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DYNAMIC LOCATION MODELS OF MOBILE SENSORS FOR TRAVEL TIME ESTIMATION ON A FREEWAY

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Travel time estimation for freeways has attracted much attention from researchers and traffic management departments. Because of various uncertain factors, travel time on a freeway is stochastic. To obtain travel time estimates for a freeway accurately, this paper proposes two traffic sensor location models that consider minimizing the error of travel time estimation and maximizing the collected traffic flow. First, a dynamic optimal location model of the mobile sensor is proposed under the assumption that there are no traffic sensors on a freeway. Next, a dynamic optimal combinatorial model of adding mobile sensors taking account of fixed sensors on a freeway is presented. It should be pointed out that the technology of data fusion will be adopted to tackle the collected data from multiple sensors in the second optimization model. Then, a simulated annealing algorithm is established to find the solutions of the proposed two optimization models. Numerical examples demonstrate that dynamic optimization of mobile sensor locations for the estimation of travel times on a freeway is more accurate than the conventional location model.

Keywords: traffic mobile sensor, dynamic location model, travel time estimation, simulated annealing algorithm, data fusion.

1. Introduction

Freeways constitute significant infrastructure for modern urban transportation networks. Traffic information of a freeway is essential for traffic management departments to detect the freeway network. In recent years, the technology of intelligent transportation systems has been adopted to monitor the operation of the traffic infrastructure and equipment to obtain freeway traffic information (Ban *et al.*, 2011). Moreover, traffic information of a freeway (e.g., traffic flow, travel time, and average speed) can be collected through a fixed sensor (e.g., coil, microwave, and video) or a mobile one (e.g., mobile phones, GPS, and wireless communication systems) (Kolosz *et al.*, 2013).

Installing traffic sensors is a large investment, and there really exist some common issues for the traffic

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information collection on freeways: (a) the number of traffic sensors is insufficient and the corresponding locations are unreasonable; (b) the real-time performance of traffic data collection is poor; (c) the accuracy of traffic data is not high; (d) the traffic sensor is prone to failures. These shortcomings will result in the information collected not reflecting the road conditions accurately.

In addition, most collection methods of dynamic traffic data for freeways still rely on the traditional fixed sensor. The fixed sensor can only be used for spot detection (Salari *et al*., 2019), and the performance of the whole freeway cannot be shown, while, the mobile sensor can have the property of "line detection." However, the mobile sensor needs to seek out a dynamic and robust location strategy (Zhu *et al*., 2014). Therefore, a single type of sensor may miss some significant information. Moreover, in reality, many freeways have set up a certain amount of traffic sensors. Under these circumstances, a combination of different types of traffic sensors is adopted to improve the accuracy of dynamic traffic data collection and safety of the freeway.

The sensor location problem addressed in this paper aims to investigate how to optimize dynamic location of a traffic mobile sensor that improves estimation accuracy of travel time for the freeway. The contributions made in this paper are two-fold:

- For a freeway without sensors, a dynamic optimization model of mobile sensor location for travel time estimation is established. The proposed bi-objective model aims to minimize the error of travel time estimation and maximize the detected traffic flow. Moreover, the simulated annealing algorithm is adopted to solve the proposed model.
- For a freeway with fixed sensors, a dynamic optimal combination location model with mobile sensors and fixed sensors is presented. Data fusion technology will be adopted to combine traffic data obtained by multi-source sensors, which makes the estimation more accurate.

The rest of the paper is organized as follows. Section 2 offers a thorough literature review related to the study of sensor location for travel time estimation on a freeway. In Section 3, a new dynamic location model of mobile sensors on the freeway is proposed for a freeway without sensors. In Section 4, for a freeway with fixed sensors, mobile sensors are added dynamically and a combined location model is presented. In Section 5, the simulated annealing algorithm is adopted for the proposed two models. In Section 6, we demonstrate the efficiency of the proposed models and algorithms based on a real freeway. Finally, a summary and conclusions are presented in Section 7.

2. Literature review

The problem of optimal sensor location aims at using the least number of sensors to acquire the most comprehensive and accurate traffic information (Geetla *et al*., 2014; Gentili and Mirchandani, 2012). According to the application background, the sensor location problem can be divided into an urban road network and a freeway network (Gentili and Mirchandani, 2018). For sensor location in urban road networks, the related studies in the literature are categorized into four different types: (i) OD estimation, which is to estimate the traffic flow between the origin and destination during a particular period (Yang and Fan, 2015; Ma and Qian, 2018); (ii) travel time estimation (Zhu *et al*., 2018; Xing *et al*., 2013); (iii) link flow inference (Ng, 2013; He, 2013); and (iv) path flow reconstruction (Fu *et al*., 2017; Li and Ouyang, 2011). On the other hand, sensor location on a freeway has different purposes such as traffic event detection (Chakraborty *et al*., 2019; Karatsoli *et al*., 2017), travel time estimation (Chaudhuri *et al*., 2010; Liu and Danczyk, 2009), and traffic state prediction (Fujito *et al*., 2006; Hong and Fukuda, 2012). In this paper, we concentrate on the problem of sensor location for travel time estimation on freeways.

Empirical studies have revealed that vehicle speed collected by sensors can be adopted to estimate the average speed of a road section on a freeway, and then the estimated travel time is obtained (Beryini and Lovell, 2009; Chang *et al*., 2019). Clearly, the more traffic sensors are used on the freeway, the smaller the travel time estimation errors. However, when a certain number of sensors is reached, the accuracy of estimation will not increase with the number of sensors increasing. In addition, there is usually a budget constraint for locating traffic sensors. Thus, how to locate sensors whose total installation cost does not exceed a given budget so that the estimation error of freeway travel time is the lowest possible is a hot area of research.

In the literature, the existing sensor location models for travel time estimation on freeways concentrate on various purposes. These models are summarized in Table 1 in terms of the sensor type, consideration of network uncertainty, the modeling approach, and the solution method. Olia *et al*. (2017) introduced a methodology for determining the optimal number and location of freeway roadside equipment (RSE) units for travel time estimation in vehicle-to-infrastructure and vehicle-to-vehicle communication environments. The proposed multi-objective problem is solved by a non-dominated sorting genetic algorithm. Kim *et al*. (2011) studied the influence of loop sensor location on the accuracy of travel time estimation based on the microscopic traffic simulation model. The results show that the estimation accuracy depends mainly on the location of sensors and less on the number of sensors. However, traffic sensors are prone to various failures. Thus, it is necessary to incorporate sensor failure into the optimal location model. Danczyk *et al*. (2016) proposed a sensor location model to minimize the overall monitoring error of the freeway, while considering a certain probability of sensor failure. The results show that the sensor fault has a great influence on monitoring accuracy. Zhu *et al*. (2017) established a two-stage stochastic conditional value at risk (CVaR) model for travel time estimation in a freeway corridor. The experimental results show that the performance of sensor location is significantly improved after considering sensor failure.

Most of the works use mathematical programming to study the problem of freeway sensor location. In addition, there are other methods.

(i) The shortest path method: Ban *et al*. (2009) used the dynamic programming model to determine the optimal location of freeway sensors, and the shortest path algorithm was adopted to solve the model. The optimal solution can be determined by the proposed algorithm with polynomial time. Danczyk and Liu (2011) focused on optimal location of loop sensors to minimize performance measurement errors. They described the sensor location problem under the constraint of the number of sensors as a constrained shortest path optimization problem, and designed an improved branch and bound algorithm to solve it.

(ii) The cluster analysis method: Bartin *et al*. (2007) transformed the problem of how to locate sensors on a freeway to estimate travel time accurately into that of clustering all small cells of the freeway into different classes according to the estimation error of travel time. Unlike ordinary clustering, each class of Bartin *et al*. (2007) consisted of contiguous small cells. To this end, Kianfar and Edara (2010) adopted the global K-means algorithm to obtain new clustering with the technology of adding new boundaries.

On the basis of the above discussion, most of the existing research results might focus on a fixed sensor. Obviously, the fixed sensor cannot be moved once it is installed on a certain road. Under these circumstances, a fixed sensor cannot fit the dynamic traffic network. In recent years, the problem of locating mobile sensors on freeways has attracted much attention. Chow (2016) presented a dynamic location strategy of an unmanned aerial vehicle (UVA) based on real-time data, which abstracted the problem into a stochastic arc inventory routing one, and an approximate dynamic programming algorithm based on the least squares Monte Carlo method is proposed to form the strategy. Park and Haghani (2015) proposed an optimal dynamic location model for bluetooth sensors on freeways as the pattern of travel time

Table 1. Literature summary of freeway sensor location for travel time estimation.

