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DECENTRALIZED SLIDING MODE CONTROL USING AN EVENT–TRIGGERED MECHANISM FOR DISCRETE INTERCONNECTED HAMMERSTEIN SYSTEMS

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An innovative control strategy addressing the complexities of discrete interconnected nonlinear Hammerstein subsystems is presented. The approach combines decentralized sliding mode control (DSMC) with an event-triggered mechanism (ETM) to efficiently manage complex systems characterized by discrete elements, nonlinear behavior, and interconnections. The event-triggered sliding mode control (ETSMC) framework offers a distributed control solution that utilizes the robustness and disturbance tolerance of sliding mode control while optimizing resource usage and network communication through an event-triggered mechanism. A comprehensive analysis of stability and robustness ensures that the proposed control strategy stabilizes the system and achieves its design objectives, even in the presence of uncertainties or disturbances. The effectiveness of the approach is demonstrated through two simulation examples.

Keywords: sliding mode control, event-triggered mechanism, discrete interconnected systems, Hammerstein models.

1. Introduction

Controlling discrete interconnected Hammerstein models poses a significant challenge in control theory due to their discrete nature and unique structure (Bai, 2010; Znidi *et al.*, 2022; Vineet and Utkal, 2022; Yiqun and Yan, 2023). These systems are characterized by nonlinear static blocks followed by linear dynamics, which introduce complexities requiring innovative control strategies. Addressing these challenges involves tackling nonlinearities, dynamics, and interconnections to optimize system stability and performance while managing uncertainties. Achieving effective control in this domain necessitates a blend of mathematical modeling and advanced control algorithms proficient in handling such complexities.

In the pursuit of enhancing the performance and efficiency of interconnected nonlinear systems, researchers in the field of control theory have explored various decentralized control strategies (Nan and Bin, 2021; Yueheng and Spurgeon, 2022a; Nagai and Oya, 2014; Kamoun and Kamoun, 2016). Among these strategies, decentralized sliding mode control (DSMC) stands out. DSMC has attracted attention due to its potential to offer robustness and adaptability in handling uncertainties and disturbances, thereby improving the control of interconnected nonlinear systems (Thien and Kim, 2018; Yueheng and Jiang, 2022).

DSMC is characterized by distributing control actions across multiple subsystems, each operating independently based on local information. This decentralized approach enhances robustness and adaptability, especially in complex systems.

DSMC emerges as a promising solution due to its inherent robust design and adaptability, rendering it suitable for managing the nonlinearities and complexities in interconnected systems (Elloumi and Kamoun, 2017; 2015). These systems face various challenges, from time varying parameters to unexpected disturbances. DSMC demonstrates exceptional proficiency in addressing these complexities, ensuring system stability and effective control through its robust and adaptable design (Yueheng and Spurgeon, 2022b; Elloumi and Kamoun, 2016; Labibi, 2005; Ordaz *et al.*, 2024).

To further enhance DSMC's performance and tackle

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challenges associated with communication delays, a novel approach called event-triggered sliding mode control has been proposed (Menghua, 2023; Yang and Yue, 2021; Xiaojie and Yong, 2017; Gong and Zheng, 2023). This innovative strategy aims to optimize resource utilization by adjusting control actions based on specific triggering events, thereby reducing communication overhead. Moreover, event-triggered sliding mode control offers a more deterministic approach by triggering updates based on predefined events, thus contributing to the overall efficiency of the system.

Numerous studies in the literature have explored the realm of ETSMC, presenting a diverse array of designed methodologies that underscore its efficacy.

The research outlined by Adamiak and Bartoszewicz (2021) introduces an innovative event-triggered quasi-sliding mode control strategy tailored for linear discrete time systems. This method merges a sliding mode control technique with a reference trajectory-based control approach to augment robustness against disturbances. Additionally, an event-triggering algorithm is incorporated to reduce the necessity for system communication and minimize delays in the digital control procedure. Moreover, Benyazid and Nouri (2023) detailed a new event-triggered integral sliding mode control approach for discrete nonlinear systems with time delay, specifically concentrating on Takagi–Sugeno fuzzy models. This method involves devising a novel integral sliding function and determining the design parameter matrix via linear matrix inequalities. A control protocol is then formulated to ensure the state trajectories of fuzzy systems with time delays.

