can be distinguished:

- *Extended horizon one-step-ahead control* (e.g. Ydstie *et al.*, 1985): the prediction horizon is restricted to one value, $n_{e2} = n_{e1} \equiv n_e$.
- *Long-range optimal control* (e.g. Clarke *et al.*, 1987): $n_{e1} < n_{e2}$.

Two sub-optimal control strategies will be considered, which simplify the calculations (for linear systems, see (Ydstie *et al.*, 1985)):

- *Strategy 1*: Only one change is taken into account in the control signal at the current time point k , and during the control horizon the control signal is constant:

$$\Delta u(k) \neq 0, \Delta u(k+1) = 0, \Delta u(k+2) = 0, \dots, \Delta u(k+n_u-1) = 0 \quad (2)$$

- *Strategy 2*: The changes in the control signal during the control horizon are considered as equal to one another:

$$\Delta u(k) = \Delta u(k+1) = \Delta u(k+2) = \dots = \Delta u(k+n_u-1) \quad (3)$$

3. Parametric Volterra Model and Its Incremental Predictive Form

A noiseless parametric Volterra model with quadratic steady-state characteristics has been chosen as the process model. As being linear-in-parameters, it is suitable for adaptive control.

The generalized Volterra model is defined as follows:

$$A(q^{-1})y(k+d) = c_0^* + B_1(q^{-1})u(k) + B_2(q_1^{-1}, q_2^{-1})u^2(k) \quad (4)$$

where c_0^* is the constant term (the asterisk is used to distinguish from the constant term c_0 of a polynomial) and the polynomials of the backward shifting operator q^{-1} are

$$\begin{cases} A(q^{-1}) = 1 + a_1q^{-1} + \dots + a_{n_a}q^{-n_a} \\ B_1(q^{-1}) = b_{10} + b_{11}q^{-1} + \dots + b_{1n_{b1}}q^{-n_{b1}} \\ B_2(q_1^{-1}, q_2^{-1}) u^2(k) = \sum_{i=0}^{n_{b2}} \sum_{j=i}^{n_{b2}} b_{2ij} u(k-i)u(k-j) \end{cases} \quad (5)$$

As can be seen, the parametric Volterra model includes the generalized Hammerstein model as a special case if the terms $u(k-i)u(k-j)$ ($i \neq j$) are missing.

The predictive form of the parametric Volterra model can be given as (Haber, 1995)

$$\hat{y}(k+d+n_e) = c_0^p + \alpha(q^{-1})y(k) + \beta_1(q^{-1})u(k+n_e) + \beta_2(q_1^{-1}, q_2^{-1}) u^2(k+n_e) \quad (6)$$

The parameters can be calculated recursively by solving the following Diophantine equation (Clarke *et al.*, 1987):

$$1 = F(q^{-1}) A(q^{-1}) + q^{-(d+n_e)} G(q^{-1}) \quad (7)$$

Here the degree of the polynomial F is $d+n_e-1$, the degree of polynomial G is n_a-1 , and

$$\begin{aligned} \alpha(q^{-1}) &= G(q^{-1}), \quad c_0^p = F(1)c_0^*, \quad \beta_1(q^{-1}) = F(q^{-1}) B_1(q^{-1}) \\ \beta_2(q_1^{-1}, q_2^{-1}) &= F(q^{-1}) B_2(q_1^{-1}, q_2^{-1}) \end{aligned} \quad (8)$$

Equation (6) uses only the output information available up to the current time k to predict the future output value.

