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GLOBAL STABILITY OF NONLINEAR FEEDBACK SYSTEMS WITH FRACTIONAL POSITIVE LINEAR PARTS

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The global (absolute) stability of nonlinear systems with fractional positive and not necessarily asymptotically stable linear parts and feedbacks is addressed. The characteristics $u = f(e)$ of the nonlinear parts satisfy the condition $k_1 e \leq f(e) \leq k_2 e$ for some positive k_1 and k_2. It is shown that the fractional nonlinear systems are globally asymptotically stable if the Nyquist plots of the fractional positive linear parts are located on the right-hand side of the circles $(-1/k_1, -1/k_2)$.

Keywords: global, stability, fractional, nonlinear, feedback, positive, system.

1. Introduction

In positive systems inputs, state variables and outputs take only nonnegative values for any nonnegative inputs and nonnegative initial conditions (Farina and Rinaldi, 2000; Kaczorek, 2002). Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollutions models. A variety of models having positive behavior can be found in engineering, management science, economics, social sciences, biology and medicine, etc. Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An overview of state of the art in positive systems theory is given in the works of Berman and Plemmons (1994), Farina and Rinaldi (2000), Kaczorek (2011b) or Kaczorek and Borawski (2017).

Positive linear systems with different fractional orders were addressed by Kaczorek (2010; 2011a) and Sajewski (2017b). The stability of standard and positive systems was investigated by Kaczorek (2019a; 2015b; 2017) and Kudrewicz (1964), and that of fractional systems by Busłowicz and Kaczorek (2009), Farina and Rinaldi (2000), Kaczorek (2015a; 2016; 2019b; 2015b; 2017), Kaczorek and Borawski (2017) or Mitkowski (2008). The stability and stabilization problems of positive fractional descriptor system were investigated by Ruszewski (2019) and Sajewski (2017a; 2017b). Descriptor positive systems were analyzed by Borawski (2017) and Kaczorek (2012). Linear positive electrical circuits with state feedbacks were addressed by Borawski (2017) as well as Kaczorek and Rogowski (2015). The global stability of nonlinear systems with negative feedbacks and positive, not necessarilily asymptotically stable linear parts was investigated by Kaczorek (2019b).

In this paper the global stability of nonlinear feedback systems with fractional positive linear parts will be addressed. The paper is organized as follows. In Section 2 the positive fractional linear systems and their transfer matrices are considered. The main result of the paper is given in Section 3 where sufficient conditions for the global stability of the fractional positive nonlinear feedback systems are established. Concluding remarks are given in Section 4.

The following notation will be used: $\mathbb{R}$, the set of real numbers; $\mathbb{R}^{n\times m}$, the set of real $n \times m$ matrices; $\mathbb{R}_+^{n\times m}$, the set of real $n \times m$ matrices with nonnegative entries and $\mathbb{R}_+^n = \mathbb{R}_+^{n\times 1}$; M_n, the set of $n \times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries); I_n, the $n \times n$ identity matrix.

2. Positive fractional linear systems and their transfer matrices

In this paper the following Caputo definition of the fractional derivative of α order will be used (Kaczorek, 2011b; Kaczorek and Rogowski, 2015; Ostalczyk, 2016; Podlubny, 1999):

$$\begin{aligned} {}_0D_t^\alpha f(t) &= \frac{\mathrm{d}^\alpha f(t)}{\mathrm{d}t^\alpha} \\ &= \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\dot{f}(\tau)}{(t-\tau)^\alpha}\,\mathrm{d}\tau, \end{aligned} \tag{1}$$

where $0 < \alpha < 1$,

$$\dot{f}(\tau) = \frac{\mathrm{d}f(\tau)}{\mathrm{d}\tau}$$

and

$$\Gamma(z) = \int_0^\infty t^{x-1} e^{-t}\,\mathrm{d}t, \quad \mathrm{Re}(x) > 0$$

is the Euler gamma function.

Consider the fractional continuous-time linear system

$$\frac{\mathrm{d}^\alpha x(t)}{\mathrm{d}t^\alpha} = Ax(t) + Bu(t), \tag{2a}$$

$$y(t) = Cx(t) + Du(t), \tag{2b}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ are the state, input and output vectors, respectively, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$.