Patel and Purwar (2023) propose an event-triggered multi-rate output feedback sliding mode control technique for load frequency control (LFC) in interconnected power systems, achieving a quasi sliding mode in discrete time with reduced resource utilization. Furthermore, Xiaojie and Yong (2017) introduced an event-triggered sliding mode control algorithm addressing load frequency control in interconnected power systems, ensuring stability and robustness through H^∞ performance and time-delay analysis. In addition, a control-based event-triggered sliding mode strategy for networked linear systems was explored by Yufei and Qiang (2023), featuring a novel control value-based event-triggering mechanism, integrating quantization policies for reduced information transmission, and ensuring robust stability with sliding mode control while addressing transmission delays.

Inspired by the rapid advancements in modern research, an innovative approach known as decentralized sliding mode control, employing an event-triggered mechanism, has been introduced. This new control strategy operates across multiple nonlinear or components without a central governing unit, ensuring stability and robustness against uncertainties. The primary objective behind integrating this mechanism lies in alleviating the challenge of load communication.

By judiciously employing event-triggered conditions, this methodology aims to mitigate communication overload within decentralized networks, thereby reducing the volume of transmitted data.

The structure of this paper is as follows. Section 2 introduces the problem formulation. Section 3 presents a new DSMC for discrete interconnected Hammerstein subsystems. To address the heightened communication and computational loads of DSMC, an event-triggered sliding mode control is proposed in Section 4. Subsequently, a detailed stability analysis is conducted using Lyapunov theory in Section 5. A comprehensive robustness analysis is provided in Section 6. The effectiveness of the proposed controller is assessed through two simulation examples in Section 7. Finally, conclusions are drawn in Section 8.

2. Problem formulation

The system to be controlled is characterized by a configuration comprising discrete Hammerstein subsystems intricately interconnected with one another. In this complex setup, each subsystem operates discretely, exhibiting Hammerstein nonlinearities, which mean a combination of static nonlinearity followed by dynamic linear behavior. These subsystems are interconnected, forming a network where their interactions play a pivotal role in the overall system dynamics, as described in the following equation:

$$\begin{cases} X_i(k+1) = A_i X_i(k) + B_i(U_i(k) + f(k)) \\ \qquad\qquad + \sum\limits_{j=1;j\neq i}^{p} \beta_{ij} X_j(k) , \\ Y_i(k) = H_i X_i(k), \\ U_i(k) = g_i(u_i(k)), \end{cases} \tag{1}$$

where X_i represents the subsystem states, $U_i(k)$ denotes the controllers of the system, and $X_{ji}(k)$ stands for the interactions between the subsystems, where $j \neq i$. External disturbances are expressed by $f(k)$. The constant matrices A_i, B_i, H_i and β_{ij} are assumed to be known, where β_{ij} denotes the matrix of interactions between the i-th and the j-th subsystems. Additionally, $g_i(\cdot)$ represent nonlinear functions. We denote by F an upper bound of the disturbance signal.

The following assumptions are made.

Assumption 1. The pairs (A_i, B_i) are controllable.

Assumption 2. The disturbances are bounded.

Assumption 3. The inverses of the nonlinear functions, $g_i^{-1}(\cdot)$, exist.

The primary objective of this work is to design an innovative discrete, decentralized sliding mode controller aimed at achieving desired performance criteria, such as accurate tracking and chattering reduction, even in the presence of external disturbances and subsystem interconnections.

3. Decentralized sliding mode control for interconnected Hammerstein systems

Sliding mode control is a powerful method employed for managing nonlinear systems under external perturbations. This section focuses on the development of a novel discrete decentralized sliding mode control. The process of designing DSMC typically involves the following steps:

- Identifying a switching function that ensures stability of the sliding mode on the switching plane.
- Establishing a control law that fulfills a reaching condition, ensuring that the system state will converge towards the switching plane from any initial state within a finite time.

