Int. J. Appl. Math. and Comp. Sci., *1999*, *Vol.9*, *No.3*, *689–709*

EVOLUTIONARY MULTI-OBJECTIVE PARETO OPTIMISATION OF DIAGNOSTIC STATE OBSERVERS

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A multi-objective Pareto-optimisation procedure for the design of residual generators which constitute a primary instrument for model-based fault detection and isolation (FDI) in systems of plant monitoring and control is considered. An evolutionary approach to the underlying multi-objective optimisation problem is utilised. The resulting robust observer detector allows for FDI, taking into account the issue of false alarms.

Keywords: diagnostics, fault detection and isolation, genetic algorithms, multi-objective optimisation, Pareto-optimality, residuals, state observers.

1. Introduction

Fault Detection and Isolation (FDI) systems are commonly used for diagnostic purposes and obtained by means of two distinct operations. With the first of them an occurrence of a defect or a fault is detected, while with the other the underlying faults are isolated from one another. Such systems ensure reliable operation of engineering assemblages for signal measurement, system monitoring and control, for instance. This issue is of special importance in systems of high safety (Patton *et al.*, 1989; Chen *et al.*, 1996; Gertler and Kowalczuk, 1997). The presence of errors in system components may be disagreeable, or even dangerous. Sometimes, after some length of time, even small system errors can have a profound effect on the system performance. Therefore, the detection and isolation of faults should usually be done as early as possible, so as to allow a human operator to take appropriate measures.

The concept of FDI, based on mathematical models of the monitored system, lies in a current comparison of measurements of the plant with certain signals predicted based on the system's model. Differences between the corresponding signals, called residuals, allow for identification of existing failures, faults or defects of the system. It is assumed that those differences are, in general, influenced by disturbances, noises and modelling errors. Fault detection is achieved by appropriate filtration of these residuals and principal diagnostic decisions are also taken on the basis of their evolution.

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In this paper, the design of a residual generator is discussed that is founded on a common scheme of state observation, which should be not only sensitive to sensor errors, but also robust to modelling uncertainty and measurement noises. Such an effect can be obtained by optimisation of a performance index, suitably describing the influence of faults, noises and modelling uncertainty. Thus the primary objectives of our considerations are design procedures for robust observers applicable in the FDI systems.

A consequent synthesis of such an observer should therefore be made by multi-objective optimisation of a vector performance index, taking into account all the above-mentioned factors. A practical solution to such a multi-objective optimisation problem can be found with the use of inequality constraints (Zakian and Al-Naib, 1973), and implemented by means of genetic algorithms (Chen *et al.*, 1996).

Another aspect of this problem yields the notion of Pareto optimality which allows for an efficacious judgement of a proposed set of solutions which are evaluated in terms of different quality measures. In particular, we suggest solving the multi-objective FDI optimisation problem by a method that incorporates both the concepts of Pareto optimality and evolutionary (genetic) search (Goldberg, 1989; Michalewicz, 1996; Viennet *et al.*, 1996). The idea of genetic searching is based on mechanisms emulating the evolution of nature. The respective genetic algorithms (GAs) constitute a major tool of optimisation in the class of random methods. They also provide a universal method of searching in multi-dimensional spaces.

2. Residual-Generator Design

Consider the following mathematical description of the monitored system:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{R}_1\boldsymbol{f}(t) + \boldsymbol{d}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) + \boldsymbol{R}_2\boldsymbol{f}(t) + \boldsymbol{n}(t) \tag{2}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is a state vector, $\boldsymbol{u}(t) \in \mathbb{R}^p$ stands for a control vector, $\boldsymbol{y}(t) \in \mathbb{R}^m$ is a measurement vector, $\boldsymbol{f}(t) \in \mathbb{R}^q$ denotes a fault vector, $\boldsymbol{d}(t) \in \mathbb{R}^n$ signifies a state disturbance vector, and $\boldsymbol{n}(t) \in \mathbb{R}^m$ represents output measurement disturbances.

The matrices appearing in the model (1)–(2) have the following dimensions: $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{n\times p}$, $\boldsymbol{C} \in \mathbb{R}^{m\times n}$, $\boldsymbol{D} \in \mathbb{R}^{m\times p}$, $\boldsymbol{R}_1 \in \mathbb{R}^{n\times q}$, $\boldsymbol{R}_2 \in \mathbb{R}^{m\times q}$. It is presumed that the pair $(\boldsymbol{A}, \boldsymbol{C})$ is completely observable. Signals $\boldsymbol{d}(t)$ and $\boldsymbol{n}(t)$ can also map a structural or non-structural system modelling uncertainty if

$$\boldsymbol{d}(t) = \Delta\boldsymbol{A}\boldsymbol{x}(t) + \Delta\boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{w}(t) \tag{3}$$

$$\boldsymbol{n}(t) = \Delta\boldsymbol{C}\boldsymbol{x}(t) + \Delta\boldsymbol{D}\boldsymbol{u}(t) + \boldsymbol{\nu}(t) \tag{4}$$

where the quadruple $(\Delta\boldsymbol{A},\ \Delta\boldsymbol{B},\ \Delta\boldsymbol{C},\ \Delta\boldsymbol{D})$ describes deviations from the nominal model $(\boldsymbol{A},\ \boldsymbol{B},\ \boldsymbol{C},\ \boldsymbol{D})$, while the signals $\boldsymbol{w}(t) \in \mathbb{R}^n$ and $\boldsymbol{\nu}(t) \in \mathbb{R}^m$ represent noisy characteristics.

It is thus postulated that the fault $\boldsymbol{f}(t)$ is an unknown (vector) time function, and that the influence of such faults on the state evolution of the system under consideration and on the measurement signals is conditioned by the choice of matrices $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, respectively. For example, in a simple scheme with actuator faults (in the control channel) it can be assumed that $\boldsymbol{R}_1 = \boldsymbol{B}$ and $\boldsymbol{R}_2 = \boldsymbol{D}$, while in the case of sensor faults (in the observation channel) we have $\boldsymbol{R}_1 = \boldsymbol{0}$ and $\boldsymbol{R}_2 = \boldsymbol{I}_m$.

The state observer has the well-known form (Brogan, 1991; Chen *et al.*, 1996; Kowalczuk and Suchomski, 1998)

$$\dot{\hat{\boldsymbol{x}}}(t) = (\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C})\hat{\boldsymbol{x}} + (\boldsymbol{B} - \boldsymbol{K}\boldsymbol{D})\boldsymbol{u}(t) + \boldsymbol{K}\boldsymbol{y}(t) \tag{5}$$

$$\hat{\boldsymbol{y}}(t) = \boldsymbol{C}\hat{\boldsymbol{x}}(t) + \boldsymbol{D}\boldsymbol{u}(t) \tag{6}$$

where $\dot{\hat{\boldsymbol{x}}}(t) \in \mathbb{R}^n$ is a state-vector estimate, $\hat{\boldsymbol{y}}(t) \in \mathbb{R}^m$ is an output-system estimate, and $\boldsymbol{K} \in \mathbb{R}^{n\times m}$ is an observer gain matrix.

Consequently, the residual signal $\boldsymbol{r}(t) \in \mathbb{R}^r$ is obtained from the following fault detector given in matrix form:

$$\boldsymbol{r}(t) = \boldsymbol{Q}\big(\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)\big) \tag{7}$$

where the weighting matrix $\boldsymbol{Q} \in \mathbb{R}^{r\times m}$ serves as a 'free' design parameter.