Literature	Type of sensor	Consideration of network uncertainty	Modeling approach	Solution methods
Olia *et al*., 2017	Loop	No	Mathematical programming	Sorting genetic algorithm
Kim *et al*., 2011	Loop	No	Microscopic traffic simulation	Genetic algorithm
Danczyk *et al*., 2016	Loop	Yes	Mathematical programming	Heuristic algorithm using k-shortest path search
Zhu *et al*., 2017	Loop	Yes	Stochastic programming	Genetic algorithm
Ban *et al*., 2009	Video	No	Dynamic programming	The shortest path algorithm
Danczyk *et al*., 2011	Loop	No	The shortest path optimization	Improved branch and bound algorithm
Bartin *et al*., 2007	Loop	No	Cluster analysis	K-means algorithm
Chow *et al*., 2016	UAV	Yes	Arc inventory routing	Least Squares Monte Carlo method
Park *et al*., 2015	Bluetooth	Yes	Mathematical programming	CPLEX software
Zhan *et al*., 2015	Probe vehicle	No	Data simulation	Trial and error method
This paper	**Mobile sensor**	**Yes**	**Mathematical programming**	**Simulated annealing algorithm**

error changes over time. However, it is not reasonable to determine the location of mobile sensors based on the time interval of each day due to the problem of sensor location considering not only the travel time error but also other traffic parameters. In this paper, we use a shorter time interval as the benchmark and concentrate on minimizing the travel time estimation error and maximizing the detected traffic flow to determine the dynamic mobile sensor location on the freeway.

In reality, many freeways have installed fixed sensors. Thus, how to add different types of sensors to obtain the minimum estimation error and the lowest cost has not attracted much attention. Zhan *et al.* (2015) investigated the required minimum number of probe vehicles to estimate travel time with the desired statistical accuracy for freeways equipped with fixed sensors. The results showed that the travel time estimation error with probe vehicles and fixed sensors is less than in the case of fixed sensors only. However, the optimal proportion of probe vehicles is determined by the trial and error method and does not have generality. To this end, this paper proposes a novel mathematical programming model for combinatorial location of mobile sensors and fixed sensors.

3. Dynamic mobile sensor location model

Attributed to both daily and seasonal variations in travel activity patterns, traffic conditions of a freeway are not deterministic but actually fluctuate from day to day. Moreover, the mobile sensor can not only move freely in the traffic network, but also record the traffic flow, vehicle speed, and license plate information. Therefore, the mobile sensor is easier to adapt to the dynamic traffic environment. Thus we will divide the freeway into several cells with an equal distance and propose a new dynamic location model of the mobile sensor in different time periods.

3.1. Model assumptions.

- The mobile sensors start from the first cell of the freeway and go back to the last one. This means that the sensors move in the same direction as the vehicles on the freeway.
- The speed of the mobile sensor is constant. This is to estimate how many cells the mobile sensor moves each time it changes the position.
- There is only one mobile sensor in a cell of the freeway at the same time period.
- The same mobile sensor can arrive at only one cell of the freeway at the same time period.
- The mobile sensor location in a cell of the freeway means the position of the midpoint of this cell.

3.2. Model establishment. As mentioned in the preceding sections, suppose the freeway is divided into N equal cells with small length l, as shown in Fig. 1. The number of mobile sensors is M ($M < N$). The total observation time is divided into R periods with the same time interval.

If a mobile sensor is located in a cell, the sensor gets the speed of all vehicles passing through this cell. Since the freeway is divided into cells of a very short length, the cell is regarded as a point relative to the entire freeway. Therefore, the average speed of all vehicles passing through a cell is taken as the point speed of the cell.

Suppose that there are J vehicles passing through the cell s in time period t, so the point speed of the cell is

$$v_s^t = \frac{1}{J}\sum_{j=1}^{J} v_{s,j}^t, \tag{1}$$

where v_s^t is the point speed of cell s in time period t, $v_{s,j}^t$ is the speed of vehicle j passing through cell s in time period t.

If two cells S and S' are occupied by mobile sensors i and i', respectively, then the two cells compose a section (see Fig. 1). It is to be noted that the number of mobile sensors is M and the freeway is always divided into $M+1$ sections for any time period (the length of these sections is not equal). Except for the first and last sections, the beginning and the end of every other section are occupied by the actual sensors, and the average value of point speed data collected by the sensors at the beginning and the end of the section is taken as the average speed of this section. The first section only has a sensor at the end, so the average speed of the first section equals the sensor data of its end. Similarly, the average speed of the last section equals the sensor data of its beginning. Therefore, the average speed of all sections can be deduced as

$$v_{ss'}^t = \begin{cases} v_{s'}^t, & ss' = 1, \\ \frac{1}{2}(v_s^t + v_{s'}^t), & 2 \le ss' \le M, \\ v_s^t, & ss' = M+1, \end{cases} \tag{2}$$

where $v_{ss'}^t$ is the average speed of section ss' between start cell s and end cell s' in time period t.

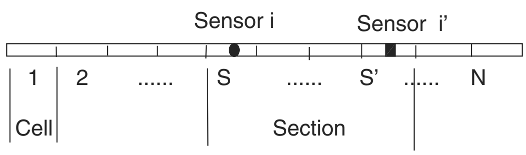


Fig. 1. Schematic diagram of a freeway.

Then, the travel time for each section is

$$E_{ss'}^t = \frac{L_{ss'}}{v_{ss'}^t} = \frac{l \cdot (s'-s)}{v_{ss'}^t}, \tag{3}$$

where $E_{ss'}^t$ is the estimated travel time of section ss' in time period t, $L_{ss'}$ is the length of section ss'.

On the other hand, there is a true travel time for each section, so the error of travel time estimation for each section is expressed as

$$e_{ss'}^t = |G_{ss'}^t - E_{ss'}^t|, \tag{4}$$

where $e_{ss'}^t$ is the error of travel time estimation for section ss' in time period t, $G_{ss'}^t$ is the true travel time of section ss' in time period t.

Once the method for estimating the travel time error has been determined, the problem of dynamic mobile sensor location on the freeway can be transformed into how to locate the mobile sensors in different cells in different time periods to minimize the sum of errors of travel time estimation of all sections. It is worth noting that the mobile sensor can not only pick up the speed of passing vehicles, but also observe the traffic flow, that is, the number of vehicles passing the cell within a time interval. The greater the traffic flow in a cell, the more accurate the point speed of that cell calculated by Eqn. (1), and thus the more accurate travel time estimation. In other words, mobile sensors located in some cells seem meaningless if one cell has a large travel time estimation error and a small traffic flow. Therefore, this paper proposes a bi-objective optimization model to minimize the error of travel time estimation and maximize the observed traffic flow on a freeway. The bi-objective model is shown below:

$$M_1: \ \min f_1 = \sum_{t=1}^{R}\sum_{s=1}^{N}\sum_{s'=s+1}^{N}\sum_{i=1}^{M}\sum_{i'=1}^{M} x_{its}x_{i'ts'}e_{ss'}^t, \tag{5}$$

$$\max f_2 = \sum_{t=1}^{R}\sum_{s=1}^{N}\sum_{i=1}^{M} x_{its}q_s^t, \tag{6}$$

$$\text{s.t.} \sum_{s=1}^{N} x_{its} = 1, \quad \forall\, i, \forall\, t, \tag{7}$$

$$\sum_{i=1}^{M} x_{its} \le 1, \quad \forall\, s, \forall\, t, \tag{8}$$

$$x_{its'} > x_{it's}, \quad \forall\, i, \forall\, t' > t, \forall\, s' > s, \tag{9}$$

$$\sum_{s=1}^{s'-1}\sum_{i=1}^{M}\sum_{i'=1}^{M} x_{its}x_{i'ts'} = \sum_{s''=s'+1}^{N}\sum_{i'=1}^{M}\sum_{i''=1}^{M} x_{i'ts'}x_{i''ts''}, \quad \forall\, s' = 2,3,\dots,N-1, \forall\, t, \tag{10}$$

$$x_{its} \in \{0,\ 1\}, \forall\, i, \forall\, t, \forall\, s, \tag{11}$$

where x_{its} is a 0–1 variable, $x_{its} = 1$ indicates that the mobile sensor i is located in cell s in time period t and otherwise zero, q_s^t is the traffic flow on cell s in time period t.