3.1. Choice of the appropriate sliding function. The first step of our development involves the design of a suitable sliding function. This function serves as a crucial element in the sliding mode control framework, defining the desired behavior of the system and guiding its response to disturbances. By carefully constructing this sliding function, we can adjust the controller's behavior to meet specific performance criteria, such as accurate tracking and reduced chattering. Thus, the discrete sliding function is chosen as follows:

$$\delta_i(k) = {G_i}^T (X_i(k) - X_{r,i}(k)), \tag{2}$$

where $G_i \in \mathbb{R}^n$ are the sliding function parameter vectors and $X_{r,i}(k)$ denote the desired state vectors, intended to be equal to zero. However, it is important to note that the initial states are different from zero.

3.2. Computation of the DSMC law. Our controller design integrates Gao's reaching law within the framework of sliding mode control, utilizing it to drive the system onto the sliding surface for robust performance. The reaching law ensures finite-time convergence, while the sliding mode control law governs the system's behavior on the sliding surface. This approach guarantees stability and robustness in achieving the desired performance objectives.

Gao's reaching law is defined as

$$\delta_i(k+1) = (1-\phi_i)\delta_i(k) - M_i \operatorname{sign}(\delta_i(k)), \tag{3}$$

where ϕ_i is a positive constant, chosen such that $0 \le \phi_i < 1$, M_i are the discontinuous terms and the signum function sign is defined by

$$\operatorname{sign}(\delta_i(k)) = \begin{cases} -1 & \text{if } \delta_i(k) < 0, \\ 1 & \text{if } \delta_i(k) > 0. \end{cases} \tag{4}$$

The sliding function at the instant $(k+1)$ is expressed as

$$\begin{aligned} \delta_i(k+1) &= {G_i}^T X_i(k+1) \\ &= {G_i}^T [A_i X_i(k) + B_i(U_i(k) + f(k)) \\ &\quad + \sum_{j=1;j\neq i}^{p} \beta_{ij} X_j(k)]. \end{aligned} \tag{5}$$

In the sliding mode, we suppose that $f(k) = 0$. Then we can write

$$\begin{aligned} \delta_i(k+1) &= {G_i}^T [A_i X_i(k) + B_i(U_i(k)) \\ &\quad + \sum_{j=1;j\neq i}^{p} \beta_{ij} X_j(k)] \\ &= (1-\phi_i)\delta_i(k) - M_i \operatorname{sign}(\delta_i(k))). \end{aligned} \tag{6}$$

The controller is then calculated as

$$\begin{aligned} U_i(k) = &-({G_i}^T B_i)^{-1} [{G_i}^T A_i X_i(k) \\ &+ {G_i}^T \sum_{j=1;j\neq i}^{p} \beta_{ij} X_j(k) \\ &- ({G_i}^T B_i)^{-1} ((1-\phi_i)\delta_i(k) \\ &+ M_i \operatorname{sign}(\delta_i(k)))]. \end{aligned} \tag{7}$$

Here we assume that $({G_i}^T B_i)$ are nonsingular.

If $g_i^{-1}(\cdot)$ exists, $u_i(k)$ is computed as

$$\begin{aligned} u_i(k) = g_i^{-1}(&-({G_i}^T B_i)^{-1} [{G_i}^T A_i X_i(k) \\ &+ {G_i}^T \sum_{j=1;j\neq i}^{p} \beta_{ij} X_j(k) \\ &- ({G_i}^T B_i)^{-1} ((1-\phi_i)\delta_i(k) \\ &+ M_i \operatorname{sign}(\delta_i(k)))]), \end{aligned} \tag{8}$$

where $M_i > \|B_i\| \, \|f(k)\|$.

Remark 1. If the inverse function $g_i^{-1}(\cdot)$ does not exist for a given nonlinear function $g_i(\cdot)$, researchers often resort to various algorithms and approximations to estimate the corresponding nonlinear inverse. Recent advancements in the field include approaches based on polynomial form approximation, which can simplify complex functions using polynomials (Rayouf and Braiek, 2019). Another method is the Bernstein–Bezier neural network (Hong and Mitchell, 2007), which utilizes neural networks to approximate the inverse function. Additionally, the De Boor algorithm, known for its efficacy in spline interpolation, can be adapted for inverse function estimation (Hong and Chen, 2012). Finally, rational B-spline model approximation offers a flexible approach to model complex nonlinear functions and their inverses with high accuracy (Chen and Harris, 2014). These methods provide a range of options for dealing with non-existent inverses in nonlinear functions, allowing more efficient and precise solutions in various applications. However, in our case, we assume that the inverse function exists for the given nonlinear function $g_i(x)$, allowing us to proceed with our analysis without requiring these approximation techniques.