Definition 1. (*Kaczorek, 2011b; Kaczorek and Rogowski, 2015*) The fractional system (2) is called (internally) *positive* if $x(t) \in \mathbb{R}_+^n$ and $y(t) \in \mathbb{R}_+^p$, $t \geq 0$ for any initial conditions $x(0) \in \mathbb{R}_+^n$ and all inputs $u(t) \in \mathbb{R}_+^m$, $t \geq 0$.

Theorem 1. (Kaczorek, 2011b; Kaczorek and Rogowski, 2015) *The fractional system (2) is positive if and only if*

$$A \in M_n, \quad B \in \mathbb{R}_+^{n\times m}, \quad C \in \mathbb{R}_+^{p\times n}, \quad D \in \mathbb{R}_+^{p\times m}. \tag{3}$$

The transfer matrix of the system (2) is given by

$$T(s^\alpha) = C[I_n s^\alpha - A]^{-1} B + D. \tag{4}$$

The matrix A is called Hurwitz if the matrix is asymptotically stable.

Theorem 2. *If the matrix $A \in M_n$ is Hurwitz and $B \in \mathbb{R}_+^{n\times m}$, $C \in \mathbb{R}_+^{p\times n}$, $D \in \mathbb{R}_+^{p\times m}$ in the fractional linear positive system (2), then all the coefficients of the transfer matrix* (4) *are positive.*

Proof. First we shall show by induction with respect to n that the matrix

$$[I_n s^\alpha - A]^{-1} \in \mathbb{R}^{n\times n}(s) \tag{5}$$

has positive coefficients.

The hypothesis is true for $n = 1$ since

$$[s^\alpha + a]^{-1} = \frac{1}{s^\alpha + a} \tag{6}$$

and for $n = 2$

$$\begin{aligned} &[I_2 s^\alpha - A_2]^{-1} \\ &= \begin{bmatrix} s^\alpha + a_{11} & -a_{12} \\ -a_{21} & s^\alpha + a_{22} \end{bmatrix}^{-1} \\ &= \frac{1}{s^{2\alpha} + a_1 s^\alpha + a_0} \begin{bmatrix} s^\alpha + a_{22} & a_{12} \\ a_{21} & s^\alpha + a_{11} \end{bmatrix}, \end{aligned} \tag{7}$$

where $a_1 = a_{11} + a_{22} \geq 0$, $a_0 = a_{11}a_{22} - a_{12}a_{21} \geq 0$. It is well-known (Kaczorek, 2016) that the positive continuous-time linear system (2) is asymptotically stable if and only if all coefficients of its characteristic polynomial are positive.

Assuming that the hypothesis is valid for $n-1$ (the matrix $[I_{n-1}s^\alpha - A_{n-1}]^{-1}$) we shall show that it is also true for n (the matrix $[I_n s^\alpha - A_n]^{-1}$). It is easy to check that the inverse matrix of the matrix

$$\begin{aligned} &[I_n s^\alpha - A_n] \\ &= \begin{bmatrix} I_{n-1}s^\alpha - A_{n-1} & u_n \\ v_n & s^\alpha + a_{nn} \end{bmatrix}, \end{aligned} \tag{8}$$

where

$$u_n = -\begin{bmatrix} a_{1n} \\ \vdots \\ a_{n-1,n} \end{bmatrix},$$

$$v_n = -\begin{bmatrix} a_{n1} & \cdots & a_{n,n-1} \end{bmatrix}$$

has the form of Eqn. (9), in which

$$a_n = (s^\alpha + a_{nn}) - v_n [I_{n-1}s^\alpha - A_{n-1}]^{-1} u_n. \tag{10}$$

By assumption, the matrix $[I_{n-1}s^\alpha - A_{n-1}]^{-1}$ has all positive coefficients and the rational function (10) has positive coefficients. Taking into account that u_n and v_n have nonnegative entries, we conclude that

$$-\frac{[I_{n-1}s^\alpha - A_{n-1}]^{-1} u_n}{a_n}$$

and

$$-\frac{v_n [I_{n-1}s^\alpha - A_{n-1}]^{-1}}{a_n}$$

are respectively column and row rational vectors with positive coefficients. By the same arguments, the matrix

$$\frac{[I_{n-1}s^\alpha - A_{n-1}]^{-1} u_n v_n [I_{n-1}s^\alpha - A_{n-1}]^{-1}}{a_n} \tag{11}$$