Evolution of the state estimation error

$$\boldsymbol{e}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t), \quad \boldsymbol{e}(t) \in \mathbb{R}^n \tag{8}$$

can be described by the following 'internal form' equation, conditioned by faults and disturbances:

$$\dot{\boldsymbol{e}}(t) = (\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C})\,\boldsymbol{e}(t) + (\boldsymbol{R}_1 - \boldsymbol{K}\boldsymbol{R}_2)\,\boldsymbol{f}(t) + \boldsymbol{d}(t) + \boldsymbol{K}\boldsymbol{n}(t) \tag{9}$$

Thus, for an asymptotically stable homogeneous error equation all the eigenvalues of $\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C}$ must have negative real parts. It can easily be shown (ibid.) that the residual vector $\boldsymbol{r}(t)$ of (7) can be expressed by the state estimation error $\boldsymbol{e}(t)$ and perturbation signals $\boldsymbol{f}(t)$ and $\boldsymbol{n}(t)$:

$$\boldsymbol{r}(t) = \boldsymbol{Q}\boldsymbol{C}\boldsymbol{e}(t) + \boldsymbol{Q}\boldsymbol{R}_2\boldsymbol{f}(t) + \boldsymbol{Q}\boldsymbol{n}(t) \tag{10}$$

The solution of (9) can be given in the Laplace domain, as shown below:

$$\begin{aligned}\boldsymbol{E}(s) = {}& [s\boldsymbol{I}_n - (\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C})]^{-1}(\boldsymbol{R}_1 - \boldsymbol{K}\boldsymbol{R}_2)\,\boldsymbol{F}(s) + [s\boldsymbol{I}_n - (\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C})]^{-1}\boldsymbol{e}(0)\\ &+ [s\boldsymbol{I}_n - (\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C})]^{-1}\boldsymbol{D}(s) - [s\boldsymbol{I}_n - (\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C})]^{-1}\boldsymbol{N}(s)\end{aligned} \tag{11}$$

where $\boldsymbol{F}(s)$, $\boldsymbol{D}(s)$ and $\boldsymbol{N}(s)$ are the Laplace transforms of the corresponding signals, while $\boldsymbol{e}(0)$ is an initial value of the state estimation error.

Consequently, the residual has the following Laplace form:

$$\boldsymbol{R}(s) = \boldsymbol{G}_{rf}(s)\boldsymbol{F}(s) + \boldsymbol{G}_{re}(s)\boldsymbol{e}(0) + \boldsymbol{G}_{rd}(s)\boldsymbol{D}(s) + \boldsymbol{G}_{rn}(s)\boldsymbol{N}(s) \tag{12}$$

where the matrix transfer functions are as follows:

$$G_{rf}(s) = Q\left\{C\left[sI_n - A_0\right]^{-1}(R_1 - KR_2) + R_2\right\} \tag{13}$$

$$G_{re}(s) = QC\left[sI_n - A_0\right]^{-1} \tag{14}$$

$$G_{rd}(s) = QC\left[sI_n - A_0\right]^{-1} \tag{15}$$

$$G_{rn}(s) = Q\left\{I_m - C\left[sI_n - A_0\right]^{-1}K\right\} \tag{16}$$

$A_0 = A - KC$ being the observation system matrix. The above transfer matrices describe the influence of all the critical factors: faults, initial conditions of the state estimation process, external disturbances and/or modelling uncertainties. The steady-state value of the residual vector is

$$r(\infty) = G_{rf}(0)f(\infty) + G_{rd}(0)d(\infty) + G_{rn}(0)n(\infty) \tag{17}$$

where

$$G_{rf}(0) = Q\left[CA_0^{-1}(R_1 - KR_2) + R_2\right] \tag{18}$$

$$G_{rd}(0) = -QCA_0^{-1} \tag{19}$$

$$G_{rn}(0) = Q\left[I_m + CA_0^{-1}K\right] \tag{20}$$

It is clear that under the assumption $\mathrm{spectr}[A_0] \subset \mathbb{C}_-$ the inverse matrix A_0^{-1} exists. As the matrices K and Q constitute the underlying parameterisation of the designed detector, it is necessary to choose the values of the entries of those matrices such that they will emphasise the influence of $F(s)$ on $R(s)$ and, at the same time, restrict the impact of the remaining factors on $R(s)$. In order to define such tasks of parametric optimisation, let us consider the following weighted partial-objective functions in the whole frequency domain (different from Chen *et al.*, 1996)

$$J_1(K,Q) = \left\|W_1(s)G_{rf}(s)\right\|_\infty \tag{21}$$

$$J_2(K,Q) = \left\|W_2(s)G_{rd}(s)\right\|_\infty \tag{22}$$

$$J_3(K,Q) = \left\|W_3(s)G_{rn}(s)\right\|_\infty \tag{23}$$

$$J_4(K,Q) = \left\|A_0^{-1}\right\|_\sigma \tag{24}$$

$$J_5(K,Q) = \left\|A_0^{-1}K\right\|_\sigma \tag{25}$$

with the following matrix norms:

$$\left\|M(s)\right\|_\infty = \sup_\omega \bar{\sigma}\left[M(j\omega)\right] \tag{26}$$

$$\left\|M\right\|_\sigma = \bar{\sigma}[M] \tag{27}$$

where $\bar{\sigma}[M]$ is the maximum singular value of a matrix M.

The weighting matrix functions $\boldsymbol{W}_1(s)$, $\boldsymbol{W}_2(s)$ and $\boldsymbol{W}_3(s)$, which represent the prior knowledge about the spectral properties of the process, introduce additional degrees of freedom of the detector design procedure. Those matrices allow for a spectral separation of the effects of faults and noises. In order to maximise the influence of faults at low frequencies and minimise the noise effect at high frequencies, the matrix function $\boldsymbol{W}_1(s)$ should have a low-pass property, and the spectral effect of $\boldsymbol{W}_3(s)$ should be opposite to that of $\boldsymbol{W}_1(s)$.

The profit index $J_1(\boldsymbol{K},\boldsymbol{Q})$ constitutes the main maximised criterion (with some hope for a similar beneficial effect on lower bounds on the minimum singular values), while $J_2(\boldsymbol{K},\boldsymbol{Q})$ and $J_3(\boldsymbol{K},\boldsymbol{Q})$ account for state and output disturbance effects, respectively. From (19) and (20), the cost functions $J_4(\boldsymbol{K},\boldsymbol{Q})$ and $J_5(\boldsymbol{K},\boldsymbol{Q})$, describing the influence of static deviations from the nominal model of the plant, represent important robustness measures.

Once we have fixed weighting matrices $\boldsymbol{W}_1(s)$, $\boldsymbol{W}_2(s)$ and $\boldsymbol{W}_3(s)$, the synthesis of the detection filter boils down to the issue of multi-objective optimisation of the pair $(\boldsymbol{K},\boldsymbol{Q}) \in \mathbb{R}^{n\times m} \times \mathbb{R}^{r\times m}$ with regard to the goal expressed by

$$\operatorname*{opt}_{(\boldsymbol{K},\boldsymbol{Q})} \boldsymbol{J}(\boldsymbol{K},\boldsymbol{Q}) = \begin{bmatrix} \max\limits_{(\boldsymbol{K},\boldsymbol{Q})} J_1(\boldsymbol{K},\boldsymbol{Q}) \\ \min\limits_{(\boldsymbol{K},\boldsymbol{Q})} J_2(\boldsymbol{K},\boldsymbol{Q}) \\ \min\limits_{(\boldsymbol{K},\boldsymbol{Q})} J_3(\boldsymbol{K},\boldsymbol{Q}) \\ \min\limits_{(\boldsymbol{K})} J_4(\boldsymbol{K}) \\ \min\limits_{(\boldsymbol{K})} J_5(\boldsymbol{K}) \end{bmatrix} \tag{28}$$

The selection of the observer gain $\boldsymbol{K}$ can be done in several ways. For example, the method of eigenstructure assignment of the observation system matrix $\boldsymbol{A}_0$, or the method based on the Kalman-Bucy filtering can be applied (in the latter case, the knowledge of covariance characteristics of noise perturbations in the considered model is necessary). In this paper, we follow the first approach (Chen *et al.*, 1996), in which the whole spectrum (eigenvalues λ_i) of the observation system $\boldsymbol{A}_0$ is placed in a required region of the complex plane. It is also necessary that this system be robust to deviations $(\Delta\boldsymbol{A}, \Delta\boldsymbol{C})$ from the nominal plant model. Therefore, the spectral synthesis of the matrix $\boldsymbol{A}_0$ should incorporate the task of robust stabilisation of the observer.