To improve the performance of the proposed controller, we suggest incorporating the saturation function, which would provide smoother control behavior,

$$\begin{aligned} u_i(k) = g_i^{-1}(&-({G_i}^T B_i)^{-1}[{G_i}^T A_i X_i(k) \\ &+ {G_i}^T \sum_{j=1;j\neq i}^{p} \beta_{ij} X_j(k) \\ &+ ({G_i}^T B_i)^{-1}((1-\phi_i)\delta_i(k) \\ &- M_i \operatorname{sat}(\delta_i(k)))]), \end{aligned} \tag{9}$$

where

$$\operatorname{sat}(\delta_i(k)) = \begin{cases} \frac{\delta_i(k)}{\xi_{i_i}} & \text{if } \|\delta_i(k)\| \leq \xi_{i_i}, \\ \operatorname{sign}(\delta_i(k) & \text{if } \|\delta_i(k)\| > \xi_{i_i}. \end{cases} \tag{10}$$

The quasi-sliding mode bands are denoted by ξ_{i_i}.

While decentralized sliding mode control offers notable advantages, it is crucial to acknowledge certain drawbacks, particularly concerning increased communication and computational loads. This limitation arises from the independent operation of each subsystem, requiring constant information exchange and potentially impacting the overall system efficiency.

4. DSMC using an event triggered mechanism

To address the aforementioned issues, we propose an enhanced control law that integrates an event-triggered mechanism, aiming to mitigate communication and computational challenges associated with decentralized sliding mode control (DSMC). This innovation strategically leverages the advantages of event-triggered systems, optimizing the timing of control updates and reducing unnecessary communication overhead. By introducing this refined control strategy, we seek to enhance the overall efficiency and adaptability of DSMC in discrete interconnected systems.

Define the state error caused by the event-triggered control scheme as

$$e_i(k) = X_i(k_s) - X_i(k), \tag{11}$$

where k_s represents the moment when the control signal was last updated.

The event triggered condition is proposed as follows:

$$k_s + 1 = \inf\{k > k_s : \|e_i(k)\| < \gamma \|X_i(k)\|\}, \tag{12}$$

where γ is a positive constant representing a threshold.

The sliding function at instant $k_s + 1$ is

$$\begin{aligned} \delta_i(k_s+1) &= {G_i}^T X_i(k_s+1) \\ &= {G_i}^T [A_i X_i(k_s) + B_i(U_i(k_s) + f(k)) \\ &\quad + \sum_{j=1;j\neq i}^{p} \beta_{ij} X_j(k_s)]. \end{aligned} \tag{13}$$

In the sliding mode, we suppose that $f(k) = 0$. Then we can write

$$\begin{aligned} \delta_i(k_s+1) &= {G_i}^T [A_i X_i(k_s) + B_i(U_i(k_s)) \\ &\quad + \sum_{j=1;j\neq i}^{p} \beta_{ij} X_j(k_s)] \\ &= (1-\phi_i)\delta_i(k_s) - M_i \operatorname{sat}(\delta_i(k_s)). \end{aligned} \tag{14}$$

The new DSMC control law using an event-triggered communication scheme is then calculated by

$$u_i(k_s) = \begin{cases} g_i^{-1}((G_i^T B_i)^{-1}[G_i^T A_i X_i(k_s) \\ +G_i^T \sum\limits_{j=1}^{p} \beta_{ij} X_j(k_s) - (1-\phi_i)\delta_i(k) \\ +M_i \operatorname{sat}(\delta_i(k_s))]) \\ \qquad\qquad \text{if} \quad \|e_i(k)\| < \gamma \|X_i(k)\|, \\ u_i(k_s-1) \quad \text{if} \quad \|e_i(k)\| > \gamma \|X_i(k)\| \end{cases} \tag{15}$$

for all $k \in [k_s,\, k_{s+1})$.

