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DECENTRALIZED FEEDBACK CONTROLLERS FOR UNCERTAIN INTERCONNECTED SYSTEMS WITH MARKOVIAN JUMPING PARAMETERS[†]

EL-KEBIR BOUKAS[*], ANDRZEJ SWIERNIAK[**], HAILANG YANG[*]

This paper addresses the problem of design of decentralized controllers for a class of interconnected linear systems with Markovian jumping parameters and unknown but structured uncertainties. Results of Trinh and Aldeen (1993) are extended to a stochastic class of piecewise deterministic systems. Under the assumption that the Markovian jump process (disturbance) is irreducible and the complete access to the system's state and its mode is realizable, we establish the conditions which enable us to obtain a robust decentralized controller. An estimation method for the difference between the cost for the real system under our control law and the optimal performance for the nominal model is also presented.

1. Introduction

Most real physical dynamic processes comprise uncertain plants. Methods for designing stabilizing controllers for systems with uncertainty have been investigated by many researchers. However, in this context, only few works have been done on the systems with Markovian jumping parameters. This class of systems has been used to model many real life systems, including manufacturing ones (see e.g. the papers by Boukas and Haurie (1990), Boukas and Yang (1993), Boukas *et al.* (1994), and the references contained therein).

For the deterministic case, many authors have considered the uncertain large-scale interconnected systems stabilized by LQ controllers, see e.g. (Ikeda and Siljak, 1990; 1992) where the problem of the decentralized stability for a class of nonlinear interconnected systems is discussed. In an interesting paper by Trinh and Aldeen (1993), a method for the design of decentralized controllers for interconnected dynamic systems with structured uncertainties is presented. The aim of this paper is to extend the results of Trinh and Aldeen to the systems with Markovian jumping parameters. Trinh and Aldeen did not make any discussion about the performance index degradation which may be very important (see e.g. Swierniak, 1982). In our

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[*] Mechanical Engineering Department, École Polytechnique de Montréal, Montréal, Québec, H3C 3A7 Canada, e-mail: boukas@anas.meca.polymtl.ca

[**] Department of Automatic Control, Silesian Technical University, 44–101 Gliwice, Poland, *e-mail:* aswier@ia.gliwice.edu.pl

paper, we establish an estimation method for the difference between the cost for the real system under our control law and the optimal performance for the nominal model.

The paper is organized as follows. In Section 2, we describe the class of systems under consideration and formulate the addressed problem. In Section 3, we state the decentralized stability result and give its proof. In Section 4, an analysis of the suboptimality of the control law is presented.

2. System Description and Problem Formulation

We consider an uncertain linear interconnected system with Markovian jumping parameters composed of N uncertain subsystems. The system may be described by the following dynamics:

$$\begin{aligned} \dot{x}(t) &= [A(\xi(t)) + \delta A(\xi(t), a)]x(t) + [B(\xi(t)) + \delta B(\xi(t), a)]u(t) \\ x(0) &= x_0 \end{aligned} \tag{1}$$

which is an interconnection of N linear subsystems of the following form:

$$\begin{aligned} \dot{x}_i(t) &= [A_{ii}(\xi(t)) + \delta A_{ii}(\xi(t), a)]x_i(t) + [B_i(\xi(t)) + \delta B_i(\xi(t), a)]u_i(t) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} [A_{ij}(\xi(t)) + \delta A_{ij}(\xi(t), a)]x_j(t) \end{aligned} \tag{2}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ stands for the state of the subsystem, $u_i(t) \in \mathbb{R}^{m_i}$ is the controllable input vector, $A_{ij}(\xi)$ and $B_i(\xi)$ are matrices of appropriate dimensions, $\delta A_{ij}(\xi(t), a)$ and $\delta B_i(\xi(t), a)$ represent the uncertainties of the subsystems, $\xi(t)$ represents a homogeneous continuous discrete-state Markov process taking values in a finite set $\mathcal{B} = \{1, 2, .., s\}$. We assume that $P_t = (P_t^1, ..., P_t^s)$ with $P_t^\alpha = \Pr\{\xi_t = \alpha\}$, $\alpha = 1, ..., s$, satisfies the Kolmogorov forward equation, i.e.