The objective function (5) ensures that the estimated travel time error of the whole freeway in the total observation time is minimum. The objective function (6) ensures that the total observed flow is maximum. Now we use a simple example (see Fig. 2) to explain the calculation of the objective functions f_1 and f_2. Suppose a freeway consists of seven cells, and there are two mobile sensors. The total observation time is divided into two periods. In the first one, sensors i_1 and i_2 are located in cells 2 and 5, respectively. That is, $x_{125} = x_{215} = 1$. Then, the freeway is divided into three sections, namely, from cell 1 to cell 2, from cell 2 to cell 5, from cell 5 to cell 7. In the second time period, sensors i_1 and i_2 are located in cells 4 and 6, respectively ($x_{124} = x_{226} = 1$). Then, the corresponding three sections are from cell 1 to cell 4, from cell 4 to cell 6, from cell 6 to cell 7.

According to Eqn. (2), the average speed of the three sections in two time periods is

$$\begin{aligned} v_{12}^1 &= v_2^1, \quad v_{25}^1 = \frac{v_2^1 + v_5^1}{2}, \quad v_{57}^1 = v_5^1, \\ v_{14}^2 &= v_4^2, \quad v_{46}^2 = \frac{v_4^2 + v_6^2}{2}, \quad v_{67}^2 = v_6^2. \end{aligned} \tag{12}$$

Then, the estimated travel time of the three sections in two

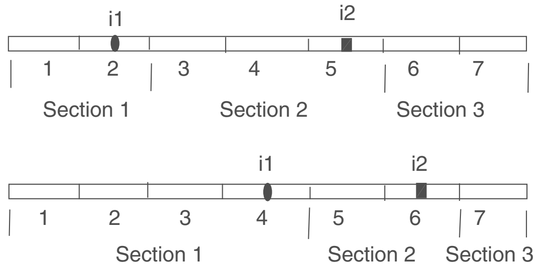


Fig. 2. Simple example of dynamic location of sensors on a freeway.

time periods by Eqn. (3) is

$$\begin{aligned} E_{12}^1 &= \frac{l}{v_{12}^1}, \quad E_{25}^1 = \frac{3l}{v_{25}^1}, \quad E_{57}^1 = \frac{2l}{v_{57}^1}, \\ E_{14}^2 &= \frac{3l}{v_{14}^2}, \quad E_{46}^2 = \frac{2l}{v_{46}^2}, \quad E_{67}^2 = \frac{l}{v_{67}^2}. \end{aligned} \tag{13}$$

Hence, the estimated error of travel time f_1 and observed traffic flow f_2 for the entire freeway in the total observation time are calculated as follows:

$$\begin{aligned} f_1 &= e_{12}^1 + e_{25}^1 + e_{57}^1 + e_{14}^2 + e_{46}^2 + e_{67}^2, \\ f_2 &= q_2^1 + q_5^1 + q_4^2 + q_6^2. \end{aligned} \tag{14}$$

The constraint (7) ensures that the same sensor can only be located in one cell at the same time period. The constraint (8) guarantees that at most one sensor is located in the same cell at the same time period. The constraint (9) ensures that the mobile sensor always moves forward along the one-way freeway. For sensor i, if it is located in cell s' in time period t, i.e., $x_{its'} = 1$, then, in any later time period t', sensor i cannot be located in all cells s in front of cell s', i.e. $x_{it's} = 0$. The constraint (10) guarantees that any given cell can only serve as the beginning of a section at most. In other words, for any cell (except for the first and the last cell), when it is the beginning of a section, it must be the end of the previous section. When it is not the beginning of a section, it is certainly not the end of the previous section. The constraint (11) is the restriction on decision variable x_{its}.

4. Combinatorial location model of mobile and fixed sensors

For most freeways, there already exist fixed sensors to collect or estimate traffic information. However, traffic estimation will not be accurate due to some uncertainty factors such as sensor failures. In addition, the distance between each fixed sensor is too big to provide high-precision traffic data effectively on account of the limitation of upfront cost. Unlike fixed sensors, mobile ones can collect real-time traffic data in a wide range. Therefore, it is more reasonable to add mobile sensors to the freeway with fixed sensors to obtain high-precision dynamic traffic data.

Suppose the number of fixed sensors that have been located on a freeway is C. The data collected by these fixed sensors include point speed and traffic flow. The freeway is divided into $C + 1$ sections according to the position of fixed sensors. The length of each section is equal to the distance between two adjacent fixed sensors. Therefore, the problem of dynamic location of mobile sensors on a freeway with fixed sensors consists in determining how to locate mobile sensors in sections in different time periods.

There will be two types of sensors (fixed sensors and mobile sensors) in a section when mobile sensors are located at this section. Since location points of different kinds of sensors are disparate, it is important to integrate multi-source traffic sensor data to obtain fusion data accurately. In the literature, there are some common information fusion approaches, including the weighted average, the Kalman filter, modulus theory, and artificial neural networks (Meng *et al.*, 2020). Among these data fusion methods, the weighted average method is suitable for the dynamic traffic problem and it is easy to operate. Thus, the weighted average method is adopted to calculate the fusion speed of the section, that is,

$$v_{kt} = \frac{1}{2}\left(\frac{v_{kt}^f + \sum\limits_{s\in S(k)}\sum\limits_{i=1}^{M} x_{its}v_s^t}{\sum\limits_{s\in S(k)}\sum\limits_{i=1}^{M} x_{its}}\right), \tag{15}$$

where v_{kt} is the fusion average speed of section k in time period t, v_{kt}^f is the average speed obtained by the fixed sensor of section k in time period t, which is calculated by Eqn. (2), $S(k)$ is the set of cells included in section k except for the beginning and end cells. Since fixed sensors have been located at the beginning and end cells of each section, according to the model assumptions that only one sensor can be located in each cell, mobile sensors can only be located in the cell without fixed sensors. The meaning of x_{its} and v_s^t is the same as before.

In Eqn. (15),

$$\frac{\sum\limits_{s\in S(k)}\sum\limits_{i=1}^{M} x_{its}v_s^t}{\sum\limits_{s\in S(k)}\sum\limits_{i=1}^{M} x_{its}}$$

represents average speed obtained by the mobile sensor of section k in time period t. Fusion average speed v_{kt} is obtained by the weighted average of the fixed and mobile speed data. Thus, the travel time estimation error for each section k in time period t can be calculated as

$$e_{kt} = \left| G_{kt} - \frac{L_k}{v_{kt}} \right|, \tag{16}$$

where G_{kt} is the true travel time of section k in time period t, L_k is the length of section k.

Similarly, the fusion traffic flow of each section can be obtained with the following equation:

$$q_{kt} = \frac{1}{2}\left(\frac{q_{kt}^f + \sum_{s\in S(k)}\sum_{i=1}^{M} x_{its}q_s^t}{\sum_{s\in S(k)}\sum\limits_{i=1}^{M} x_{its}}\right), \tag{17}$$

where q_{kt} is the fusion flow of section k in time period t. q_{kt}^f is the fixed sensor flow of section k in time period t. The meaning of q_s^t is the same as before.

On the basis of the above discussion, in the case of fixed sensors on a freeway, the dynamic location model of adding mobile sensors is described as follows:

$$M_2: \ \min f_3 = \sum_{t=1}^{R}\sum_{k=1}^{C+1} e_{kt}, \tag{18}$$

$$\max f_4 = \sum_{t=1}^{R}\sum_{k=1}^{C+1} q_{kt}, \tag{19}$$

$$\text{s.t.} \sum_{s\in S} x_{its} = 1, \quad \forall\, i,\, \forall\, t, \tag{20}$$

$$\sum_{i=1}^{M} x_{its} \le 1, \quad \forall\, s,\, \forall\, t, \tag{21}$$

$$x_{its'} > x_{it's}, \quad \forall\, i,\, \forall\, t' > t,\, \forall\, s' > s, \tag{22}$$

$$x_{its} \in \{0,\, 1\}, \quad \forall\, i,\, \forall\, t,\, \forall\, s, \tag{23}$$

where $S \in \{S(k)\},\ k = 1, 2, \dots, C+1$.