The structure of the event triggered SMC is presented in Fig. 1.

5. Stability analysis

Theorem 1. *Given the system described by (1), the control defined by Eqn. (15), and the event-triggering condition (12), the state trajectories of each subsystem can be kept in the sliding region within a finite time, that is, the stability and robustness of the system in each area can be guaranteed.*

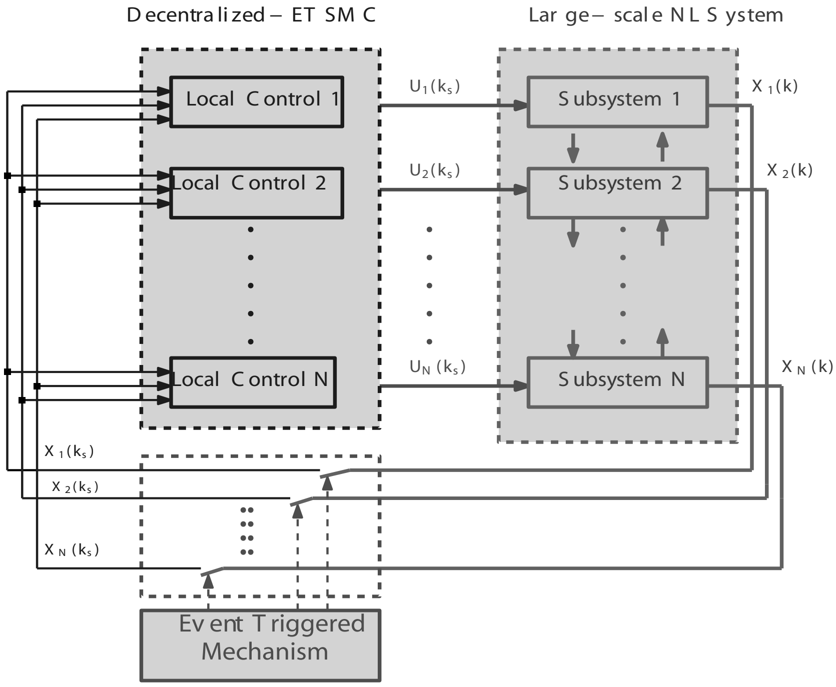

Fig. 1. ETDSMC structure for the discrete interconnected system.

Proof. In order to prove the stability of the interconnected subsystems, the Lyapunov function is chosen as follows:

$$V_i(k) = \frac{1}{2}\delta_i{}^T(k_s)\delta_i(k_s). \tag{16}$$

We have

$$\begin{aligned}\Delta\delta_i(k_s) &= \delta_i(k_s+1) - \delta_i(k_s)\\ &= B_i f(k) - \phi_i\delta_i(k) - M_i\,\mathrm{sat}(\delta_i(k_s)),\end{aligned} \tag{17}$$

$$\begin{aligned}\Delta V_i(k) &= V_i(k+1) - V_i(k)\\ &= \delta_i{}^T(k_s)\Delta\delta_i(k_s) + \frac{1}{2}\Delta^T\delta_i(k_s)\Delta\delta_i(k_s)\\ &= \delta_i{}^T(k_s)[B_i f(k) - \phi_i\delta_i(k) - M_i\,\mathrm{sat}(\delta_i(k_s))]\\ &\quad + \frac{1}{2}\Delta^T\delta_i(k_s)\Delta\delta_i(k_s)\\ &\le -\phi_i\delta_i{}^T(k_s)\delta_i(k_s) + \frac{1}{2}\Delta^T\delta_i(k_s)\Delta\delta_i(k_s)\\ &\quad + \left\|\delta_i{}^T(k_s)\right\|\|B_i\|\|f(k)\|\\ &\quad - M_i\delta_i{}^T(k_s)\,\mathrm{sat}(\delta_i(k_s)).\end{aligned} \tag{18}$$

Thus, we obtain

$$\begin{aligned}\Delta V_i(k) &\le -\phi_i\delta_i{}^T(k_s)\delta_i(k_s) + \frac{1}{2}\Delta^T\delta_i(k_s)\Delta\delta_i(k_s)\\ &\quad + M_i\left\|\delta_i{}^T(k_s)\right\| - M_i\delta_i{}^T(k_s)\,\mathrm{sat}(\delta_i(k_s)).\end{aligned} \tag{19}$$