3. Genetic Multi-Objective Optimisation in the Pareto Sense

In many practical decision processes it is essential to totally optimise several objective functions (Goldberg, 1989; Michalewicz, 1996; Viennet *et al.*, 1996). For integrating those objectives into one, it is necessary to determine the relations among the partial objectives considered. With multi-objective optimisation in mind, the notion of optimality is not obvious. If one is not going to weigh different objectives arbitrarily, it is prerequisite to define a suitable measure of optimality. This is the notion of the optimality in the sense of Pareto (Goldberg, 1989; Michalewicz, 1996; Viennet *et al.*, 1996) that becomes suitable in such cases. With this, a useful classification of the (*dominated* and *non-dominated*) solutions resulting from (multi-objective) optimisation is possible. The condition of Pareto optimality for a maximisation* task can be formulated as follows.

Consider two vectors $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^k$. Vector $\boldsymbol{a}$ is partially smaller that vector $\boldsymbol{b}$ if and only if (Goldberg, 1989; Michalewicz, 1996) for all their co-ordinates $i = 1, 2, \ldots, k$ we have

$$\forall i \ (a_i \leq b_i) \wedge \exists i \ (a_i < b_i) \tag{29}$$

Thus in the Pareto sense, a solution $\boldsymbol{a}$ is dominated if there exists a solution $\boldsymbol{b}$ partially 'better' than $\boldsymbol{a}$ in terms of the definition (29). If a solution is not dominated, then it is called a Pareto-optimal one (Goldberg, 1989; Michalewicz, 1996).

Such a concept of optimality does not give any directions as for the choice of a single solution from amongst several Pareto-optimal solutions found. Therefore, in such cases, it is the designer who has a chance to make an independent judgement of the whole range of offers.

The Pareto-optimal solutions of the described optimisation task are obtained with the aid of genetic algorithms (Michalewicz, 1996), with which a global search in the space of optimised parameters is possible that is immune to a possible discontinuity or multimodality of the partial objective functions. Genetic algorithms are principally characterised by: processing encoded forms of parameters values, simultaneous search of a larger number of regions, and stochastic rules of a genetic expansion.

A set of optimised parameters is called an *individual*. Genetic algorithms (GAs) usually operate on tenths of individuals. A group of individuals makes a *population*. In GAs the population of individuals is subject to a simulated evolution. This means that in each cycle of the algorithm, certain 'good' solutions reproduce, while 'bad' ones die out. The population obtained after one cycle of a GA is called a (*new*) *generation*. Evaluation of generations is carried out on the basis of an objective function, according to which the value of a *fitness degree* of each individual is evaluated.

The genetic mechanism of GA works as follows. For each generation, a set of most suitable solutions constitutes a group of individuals of special meaning that is called a *parential pool*. Through *genetic operations* the parental pool generates a set of new individuals called *offspring*. There are two basic types of genetic operations: *crossover* and *mutation*. Simulated evolution cycles are repeated until a desired termination condition is fulfilled (for instance, when an assumed number of cycles is arrived at).

In the process of selection of individuals a *method of ranking* is applied that assigns a scalar rank to the vector fitness of each individual (Man *et al.*, 1997). This rank directly relates to the number of individuals in the current population by which

* For simplicity, we define the maximisation task that is directly implemented in genetic algorithms.

the individual considered is dominated in the sense of Pareto. Namely, the rank of an individual ν_i in the population is set according to the following formula:

$$\rho(\nu_i) = \mu_{\max} - \mu(\nu_i) \tag{30}$$

where $\mu(\nu_i)$ is the number of individuals by which ν_i is dominated in the same population, while $\mu_{\max}$ is the maximum value among all $\mu(\nu_i)$, $i = 1, 2, \ldots, N$.

Figure 1 illustrates an example of a two-objective ranking in the Pareto sense for a vector fitness degree $\Phi = [\varphi_1, \varphi_2]$ which is calculated from an objective function $J = [J_1, J_2]$. The individuals $\{\nu_1, \nu_2, \nu_3, \nu_4\}$ constitute the Pareto-optimal set, or the primary Pareto front, while ν_5 and ν_6 constitute the secondary Pareto front.

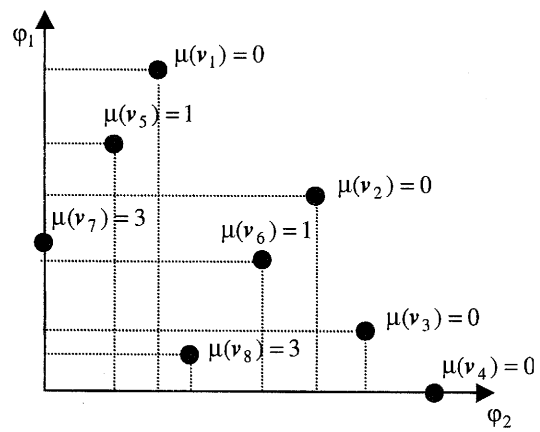

Fig. 1. Exemplary domination in the Pareto sense for a two-objectives function.

The selection of individuals is carried out with the use of a proportional method founded on a deterministic procedure and supplemented by a 'stochastic-remainder' choice. The latter is also based on a proportional method which imitates a roulette wheel whose angle sectors are proportional to the individual ranks, taken as 'uniform' fitness degrees (see Appendix A). Thus both the deterministic and stochastic mechanisms relate to the ranks of individuals. It is also clear from (30) that in this selection procedure the lowest Pareto-front is completely ruled out, as it is e.g. in the case of the individuals ν_7 and ν_8 characterised by the zero rank (and $\mu_{\max}$) in the exemplary characterisation of the population in Fig. 1.

4. Genetic Niching Mechanism

In order to sustain a diversity of individuals in the population 'processed' by a GA, an additional 'niching' mechanism (Goldberg, 1989; Michalewicz, 1996) can be utilised. This method gives rise to creation of *niches* and *species* in the population. The niche is a finite 'ball' region in the parameter space, in which at least one individual is situated. As close individuals can also have similar characteristics with respect to the degree of fitness, they can be recognised as species (of a distinct sort), as well.

Let us consider the following degree of closeness (kinship) between two individuals $\boldsymbol{\nu}_i,\ \boldsymbol{\nu}_j \in \mathbb{R}^k,\ i,j = 1,2,\dots,N$, which is expressed by a *closeness function* within a niche, also called a 'sharing' function in (Goldberg, 1989):

$$\delta_{ij} = \begin{cases} 1 - \|\boldsymbol{\nu}_i - \boldsymbol{\nu}_j\|_P & \text{if} \quad 0 \le \|\boldsymbol{\nu}_i - \boldsymbol{\nu}_j\|_P < 1 \\ 0 & \text{if} \quad \|\boldsymbol{\nu}_i - \boldsymbol{\nu}_j\|_P \ge 1 \end{cases} \tag{31}$$

where

$$\|\boldsymbol{\nu}\|_P = \sqrt{\boldsymbol{\nu}^T \boldsymbol{P}^{-1} \boldsymbol{\nu}}, \quad \boldsymbol{P} = \operatorname{diag}\left\{\Delta_1^2/36, \dots, \Delta_k^2/36\right\} \tag{32}$$

while $\Delta_l,\ l = 1,2,\dots,k$ is the range of the l-th searched parameter whose third part constitutes the l-th diameter of the niche. In such a way the niche is a hyperellipsoid centred on the i-th chosen individual.

The niching method itself consists in adjusting the magnitude of the fitness-degree vector of each individual in its own niche according to the following niche-related prescription:

$$\widetilde{\Phi}(\boldsymbol{\nu}_i) = \frac{\Phi(\boldsymbol{\nu}_i)}{\sum_{j=1}^{N} \delta_{ij}} \tag{33}$$

where $\Phi(\boldsymbol{\nu}_i)$ is the vector fitness of the i-th individual and $\widetilde{\Phi}(\boldsymbol{\nu}_i)$ is its 'niche-adjusted' vector of fitness. The sum in the denominator concerns the whole set of individuals in the dynamically-determined niche centred on the i-th individual. If the individual is the only member of its own niche, then its fitness degree is not decreased, as $\sum \delta_{ij} = 1$. In other cases, the fitness (degree) is decreased according to the number of neighbours in the niche.