$$\begin{aligned} \frac{dP_t}{dt} &= \Lambda P_t, \quad 0 \le t \le T \\ P_0 &= \bar{P} \end{aligned} \tag{3}$$

where $\bar{P}$ is the initial probability of the process $\{\xi_t\}$, $\Lambda = \left[q_{\alpha\beta}\right]^T$ is the stationary transition rate matrix of the process $\{\xi_t, t \in [0, T]\}$, $q_{\alpha\beta}$ stands for the transition probability rate from state α to state β and satisfies the following relations:

$$q_{\alpha\beta} \ge 0 \tag{4}$$

$$q_\alpha = -q_{\alpha\alpha} = \sum_{\substack{\beta\in\mathcal{B} \\ \alpha\neq\beta}} q_{\alpha\beta} \tag{5}$$

The vector parameter a lies within a prescribed bounded and connected set $\mathcal{Q} \subset \mathbb{R}^q$.

In eqns. (1)–(2), $x(t) = (x_1^T, ..., x_N^T)^T \in \mathbb{R}^n$ stands for the state of the system, $n = \sum_{i=1}^N n_i$, $u(t) = (u_1^T, ..., u_N^T)^T \in \mathbb{R}^m$ is the controllable input vector of the interconnected system, $m = \sum_{i=1}^N m_i$, $A(\xi) = (A_{ij}(\xi))$ and $B(\xi) = \mathrm{diag}(B_1(\xi), ..., B_N(\xi))$.

Let the uncertainties satisfy the following matching conditions (see e.g. Leitmann, 1979).

Assumption 1. For each $\alpha \in \mathcal{B}$, let the uncertainties satisfy the following equations:

$$\delta A(\xi(t),a) = B(\xi(t))G(\xi(t),a) \tag{6}$$

$$\delta B(\xi(t),a) = B(\xi(t))H(\xi(t),a) \tag{7}$$

where

$$\begin{aligned}
\delta A(\xi(t),a) &= (\delta A_{ij}(\xi(t),a)) \in \mathbb{R}^{n\times n} \\
\delta B(\xi(t),a) &= \mathrm{diag}(\delta B_1(\xi(t),a),\ldots,\delta B_N(\xi(t),a)) \in \mathbb{R}^{n\times m} \\
G(\xi(t),a) &= (G_{ij}(\xi(t),a)) \in \mathbb{R}^{m\times n} \\
H(\xi(t),a) &= \mathrm{diag}(H_1(\xi(t),a),\ldots,H_N(\xi(t),a)) \in \mathbb{R}^{m\times m}
\end{aligned}$$

From now on, when we say that $A > B$, we mean that $A - B$ is a positive definite matrix, and we use the Euclidean matrix norm. We also need the following assumption.

Assumption 2. $H_i(\xi(t),a) \in \mathbb{R}^{m_i\times m_i}$ satisfies the following condition:

$$\min_{\alpha\in\mathcal{B},a\in\mathcal{Q}}\left\{H_i^T(\alpha,a)+H_i(\alpha,a)\right\}+I_{m_i} > 0 \tag{8}$$

In the literature, many definitions for the stochastic stability have been proposed. In this paper, we will use the one given by the following definition.

Definition 1. The system (1)–(2) is said to be stochastically stabilizable if, for all finite $x_0 \in \mathbb{R}^n$ and $\alpha \in \mathcal{B}$, there exists a control $u = u^*(x,\alpha,t)$ such that there exists a symmetric positive definite matrix $\tilde{P}$ satisfying

$$\lim_{T\to\infty} E_{u(\cdot)}\left\{\int_0^T x^T(t,x_0,\alpha,a,u)x(t,x_0,\alpha,a,u)\,\mathrm{d}t|x_0,\alpha\right\} \le x_0^T\tilde{P}x_0 \tag{9}$$

where $x(t,x_0,\alpha,a,u)$ represents the corresponding solution of system (1) at time t when the control $u(\cdot)$ is used and the initial conditions are respectively x_0 and α.