Utilizing the saturation function $\mathrm{sat}(\delta_i(k_s))$ allows us to deduce that, if $\|\delta_i(k)\| \le \xi_i$, when using the designed controller in Eqn. (15), we obtain

$$\begin{aligned}\Delta V_i(k) &\le -\phi_i\delta_i{}^T(k_s)\delta_i(k_s) + \frac{1}{2}\Delta^T\delta_i(k_s)\Delta\delta_i(k_s)\\ &\quad + M_i\Big(\left\|\delta_i{}^T(k_s)\right\| - \frac{\left\|\delta_i{}^T(k_s)\right\|^2}{\xi_i}\Big)\\ &\le -\phi_i\delta_i{}^T(k_s)\delta_i(k_s) + \frac{1}{2}\Delta^T\delta_i(k_s)\Delta\delta_i(k_s).\end{aligned} \tag{20}$$

If $\|\delta_i(k)\| > \xi_i$, using the controller specified in Eqn. (15), we obtain

$$\Delta V_i(k) \le -\phi_i\delta_i{}^T(k_s)\delta_i(k_s) + \frac{1}{2}\Delta^T\delta_i(k_s)\Delta\delta_i(k_s). \tag{21}$$

It should be noted that ϕ_i can be selected appropriately such that $\Delta V_i(k) \le 0$. ■

6. Robustness analysis

Theorem 2. *For the system (1) under the control of the proposed decentralized event-triggered sliding mode control represented by (15), the utilized sliding function (13) converges to a quasi-sliding mode when the following parameters are chosen:*

$$\xi_i > M_i + F_0, \tag{22}$$

$$M_i > F_0, \tag{23}$$

where ξ_i represents a small positive constant, and $F_0 = \|B_i\|\,\|f(k)\|$.

Proof. Replacing the expression of the control law (15) in the equation of the sliding function (13), we obtain

$$\begin{aligned}\delta_i(k_s+1) = (1-\phi_i)\delta_i(k_s) - M_i \operatorname{sat}(\delta_i(k_s)) \\ + B_i f(k).\end{aligned} \tag{24}$$

A quasi-sliding mode in discrete DETSMC exists if and only if

$$|\delta_i(k_s+1)| < |\delta_i(k_s)| \quad \text{if } |\delta_i(k_s)| > \xi_i, \tag{25}$$

$$|\delta_i(k_s+1)| \le \xi_i \quad \text{if } \delta_i(k_s) > \xi_i. \tag{26}$$

Case 1. $\delta_i(k_s) > \xi_i$
We have

$$\begin{aligned}\delta_i(k_s+1) - \delta_i(k_s) &= -\phi_i\delta_i(k_s) - M_i \operatorname{sat}(\delta_i(k_s)) \\ &\quad + B_i f(k) \\ &\le -\phi_i\xi_i - M_i + F_0.\end{aligned} \tag{27}$$

Using Eqns. (22) and (23), we can write $\delta_i(k_s+1) - \delta_i(k_s) < 0$. Thus,

$$\begin{aligned}\delta_i(k_s+1) + \delta_i(k_s) &= (2-\phi_i)\delta_i(k_s) \\ &\quad - M_i + B_i f(k) \\ &\ge (2-\phi_i)\xi_i - M_i + F_0.\end{aligned} \tag{28}$$

Making of (22) and (23), we can write $\delta_i(k_s+1) + \delta_i(k_s) > 0$.