In fact, the proposed two models M_1 and M_2 are very similar. Obviously, the first objective functions of both models (Eqns. (5) and (18)) are to minimize the error of travel time estimation, and the second objective functions (Eqns. (6) and (19)) are to maximize the observed traffic flow. The only difference between the two models is the candidate cell for mobile sensor location. Specifically, model M_1 needs to consider how to locate mobile sensors in all N cells of the whole freeway, while model M_2 only needs to consider the remaining $N - C$ cells except C fixed sensors.

5. Solution algorithm

The proposed two models M_1 and M_2 are bi-objective optimization problems. There usually exist contradictions among multiple sub-objectives, so it is necessary to coordinate and trade-off among the multiple sub-objectives so that each sub-objective can reach an ideal value as far as possible, that is, to find its Pareto solution (Chou *et al*., 2019). In order to deal with bi-objective optimization, the linear weighting method is adopted to convert a bi-objective problem into a single-objective one (Kolak *et al*., 2018). It is noted that, for a model M_1, f_1 is minimization, f_2 is maximization, and the magnitude of the two objectives is quite different. Therefore, objectives f_1 and f_2 are first normalized according to the following formula:

$$f_i' = \frac{f_i - f_{i,\min}}{f_{i,\max} - f_{i,\min}}, \tag{24}$$

where f_i' is the normalized function of objective function f_i, $f_{i,\max},\ f_{i,\min}$ is the maximum and the minimum of objective function f_i, respectively.

Then, normalized functions f_1' and f_2' are converted using the linear weighting method into the following single-objective optimization:

$$\begin{aligned} M_1: \quad & \min\ f = f_1' - f_2', \\ & \text{s.t. } (7) \sim (11). \end{aligned} \tag{25}$$

Similarly, model M_2 is modified to the following single-objective optimization:

$$\begin{aligned} M_2: \quad & \min\ f = f_3' - f_4', \\ & \text{s.t. } (20) \sim (23). \end{aligned} \tag{26}$$

Obviously, models M_1 and M_2 are also integer programming problems. Some conventional algorithms can be adopted to solve integer programming problems, such as branch and bound, cut plane, and enumerations (Fischetti and Monaci, 2020). However, these conventional algorithms are only suitable for smaller scale problems. In other words, these algorithms are less efficient when there are more variables. In recent years, some intelligent algorithms have been frequently used to solve large-scale integer programming problems, such as genetic algorithms, particle swarm optimization, the tabu search algorithm, and the simulated annealing algorithm (Nemati *et al*., 2018). These algorithms can produce an approximate optimal solution within acceptable time. Compared with other intelligent algorithms, the simulated annealing algorithm converges to the optimal solution with probability 1 theoretically. Meanwhile, the simulated annealing algorithm has the advantages of a simple description, using flexible, high running efficiency, and is less affected by the initial solution (Song *et al*., 2020). Moreover, models M_1 and M_2 are very similar. Thus, the simulated annealing algorithm is employed to solve the two models proposed in this paper.

Simulated annealing is an algorithm that simulates the solid annealing process. Such a process includes the coding of the solution, an annealing temperature, the energy function, generation and acceptance of a new solution, and the length of the Markov chain (Liu *et al*., 2019). Detailed descriptions are as follows.

Solution coding: The natural number coding and a matrix are used to express the dynamic location scheme of mobile sensors. This is illustrated by a simple example in Fig. 2, which can be coded using the following matrix:

$$X = \begin{pmatrix} 2 & 5 \\ 4 & 6 \end{pmatrix}.$$

The element a_{ti} of this matrix represents the cell number where mobile sensor i is located in time period t. For example, $a_{12} = 5$ represents that sensor 2 is located in cell 5 in time period 1. Thus, each row of the matrix represents the distribution of all sensors in each time period, and each column represents the path of each sensor over all time periods.

Annealing temperature: The annealing temperature controls the solution process. As long as the initial temperature is high enough, the cooling process is slow enough, and the termination temperature is low enough, so that the algorithm can converge to the global optimal solution. Therefore, the initial temperature is generally set to 95$\sim$100, and the termination temperature is 1$\sim$5. The cooling function always adopts the equation $T_{k+1} = \alpha T_k$, $k = 0, 1, 2, \ldots$, where α is a constant, usually ranging from 0.5 to 0.99.

Energy function: The energy function represents the pros and cons of the current solution. The lower the energy, the closer the solution to the optimal one. Since the objective function in Eqns. (25) and (26) aims at finding the minimum, the energy function is defined as $E = f$.

Generation of a new solution: A new feasible solution is obtained by randomly changing the value of some decision variables corresponding to the current solution. In this paper, we randomly select an altered element for each row in the coding matrix of the current solution. Then for the same row, one of the cells unoccupied by sensors is randomly picked to replace the altered element. For example, the current solution shown in Fig. 2 produces a new solution as follows:

$$\begin{pmatrix} 2 & \boxed{\mathbf{5}} \\ \boxed{4} & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & \boxed{\mathbf{3}} \\ \boxed{5} & 6 \end{pmatrix}$$

Current solution New solution

Acceptance of the new solution: The acceptance of the new solution is divided into unconditional acceptance and conditional acceptance. If the objective value of the new solution is less than that of the current solution, the new solution is accepted. Otherwise, the new solution is accepted with a certain probability.

Length of the Markov chain: The length of the Markov chain is the number of iterations at any temperature. In other words, it is the number of inner cycles of the algorithm, which is generally regarded as $L_k = 100n$, where n is the scale of problem.

Therefore, the simulated annealing algorithm is to gradually reduce the temperature and repeat the following process: generate a new solution, calculate the difference of the objective function, accept or discard. The algorithm steps are shown in Algorithm 1.

Algorithm 1. Simulated annealing.

Step 1. Initialization. Set initial solution X_0, initial temperature T_0, termination temperature T_f, cooling function T_k, energy function B, and length of Markov chain L_k. Set iteration number $k = 0$, current temperature $T_k = T_0$ and current solution $X_k = X_0$.

Step 2. Disturb current solution X_k, and randomly generate new solution X_i. Calculate the increment of energy function $\triangle B = B(X_i) - B(X_k)$.

Step 3. If $\triangle B < 0$, accept new solution X_i as the new current solution. Otherwise, accept new solution X_i with a probability of $\exp(-\triangle B/T_k)$.

Step 4. At temperature T_k, repeat the disturbance and acceptance L_k times, i.e., repeat Steps 2 and 3.

Step 5. Decrease temperature T_k according to the cooling function. If $T_k < T_f$, stop the iteration, otherwise, go to Step 2.

6. Simulation experiment

In this section, the He-huai-fu Freeway in the Anhui Province, China, is used to illustrate the effectiveness of the proposed models and algorithms. This freeway starts at the Shushan of Hefei in the south and ends at the Sishipu of Fuyang in the north. Since its opening in 2008, the He-huai-fu Freeway has effectively strengthened exchanges between the provincial capital and northwest Anhui, and it is an important road from East China to Central Plain. The entire length of the He-huai-fu Freeway is 191 km and it passes through four cities, namely, Hefei, Lu'an, Huainan and Fuyang. The section from Hefei to Huainan is a two-way six-lane freeway, and the section from Huainan to Fuyang is a two-way four-lane freeway. The He-huai-fu Freeway includes 14 interchanges and 4 service areas. Their lanes and pile numbers are shown in Table 2. In this paper, the pile number of the He-huai-fu Freeway from K225+954 to K247+213 is selected as the research road, whose total length is 21.26 km. There already exist 6 fixed sensors on the freeway with non-equal intervals, and the related parameters are shown in Fig. 3.

The freeway under study should be divided into several cells. On the one hand, the length of each cell cannot be too long. Different cells represent different locations of the freeway, i.e., the sensor gets the traffic information of the cell where it is located. The shorter the cell, the more accurate the information collected. But the cell length should not be too short, because the shorter the cell length, the bigger the number of cells, which leads to too many decision variables of the model. For illustration, the freeway in Fig. 3 is divided into 212 cells with 100 meters in length.

Because traffic data of the cell are difficult to obtain in practice, this paper uses the microscopic traffic simulation program VISSIM to generate the required speed data and traffic flow data. In order to approximate the actual environment, the simulation input data the based on the actual traffic flow. The simulation traffic flow is set as shown in Table 3. Here, q_1 is the traffic flow of the main line, q_2, q_3, q_4, q_5 represent the traffic

Table 2. Road information of the He-huai-fu Freeway.