The purpose of this paper is to find a robust stabilizing decentralized state controllers of the form

$$u(t) = -K(\xi)x(t) \tag{10}$$

where $K(\xi) = \mathrm{diag}(K_1(\xi),\ldots,K_N(\xi)) \in \mathbb{R}^{m\times n}$ is block diagonal, and $K_i(\xi) \in \mathbb{R}^{m_i\times n_i}$ are feedback gain matrices.

In the sequel, we will refer to the system

$$\dot{x}(t) = A(\xi(t))x(t) + B(\xi(t))u(t) \tag{11}$$

as a nominal system of (1) and let $P(\alpha)$ a symmetric, positive definite solution of the following set of the coupled matrix Riccati equations

$$A^T(\alpha)P(\alpha) + P(\alpha)A(\alpha) \\ -P(\alpha)B(\alpha)R^{-1}(\alpha)B^T(\alpha)P(\alpha) + Q(\alpha) + \sum_{\beta \in \mathcal{B}} q_{\alpha\beta}P(\beta) = 0 \qquad (12)$$

We will assume that the nominal system is stochastically stabilizable under the control law $u(t) = -R^{-1}(\alpha)B^T(\alpha)P(\alpha)x(t)$.

3. Decentralized Stability

The following theorem states the main result of this paper.

Theorem 1. *For each α, let $L(\alpha) \in \mathbb{R}^{m\times n}$ be an arbitrary matrix and $K(\alpha) \in \mathbb{R}^{m\times n}$ be a block diagonal matrix, satisfying the following relationship for each $\alpha \in \mathcal{B}$*

$$K(\alpha) + L(\alpha) = R^{-1}(\alpha)B^T(\alpha)P(\alpha) \qquad (13)$$

where $P(\alpha)$ is a symmetric, positive definite solution of (12) and the matrices $R(\alpha) > 0$ and $Q(\alpha) > 0$ (symmetric) are chosen to satisfy the following conditions:

$$R(\alpha) = \frac{\eta(\alpha)}{1 + 2\chi(\alpha)} I_m \qquad (14)$$

and

$$Q(\alpha) > G^T(\alpha, a)G(\alpha, a) + \frac{1}{\chi(\alpha)}(1 + \bar{H}^2(\alpha))L^T(\alpha)L(\alpha) \qquad (15)$$

where $\chi(\alpha) > 1$ is a real scalar, $\eta(\alpha)$ and $\bar{H}^2(\alpha)$ are respectively defined by:

$$\eta(\alpha) = \min_{a \in \mathcal{Q}} \lambda_{min}\left[H^T(\alpha, a) + H(\alpha, a) + I\right] \qquad (16)$$

$$\bar{H}^2(\alpha) = \max_{a \in \mathcal{Q}} ||H(\alpha, a)||^2 \qquad (17)$$

Then the system (1)-(2) is stochastically stabilizable by the control law $u(t) = -K(\alpha)x(t)$.

Proof. Due to the assumption that the nominal system is stochastically stabilizable by the control law $u(t) = -R^{-1}(\alpha)B^T(\alpha)P(\alpha)x(t)$, from (Ji and Chizeck, 1990), we have that the matrix $P(\alpha)$, i.e. the solution of (12), is positive definite and symmetric. Let the Lyapunov function be defined by the following expression:

$$V(\alpha, x) = x^T P(\alpha)x \qquad (18)$$

Consider the weak infinitesimal operator $\tilde{A}$ of the process $\{\xi, x(t), t \in [0, T]\}$, which is given by

$$\tilde{A}V(\alpha, x) = \Big[[A(\alpha) + \delta A(\alpha, a)]x(t) + [B(\alpha) + \delta B(\alpha, a)]u(t)\Big]^T P(\alpha)x(t) \\ +x^T P(\alpha)\Big[[A(\alpha) + \delta A(\alpha, a)]x(t) + [B(\alpha) + \delta B(\alpha, a)]u(t)\Big] + \sum_{\beta \in \mathcal{B}} q_{\alpha\beta}V(\beta, x)$$