For illustrative purposes, consider two exemplary processes (A and B) of niching. In Case A, the placement of 12 individuals in a two-dimensional space is shown in Fig. 2(a) with their corresponding vector of fitness given in Fig. 2(b). Note that a single Pareto-optimal solution is marked with a 'star.' Figure 2(c) presents the niche-adjusted fitness vectors of individuals prepared for the ranking selection. As a result of the niching mechanism, the fitness degree of ten individuals (the star and circles) have been decreased (see the arrows) and the fitness of two individuals (the cross and one dot) remain the same. Figure 3(a) depicts Case B of the population's state, where the numbers denote the multiplicities of individuals in the population. With the population's characterisation given in Figs. 3(a) and 3(b), the effect of magnitude adjustment on the fitness degree vector is illustrated by Fig. 3(c). In this case, the fitness degree of all the individuals in the population is decreased. As the niching operation allows for both increasing probability of survival (i.e. an appearance in the next generation) for species of 'sparse' niches and decreasing it for 'dense' niches, the mechanism has a nature of 'uniformly breeding.' Nevertheless, it is important that in spite of the uniformly breeding policy, as a global effect of genetic expansion and selection procedures, one observes that there are constant densities sustained in certain niches. This can eventually be interpreted in terms of their robustness to changes in the fitness measure.

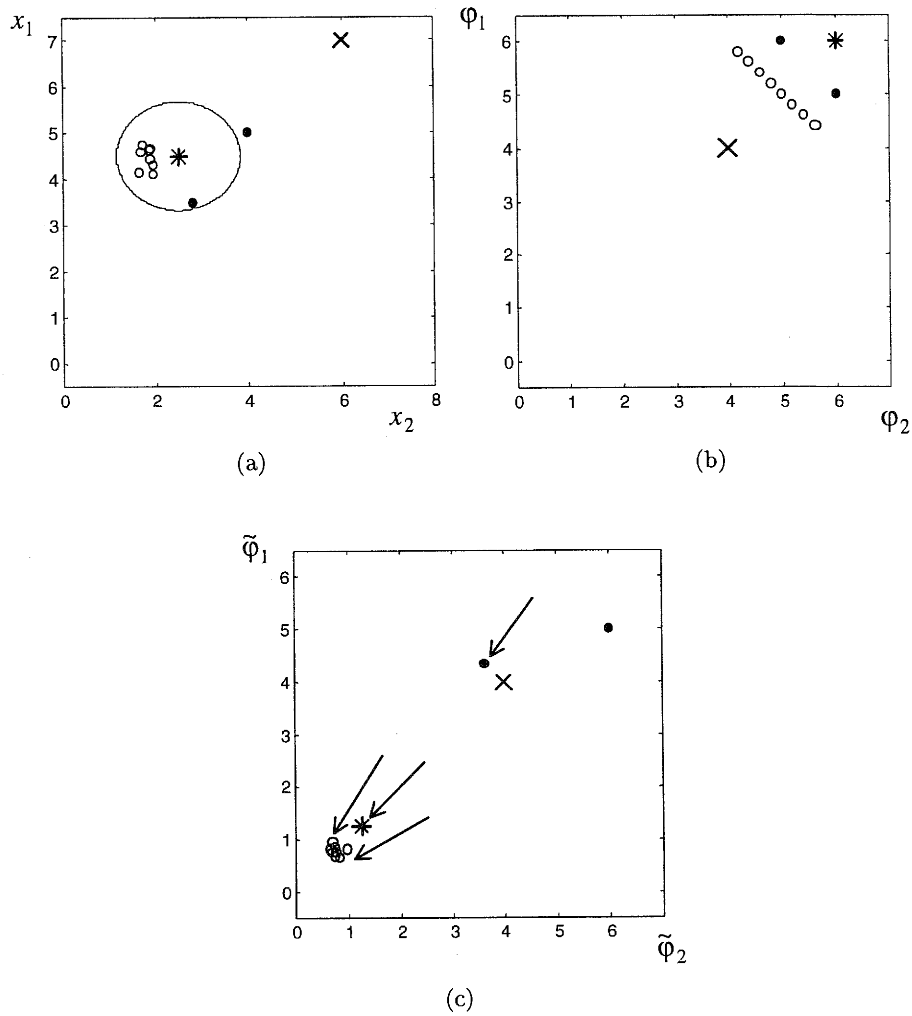

Fig. 2. Niching mechanism, Case A: the population and ellipse niche of the optimal solution (a), the true fitness amongst the population (b) and the niche-adjusted fitness (c).

5. Evolutionary Multi-Objective Otimisation

Chen *et al.* (1996) proposed the method of sequential inequalities (Zakian and Al-Naib, 1973) in the multi-objective optimisation procedure performed by evolutionary (genetic) algorithms. In their approach, the cost indices are expressed in the frequency domain and all the performance objectives are used in a set of inequality constraints tested in a limited frequency range. Finally, GAs are used in searching for optimal solutions satisfying all the inequality constraints. The eigenstructure assignment approach (Liu and Patton, 1996; Chen *et al.*, 1996) is employed in order to get an appropriate parameterisation of the gain matrix $\boldsymbol{K}$. What is important, in our approach the multi-objective optimisation problem is solved by a method that incorporates both the concepts of Pareto optimality and genetic search in the whole frequency domain.

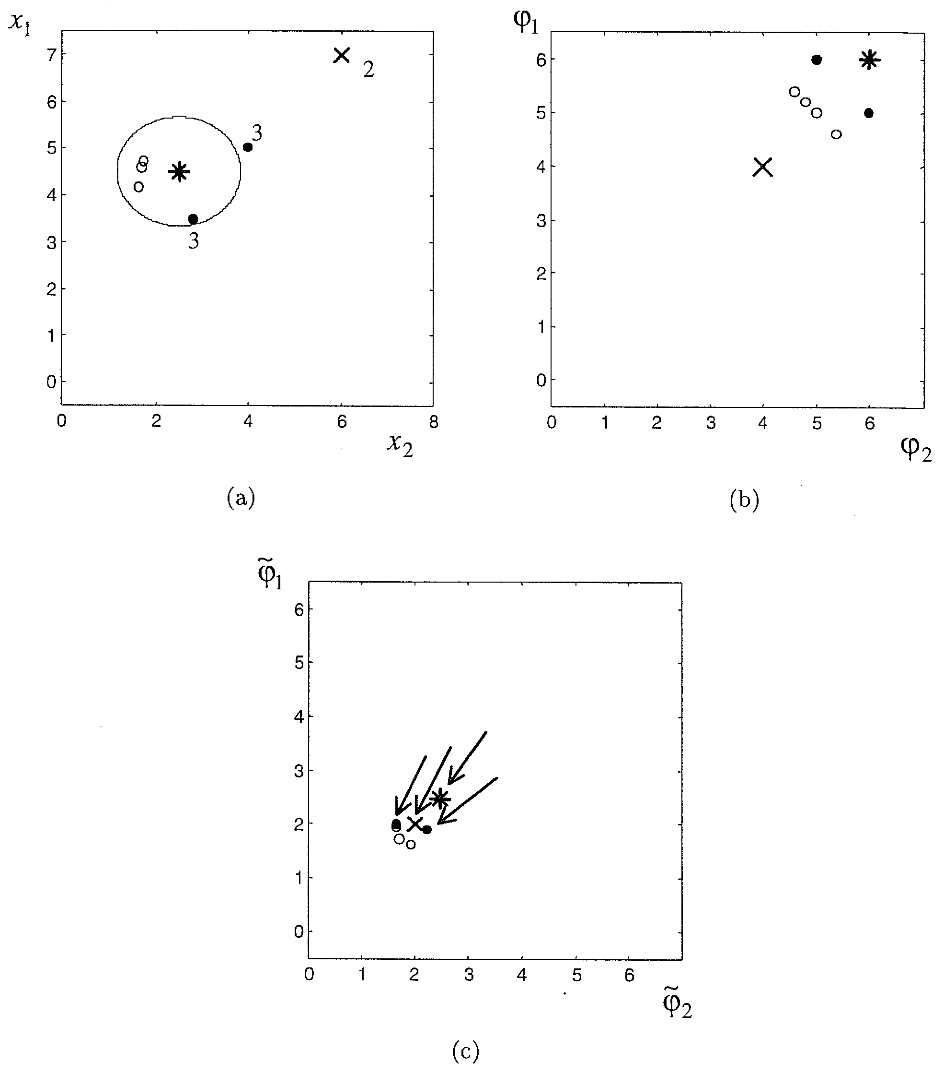

Fig. 3. Niching mechanism, Case B: the population and ellipse niche of the optimal solution (a), the true fitness amongst the population (b) and the niche-adjusted fitness (c).