The control algorithm can be derived easier if the predictive form depends on the input signal increments, rather than on the input signal itself. The current and the future control signals are expressed with the current and future control increments and $u(k-1)$ as follows:

$$u(k+i) = u(k-1) + \sum_{j=0}^i \Delta u(k+j), \quad i = 0, 1, \dots, n_e \quad (9)$$

Let us define $\Delta u^*(k)$ as

$$\Delta u^*(k) = \begin{cases} u(k) - u(k-1) & \text{if } k \geq 0 \\ u(k) & \text{if } k < 0 \end{cases} \quad (10)$$

Taking (9) and (10) into consideration in the predictive equation (6), the incremental predictive equation can be derived in the following form:

$$\hat{y}(k+d+n_e | k) = c_0^p + \delta(q^{-1})y(k) + \gamma_1(q^{-1})\Delta u^*(k+n_e) + \gamma_2(q_1^{-1}, q_2^{-1})\Delta u^{*2}(k+n_e) \quad (11)$$

where

$$\delta(q^{-1}) = \alpha(q^{-1})$$

$$\gamma_{1i} = \sum_{\nu=0}^{\min(i, n_{b1}+d+n_e-1)} \beta_{1\nu}, \quad i = 0, 1, \dots, n_{b1} + d + n_e - 1$$

$$\gamma_{2ii} = \sum_{\nu=0}^{\min(i, n_{b2}+d+n_e-1)} \sum_{\mu=\nu}^{\min(i, n_{b2}+d+n_e-1)} \beta_{2\nu\mu}, \quad i = 0, 1, \dots, n_{b2} + d + n_e - 1$$

$$\gamma_{2ij} = \sum_{\nu=0}^{\min(i, n_{b2}+d+n_e-1)} \left[\sum_{\mu=\nu}^{\min(i, n_{b2}+d+n_e-1)} \beta_{2\nu\mu} + \sum_{\mu=\nu}^{\min(j, n_{b2}+d+n_e-1)} \beta_{2\nu\mu} \right],$$

$$i = 0, 1, \dots, n_{b2} + d + n_e - 1, \quad j = i + 1, i + 2, \dots, n_{b2} + d + n_e - 1 \quad (12)$$

Equation (11) includes product terms of the past, current and future control increments. As the past increments are known when the control algorithm has to be solved, such product terms can be handled as linear functions of the unknown (current or future) control increments. The incremental predictive form of the parametric Volterra model can be expressed as a function of the current and future control increments:

$$\hat{y}(k+d+n_e | k) = p_0^{(n_e)} + P_1^{(n_e)}(q^{-1})\Delta u(k+n_e) + P_2^{(n_e)}(q_1^{-1}, q_2^{-1})\Delta u^2(k+n_e) \quad (13)$$

The degrees of the polynomials are $\deg(P_1) = n_e$ and $\deg(P_2) = [n_e, n_e]$. The coefficients are calculated as follows:

$$p_0^{(n_e)} = c_0^p + \delta(q^{-1})y(k) + \sum_{i=n_e+1}^{n_e+n_{b2}+d-1} \left[\gamma_{1i} + \sum_{j=i}^{n_e+n_{b2}+d-1} \gamma_{2ij}\Delta u(k+n_e-j) \right] \times \Delta u(k+n_e-i) \quad (14a)$$

$$p_{1i}^{(n_e)} = \gamma_{1i} + \sum_{j=n_e+1}^{n_e+n_{b1}+d-1} \gamma_{2ij}\Delta u(k+n_e-j), \quad i = 0, 1, \dots, n_e \quad (14b)$$

$$p_{2ij}^{(n_e)} = \gamma_{2ij}, \quad i = 0, 1, \dots, n_e, \quad j = 1, 2, \dots, n_e \quad (14c)$$

The upper index (n_e) means that the coefficients depend on the prediction step.

4. Control Algorithm

With a quadratic process model the cost function (1) is a fourth-degree function of the control increments in the control horizon. The minimization can be performed numerically with and without constraints. If, however, we assume any sub-optimal strategy (2) or (3), then only the current control increment has to be searched. Assuming hard input constraints the one-dimensional optimization can be performed numerically. Without constraints the minimization of the cost function (1) leads to the solution of a cubic polynomial equation

$$k_0 + k_1 \Delta u(k) + k_2 \Delta u^2(k) + k_3 \Delta u^3(k) = 0 \quad (15)$$

The solution to (15) depends on the control strategy chosen (Haber *et al.*, 1998):

