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ADDENDUM TO "THE WELL–POSEDNESS OF A SWIMMING MODEL IN THE 3–D INCOMPRESSIBLE FLUID GOVERNED BY THE NONSTATIONARY STOKES EQUATION"

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In this addendum we address some unintentional omission in the description of the swimming model in our recent paper (Khapalov, 2013) .

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In our recent work (Khapalov, 2013) we introduced the following mathematical model for a swimmer whose body consists of finitely many subsequently connected parts $S_i(z_i(t)$ linked by rotational and elastic Hooke forces:

$$\frac{\partial y}{\partial t} = \nu \Delta y \; + \; F(z,v) \; - \; \nabla p \quad \text{in} \quad Q_T = \Omega \times (0,T), \tag{1}$$

$$\operatorname{div} y = 0 \;\text{ in }\; Q_T, \quad y = 0 \quad \text{in } \Sigma_T = \partial\Omega \times (0,T),$$

$$y = (y_1, y_2, y_3), \quad y\,|_{t=0} = y_0 \;\text{ in } \Omega,$$

$$\frac{\mathrm{d}z_i}{\mathrm{d}t} = \frac{1}{\operatorname{mes}\{S_i(0)\}} \int_{S_i(z_i(t))} y(x,t)\,\mathrm{d}x, \quad z_i(0) = z_{i0}, \tag{2}$$

$i = 1, \dots, n, \;\; n > 2$, where, for $t \in [0,T]$,

$$z(t) = (z_1(t), \dots, z_n(t)),$$

$$z_i(t) \in \mathbb{R}^3, \quad i = 1, \dots, n,$$

$$v(t) = (v_1(t), \dots, v_{n-2}(t)) \in \mathbb{R}^{n-2},$$

$$\begin{aligned}
&F(z,v) \\
&= \sum_{i=2}^{n} [\xi_{i-1}(x,t) k_{i-1} \\
&\quad \times \frac{(\|z_i(t) - z_{i-1}(t)\|_{\mathbb{R}^3} - l_{i-1})}{\|z_i(t) - z_{i-1}(t)\|_{\mathbb{R}^3}} (z_i(t) - z_{i-1}(t)) \\
&\quad + \xi_i(x,t) k_{i-1} \frac{(\|z_i(t) - z_{i-1}(t)\|_{\mathbb{R}^3} - l_{i-1})}{\|z_i(t) - z_{i-1}(t)\|_{\mathbb{R}^3}} \\
&\quad \times (z_{i-1}(t) - z_i(t))] \\
&\quad + \sum_{i=2}^{n-1} v_{i-1}(t) \{ \xi_{i-1}(x,t) \, (A_i(z_{i-1}(t) - z_i(t))) \\
&\quad - \xi_{i+1}(x,t) \frac{\|z_{i-1}(t) - z_i(t)\|^2_{\mathbb{R}^3}}{\|z_{i+1}(t) - z_i(t)\|^2_{\mathbb{R}^3}} \, (B_i(z_{i+1}(t) - z_i(t))) \} \\
&\quad - \sum_{i=2}^{n-1} \xi_i(x,t) v_{i-1}(t) \{ (A_i(z_{i-1}(t) - z_i(t))) \\
&\quad - \frac{\|z_{i-1}(t) - z_i(t)\|^2_{\mathbb{R}^3}}{\|z_{i+1}(t) - z_i(t)\|^2_{\mathbb{R}^3}} \, (B_i(z_{i+1}(t) - z_i(t))) \}.
\end{aligned} \tag{3}$$

In the above, Ω is a bounded domain in R^3 with boundary $\partial\Omega$ of class C^2, $y(x,t)$ and $p(x,t)$ are respectively the velocity and the pressure of the fluid at point $x = (x_1, x_2, x_3) \in \Omega$ at time t, while ν is the kinematic viscosity constant.