Substituting the control law and using eqn. (12) in conjunction with the matching conditions gives

$$\tilde{A}V(\alpha,x) = x^T\Big[[A^T(\alpha)P(\alpha) + G^T(\alpha,a)B^T(\alpha)P(\alpha)$$

$$-2P(\alpha)B(\alpha)R^{-1}(\alpha)B^T(\alpha)P(\alpha) + L^T(\alpha)B^T(\alpha)P(\alpha)$$

$$-P(\alpha)B(\alpha)R^{-1}(\alpha)H^T(\alpha,a)B^T(\alpha)P(\alpha) + L^T(\alpha)H^T(\alpha,a)B^T(\alpha)P(\alpha)$$

$$+P(\alpha)A(\alpha) + P(\alpha)B(\alpha)G(\alpha,a) + P(\alpha)B(\alpha)L(\alpha)$$

$$-P(\alpha)B(\alpha)H(\alpha,a)R^{-1}(\alpha)B^T(\alpha)P(\alpha)$$

$$+P(\alpha)B(\alpha)H(\alpha,a)L(\alpha) + \sum_{\beta\in\mathcal{B}} q_{\alpha\beta}P(\beta)\Big]x$$

Using the matrix inequality $EF^T + FE^T \leq \frac{1}{\theta}FF^T + \theta EE^T$, where $\theta > 0$, E and F are matrices of appropriate dimensions, the fact that $L^T(\alpha)H^T(\alpha,a)H(\alpha,a)L(\alpha) \leq \bar{H}^2(\alpha)L^T(\alpha)L(\alpha)$ (see Zhou and Khargonekar, 1988) and taking into account eqn. (12), we have that

$$\tilde{A}V(\alpha,x) \leq x^T\Big[-Q(\alpha) + G^T(\alpha,a)G(\alpha,a) + \frac{1}{\chi(\alpha)}(1+\bar{H}^2(\alpha))L^T(\alpha)L(\alpha)$$

$$-P(\alpha)B(\alpha)R^{-1}(\alpha)B^T(\alpha)P(\alpha) - P(\alpha)B(\alpha)H(\alpha,a)R^{-1}(\alpha)B^T(\alpha)P(\alpha)$$

$$-P(\alpha)B(\alpha)R^{-1}(\alpha)H^T(\alpha,a)B^T(\alpha)P(\alpha) + (1+2\chi(\alpha))P(\alpha)B(\alpha)B^T(\alpha)P(\alpha)\Big]x$$

Furthermore, we have:

$$x^T\Big[-P(\alpha)B(\alpha)H(\alpha,a)R^{-1}(\alpha)B^T(\alpha)P(\alpha)$$

$$-P(\alpha)B(\alpha)R^{-1}(\alpha)H^T(\alpha,a)B^T(\alpha)P(\alpha) - P(\alpha)B(\alpha)R^{-1}(\alpha)B^T(\alpha)P(\alpha)$$

$$+(1+2\chi(\alpha))P(\alpha)B(\alpha)B^T(\alpha)P(\alpha)\Big]x$$

$$= -\frac{1+2\chi(\alpha)}{\eta(\alpha)}\Big[B^T(\alpha)P(\alpha)x\Big]^T\Big[H(\alpha,a) + H^T(\alpha,a) + I - \eta(\alpha)I\Big]$$

$$\times\Big[B^T(\alpha)P(\alpha)x\Big] \quad \leq \quad 0$$

The last inequality follows from eqn. (16).

Let

$$\gamma(\alpha) = \frac{\min_{a\in\mathcal{Q}} \lambda_{min}\Big[Q(\alpha) - G^T(\alpha,a)G(\alpha,a) - \frac{1}{\chi(\alpha)}[1+\bar{H}^2(\alpha)]L^T(\alpha)L(\alpha)\Big]}{\lambda_{max}[P(\alpha)]}$$

which is positive due to (15). Then it follows that

$$\tilde{A}V(\alpha,x) \leq -\gamma(\alpha)V(\alpha,x)$$

where $\gamma := \min_{\alpha\in\mathcal{B}} \gamma(\alpha)$. Then, by Dynkin's formula and the Gronwall-Bellman lemma, we obtain for all $\alpha \in \mathcal{B}$,

$$E[V(\alpha, x)] \leq \exp(-\gamma t)V(\alpha, x_0) \tag{19}$$

Considering the conditional expectation, we have that

$$E[V(\alpha, x)|x_0, \xi(0) = \alpha] \leq \exp(-\gamma t)x_0^T P(\alpha)x_0 \tag{20}$$