has also all rational entries in s^α with positive coefficients and the matrix (5) has positive coefficients. Therefore, if $B \in \mathbb{R}_+^{n\times m}$, $C \in \mathbb{R}_+^{p\times n}$, $D \in \mathbb{R}_+^{p\times m}$ then all coefficients of the transfer matrix (4) are positive. ■

$$[I_n s^\alpha - A_n]^{-1} = \begin{bmatrix} [I_{n-1}s^\alpha - A_{n-1}]^{-1} + \dfrac{[I_{n-1}s^\alpha - A_{n-1}]^{-1}u_n v_n [I_{n-1}s^\alpha - A_{n-1}]^{-1}}{a_n} & -\dfrac{[I_{n-1}s^\alpha - A_{n-1}]^{-1}u_n}{a_n} \\ -\dfrac{v_n[I_{n-1}s^\alpha - A_{n-1}]^{-1}}{a_n} & \dfrac{1}{a_n} \end{bmatrix}. \tag{9}$$

Example 1. Consider the fractional positive linear system (2) with the matrices

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad C = [\ 2 \quad 1\ ], \qquad D = [2]. \tag{12}$$

The matrix A is Hurwitz. Using (4) and (12), we obtain

$$\begin{aligned} T(s^\alpha) &= C[I_2 s^\alpha - A]^{-1}B + D \\ &= [\ 2 \quad 1\ ] \begin{bmatrix} s^\alpha + 2 & -1 \\ -1 & s^\alpha + 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + [2] \\ &= \frac{2s^{2\alpha} + 11s^\alpha + 15}{s^{2\alpha} + 4s^\alpha + 3}. \end{aligned} \tag{13}$$

The transfer function (13) has positive coefficients. ♦

3. Asymptotic stability

Consider the fractional nonlinear feedback system shown in Fig. 1 consisting of the fractional linear part described by the equations

$$\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = Ax + bu, \tag{14a}$$

$$y = cx, \tag{14b}$$

where $x = x(t) \in \mathbb{R}^n$, $u = u(t) \in \mathbb{R}$, $y = y(t) \in \mathbb{R}$, $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, $c \in \mathbb{R}^{1\times n}$, and of the nonlinear element with the characteristic $u = f(e)$ (Fig. 2) satisfying the conditions

$$f(0) = 0 \quad \text{and} \quad 0 \le \frac{f(e)}{e} \le k, \quad k < +\infty. \tag{15}$$

It is assumed that the fractional linear part (14) is positive, i.e.,

$$A \in M_n, \quad b \in \mathbb{R}^n_+, \quad c \in \mathbb{R}^{1\times n}_+, \tag{16}$$

but not necessarily asymptotically stable.

In many cases, if the linear part is unstable, then by a suitable choice of the gain k_1 we may obtain (Fig. 3) an asymptotically stable positive linear part with the transfer function

$$T_1(s^\alpha) = \frac{T(s^\alpha)}{1 + k_1 T(s^\alpha)} \tag{17}$$

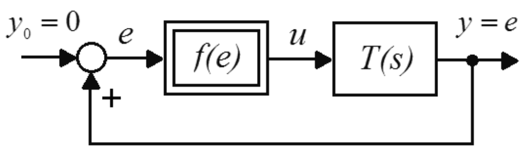

Fig. 1. Fractional nonlinear feedback system.

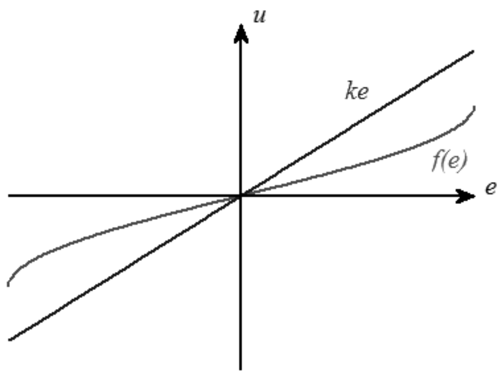

Fig. 2. Characteristic of the nonlinear element.

and a nonlinear element with the characteristic $f_1(e) = f(e) - k_1 e$ satisfying the condition (Fig. 4)

$$f_1(0), k_1 \le \frac{f_1(e)}{e} \le k_2 = k - k_1, \tag{18}$$

e.g., the characteristic is located between the straight lines $k_1 e$ and $k_2 e$.

Definition 2. The fractional nonlinear system is called globally (or absolutely) *asymptotically stable* if $\lim_{t\to\infty} x(t) = 0$ for any $x(0) \in \mathbb{R}^n_+$.