The swimmer in (1)–(3) is modeled as a collection of n open connected bounded sets $S_i(z_i(t)), i = 1, \dots, n$, of non-zero measure, identified with the fluid within the space they occupy. The points $z_i(t)$ are their respective centers of mass. The sets $S_i(z_i(t))$ are viewed as the given sets $S_i(0)$ ("0" stands for the origin) that have been shifted to the respective positions $z_i(t)$ without changing their orientation in space. Respectively, for $i = 1, \dots, n$,

$$\xi_i(x,t) = \begin{cases} 1, & \text{if } x \in S_i(z_i(t)), \\ 0, & \text{if } x \in \Omega \backslash S_i(z_i(t)). \end{cases}$$

Our goal in this addendum is to address the following unintentional omission in the description of the force term in (3). Namely, the forces described in this term are intended to be internal relative to the swimmer. However, the form of (3) satisfies this condition in terms of forces applied to $z_i(t)$ as the centers of mass of $S_i(z_i(t))$ only if the sets $S_i(0)$'s have identical measure (for details, see Khapalov, 2010; 2013).

In the general case, one needs to make some additional normalizing adjustments in the magnitudes of the respective terms in (3) which take into account the size of supports $S_i(z_i(t))$, for example, as follows:

$$\begin{aligned}
&F(z,v)\\
&=\sum_{i=2}^{n}[(\mathrm{mes}\,(S_{i-1}(0))^{-1}\xi_{i-1}(x,t)k_{i-1}\\
&\quad\times\frac{(\|z_i(t)-z_{i-1}(t)\|_{\mathbb{R}^3}-l_{i-1})}{\|z_i(t)-z_{i-1}(t)\|_{\mathbb{R}^3}}(z_i(t)-z_{i-1}(t))\\
&\quad+(\mathrm{mes}\,(S_i(0))^{-1}\xi_i(x,t)k_{i-1}\\
&\quad\times\frac{(\|z_i(t)-z_{i-1}(t)\|_{\mathbb{R}^3}-l_{i-1})}{\|z_i(t)-z_{i-1}(t)\|_{\mathbb{R}^3}}(z_{i-1}(t)-z_i(t))]\\
&\quad+\sum_{i=2}^{n-1}v_{i-1}(t)\{(\mathrm{mes}\,(S_{i-1}(0))^{-1}\xi_{i-1}(x,t)\\
&\quad\times(A_i(z_{i-1}(t)-z_i(t)))\\
&\quad-(\mathrm{mes}\,(S_{i+1}(0))^{-1}\xi_{i+1}(x,t)\frac{\|z_{i-1}(t)-z_i(t)\|^2_{\mathbb{R}^3}}{\|z_{i+1}(t)-z_i(t)\|^2_{\mathbb{R}^3}}\\
&\quad\times(B_i(z_{i+1}(t)-z_i(t)))\}\\
&\quad-\sum_{i=2}^{n-1}(\mathrm{mes}\,(S_i(0))^{-1}\xi_i(x,t)v_{i-1}(t)\\
&\quad\times\{(A_i(z_{i-1}(t)-z_i(t)))\\
&\quad-\frac{\|z_{i-1}(t)-z_i(t)\|^2_{\mathbb{R}^3}}{\|z_{i+1}(t)-z_i(t)\|^2_{\mathbb{R}^3}}(B_i(z_{i+1}(t)-z_i(t)))\}.
\end{aligned}\tag{4}$$

The added extra coefficient $(\mathrm{mes}\,(S_i(0))^{-1}$ at each characteristic function $\xi_i(x,t)$ ensures that all the forces of the swimmer are internal (see also the respective discussion in the end of Chapter 11 of the book by Khapalov (2010) for the 2-D case). In the case of sets $S_i(z_i(t))$ of identical measure, the aforementioned (identical) extra coefficients can be viewed as included in k_i's and v_i's.

The above change of the forcing term in (3) to (4) is essentially a typing error, which does not affect the rest of our previous work (Khapalov, 2013).

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