Traffic direction	Name	Lanes	Pile numbers: Entrance	Pile numbers: Exit
South ↓ North	Shushan interchange	6	K60+830	
	Hefei west interchange			K73+130
			K73+580	
				K73+750
			K73+940	
	Wushan interchange			K85+860
				K86+460
			K86+960	
	Longmensi service area			K98+930
			K99+130	
	Yangmiao interchange			K106+630
			K107+030	
	Changfeng south interchange			K131+829
			K132+210	
	Changfeng interchange			K136+410
			K136+910	
	Caoan interchange			K142+660
			K143+210	
	Huainan south interchange			K148+860
			K149+660	
			K150+310	
	Bagongshan service area			K156+230
			K156+630	
	Huainan west interchange			K165+360
			K167+060	
	Jiaogang lake service area			K185+080
			K185+580	
	Maoji interchange			K202+860
			K203+130	
	Yingshang east interchange	4		K212+780
			K213+230	
	Yingshang service area			K222+880
			K223+380	
	Yingshang west interchange			K234+240
			K234+740	
	Fuyang south interchange			K243+250
			K243+540	
	Sishipu interchange			K251+330
			K251+930	

Note: The unit of pile numbers in front of '+' is kilometer and the back is meter.

flow of the two on-ramps, respectively. According to the classification statistics of vehicles collected in Table 3, the ratio of cars to trucks is set at 10:1. The speed range of the car is set to be (60, 120) km/h, and that of the trucks is set to be (60, 100) km/h.

The simulation time is 0–21600 s, where the first 3600 s are the warm-up time and the last 18000 s are the normal running time. The time interval of data acquisition is 10 minutes, so the running time is divided into $R = 18000 \div (10 \times 60) = 30$ periods.

The true travel time is obtained from the VISSIM file 'Travel Time'. The initial speed data from the VISSIM file 'Data Point' are regarded as the true speed. There are two types of sensors on the studied freeway: fixed sensors

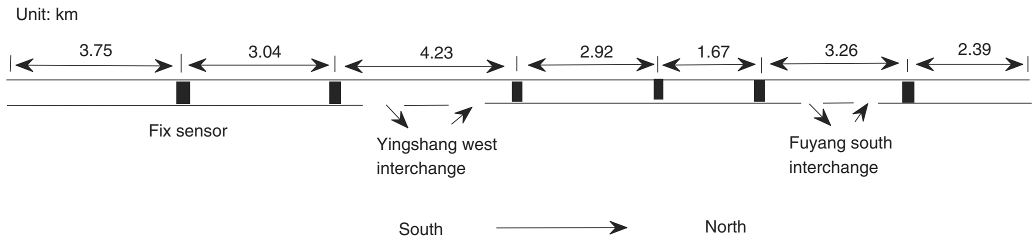


Fig. 3. Location of fixed sensors on the study road of the He-huai-fu Freeway.

Table 3. Simulation settings of traffic flow.

Simulation time	Traffic flow				
	q_1	q_2	q_3	q_4	q_5
0 ~ 3600 s	2120	157	126	53	84
3600 ~ 7200 s	2520	189	151	63	101
7200 ~ 10800 s	2678	199	158	66	115
10800 ~ 14400 s	2625	197	162	57	105
14400 ~ 18000 s	2436	183	147	65	100
18000 ~ 21600 s	1837	136	110	48	78

and mobile sensors, which have different precisions in collecting vehicle speed data. Generally, the error of the fixed sensor is 5%, and that of the mobile sensor is 10%. Therefore, in order to distinguish the speed data of the two sensors, we multiply the initial speed data by (1 ± 0.05) and (1 ± 0.1), respectively, where the positive and negative signs are randomly selected.

The parameters of the simulated annealing algorithm are set as follows: initial temperature $T_0 = 97$, termination temperature $T_f = 3$, coefficient of the cooling function $\alpha = 0.95$ and length of the Markov chain $L_k = 10000$.

6.1. Location of the mobile sensor. In this section, we solve model M_1 for the investigated freeway, that is, how to dynamically locate mobile sensors on the premise where that there is no sensor on the freeway.

6.1.1. Comparison of the mobile sensor and the fixed sensor. In model M_1, the number of mobile sensors ranges from 6 to 18, and the corresponding normalized objective function values f_1 (Eqn. (5)), f_2 (Eqn. (6)), f (Eqn. (25)) can be obtained. For comparison, we also evaluate the objective function when locating the same number of fixed sensors. Since the fixed sensor cannot move, the total objective function value is the sum of the objective function values for all time periods. The detailed results are shown in Fig. 4.

In Fig. 4(a), when the same number of sensors is located, the travel time error f_1 estimated by the mobile sensor is always smaller than that of the fixed sensor. This is mainly because the position of the mobile sensor varies in each time period while that of the fixed sensor is constant, so the optimal solution of the mobile sensor is also the optimal solution in each time period, but the fixed sensor does not guarantee that. In addition, with an increase in the number of sensors, the travel time errors of mobile and fixed sensors are both reduced. This is because the freeway is divided into sections by the sensors. The greater the number of sensors, the greater the number of sections, and the more accurate the estimation of the entire freeway. It should be noted, however, that this downward trend is mitigated when the number of sensors reaches 13. In other words, increasing the number of sensors at this point has little effect on the accuracy of travel time. Therefore, it is not always best to have as many sensors as possible, and there are cost constraints in real life.

In Fig. 4(b), traffic flow f_2 observed by the mobile sensor is always greater than that observed by the fixed sensor, because the mobile sensor can move flexibly. In addition, no matter what kind of sensor is used, with an increase in the number of sensors, the observed traffic flow is increased.

Figure 4(c) shows that the mobile sensor still performs better than the fixed sensor for the objective function $f = f_1 - f_2$. For instance, the objective function values of the mobile sensor and the fixed sensor are -0.4877 and -0.3563 when the number of sensors is 9, respectively. However, the advantages of mobile sensors over fixed sensors decrease as the number of sensors increases. For example, the objective function values of mobile and fixed sensors are almost the same when the number of sensors is 18. This is partly because when the number of mobile sensors increases, the possible locations for mobile sensors decrease. On the other hand, model M_1 is originally bi-objective including f_1 and f_2 with possible conflicts. The objective function f is a linear combination of f_1 and f_2 into a single objective, so it represent a trade-off between the two objectives to some extent.

6.1.2. Sensitivity analysis of the model parameter. From the above experimental results, we have seen that the number of sensors has a certain impact on the objective function value of model M_1. In fact, the key to determine

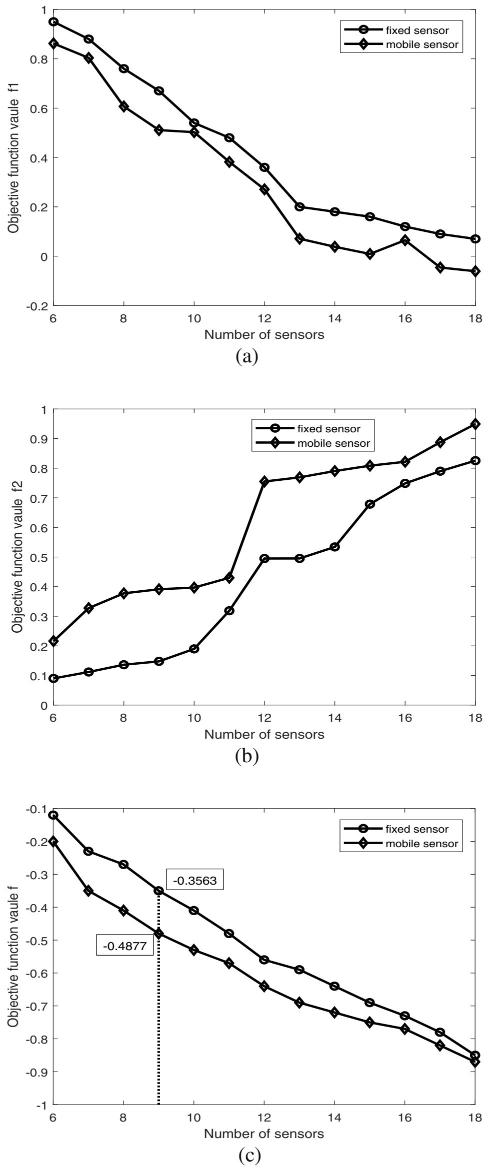


Fig. 4. Comparison of different objective function values for mobile and fixed sensors.