Thus we have

$$\begin{aligned} E\Big\{\int_0^T x^T(t)P(\alpha)x(t)\,\mathrm{d}t|x_0, \alpha\Big\} &\leq \Big(\int_0^T \exp(-\gamma t)dt\Big)x_0^T P(\alpha)x_0 \\ &\leq -\frac{1}{\gamma}\Big[\exp(-\gamma T) - 1\Big]x_0^T P(\alpha)x_0 \end{aligned} \tag{21}$$

Letting $T \to \infty$ and

$$\tilde{P} = \max_{\alpha\in\mathcal{B}} \frac{P(\alpha)}{\gamma||P(\alpha)||} \tag{22}$$

we get

$$\lim_{T\to\infty} E_{u(\cdot)}\left\{\int_0^T x^T(t)x(t)\,\mathrm{d}t|x_0, \alpha\right\} \leq x_0^T \tilde{P} x_0 \tag{23}$$

This completes the proof of the theorem. ■

4. Suboptimality of the Control Law

In the above discussion, we have not used the optimal control law to control the large scale system. For the nominal system described by eqn. (11), the optimization problem consists of minimizing the cost function

$$J = E\left\{\int_0^\infty x^T(t)Q(\xi(t))x(t) + u^T(t)R(\xi(t))u(t)\,\mathrm{d}t\right\} \tag{24}$$

We know that the control law $u(t) = -R^{-1}(\alpha)B^T(\alpha)P(\alpha)x(t)$ minimizes the cost (24) and the minimum cost for the nominal model is given by

$$J^0 = x_0^T P(\alpha)x_0 \tag{25}$$

where $P(\alpha)$ is the solution of (12). The control law, $u(t) = -K(\alpha)x(t)$, we used is not an optimal one. In the sequel, we try to estimate the difference between the optimal cost and the cost corresponding to our control law. Since our control law $u(t) = -K(\alpha)x(t)$ also stabilizes the nominal model, the corresponding cost for the nominal model is given by

$$\begin{aligned} J^* &= E\Big\{\int_0^\infty x^T(t)[Q(\xi(t)) + K^T(\xi(t))R(\xi(t))K(\xi(t))]x(t)\,\mathrm{d}t\Big\} \\ &= x_0^T M(\alpha)x_0 \end{aligned} \tag{26}$$

where $M(\alpha)$ satisfies the following equation (see e.g. Ji and Chizek, 1990):

$$[A(\alpha) - B(\alpha)K(\alpha)]^T M(\alpha) + M(\alpha)[A(\alpha) - B(\alpha)K(\alpha)]^T$$
$$+Q(\alpha) + K^T(\alpha)R(\alpha)K(\alpha) + \sum_{\beta \in \mathcal{B}} q_{\alpha\beta} M(\beta) = 0 \tag{27}$$

Therefore,

$$J^* - J^0 = x_0^T [M(\alpha) - P(\alpha)] x_0 \tag{28}$$

$$\frac{J^* - J^0}{J^0} \le \frac{\lambda_{max}[M(\alpha)] - \lambda_{min}[P(\alpha)]}{\lambda_{min}[P(\alpha)]} = \frac{\lambda_{max}[M(\alpha)]}{\lambda_{min}[P(\alpha)]} - 1 \tag{29}$$

Next, we will estimate the difference between the optimal cost of the nominal system and the cost corresponding to the real system (1) using our control law. Since our control law $u(t) = -K(\alpha)x(t)$ also stabilizes the real system, the cost for the real system may be found as

$$\bar{J} = E\left\{ \int_0^\infty x^T(t)[Q(\xi(t)) + K^T(\xi(t))R(\xi(t))K(\xi(t))]x(t)\, dt \right\}$$
$$= x_0^T N(\alpha, a) x_0 \tag{30}$$

where $N(\alpha, a)$ satisfies the following equation (see e.g. Ji and Chizek, 1990):

$$[A(\alpha) + \delta A(\alpha, a) - B(\alpha)K(\alpha) - \delta B(\alpha, a)K(\alpha)]^T N(\alpha, a)$$
$$+N(\alpha, a)[A(\alpha) + \delta A(\alpha, a) - B(\alpha)K(\alpha) - \delta B(\alpha, a)K(\alpha)]$$
$$+Q(\alpha) + K^T(\alpha)R(\alpha)K(\alpha) + \sum_{\beta \in \mathcal{B}} q_{\alpha\beta} N(\beta, a) = 0 \tag{31}$$