A Pareto-optimal observer is obtained via optimisation of the vector index $\boldsymbol{J}(\boldsymbol{K}, \boldsymbol{Q})$ of (28) whose coordinates are the (weighted) partial objectives: profit function (21) and cost functions (22)–(25). These objectives are composed in a way that allows for an achievement of a robust observer. In order to simplify the optimisation procedure, the method of assignment of the spectrum $\{\lambda_i\}$ of the observation system matrix $\boldsymbol{A}_0$ is applied with the aim of yielding a suitable parameterisation of the gain matrix $\boldsymbol{K} = \boldsymbol{K}(\lambda)$, where $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]^T$.

In the evolutionary algorithm discussed here, each individual is represented by a sequence of real numbers. Such a description allows for searching in the continuous domain of real numbers. Thus each individual is simply a vector whose coordinates are the optimised parameters.

The procedural steps of the evolutionary algorithm, tailored to the considered design case, are as follows:

Optimisation Procedure:

1. Randomly generate an initial population of N individuals, i.e. $\boldsymbol{V} = [\boldsymbol{\nu}_1, \boldsymbol{\nu}_2, \ldots, \boldsymbol{\nu}_N]$ where $\boldsymbol{\nu}_j = [\nu_1^i, \nu_2^j, \ldots, \nu_n^j]^T \in \mathbb{R}^n$ is the j-th individual, $j = 1, 2, \ldots, N$, and ν_k^j is its k-th coordinate, $k = 1, 2, \ldots, n$.

2. For each individual $\boldsymbol{\nu}_j$ compute its fitness vector $\Phi(\boldsymbol{\nu}_j)$ whose coordinates are defined by

$$\varphi_i(\boldsymbol{\nu}_j) = \begin{cases} J_i(\boldsymbol{\nu}_j), & i = 1 \\ C_{\max_i} - J_i(\boldsymbol{\nu}_j), & i = 2, 3, 4, 5 \end{cases} \tag{34}$$

   where $J_i(\boldsymbol{\nu}_j)$ is the i-th cost function (21)–(25) and the coefficient $C_{\max_i}$ can be equal to a maximum value of the i-th cost criterion function in the current population.

3. Apply the niching mechanism in order to obtain the niche-adjusted fitness of all the individuals.

4. Estimate the individuals' ranks in the sense of Pareto.

5. Perform the selection based on the population ranking in order to create the parental pool.

6. Create a new generation of individuals $\boldsymbol{V}'$ through the following operations:

   (a) Execute the arithmetical crossover (a linear combination of two vectors) to produce offspring. Given a pair of parents $\boldsymbol{\nu}_r$ and $\boldsymbol{\nu}_s'$, their offspring $\boldsymbol{\nu}_r'$ and $\boldsymbol{\nu}_s'$ are set by

$$\boldsymbol{\nu}_r' = a\boldsymbol{\nu}_r + (1-a)\boldsymbol{\nu}_s, \quad \boldsymbol{\nu}_s' = a\boldsymbol{\nu}_s + (1-a)\boldsymbol{\nu}_r$$

   where $a \in [0, 1]$ is a real value randomly chosen for each mating pair.

(b) Execute mutation which introduces an appropriate random 'perturbation' of the newly-formed solution. A coordinate of the vector $\boldsymbol{\nu}_j$, chosen for a change with a mutation probability, is replaced with a random value α from the coordinate domain:

$$\boldsymbol{\nu}'_j = \begin{bmatrix} \nu_1^j & \nu_2^j & \dots & \nu_k^j & \dots & \nu_n^j \end{bmatrix}^T \Big|_{\nu_k^j=\alpha}$$

where $\alpha \in [\underline{\nu}_k^j, \bar{\nu}_k^j]$, while $\underline{\nu}_k^j$ and $\bar{\nu}_k^j$ are lower and upper bounds of the coordinate ν_k^j, respectively. The value α is generated with a uniform distribution of probability.

7. Replace population $\boldsymbol{V}$ with population $\boldsymbol{V}'$.

8. When the assumed number of generations is obtained, Pareto-optimal solutions are ultimately selected from the most recent population.

6. Illustrative Example

Consider the unstable state-space plant model (1)–(2) characterised by the following matrices (Mudge and Patton, 1989):

$$\boldsymbol{A} = \begin{bmatrix} -0.277 & 0 & -32.9 & 9.81 & 0 \\ -0.1033 & -8.525 & 3.75 & 0 & 0 \\ 0.3649 & 0 & -0.639 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} -5.432 & 0 \\ 0 & -28.64 \\ -9.49 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The quadruple $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$ describes a linearised lateral control system of a remotely piloted aircraft.

For simplicity of computations, the weighting matrix $\boldsymbol{Q}$ is assumed to be the 3×3 identity matrix. Thus the search goal can be expressed as

$$\underset{(\boldsymbol{K},\boldsymbol{Q})}{\text{opt}}\ \boldsymbol{J}(\boldsymbol{K}, \boldsymbol{Q}) = \underset{\boldsymbol{K}}{\text{opt}}\ \boldsymbol{J}(\boldsymbol{K}) = \underset{\lambda}{\text{opt}}\ \boldsymbol{J}\left(\boldsymbol{K}(\lambda)\right) = \underset{\boldsymbol{\nu}_j}{\text{opt}}\ \boldsymbol{J}\Big(\boldsymbol{K}\left(\lambda(\boldsymbol{\nu}_j)\right)\Big)$$

where

$$\lambda(\boldsymbol{\nu}_j) = \begin{bmatrix} \nu_1^j & \nu_2^j & \nu_3^j + \mathrm{j}\nu_4^j & \nu_3^j - \mathrm{j}\nu_4^j & \nu_5^j \end{bmatrix}^T \in \mathbb{C}^5$$

and the j-th instrumental individual has the form

$$\nu_j = \begin{bmatrix} \nu_1^j & \nu_2^j & \nu_3^j & \nu_4^j & \nu_5^j \end{bmatrix}^T \in \mathbb{R}^5$$

It is presumed that a fault may occur in the control channel ($\boldsymbol{R}_1 = \boldsymbol{B}$, $\boldsymbol{R}_2 = \boldsymbol{D}$). Furtheremore, the hypercube of the optimised parameters $\{\nu_j\}$ is specified as follows (Chen *et al.*, 1996):

$$\nu_1^j \in [-5, -0.2], \quad \nu_2^j \in [-15, -3], \quad \nu_3^j \in [-10, -2], \quad \nu_4^j \in [0.2, 4], \quad \nu_5^j \in [-30, -8]$$

The 3×3 weighting transfer function matrices of the indices (21)–(23) are of the following forms:

$$\begin{aligned} \boldsymbol{W}_1(s) &= \text{diag}\left\{\frac{1}{(0.1s+1)(0.02s+1)}\right\} \\ \boldsymbol{W}_2(s) &= \text{diag}\{1\} \\ \boldsymbol{W}_3(s) &= \text{diag}\left\{\frac{(0.1s+1)(0.02s+1)}{(0.005s+1)^2(0.001s+1)}\right\} \end{aligned}$$

The residual generator has been designed by means of the evolutionary optimisation procedure based on the index $\boldsymbol{J}(\boldsymbol{K}, \boldsymbol{Q})$ of (28) and described above. Some practical and implementation thoughts concerning the procedure itself are given in Appendix B.