- Strategy 1 (the control signal is kept constant in the control horizon)

$$k_0 = \sum_{n_e=n_{e1}}^{n_{e2}} \gamma_{y n_e} \left[p_0^{(n_e)} - w(k+d+n_e) \right] p_{1 n_e}^{(n_e)} \quad (16a)$$

$$k_1 = 2 \sum_{n_e=n_{e1}}^{n_{e2}} \left[p_0^{(n_e)} - w(k+d+n_e) \right] p_{2 n_e n_e}^{(n_e)} + \sum_{n_e=n_{e1}}^{n_{e2}} \left[p_{1 n_e}^{(n_e)} \right]^2 + \gamma_{u0} \quad (16b)$$

$$k_2 = 3 \sum_{n_e=n_{e1}}^{n_{e2}} p_{1 n_e}^{(n_e)} p_{2 n_e n_e}^{(n_e)} \quad (16c)$$

$$k_3 = 2 \sum_{n_e=n_{e1}}^{n_{e2}} \left[p_{2 n_e n_e}^{(n_e)} \right]^2 \quad (16d)$$

- Strategy 2 (the control increments are kept constant in the control horizon)

$$k_0 = \sum_{n_e=n_{e1}}^{n_{e2}} \gamma_{y n_e} \left[p_0^{(n_e)} - w(k+d+n_e) \right] \left[\sum_{i=0}^{n_e} p_{1i}^{(n_e)} \right] \quad (17a)$$

$$k_1 = 2 \sum_{n_e=n_{e1}}^{n_{e2}} \left[p_0^{(n_e)} - w(k+d+n_e) \right] \left[\sum_{i=0}^{n_e} \sum_{j=i}^{n_e} p_{2ij}^{(n_e)} \right] + \sum_{n_e=n_{e1}}^{n_{e2}} \left[\sum_{i=0}^{n_e} p_{1i}^{(n_e)} \right]^2 + \sum_{j=0}^{n_{e2}} \gamma_{uj} \quad (17b)$$

$$k_2 = 3 \sum_{n_e=n_{e1}}^{n_{e2}} \left[\sum_{i=0}^{n_e} p_{1i}^{(n_e)} \right] \left[\sum_{i=0}^{n_e} \sum_{j=i}^{n_e} p_{2ij}^{(n_e)} \right] \quad (17c)$$

$$k_3 = 2 \sum_{n_e=n_{e1}}^{n_{e2}} \left[\sum_{i=0}^{n_e} \sum_{j=i}^{n_e} p_{2ij}^{(n_e)} \right]^2 \quad (17d)$$

Equations (16) and (17) are valid for the long-range optimum case. With one-step-ahead extended horizon we have $n_{e2} = n_{e1} \equiv n_e$.

The control increment is to be chosen from among the solutions to eqn. (15). From among three real roots that one is chosen for which the value of the cost function is minimal. The real root is chosen if conjugate complex pairs exist too. It has to be checked whether the control signal is inside the control limits, otherwise the weighting factor of the control increments should be increased.

The tuning variables are the prediction horizon $n_{e2} - n_{e1}$, the extension of the prediction horizon beyond the dead time (n_{e1}), and the weighting factor γ_u penalizing large values of the control increments. The control aim is a fast aperiodic behavior. Long-range control is generally smoother and slower than one-step-ahead extended horizon control. The control behavior is smoother and slower with Strategy 2 than with Strategy 1. The extension is suggested to be higher or equal to the order of the linear(ized) part of the process otherwise inter-sampling oscillations may occur. The smaller the weighting factor of the control increments, the higher the control effort could be and the faster the system output is. Simulations show the effect of the tuning parameters.