Therefore,

$$\bar{J} - J^0 = x_0^T [N(\alpha, a) - P(\alpha)] x_0 \tag{32}$$

$$\frac{\bar{J} - J^0}{J^0} \le \frac{\lambda_{max}[N(\alpha, a)] - \lambda_{min}[P(\alpha)]}{\lambda_{min}[P(\alpha)]} = \frac{\lambda_{max}[N(\alpha, a)]}{\lambda_{min}[P(\alpha)]} - 1 \tag{33}$$

In order to estimate the maximum eigenvalue of $N(\alpha, a)$, we need the following lemma.

Lemma 1. *Under the assumptions of Section 3, the system*

$$\dot{x}(t) = A(\xi(t))x(t) + [B(\xi(t)) + \delta B(\xi(t), a)]u(t) \tag{34}$$

is stochastically stable under the control law $u(t) = -R^{-1}(\alpha)B^T(\alpha)P(\alpha)x(t)$.

Proof. Let the Lyapunov function be defined by the following expression:

$$V(\alpha, x) = x^T P(\alpha) x \tag{35}$$

Consider the weak infinitesimal operator $\tilde{A}$ of the process $\{\xi, x(t), t \in [0,T]\}$, which is given by

$$\begin{aligned}\tilde{A}V(\alpha,x) &= x^T\Big[A^T(\alpha)P(\alpha) - 2P(\alpha)B(\alpha)R^{-1}(\alpha)B^T(\alpha)P(\alpha)\\ &\quad -P(\alpha)B(\alpha)R^{-1}(\alpha)H^T(\alpha,a)B^T(\alpha)P(\alpha) + P(\alpha)A(\alpha)\\ &\quad -P(\alpha)B(\alpha)H(\alpha,a)R^{-1}(\alpha)B^T(\alpha)P(\alpha) + \sum_{\beta\in\mathcal{B}} q_{\alpha\beta}P(\beta)\Big]x\\ &= x^T\Big[-Q(\alpha) - P(\alpha)B(\alpha)R^{-1}(\alpha)B^T(\alpha)P(\alpha)\\ &\quad -P(\alpha)B(\alpha)R^{-1}(\alpha)H^T(\alpha,a)B^T(\alpha)P(\alpha)\\ &\quad -P(\alpha)B(\alpha)H(\alpha,a)R^{-1}(\alpha)B^T(\alpha)P(\alpha)\Big]x\end{aligned}$$

From Assumption 2, we have

$$\begin{aligned}& x^T\Big[-P(\alpha)B(\alpha)H(\alpha,a)R^{-1}(\alpha)B^T(\alpha)P(\alpha)\\ &\quad -P(\alpha)B(\alpha)R^{-1}(\alpha)H^T(\alpha,a)B^T(\alpha)P(\alpha)\\ &\quad -P(\alpha)B(\alpha)R^{-1}(\alpha)B^T(\alpha)P(\alpha)\Big]x\\ &= -\frac{1+2\chi(\alpha)}{\eta(\alpha)}\Big[B^T(\alpha)P(\alpha)x\Big]^T\\ &\quad \times\Big[H(\alpha,a) + H^T(\alpha,a) + I\Big]\Big[B^T(\alpha)P(\alpha)x\Big] \le 0\end{aligned}$$

Then it follows that

$$\tilde{A}V(\alpha,x) \le -\lambda_{min}[Q(\alpha)]\|x\|^2$$

The same approach as in the proof of Theorem 1 can be used to finish this proof. ■

The following theorem gives an estimate on the maximum eigenvalue of $N(\alpha,a)$.