Fifty five Pareto-optimal solutions (out of a population of eighty individuals) for the observer gain matrices, characterised in Fig. 4 in terms of their partial objectives,

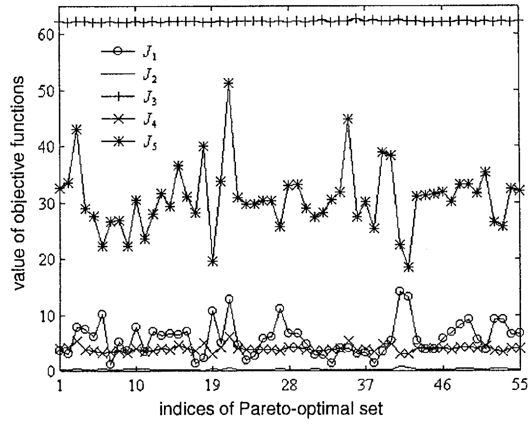

Fig. 4. The objective functions of 55 Pareto-optimal solutions.

have been obtained as a final effect of this optimisation process. The corresponding set of the instrumental Pareto-optimal parameters $\{\nu_j\}$, $j = 1, 2, \ldots, 55$, describing the spectrum of the observation system matrix, is graphically presented in Fig. 5.

Distribution of two selected coordinates (ν_1^j, ν_2^j) of all the Pareto-optimal solutions ($j = 1, 2, \ldots, 55$) in their respective two-dimensional (ν_1, ν_2) subspace is depicted in Fig. 6, where the numbers (next to some 'circles') correspond to the indices of the Pareto-optimal observers indicated in Fig. 5. The corresponding values of two chosen objective functions (profit J_1 and cost J_5) are presented in Fig. 7.

There is an illustrative dense niche shown in Fig. 6 (the distinguished ellipse centred on the 'last' (55-th) solution, with radii of 0.8 and 2, respectively). The demonstrated niche (species) can be interpreted in terms of robust (confirmed) Pareto-

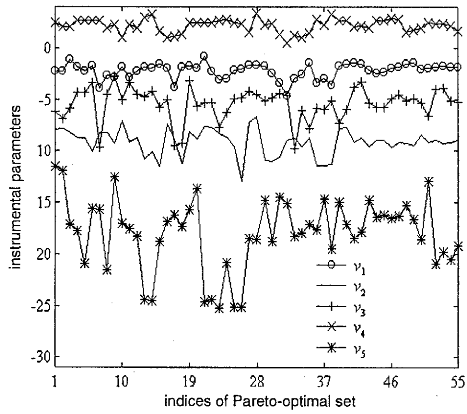

Fig. 5. The considered 55 instrumental Pareto-optimal solutions.

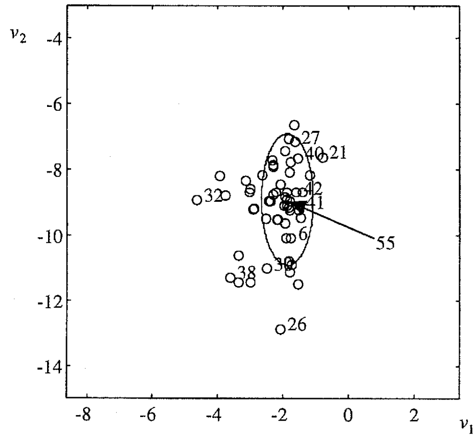

Fig. 6. The optimal solutions against the niche of solution $\boldsymbol{K}_{55}$ (in the selected two-dimensional subspace).

optimality of the final solution: The individuals bred in this niche are immune enough to survive in the cyclic genetic evolution process. In the example under consideration, a natural jeopardy appears that is connected with the solutions which are optimal in terms of certain criteria and completely unfavourable from the viewpoint of the other co-ordinates of the vector quality index (28). For example, the non-dominated solution #21 which is characterised (as shown in Fig. 7) by one of the most high profits (J_1) has, at the same time, the highest cost (J_5).

From among all the Pareto-optimal solutions, the observers with the gain matrices $\boldsymbol{K}_{38}$, $\boldsymbol{K}_{42}$ and $\boldsymbol{K}_{55}$ given in Table 1 were chosen for the further study. The unstable system under consideration was stabilised with the aid of a state feedback controller using knowledge about the estimated system state. The first coordinate of the fault vector $\boldsymbol{f}(t)$ of (1)–(2) was subjected to an additive fault in a 'sigmoid' form shown in Fig. 8.

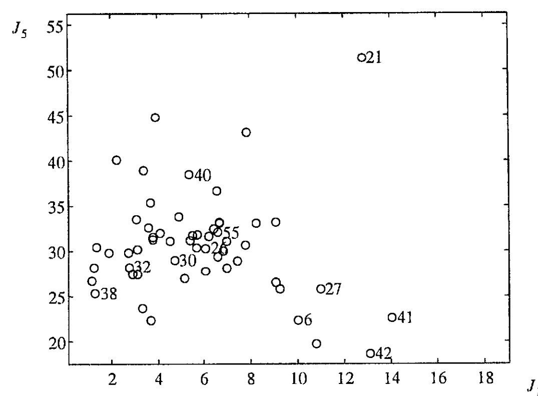

Fig. 7. The selected two-objective characterisation of the Pareto-optimal solutions.

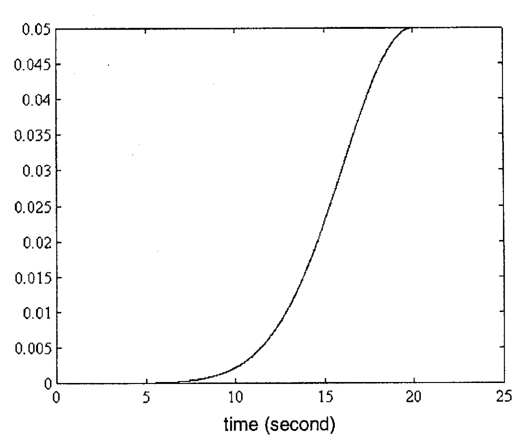

Fig. 8. The applied fault signal $\boldsymbol{f}(t)$.

Table 1. Examples of the observer-gain matrices.

Observer gain matrix	Eigenvalues	Objective functions
$\boldsymbol{K}_{38} = \begin{bmatrix} -51.669 & 9.810 & 3.728 \\ 9.972 & 0.000 & -5.008 \\ 21.916 & 0.000 & -5.788 \\ 1.000 & 19.479 & 0.000 \\ 1.685 & 0.000 & 5.826 \end{bmatrix}$	$\lambda_1 = -3.603$ $\lambda_2 = -11.308$ $\lambda_{3,4} = -5.164 \pm j3.315$ $\lambda_5 = -19.479$	$J_1 = 1.343$ $J_2 = 0.175$ $J_3 = 62.616$ $J_4 = 3.437$ $J_5 = 25.302$
$\boldsymbol{K}_{42} = \begin{bmatrix} -47.222 & 9.810 & -19.461 \\ -0.517 & 0.000 & -0.275 \\ 0.012 & 0.000 & 1.861 \\ 1.000 & 17.823 & 0.000 \\ -1.259 & 0.000 & 7.919 \end{bmatrix}$	$\lambda_1 = -1.568$ $\lambda_2 = -8.712$ $\lambda_{3,4} = -3.281 \pm j2.175$ $\lambda_5 = -17.823$	$J_1 = 13.079$ $J_2 = 0.508$ $J_3 = 62.260$ $J_4 = 3.114$ $J_5 = 18.501$
$\boldsymbol{K}_{55} = \begin{bmatrix} -75.242 & 9.810 & -17.838 \\ 2.303 & 0.000 & 2.080 \\ 3.359 & 0.000 & 5.517 \\ 1.000 & 19.208 & 0.000 \\ 0.183 & 0.000 & 9.445 \end{bmatrix}$	$\lambda_1 = -1.870$ $\lambda_2 = -8.917$ $\lambda_{3,4} = -5.201 \pm j1.606$ $\lambda_5 = -19.208$	$J_1 = 6.589$ $J_2 = 0.245$ $J_3 = 62.269$ $J_4 = 4.057$ $J_5 = 31.996$

Simulations were performed in the presence of system and measurement disturbances, $\boldsymbol{d}(t)$ and $\boldsymbol{n}(t)$, respectively, both modelled as zero-mean Gaussian white-noise processes. The system parameters of the simulated ('true') plant were (multiplicatively) perturbed by uniformly-distributed $\pm 10\%$ deviations from the parameters' nominal values.