Definition 3. The circle in the plane $(P(\omega), Q(\omega))$ with center at point

$$\left(-\frac{k_1 + k_2}{2k_1 k_2}, 0\right)$$

and radius

$$\frac{k_2 - k_1}{2k_1 k_2}$$

will be called the $(-1/k_1, -1/k_2)$ *circle*.

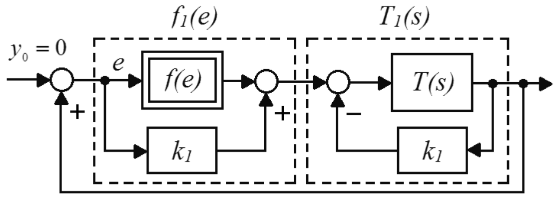

Fig. 3. Nonlinear feedback system with a gain.

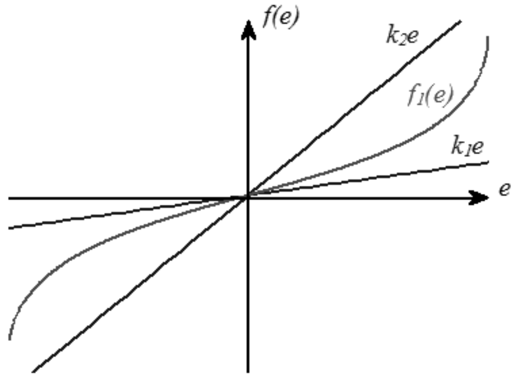

Fig. 4. Characteristic of the nonlinear element with the gain k_1.

Theorem 3. *The fractional nonlinear feedback system (Fig. 3) consisting of a positive linear asymptotically stable part with the transfer function $T_1(s)$ and of a nonlinear element with the characteristic satisfying the condition* (18) *is globally asymptotically stable if the Nyquist plot of $T_1(j\omega) = P(\omega) + jQ(\omega)$ of the linear part is located on the right-hand side of the circle $(-1/k_1, -1/k_2)$, (e.g., the plot $T_1(j\omega)$ does not cross the asymptote $P(\omega) = -1/k_2$).*

Proof. It is based on the application of the Lyapunov method to the fractional positive nonlinear system (Kaczorek, 2015b; 2017; Lyapunov, 1963; Leipholz, 1970). As the Lyapunov function we choose the time function

$$V(t) = \lambda^T E_\alpha(A_1 t^\alpha) b > 0, \quad t \in [0, +\infty), \tag{19a}$$

where $\lambda^T = \begin{bmatrix} \lambda_1 & \cdots & \lambda_n \end{bmatrix}^T$ is a strictly positive vector, i.e., $\lambda_k > 0$, $k = 1, \dots, n$, A_1 is the state matrix of the linear part with $T_1(s)$ and

$$E_\alpha(A_1 t^\alpha) = \sum_{k=0}^{\infty} \frac{A_1^k t^{ka}}{\Gamma(ka+1)} \tag{19b}$$

is the Mittag-Leffler function.

The function $V(t) > 0$ for $t \in [0, +\infty)$ since $A_1 \in M_n$ is asymptotically stable and $b \in \mathbb{R}_+^n$.

From (19) we have

$$\frac{\mathrm{d}^\alpha V(t)}{\mathrm{d}t^\alpha} = \lambda^T A_1 E_\alpha(A_1 t^\alpha) b < 0 \tag{20}$$

for $t \in [0, +\infty)$ since $\lambda^T A_1 < 0$ for the Hurwitz matrix $A_1 \in M_n$ and

$$c E_\alpha(A_1 t^\alpha) b \geq c e^{A_1 t} b > 0 \tag{21}$$

for $t \in [0, +\infty), 0 < \alpha < 1$.

Therefore, by the Lyapunov theorem the fractional positive nonlinear system is asymptotically stable if the conditions (19a) and (20) are satisfied.