Theorem 2. *Under the assumptions of Section 3, the maximum eigenvalue of the solution matrix $N(\alpha,a)$ of (31) has an upper bound given by*

$$\begin{aligned}\max_{\alpha\in\mathcal{B}}\{\lambda_{max}[N(\alpha,a)]\} &\le \max_{\alpha\in\mathcal{B}}\Big\{\lambda_{min}^{-1}\Big[-[Q(\alpha) + (1+2\chi(\alpha))B^T(\alpha)B(\alpha)]\\ &\quad \times[Q(\alpha) + K^T(\alpha)R(\alpha)K(\alpha)]^{-1}\Big]\Big\} \qquad (36)\end{aligned}$$

where $\chi(\alpha)$ is the same as in (14).

Proof. For $\zeta \in \mathbb{C}^n$, we have

$$\begin{aligned}&\zeta^T[A(\alpha)+\delta A(\alpha,a)-B(\alpha)K(\alpha)-\delta B(\alpha,a)K(\alpha)]^T N(\alpha,a)\zeta\\&+\zeta^T N(\alpha,a)[A(\alpha)+\delta A(\alpha,a)-B(\alpha)K(\alpha)-\delta B(\alpha,a)K(\alpha)]\zeta\\&+\zeta^T[Q(\alpha)+K^T(\alpha)R(\alpha)K(\alpha)]\zeta+\sum_{\beta\in\mathcal{B}}q_{\alpha\beta}\zeta^T N(\beta,a)\zeta=0\end{aligned} \tag{37}$$

which can be rewritten as

$$\begin{aligned}&2\lambda_i[N(\alpha,a)]\zeta^T S(\alpha,a)\zeta\\&+\zeta^T[Q(\alpha)+K^T(\alpha)R(\alpha)K(\alpha)]\zeta+\sum_{\beta\in\mathcal{B}}q_{\alpha\beta}\zeta^T N(\beta,a)\zeta=0\end{aligned} \tag{38}$$

where

$$\begin{aligned}S(\alpha,a) &= \frac{1}{2}\Big[A(\alpha)+\delta A(\alpha,a)-B(\alpha)K(\alpha)-\delta B(\alpha,a)K(\alpha)\\&\quad+[A(\alpha)+\delta A(\alpha,a)-B(\alpha)K(\alpha)-\delta B(\alpha,a)K(\alpha)]^T\Big]\end{aligned}$$

Since the system is stable under control law $u(t) = -K(\alpha)x(t)$, $S(\alpha,a)$ is negative definite ($S(\alpha,a) < 0$). Let us notice that $\lambda_i[N(\alpha,a)] > 0$ and $\zeta^T[Q(\alpha) + K^T(\alpha)R(\alpha)K(\alpha)]\zeta > 0$ ($\zeta \neq 0$). If $\alpha_0 \in \mathcal{B}$ is such that $\lambda_{max}[N(\alpha_0,a)] = \max_{\alpha\in\mathcal{B}}\{\lambda_{max}[N(\alpha,a)]\}$, then we have

$$\begin{aligned}\frac{1}{\lambda_{max}[N(\alpha_0,a)]} &= -\frac{2\zeta^T S(\alpha_0,a)\zeta}{\zeta^T[Q(\alpha_0)+K^T(\alpha_0)R(\alpha_0)K(\alpha_0)]\zeta}\\&\quad-\sum_{\beta\in\mathcal{B}}q_{\alpha_0\beta}\frac{\zeta^T N(\beta,a)\zeta}{\lambda_{max}[N(\alpha_0,a)]\zeta^T[Q(\alpha_0)+K^T(\alpha_0)R(\alpha_0)K(\alpha_0)]\zeta}\\&\geq -\frac{2\zeta^T S(\alpha_0,a)\zeta}{\zeta^T[Q(\alpha_0)+K^T(\alpha_0)R(\alpha_0)K(\alpha_0)]\zeta}\\&\geq \lambda_{min}\left[-2S(\alpha_0,a)[Q(\alpha_0)+K^T(\alpha_0)R(\alpha_0)K(\alpha_0)]^{-1}\right]\end{aligned} \tag{39}$$