Results of simulated performance of the residual generator are shown in Figs. 9–11 for the observers using the gain matrices $\boldsymbol{K}_{38}$, $\boldsymbol{K}_{42}$ and $\boldsymbol{K}_{55}$, respectively. They clearly exhibit a detectable slowly-growing error.

As shown in Figs. 9(c), 10(c) and 11(c), the two fault-dependent coordinates (r_1, r_3) of the residual vector $\boldsymbol{r}(t)$ demonstrate significant changes analogous to the generic fault signal applied. It can easily be seen that the system observations obtained with the gain matrix $\boldsymbol{K}_{42}$ have a strongest detection ability. In the case of the gain matrix $\boldsymbol{K}_{38}$, the second (r_2) and third (r_3) co-ordinates of the residual vector $\boldsymbol{r}(t)$ are practically non-distinguishable. Also, the sensitivity of these residuals to the simulated fault is lacking. The system observations with the gain matrix $\boldsymbol{K}_{55}$ have similar characteristics to the ones obtained by means of the matrix $\boldsymbol{K}_{42}$.

The observation system can thus be used for reliably detecting sensor faults from noisy measurements. The ultimate fault detection can be achieved, e.g. by using appropriate thresholds.

7. Conclusions

Within this work the diagnostic (FDI) considerations have been restricted to a robust-state-observer design-problem. The presented synthesis of such a detection filter is based on a multi-objective optimisation of a vector profit-and-cost index, taking into consideration faults, initial conditions of the state estimation process, and external disturbances. These objectives are used with the purpose of achieving a robust residual generator. The multi-objective optimisation task is solved with the use of the concept of Pareto optimality. The search for Pareto-optimal solutions is performed by means of evolutionary (genetic) algorithms. Such an approach allows for carrying out a global search in the space of optimised parameters that is immune to possible

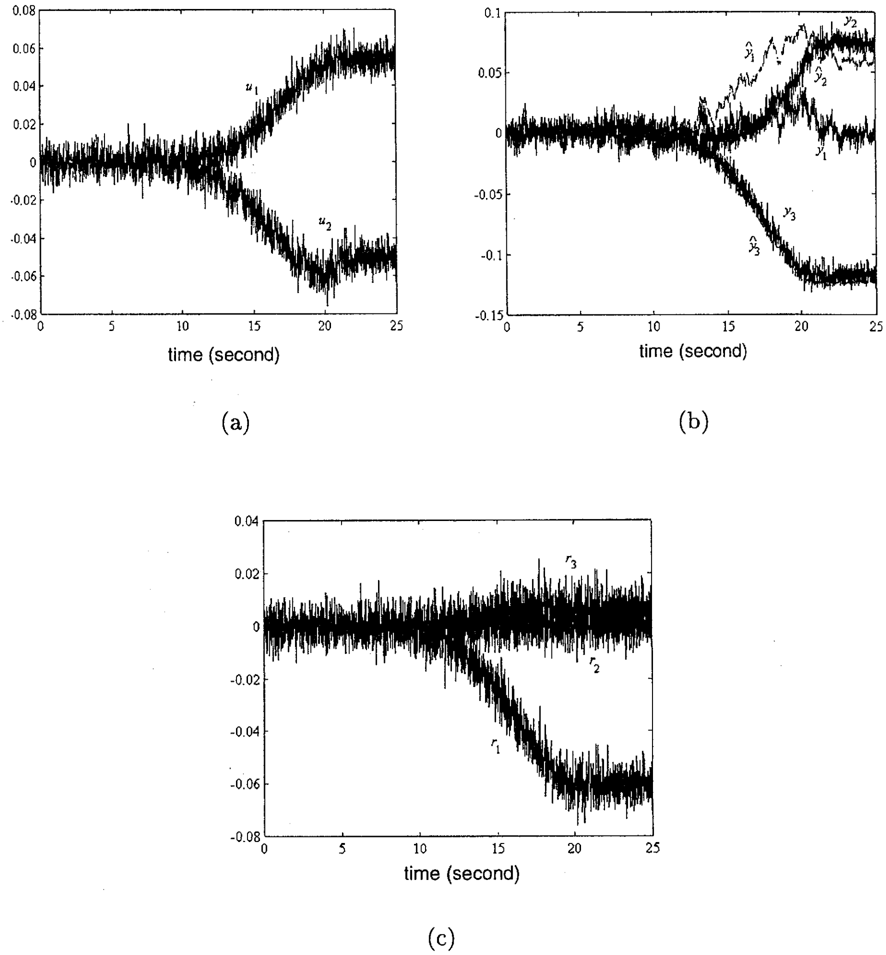

Fig. 9. Observation signals for the gain matrix K_{38}: (a) controls $u(t)$, (b) measurements $y(t)$ and their estimates, (c) residuals $r(t)$.

multimodality of partial objective functions. At the same time, the multiple solutions yielded by the Pareto-optimal approach, which are generally criticised for their ambivalence or non-uniqueness, are applied in the process of ranking the individuals of parental pools. Within the ranking selection a niching mechanism is employed, the result of which has its simple robustness interpretation, connected to sustenance of a high population-density in niches, with respect to which a general policy of 'uniformly breeding' is implemented. An exemplary synthesis of a residual generator, based on a diagnostic state observer, is presented and suitable simulation results are shown, which confirm the efficiency of the Pareto-optimal approach in the residual generator design.