5. Simulations

Example 1. (Adaptive predictive control of the parametric Volterra model) The system is given by the following Volterra model:

$$y(k) - 0.9y(k-1) + 0.2y(k-2) = 1 + 0.2u(k-1) + 0.1u(k-2) + 0.1u^2(k-1) + 0.2u(k-1)u(k-2) + 0.05u^2(k-2)$$

The reference signal was changed stepwise in each 20 steps first to 5, then to 6, 4.5, 4, and finally to 3.5. For the first 20 steps a stochastic excitation was applied and off-line LS (Least Squares) identification was executed. After that recursive LS parameter estimation was used with a forgetting factor of 0.9 and the control algorithm was executed in each sampling step. The weighting factors of the control error and the control increments were $\gamma_y = 1$ and $\gamma_u = 0.001$, respectively.

Figures 1 and 2 compare the long-range optimal control (with $n_{e1} = 1, n_{e2} = 4$) and the one-step-ahead extended horizon control (with $n_{e1} = n_{e2} = n_e = 4$) for Strategy 1 (only one change of the control signal during the control horizon). It is seen that the long-range control has a smaller control error than the one-step-ahead control after the initial transients of the adaptive tuning were decayed.

Figures 3 and 4 compare Strategy 1 (only one change of the control signal during the control horizon) and Strategy 2 (equal control increments in the control horizon) for the long-range optimal control (with $n_{e1} = 1, n_{e2} = 4$). As can be seen, the control is smoother and slower with Strategy 2 than with Strategy 1 already at the first set point change after the adaptive tuning.

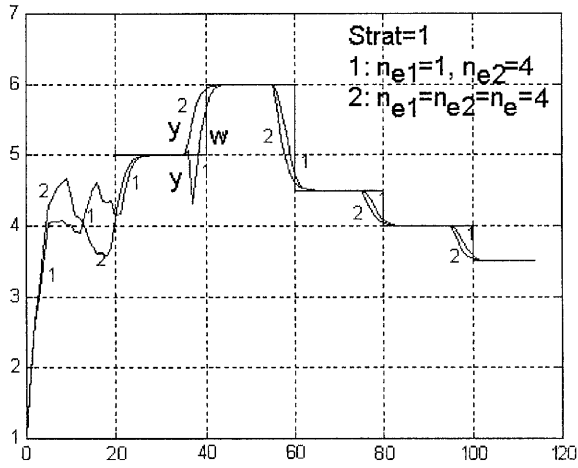


Fig. 1. Reference and output signals for adaptive long-range and extended horizon control of a parametric Volterra model with Strategy 1.

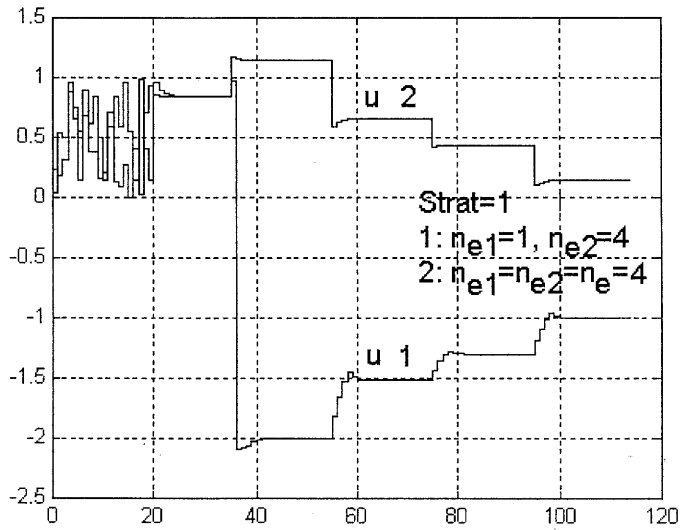


Fig. 2. Control signals for adaptive long-range and extended horizon control of a parametric Volterra model with Strategy 1.