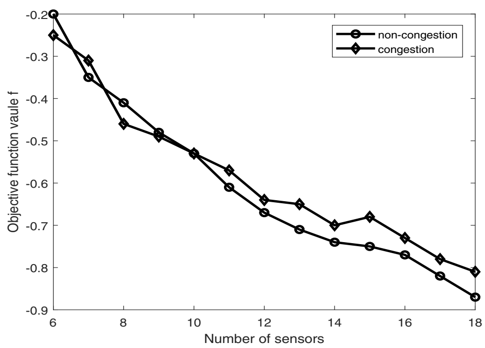


Fig. 5. Comparison of mobile sensors for different road conditions.

the solution of the model lies in the vehicle data collected by the sensor, and these data are varied in different road conditions, sampling time intervals and sensor precision. Therefore, this section makes some sensitivity analysis of model M_1 in the following aspects:

Traffic congestion: In order to study the impact of road conditions on the model, two cases of congestion and non-congestion are discussed. We take the traffic flow in Table 3 as the non-congestion data, and the speed range of the car and truck is set to be (60, 120) and (60, 100) km/h, respectively. The data in Table 3 are multiplied by 1.5 for the congestion case. Because of the heavy traffic, the vehicle speed should be reduced. To this end, the speed range of the car and truck is set to be (60, 100) and (60, 80) km/h, respectively. In the case of congestion and non-congestion, the objective function value f (Eqn. (25)) changes with the number of mobile sensors, as shown in Fig. 5.

As can be seen from Fig. 5, when the number of mobile sensors is less than 10, the objective function value of congestion is better or worse than that of non-congestion. That is to say, when there are few sensors, the road condition has little effect on the model. However, when the number of sensors exceeds 10, the objective function of non-congestion is obviously smaller than that of congestion. This is mainly because too many mobile sensors cannot move freely in a crowded environment.

We also provide the dynamic location scheme of mobile sensors for congestion and non-congestion when the number of mobile sensors is 6, that is, the cell labels located with 6 sensors in each time period. The results are shown in Table 4.

Comparing the same row in Table 4, the cell labels associated with 6 mobile sensors at the same time period are completely different for congestion and non-congestion. In other words, the sensor location is very sensitive to the degree of road congestion. And different road conditions will lead to different vehicle speeds, traffic flows and travel times, so the optimal scheme of sensor location is dependent on simulation data.

In addition, the objective function with different numbers of mobile sensors and fixed sensors for congestion and non-congestion is compared as shown in Fig. 6.

Apparently, Fig. 6(a) shows that the mobile sensor performs better than the fixed one in non-congestion networks. However, under the congestion condition in Fig. 6(b), it is obvious that the intersection point of two curves is 15. When the number of sensors is less than 15, the mobile sensor performs better than the fixed one. On the contrary, the fixed sensor has better performance when the number of sensors is more than 15. That is to say, the advantages of mobile sensors over fixed ones

Table 4. Dynamic location scheme of 6 mobile sensors with different road conditions.

Time period	Non-congestion						Congestion					
	i_1	i_2	i_3	i_4	i_5	i_6	i_1	i_2	i_3	i_4	i_5	i_6
1	2	8	17	22	39	47	3	13	26	38	41	45
2	2	23	38	56	76	91	4	19	39	54	70	86
3	9	27	47	65	80	101	5	21	39	59	77	96
4	9	31	48	71	95	110	8	23	46	62	78	101
5	12	31	51	73	95	111	8	30	53	74	89	106
6	18	38	60	80	99	115	13	35	55	76	97	117
7	21	43	61	81	100	118	16	35	59	77	97	120
8	25	43	62	83	103	122	20	43	61	81	98	121
9	25	43	63	85	103	124	24	44	65	88	103	122
10	26	45	65	86	104	125	29	50	72	91	106	123
11	28	46	70	88	108	129	31	53	72	96	118	134
12	32	50	72	93	112	132	38	58	82	104	126	144
13	33	51	73	93	114	132	45	64	83	104	127	144
14	38	59	75	99	120	135	45	66	84	105	127	145
15	43	65	86	102	122	140	50	68	86	107	128	148
16	48	67	87	105	124	144	51	69	86	107	128	149
17	55	77	98	117	133	151	53	70	88	108	129	150
18	59	79	98	117	134	152	55	71	93	113	135	155
19	60	80	99	117	134	153	62	84	102	117	135	157
20	62	81	100	122	142	157	68	85	108	125	144	166
21	65	85	101	124	143	167	70	91	110	133	155	171
22	72	88	109	128	146	169	71	94	115	135	156	174
23	72	89	112	132	154	175	72	96	120	138	157	177
24	72	94	115	136	157	175	77	97	121	138	161	180
25	78	101	117	137	159	176	84	105	128	151	171	190
26	82	102	123	140	161	177	91	113	133	152	172	190
27	84	105	124	143	162	179	94	115	136	152	174	191
28	86	105	125	144	165	181	99	121	138	156	174	195
29	90	108	125	145	169	185	106	125	142	166	183	200
30	92	111	129	146	169	191	106	129	150	167	187	204

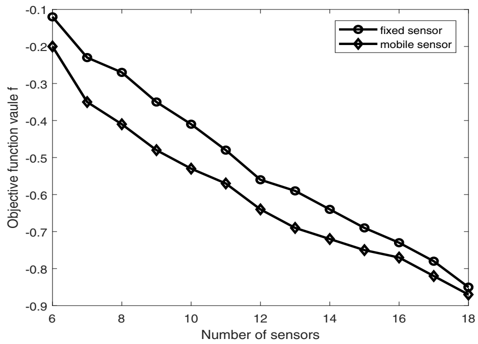
gradually decrease with traffic congestion. This is because the mobile sensor cannot move in congestion.

In summary, when the number of sensors is small, mobile sensors perform better than fixed ones regardless of road conditions. However, the advantages of mobile sensors gradually decrease when the number of sensors increases, especially in the congestion case. Therefore, how to choose different kinds of sensors on a freeway depends on traffic congestion and the number of sensors.

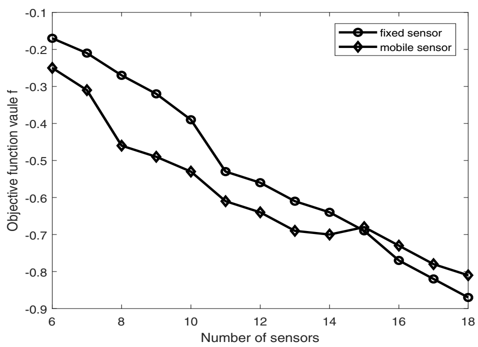
Sampling time interval: In order to illustrate the impact of different time intervals on dynamic location of mobile sensors, the time intervals are set to be 10 min, 15 min, 20 min, and 30 min, respectively. The number of sensors is 10. Then, the objective function value of model M_1 compared to with fixed sensors is shown in Fig. 7.

It is obvious that, when the time interval increases, the objective function value increases. This is because the shorter time interval can collect more traffic information. Moreover, the results with mobile sensors always perform better than the solutions with fixed sensors. Specifically, when a time interval increases, the gap between the objective function values with fixed sensors and mobile sensors decreases. For example, when the time interval increases from 10 minutes to 30 minutes, the gap between the objective function values with fixed sensors and mobile sensors decreases from 0.12 to 0.02. This is because the shorter the time interval, the more frequently the location of mobile sensor changes, which can better reflect dynamic data on the freeway.

Mobile sensor precision: Now, we study the effects of sensors with different precisions on model M_1. The detection precisions of the mobile sensor are set to be 97%, 95%, and 90%, respectively. For each precision, the number of sensors ranges from 6 to 18. The objective



(a)



(b)

Fig. 6. Comparison of mobile and fixed sensors for different road conditions (non-congestion (a), congestion (b)).

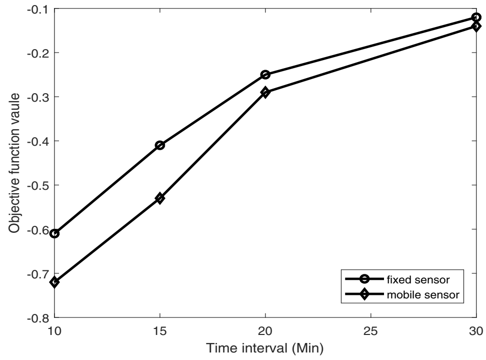


Fig. 7. Comparison of results for different time intervals.