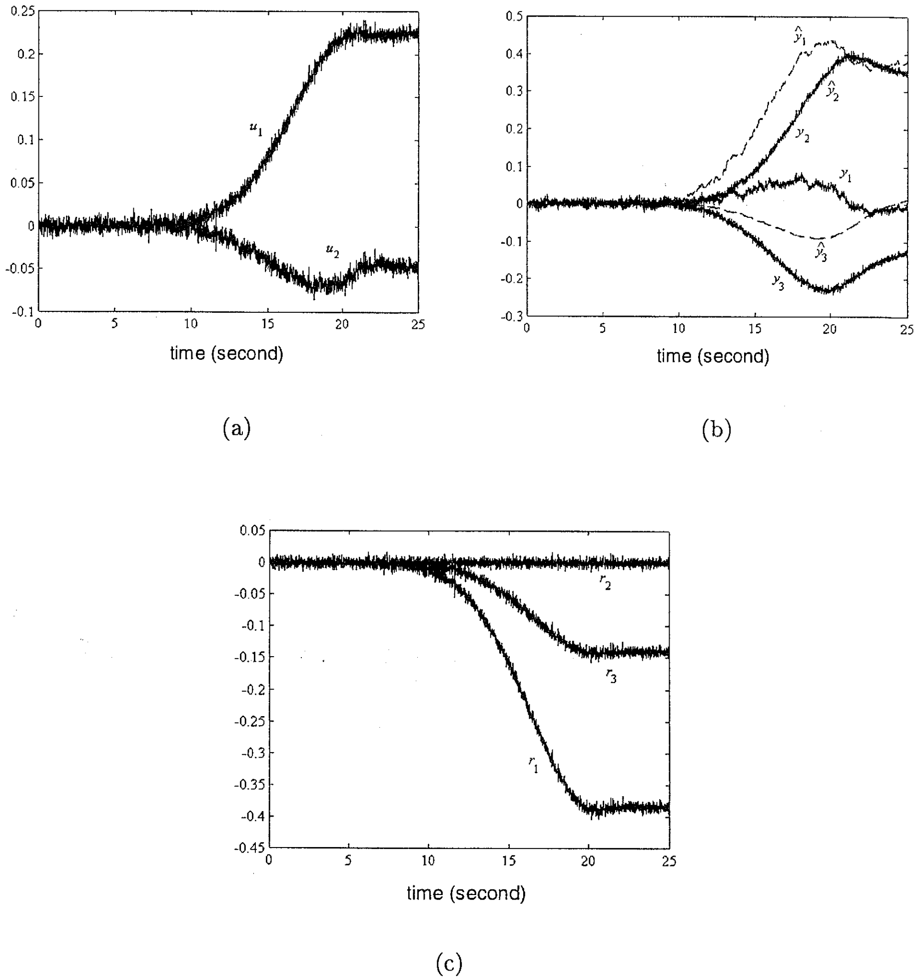

Fig. 10. The system's observations with the gain matrix K_{42}: (a) controls $u(t)$, (b) measurements $y(t)$ and their estimates, (c) residuals $r(t)$.

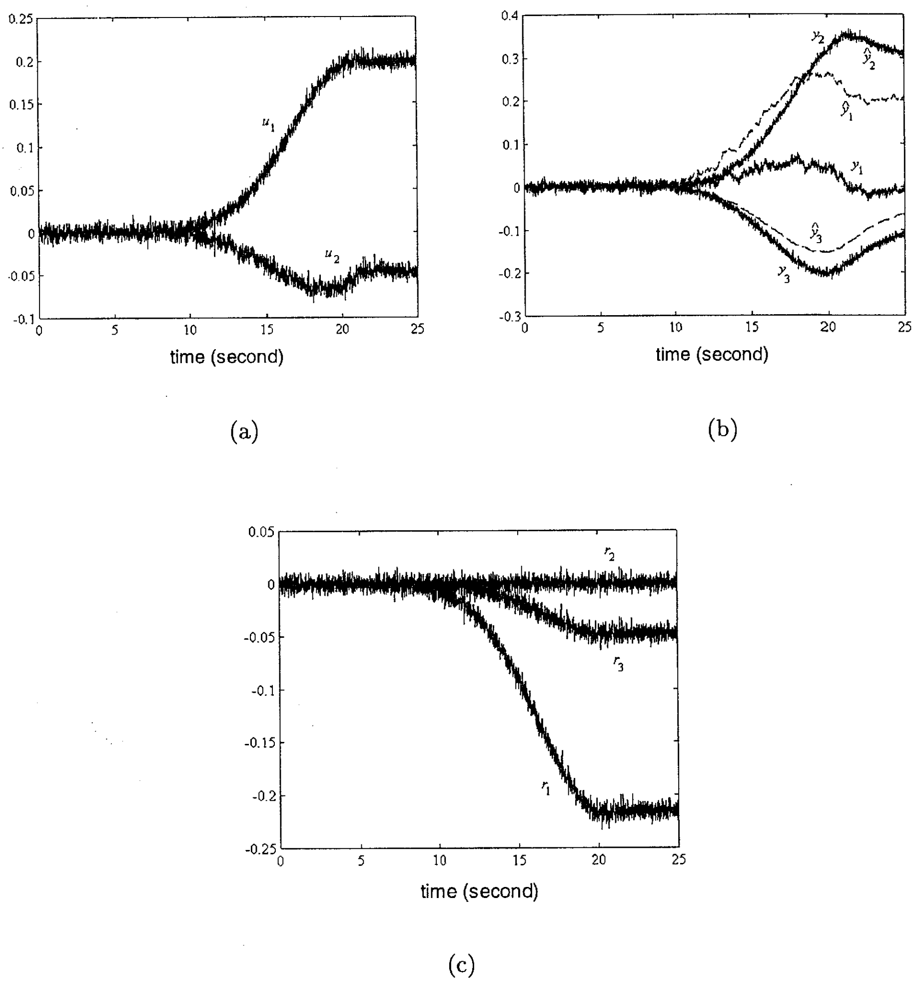

Fig. 11. The system's observations for the gain matrix $\boldsymbol{K}_{55}$: (a) controls $\boldsymbol{u}(t)$, (b) measurements $\boldsymbol{y}(t)$ and their estimates, (c) residuals $\boldsymbol{r}(t)$.

Appendices

A. Selection of the Parental Pool

A proportional deterministic (1–5) and stochastic-remainder (6–7) method for the selection of individuals based on their ranks (scalar uniform-fitness degrees) can be described as follows:

1. Assign the fitness degree to each individual in the population $\rho(\boldsymbol{\nu}_i)$, $i = 1, 2, \ldots, N$.

2. Calculate the total fitness

$$\rho_{\mathrm{sum}} = \sum_{i=1}^{N} \rho(\boldsymbol{\nu}_i), \quad i = 1, 2, \ldots, N$$

3. Assign individual probabilities of selection

$$\rho_{\mathrm{select}}(\boldsymbol{\nu}_i) = \frac{\rho(\boldsymbol{\nu}_i)}{\rho_{\mathrm{sum}}}, \quad i = 1, 2, \ldots, N$$

4. Assign the 'proportional' number of selected individuals

$$e(\boldsymbol{\nu}_i) = \rho_{\mathrm{select}}(\boldsymbol{\nu}_i)N, \quad i = 1, 2, \ldots, N$$

5. Copy $N_{\mathrm{int}} = \sum_{i=1}^{N} \lfloor e(\boldsymbol{\nu}_i) \rfloor$ individuals to a parental pool (according to the integer part of $e(\boldsymbol{\nu}_i)$).

6. Calculate the 'distribution' of a sequence of individuals according to the fractional part of $e(\boldsymbol{\nu}_i)$

$$q(\boldsymbol{\nu}_i) = \sum_{l=1}^{j} \Big\{ e(\boldsymbol{\nu}_l) - \lfloor e(\boldsymbol{\nu}_l) \rfloor \Big\}$$

7. Perform the multiple ($\tilde{N} = N - N_{\mathrm{int}}$) 'turning' of the simulated 'roulette wheel' according to the following steps:

   (a) generating a random number $r \in [0, 1]$;

   (b) selecting an individual that fulfils the condition

   $$r \leq \frac{q(\boldsymbol{\nu}_i)}{q(\boldsymbol{\nu}_N)}$$

   (c) copying the selected individual to the parental pool.

B. Some Implementation Annotations

In the first attempt, in order to reduce the computational load, it is the gain matrix $\boldsymbol{K}$ that was solely subject to optimisation in the considered case.

In the implemented optimisation procedure, it was assumed that the crossover probability was equal to 0.8 and the mutation probability was 0.09. These values were applied based on the general advice that the crossover probability should be in the range $[0.6, 1]$, while the mutation probability should be 'small enough.' As regards the genetic algorithms with binary parameter representations, Goldberg (1989) suggests that the mutation probability should be inversely proportional to the number of individuals in the population. With the floating-point representation applied, in order to compensate for a singular operation of mutation (with respect to each individual coordinate), this basic probability of change was increased by a factor of ten.

It is convenient that there are several procedures in the popular MATLAB/SIMULINK* package that can be used for the purpose of the discussed optimisation problem. The procedure PLACE, for instance, can compute an observation gain matrix $\boldsymbol{K}$ in such a way that all the eigenvalues of $\boldsymbol{A} - \boldsymbol{KC}$ comply with the specification represented by vector $\boldsymbol{\nu}_j$. The procedure INFNORM can calculate the infinite norms (21)–(23) for the state-space models of the respective transfer functions. The procedure NORM, or SVD, can be utilised to estimate the maximum singular values according to (24), (25). The procedure PLACE is available from the CONTROL TOOLBOX sub-package. Simulation of the designed continuous-time observation system can be carried out in the SIMULINK platform.

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Received: 2 February 1998
Revised: 29 July 1998

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