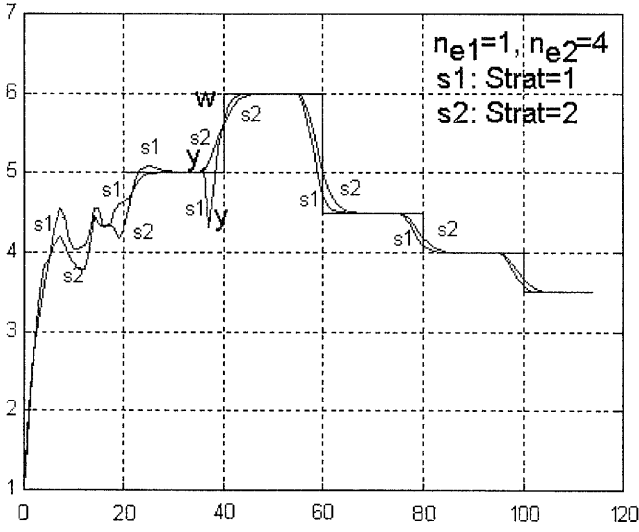


Fig. 3. Reference and output signals for adaptive long-range control of a parametric Volterra model with Strategies 1 and 2.

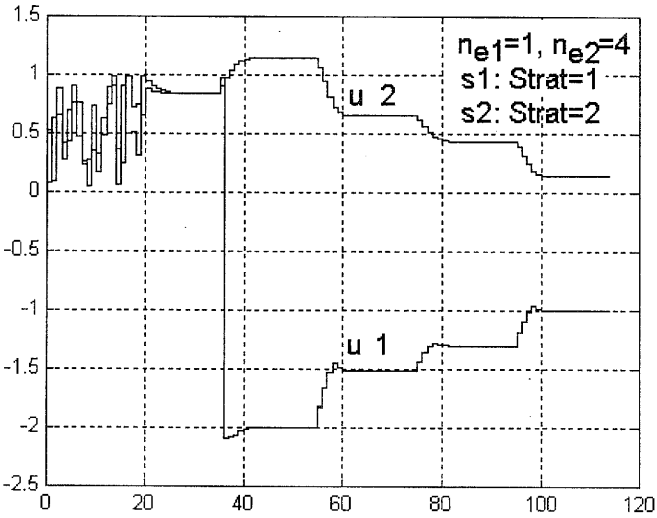


Fig. 4. Control signals for adaptive long-range control of a parametric Volterra model with Strategies 1 and 2.

Example 2. (Adaptive predictive control of a simple Wiener cascade model based on the generalized Hammerstein and the parametric Volterra model) The simple Wiener model is shown in Fig. 5. The static characteristics of the cascade Wiener model is again $2 + U + 0.5U^2$. The transfer function of the continuous linear dynamic part preceding the static nonlinearity is $1/(50s^2 + 15s + 1)$. The sampling time was 5. The extended horizon control strategy is applied for $n_e = 2$ with Strategy 1. Adaptive control is used. For the first 150 s a stochastic excitation was applied and off-line LS identification was executed. After that recursive LS parameter estimation was used with a forgetting factor of 0.9 and the control algorithm was executed in each sampling step.

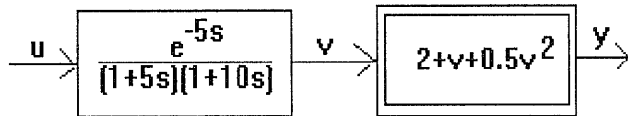


Fig. 5. Simple Wiener cascade model.

Figure 6 shows the reference, the output and the control signals for the one-step-ahead extended horizon algorithm based on the generalized Hammerstein model, while Fig. 7 gives the results based on the parametric Volterra model. Both the algorithms give acceptable results, but for higher jumps of the reference signal the Volterra algorithm gives much better results. (See the plots between the discrete times 200 and 400).

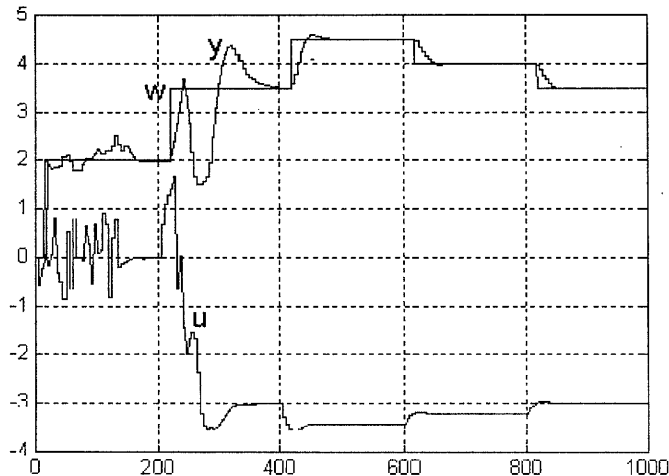


Fig. 6. Extended horizon adaptive control of a Wiener model based on the generalized Hammerstein model.