Note that

$$T_1(s) = c\mathcal{L}[e^{A_1 t}]b = c[I_n s - A_1]^{-1} b, \tag{22}$$

where $\mathcal{L}$ is the Laplace transform operator. From (19) and (22) we have

$$\operatorname{Re} T_1(j\omega) + \frac{1}{k} > 0 \tag{23}$$

for $\omega \geq 0$ and $k = k_2 - k_1 > 0$. Taking into account that

$$\begin{aligned} \operatorname{Re} T_1(j\omega) + \frac{1}{k_2 - k_1} &= \operatorname{Re}\left[\frac{T(j\omega)}{1 + k_1 T(j\omega)} + \frac{1}{k_2 - k_1}\right] \\ &= \frac{1}{k_2 - k_1} \operatorname{Re}\left[\frac{1 + k_2 T(j\omega)}{1 + k_1 T(j\omega)}\right] \end{aligned} \tag{24}$$

and that the border of asymptotic stability is the $j\omega$ axis, we obtain

$$j\omega = \frac{1 + k_2[P(\omega) + jQ(\omega)]}{1 + k_1[P(\omega) + jQ(\omega)]} \tag{25a}$$

or

$$\begin{aligned} & j\omega\{1 + k_1[P(\omega) + jQ(\omega)]\} \\ & \qquad = 1 + k_2[P(\omega) + jQ(\omega)] \end{aligned} \tag{25b}$$

From (25b) we have

$$-\omega k_1 Q(\omega) = 1 + k_2 P(\omega), \tag{26a}$$

$$\omega[1 + k_1 P(\omega)] = k_2 Q(\omega) \tag{26b}$$

and after elimination of ω we obtain

$$[1 + k_1 P(\omega)][1 + k_2 P(\omega)] + k_1 k_2 Q^2(\omega) = 0 \tag{27a}$$

or

$$\frac{1}{k_1 k_2} + \frac{k_1 + k_2}{k_1 k_2} P(\omega) + P^2(\omega) + Q^2(\omega) = 0. \tag{27b}$$

Note that (27b) can be rewritten in the form of the equation

$$\left[P(\omega) + \frac{k_1 + k_2}{2k_1 k_2}\right]^2 + Q^2(\omega) = \left(\frac{k_2 - k_1}{2k_1 k_2}\right)^2 \tag{28}$$

which describes the circle $(-1/k_1, -1/k_2)$ (see Fig. 5). This completes the proof. ■

This theorem can be considered an extension of the fractional nonlinear systems with positive linear parts of the Kudrewicz theorem (Kudrewicz, 1964) for nonlinear systems with standard linear parts.

Example 2. Consider the fractional nonlinear system with the unstable linear part with

$$T(s^\alpha) = \frac{L(s^\alpha)}{M(s^\alpha)} = \frac{s^\alpha + 5.5}{s^{2\alpha} + 1.8s^\alpha - 0.1} \tag{29}$$

and the nonlinear element with the characteristic $u = f(e)$ shown in Fig. 6.

To obtain the nonlinear system with an asymptotically stable linear part we choose $k_1 = 0.2$ and obtain

$$\begin{aligned} T_1(s^\alpha) &= \frac{T(s^\alpha)}{1 + k_1 T(s^\alpha)} \\ &= \frac{L(s^\alpha)}{M(s^\alpha) + k_1 L(s^\alpha)} \\ &= \frac{s^\alpha + 5.5}{s^{2\alpha} + 1.8s^\alpha - 0.1 + 0.2(s^\alpha + 5.5)} \\ &= \frac{s^\alpha + 5.5}{s^{2\alpha} + 2s^\alpha + 1}. \end{aligned} \tag{30}$$

Note that the characteristic of the nonlinear element $u = f(e)$ (Fig. 6) satisfies the condition

$$0.2 < \frac{f(e)}{e} < 2. \tag{31}$$

In this case, for (30) we have

$$\begin{aligned} T_1((j\omega)^\alpha) &= \frac{(j\omega)^\alpha + 5.5}{(j\omega)^{2\alpha} + 2(j\omega)^\alpha + 1} \\ &= P(\omega) + jQ(\omega), \end{aligned} \tag{32}$$

where $P(\omega)$ and $Q(\omega)$ are given by (33).

The circle $(-5, -0.5)$ and the Nyquist plots for $\alpha = 0.5$ and $\alpha = 0.8$ are shown in Fig. 7. By Theorem 3 the fractional nonlinear system is globally stable.

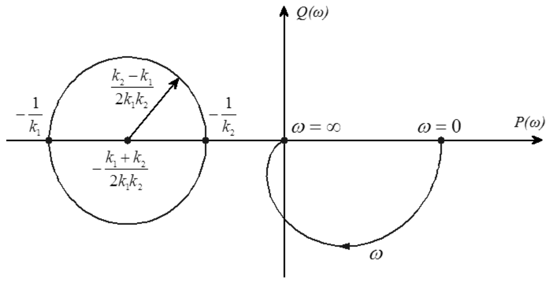

Fig. 5. Nyquist plot with the circle $(-1/k_1, -1/k_2)$.