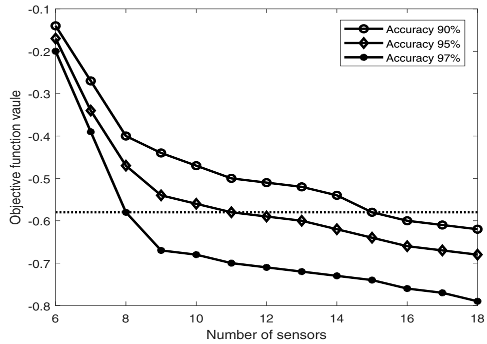


Fig. 8. Comparison of results for different precisions.

function of models with different numbers of sensors and precisions is shown in Fig. 8.

Apparently, when the number of mobile sensors increases, the total objective function value with different precisions decreases. Moreover, the higher the precision of mobile sensors, the smaller the total objective function value with the same number of sensors. For a low-precision sensor, we can increase the number of sensors to improve the objective function value. For example, in order to reach the objective function value of -0.58, the number of mobile sensors needs to be set as 8, 11, and 15 with a 97%, 95%, and 90% precision, respectively. This indicates that the precision of mobile sensors plays an influential role in sensor location.

6.2. Combination location of mobile and fixed sensors. In this section, we solve model M_2 for the investigated freeway, that is, how to dynamically add mobile sensors on the premise where 6 fixed sensors have been located on the freeway.

6.2.1. Comparison of adding mobile sensors and adding fixed sensors. In model M_2, the number of mobile sensors ranges from 1 to 10. Moreover, we also calculate the objective function value of adding the same number of fixed sensors for comparison. The detailed results are shown in Fig. 9.

It can be seen from Fig. 9 that the objective function value decreases as the number of sensors increases, regardless of which type of sensor is added. In addition, when the number of added sensors is 4, the trend of rapid decline slows down. This shows that when the number of added sensors reaches 4, it will not significantly improve the results if adding more sensors. Thus, the following experiments only discuss the case of adding 4 mobile sensors. Moreover, the mobile sensor performs better than the fixed one for the same number of added sensors. For instance, the objective function values for mobile sensor and fixed sensor are -0.6423 and -0.5689 in the case of adding 4 sensors, respectively. On the basis of the above discussion, a combination of mobile and fixed sensors can be adopted to obtain more accurate traffic information in reality.

6.2.2. Position analysis of added mobile sensors. It is assumed that the added number of mobile sensors is 4 and the time interval is 10 minutes. The position of each mobile sensor in 30 time periods can be obtained by solving model M_2. The whole freeway is divided into 7 sections by 6 fixed sensors and counts the total number of mobile sensors located in each section in 30 time periods.

Then, the number of mobile sensors on each section and the distance of each section are arranged in descending order. The detailed descriptions are shown in Table 5.

It can be seen from Table 5 that the number of mobile sensors is proportional to their distance for most sections. For example, the number of sensors and the distance of Section 2 are larger than those of Section 5. This indicates that the longer the distance, the more sensors need to be located in this section. This aims at ensuring more accurate estimation. However, the numbers of mobile sensors in some sections are inconsistent with the orders of their distances. For example, although the distance of Section 6 is shorter than that of Section 1, the number of sensors in Section 6 is obviously greater than that of Section 1. There are two reasons that may lead to such a phenomenon. First, it can be seen from Fig. 3 that Section 6 contains entrance and exit ramps, which may make travel time and traffic flow vary greatly. Therefore, the sensors should be located in a section with relatively large fluctuation information, so as to collect more valuable information. Secondly, this paper assumes that the mobile sensor starts from the beginning point of the freeway and returns to the ending point. Thus, Section 1 is regarded as the starting point and will not have more opportunities to locate sensors. In summary, the location of mobile sensors will consider the length and position of each section.

6.2.3. Data fusion comparison. For model M_2, the speed and flow data of the fixed and mobile sensors should be fused. In this section, we examine the effect of data fusion on Section 3 of the freeway. The reason for choosing Section 3 is the longest distance among all 7 sections, and the fact that it contains an entrance and exit ramp. Moreover, Section 3 is in the middle of the entire freeway. The comparison of speed data and flow data in Section 3 is shown in Figs. 10 and 11.

In Figs. 10 and 11, the fused data are very close to the actual data and the error is near zero. Moreover, we know that the number of mobile sensors located in Section 3 is 26 in 30 time periods from Table 5. Apparently, there are 4 times when mobile sensor data are zero, and the error of 4 points on the line that represents the mobile sensor is very large, as shown in Figs. 10 and 11. In addition, the fusing error of speed is smaller than that of the flow, which is mainly due to the fact that the speed of vehicles passing through Section 3 is stable at about 80 km/h and the flow ranges from 120 to 430 pcu/hour. Thus the combination of fixed and mobile sensors is more effective for travel time estimation. In summary, linear weighting is a simple and effective fusion data method, which can make the fusion data closer to their true values.

7. Conclusions

In order to obtain travel time estimates for a freeway accurately, this paper proposed two sensor location models that consider minimizing the error of travel time estimation and maximizing the collected traffic flow. First, a dynamic optimization model for mobile sensor location was proposed under the assumption that there are no traffic sensors on the freeway. Next, a dynamic optimal combinatorial model of mobile sensors and fixed sensors was presented taking account of fixed sensors on the freeway. The proposed models were optimized with the simulated annealing algorithm. In the simulation experiment, we verified the applicability of the model and the robustness of the algorithm. The experimental results show that the mobile sensor is better than the fixed sensor in most cases, and the combination of both sensors is best. However, this paper has several limitations, and thus we suggest the following future research works.

- The proposed models need to know the traffic flow and all vehicles' speed in each cell, but it is not practical to locate sensors in each cell to get this information. Therefore, this paper simulates the data needed in the simulation experiment. How to use special technology such as the Freeway Performance Measurement System to obtain vehicle information in reality is worth further studies.

- It is an important research direction how to consider sensor failures in the proposed model to guarantee its reliability.

- The linear weighting method is used to fuse the data of mobile and fixed sensors in this paper. In future research, some artificial intelligence fusion algorithms will be adopted to compare the effect of various algorithms on traffic data fusion.

Acknowledgment

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References

Ban, X., Chu, L., Herring, R. and Margulici, J. (2011). Sequential modeling framework for optimal sensor placement for multiple intelligent transportation system applications, *Journal of Transportation Engineering* **137**(2): 112–120.

Table 5. Comparison of the number of mobile sensors and distance of sections.

	Number of sensors	Order	Distance (km)	Order
Section 1	13	5	3.75	2
Section 2	**20**	3	**3.04**	4
Section 3	26	1	4.23	1
Section 4	19	4	2.92	5
Section 5	**12**	6	**1.67**	7
Section 6	21	2	3.26	3
Section 7	9	7	2.39	6

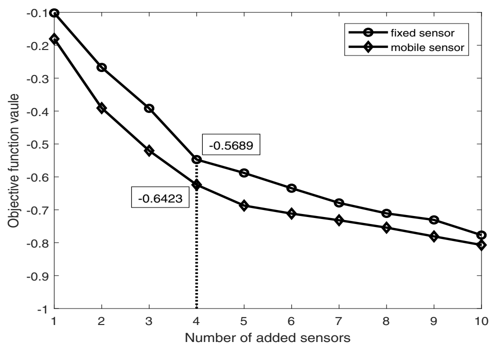

Fig. 9. Comparison of results with different numbers of added sensors.

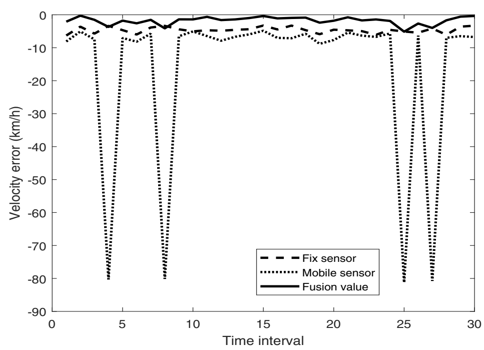

Fig. 10. Comparison of speed data in Section 3.

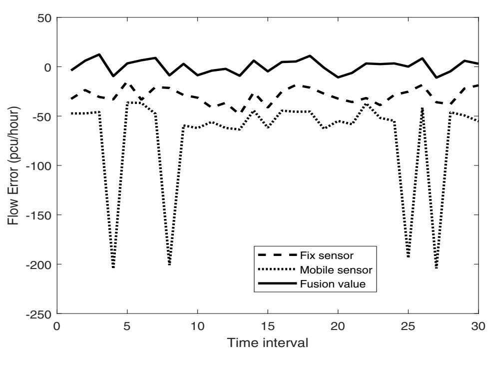

Fig. 11. Comparison of flow data on Section 3.