since

$$\begin{aligned}2S(\alpha,a) &= A(\alpha)+\delta A(\alpha,a)-B(\alpha)K(\alpha)-\delta B(\alpha,a)K(\alpha)\\&\quad+[A(\alpha)+\delta A(\alpha,a)-B(\alpha)K(\alpha)-\delta B(\alpha,a)K(\alpha)]^T\\&= A(\alpha)+A^T(\alpha)+B(\alpha)G(\alpha,a)+G^T(\alpha,a)B^T(\alpha)-B(\alpha)K(\alpha)\\&\quad-K^T(\alpha)B^T(\alpha)-B(\alpha)H(\alpha,a)K(\alpha)-K^T(\alpha)H^T(\alpha,a)B^T(\alpha)\\&= A(\alpha)+A^T(\alpha)+B(\alpha)G(\alpha,a)+G^T(\alpha,a)B^T(\alpha)-B(\alpha)P^*(\alpha)\\&\quad+B(\alpha)L(\alpha)-P^{*T}(\alpha)B^T(\alpha)+L^T(\alpha)B^T(\alpha)\end{aligned}$$

$$-B(\alpha)H(\alpha,a)P^*(\alpha)+B(\alpha)H(\alpha,a)L(\alpha)$$
$$-P^{*T}(\alpha)H^T(\alpha,a)B^T(\alpha)+L^T(\alpha)H^T(\alpha,a)B^T(\alpha)$$

where $P^*(\alpha)=R^{-1}(\alpha)B^T(\alpha)P(\alpha)$.

Once again, using the matrix inequlity $EF^T+FE^T\le\frac{1}{\theta}FF^T+\theta EE^T$, where $\theta>0$, E and F are matrices of appropriate dimensions, and the fact that

$$L^T(\alpha)H^T(\alpha,a)H(\alpha,a)L(\alpha)\le\bar{H}^2(\alpha)L^T(\alpha)L(\alpha)$$

we have

$$\begin{aligned}2S(\alpha,a) &\le A(\alpha)+A^T(\alpha)+B^T(\alpha)B(\alpha)+G^T(\alpha,a)G(\alpha,a)-B(\alpha)P^*(\alpha)\\ &\quad -P^{*T}(\alpha)B^T(\alpha)+2\chi(\alpha)B^T(\alpha)B(\alpha)+\frac{1}{\chi(\alpha)}L^T(\alpha)L(\alpha)\\ &\quad +\frac{1}{\chi(\alpha)}\bar{H}^2L^T(\alpha)L(\alpha)-B(\alpha)H(\alpha,a)P^*(\alpha)-P^{*T}(\alpha)H^T(\alpha,a)B^T(\alpha)\\ &\le A(\alpha)+A^T(\alpha)+(1+2\chi(\alpha))B^T(\alpha)B(\alpha)+Q(\alpha)-B(\alpha)P^*(\alpha)\\ &\quad -P^{*T}(\alpha)B^T(\alpha)-B(\alpha)H(\alpha,a)P^*(\alpha)-P^{*T}(\alpha)H^T(\alpha,a)B^T(\alpha)\end{aligned}$$

The last inequality is valid because of (15). Taking into account the fact that the system (34) is stable under the control law $u(t)=-P^*(\alpha)x(t)$, we have

$$2S(\alpha,a)\le Q(\alpha)+(1+2\chi(\alpha))B^T(\alpha)B(\alpha)$$

Therefore,

$$\begin{aligned}\frac{1}{\lambda_{max}[N(\alpha_0,a)]} &\ge \lambda_{min}\Big[-[Q(\alpha_0)+(1+2\chi(\alpha_0))B^T(\alpha_0)B(\alpha_0)]\\ &\qquad \times[Q(\alpha_0)+K^T(\alpha_0)R(\alpha_0)K(\alpha_0)]^{-1}\Big]\end{aligned} \tag{40}$$

From this we conclude our result. ∎

5. Conclusion

In this paper, the decentralized controller design method for large-scale uncertain linear systems with Markovian jumping parameters is presented and an estimation method for the difference between the result of our control law and the optimal performance for nominal model is given. The result is obtained under an assumption on the matching conditions. The weighting matrices in the performance index are chosen to ensure robust stability for the overall system and the performance deterioration resulting from the uncertainties is estimated.

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