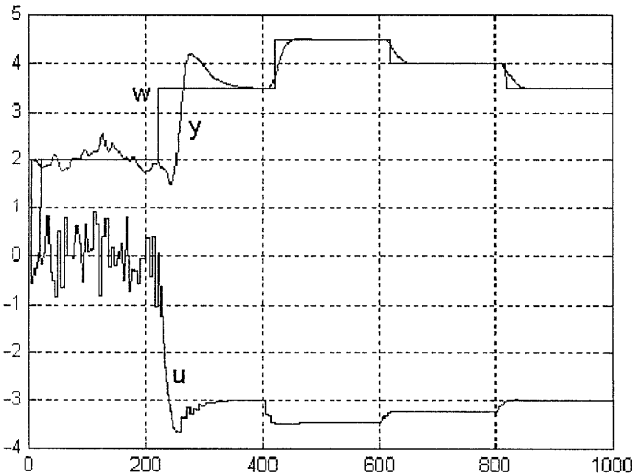


Fig. 7. Extended horizon adaptive control of a Wiener model by the parametric Volterra model.

6. Conclusions

Sub-optimal long-range and extended horizon one-step-ahead predictive control algorithms have been derived for the nonlinear parametric Volterra model (including the generalized Hammerstein model). Using a restriction on the control increments during the control horizon, the multi-dimensional optimization could be reduced to a one-dimensional minimization. Two sub-optimal strategies were introduced: a constant control signal or equal control increments during the control horizon. The sub-optimal algorithms require fewer computations and they are suitable for real-time applications and adaptive control. Simulations illustrate the new algorithms.

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References

- Bars R. and Haber R. (1988): *Long-range predictive control of nonlinear systems given by Volterra series*. — Prep. 8-th IFAC/IFORS Symp. *Identification and System Parameter Estimation*, Beijing, China, pp.1313–1317.

- Clarke D.W., Mohtadi C. and Tuffs P.S. (1987): *Generalized predictive control. Part 1. Basic algorithm, Part 2. Extensions and interpretations.* — *Automatica*, Vol.23, No.2, pp.137–160.
- Haber R. (1995): *Predictive control of nonlinear dynamic processes.* — *Appl. Math. Comp.*, Vol.70, No.2–3, pp.169–184.
- Haber R., Bars R. and Abufaris A. (1997): *Suboptimal and optimal extended horizon predictive control of the Hammerstein model.* — *Proc. European Control Conference ECC'97*, Brussels, Belgium, paper summaries, Vol.2., TH-A B3/1-6.
- Haber R., Bars R. and Lengyel O. (1998): *Long-range predictive control of the parametric Hammerstein model.* — *Proc. IFAC Symp. Nonlinear Control System Design*, Enschede, the Netherlands, Vol.2, pp.434–439.
- Zheng A. (1998): *Some practical issues and possible solutions for nonlinear model predictive control.* — *Abstract, Int. Symp. Nonlinear Model Predictive Control: Assessment and Future Directions*, Ascona, Switzerland, p.32.
- Zhu Q.M., Warwick K. and Douce J.L. (1991): *Adaptive general predictive controller for nonlinear systems.* — *IEE Proc.*, Part D, Vol.138, No.1, pp.33–40.
- Ydstie B.E., Kershenbaum L.S. and Sargent R.H. (1985): *Theory and application of an extended horizon self tuning controller.* — *AIChE J.*, Vol.31, No.11, pp.1771–1780.