Ban, X., Herring, R., Margulici, J. and Bayen, A. (2009). Optimal sensor placement for travel time estimation, *Transportation and Traffic Theory* **2009**: 697–721.

Bartin, B., Ozbay, K. and Iyigun, C. (2007). A clustering based methodology for determining optimal roadway configuration of detectors for travel time estimation, *Transportation Research Record* **2000**: 98–105.

Beryini, R. and Lovell, D. (2009). Impacts of sensor spacing on accurate freeway travel time estimation for traveler information, *Journal of Intelligent Transportation Systems* **13**(2): 97–110.

Chakraborty, P., Hegde, C. and Sharma, A. (2019). Data-driven parallelizable traffic incident detection using spatio-temporally denoised robust thresholds, *Transportation Research C* **105**: 81–99.

Chang, B.-J., Hwang, R.-H., Tsai, Y.-L., Yu, B.-H. and Liang, Y.-H. (2019). Cooperative adaptive driving for platooning autonomous self driving based on edge computing, *International Journal of Applied Mathematics and Computer Science* **29**(2): 213–225, DOI: 10.2478/amcs-2019-0016.

Chaudhuri, P., Martin, P.T., Stevanovic, A.Z. and Zhu, C. (2010). The effects of detector spacing on travel time prediction on freeways, *World Academy of Science, Engineering and Technology* **42**(6): 1–10.

Chou, J.-J., Shih, C.-S., Wang, W.-D. and Huang, K.-C. (2019). IoT sensing networks for gait velocity measurement, *International Journal of Applied Mathematics and Computer Science* **29**(2): 245–259, DOI: 10.2478/amcs-2019-0018.

Chow, J. (2016). Dynamic UAV-based traffic monitoring under uncertainty as a stochastic arc-inventory routing policy, *International Journal of Transportation Science and Technology* **5**(3): 167–185.

Danczyk, A., Di, X. and Liu, H. (2016). A probabilistic optimization model for allocating freeway sensors, *Transportation Research C* **67**: 378–398.

Danczyk, A. and Liu, H. (2011). A mixed-integer linear program for optimizing sensor locations along freeway corridors, *Transportation Research Part B* **45**(1): 208–217.

Fischetti, M. and Monaci, M. (2020). A branch-and-cut algorithm for mixed-integer bilinear programming, *European Journal of Operational Research* **282**(2): 506–514.

Fu, C., Zhu, N. and Ma, S. (2017). A stochastic program approach for path reconstruction oriented sensor location model, *Transportation Research Part B* **102**: 210–237.

Fujito, I., Margiotta, R., Huang, W. and Perez, W.A. (2006). Effect of sensor spacing on performance measure calculations, *Journal of the Transportation Research Board* **1945**: 1–11.

Geetla, T., Batta, R., Blatt, A., Flanigan, M. and Majka, K. (2014). Optimal placement of omnidirectional sensors in a transportation network for effective emergency response and crash characterization, *Transportation Research C* **45**: 64–82.

Gentili, M. and Mirchandani, P. (2012). Locating sensors on traffic networks: Models, challenges and research opportunities, *Transportation Research C* **24**: 227–255.

Gentili, M. and Mirchandani, P. (2018). Review of optimal sensor location models for travel time estimation, *Transportation Research C* **90**: 74–96.

He, S. (2013). A graphical approach to identify sensor locations for link flow inference, *Transportation Research B* **51**: 65–76.

Hong, Z. and Fukuda, D. (2012). Effects of traffic sensor location on traffic state estimation, *Procedia-Social and Behavioral Sciences* **54**(2290): 1186–1196.

Karatsoli, M., Margreiter, M. and Spangler, M. (2017). Bluetooth-based travel times for automatic incident detection-a systematic description of the characteristics for traffic management purposes, *Transportation Research Procedia* **24**: 204–211.

Kianfar, J. and Edara, P. (2010). Optimizing freeway traffic sensor locations by clustering global-positioning-system-derived speed patterns, *IEEE Transactions on Intelligent Transportation Systems* **11**(3): 738–747.

Kim, J., Park, B., Lee, J. and Won, J. (2011). Determining optimal sensor locations in freeway using genetic algorithm-based optimization, *Engineering Applications of Artificial Intelligence* **24**(2): 318–324.

Kolak, O., Feyzioğlu, O. and Noyan, N. (2018). Bi-level multi-objective traffic network optimisation with sustainability perspective, *Expert Systems with Applications* **104**(15): 294–306.

Kolosz, B., Grant-Muller, S. and Djemame, K. (2013). Modelling uncertainty in the sustainability of intelligent transport systems for highways using probabilistic data fusion, *Environmental Modelling & Software* **49**: 78–97.

Li, X. and Ouyang, Y. (2011). Reliable sensor deployment for network traffic surveillance, *Transportation Research B* **45**: 218–231.

Liu, F. L., Wang, Y., Bai, Y. and Yu, J. (2019). Study on stealth characteristics of metamaterials based on simulated annealing algorithm, *Procedia Computer Science* **147**: 221–227.

Liu, H. and Danczyk, A. (2009). Optimal sensor locations for freeway bottleneck identification, *Computer-Aided Civil and Infrastructure Engineering* **24**(8): 535–550.

Ma, W. and Qian, Z. (2018). Statistical inference of probabilistic origin-destination demand using day-to-day traffic data, *Transportation Research C* **88**: 227–256.

Meng, T., Jing, X., Yan, Z. and Pedrycz, W. (2020). A survey on machine learning for data fusion, *Information Fusion* **57**: 115–229.

Nemati, M., Braun, M. and Tenbohlen, S. (2018). Optimization of unit commitment and economic dispatch in microgrids based on genetic algorithm and mixed integer linear programming, *Applied Energy* **210**: 944–963.

Ng, M. (2013). Partial link flow observability in the presence of initial sensors: Solution without path enumeration, *Transportation Research E* **51**: 62–66.

Olia, A., Abdelgawad, H., Abdulhai, B. and Razavi, S. (2017). Optimizing the number and locations of freeway roadside equipment units for travel time estimation in a connected vehicle environment, *Journal of Intelligent Transportation Systems* **21**(4): 296–309.

Park, H. and Haghani, A. (2015). Optimal number and location of bluetooth sensors considering stochastic travel time prediction, *Transportation Research C* **55**: 203–216.

Salari, M., Kattan, L., Lam, W., Lo, H. and Esfeh, M. (2019). Optimization of traffic sensor location for complete link flow observability in traffic network considering sensor failure, *Transportation Research Part B* **121**: 216–251.

Song, Z.R., Zang, L.L. and Zhu, W.X. (2020). Study on minimum emission control strategy on arterial road based on improved simulated annealing genetic algorithm, *Physica A* **537**: 1–11.

Xing, T., Zhou, X. and Taylor, J. (2013). Designing heterogeneous sensor networks for estimating and predicting path travel time dynamics: An information-theoretic modeling approach, *Transportation Research B* **57**: 66–90.

Yang, Y. and Fan, Y. (2015). Data dependent input control for origin-destination demand estimation using observability analysis, *Transportation Research B* **78**: 385–403.

Zhan, F., Wan, X., Zhang, J., Li, R. and Ran, B. (2015). Sample size reduction method based on data fusion for freeways with fixed detectors, *Transportation Research Record* **2528**: 18–26.

Zhu, N., Fu, C. and Ma, S. (2018). Data-driven distributionally robust optimization approach for reliable travel-time-information-gain-oriented traffic sensor location model, *Transportation Research B* **113**: 91–120.

Zhu, N., Liu, Y., Ma, S. and He, Z. (2014). Mobile traffic sensor routing in dynamic transportation systems, *IEEE Transactions on Intelligent Transportation Systems* **15**(5): 2273–2285.

Zhu, N., Ma, S. and Zheng, L. (2017). Travel time estimation oriented freeway sensor placement problem considering sensor failure, *Journal of Intelligent Transportation Systems* **21**(1): 26–40.

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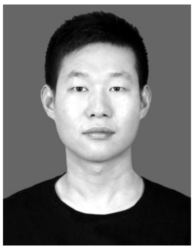

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