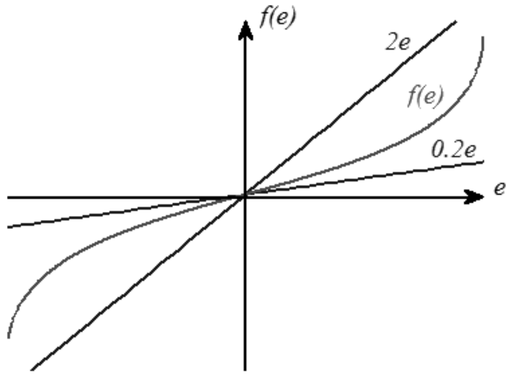

Fig. 6. Characteristic of the nonlinear element of Example 2.

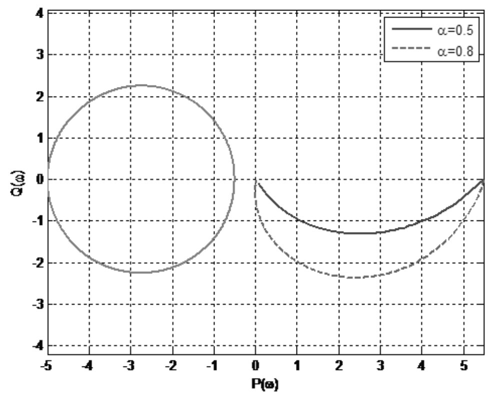

Fig. 7. Circle $(-5, -0.5)$ and the Nyquist plots for $\alpha = 0.5$ and $\alpha = 0.8$.

4. Concluding remarks

The global stability of nonlinear systems with fractional unstable positive linear parts has been analyzed. The characteristics $u = f(e)$ of the nonlinear element satisfy the assumption (15) and the fractional linear parts described by the equations (14) are not necessarily asymptotically stable. The gain k_1 of the positive linear part has been chosen so that the transfer function (17) is asymptotically stable and the characteristic $u = f(e)$ satisfies the condition (18).

It has been shown that the fractional nonlinear systems are globally asymptotically stable if the Nyquist plots of the linear parts are located on the right-hand side of the circles $(-1/k_1, -1/k_2)$. This theorem is an extension of the Kudrewicz theorem (Kaczorek and Borawski, 2017) for nonlinear systems with standard linear parts.

The discussion has been illustrated by numerical examples. It can be extended to the fractional nonlinear systems with positive descriptor linear parts.

$$P(\omega) = \frac{5.5 + 2\omega^{2\alpha} + 5.5\omega^{2\alpha}\cos(\pi\alpha) + \omega^{3\alpha}\sin(\pi\alpha)\sin\left(\frac{\pi}{2}\alpha\right) + \left(12\omega^{\alpha} + \omega^{3\alpha}\cos(\pi\alpha)\right)\cos\left(\frac{\pi}{2}\alpha\right)}{1 + \omega^{4\alpha} + 4\omega^{2\alpha} + 2\omega^{2\alpha}\cos(\pi\alpha) + 4\omega^{3\alpha}\sin(\pi\alpha)\sin\left(\frac{\pi}{2}\alpha\right) + \left(4\omega^{\alpha} + 4\omega^{3\alpha}\cos(\pi\alpha)\right)\cos\left(\frac{\pi}{2}\alpha\right)},$$
$$Q(\omega) = \frac{-5.5\omega^{2\alpha}\sin(\pi\alpha) - \omega^{3\alpha}\sin(\pi\alpha)\cos\left(\frac{\pi}{2}\alpha\right) - \left(10\omega^{\alpha} - \omega^{3\alpha}\cos(\pi\alpha)\right)\sin\left(\frac{\pi}{2}\alpha\right)}{1 + \omega^{4\alpha} + 4\omega^{2\alpha} + 2\omega^{2\alpha}\cos(\pi\alpha) + 4\omega^{3\alpha}\sin(\pi\alpha)\sin\left(\frac{\pi}{2}\alpha\right) + \left(4\omega^{\alpha} + 4\omega^{3\alpha}\cos(\pi\alpha)\right)\cos\left(\frac{\pi}{2}\alpha\right)}. \tag{33}$$

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