Thus,

$$|\delta_i(k_s+1)| < |\delta_i(k_s)| \quad \text{if } \delta_i(k_s) > \xi_i. \tag{29}$$

Case 2. $\delta_i(k_s) < -\xi_i$
In this scenario, we get

$$\begin{aligned}\delta_i(k_s+1) - \delta_i(k_s) &= -\phi_i\delta_i(k_s) + M_i + B_i f(k) \\ &\ge \phi_i\xi_i + F_0 + M_i > 0,\end{aligned} \tag{30}$$

$$\begin{aligned}\delta_i(k_s+1) + \delta_i(k_s) &= (2-\phi_i)\delta_i(k_s) + B_i f(k) + M_i \\ &\le (\phi_i - 2)\xi_i + F_0 + M_i.\end{aligned} \tag{31}$$

Since $0 \le \phi_i < 1$, we can write

$$\delta_i(k_s+1) + \delta_i(k_s) \le (\phi_i - 1)\xi_i < 0. \tag{32}$$

Equations (22) and (23) lead to

$$|\delta_i(k_s+1)| < |\delta_i(k_s)| \quad \text{if } \delta_i(k_s) < -\xi_i. \tag{33}$$

Case 3. $0 \le \delta_i(k_s) \le \xi_i$
We can write

$$\begin{aligned}\delta_i(k_s+1) &= (1-\phi_i)\delta_i(k_s) \\ &\quad - M_i \operatorname{sat}(\delta_i(k_s)) + B_i f(k) \\ &\le \xi_i - M_i + F_0 < \xi_i.\end{aligned} \tag{34}$$

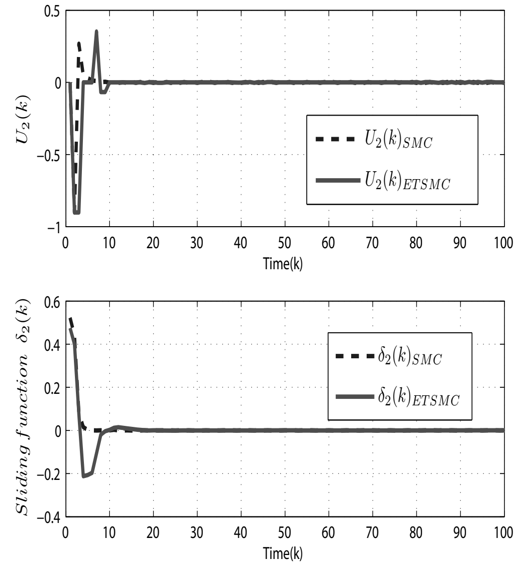

Fig. 2. Evolution of the control signal $U_2(k)$ and the sliding surface $\delta_2(k)$ (Example 1).

Additionally, $\delta_i(k_s)$ can satisfy the following inequality:

$$\begin{aligned}\delta_i(k_s+1) &= (1-\phi_i)\delta_i(k_s) - M_i + B_i f(k) \\ &\ge -M_i - F_0 > -\xi_i.\end{aligned} \tag{35}$$

Therefore,

$$|\delta_i(k_s+1)| \le \xi_i \quad \text{if} \quad 0 \le \delta_i(k_s) \le \xi_i. \tag{36}$$

Case 4. $-\xi_i \le \delta_i(k_s) \le 0$
We assume that $\delta_i(k)$ fulfills the requirements set by the following two inequalities:

$$\begin{aligned}\delta_i(k_s+1) &= (1-\phi_i)\delta_i(k_s) \\ &\quad - M_i \operatorname{sat}(\delta_i(k_s)) + B_i f(k) \\ &\ge -(1-\phi_i)\xi_i + M_i + F_0 \\ &\ge -\xi_i,\end{aligned} \tag{37}$$

$$\begin{aligned}\delta_i(k_s+1) &= (1-\phi_i)\delta_i(k_s) \\ &\quad - M_i \operatorname{sat}(\delta_i(k_s)) + B_i f(k) \\ &= (1-\phi_i)\delta_i(k_s) - M_i\frac{\delta_i(k_s)}{\xi_i} \\ &\quad + B_i f(k) \\ &\le F_0 < \xi_i.\end{aligned} \tag{38}$$

Thus,

$$|\delta_i(k_s+1)| \le \xi_i \quad \text{if} \quad -\xi_i \le \delta_i(k_s) \le 0. \tag{39}$$

Afterward, we show that, by choosing appropriate parameters for the control law, our method ensures the existence of the quasi-sliding mode. ■

7. Simulation

In this section, we provide two simulation examples to assess how well the suggested controller performs.

7.1. Example 1. A complex system, S, comprises two interconnected subsystems. These subsystems $S1$ and $S2$, are defined by sets of linear discrete-time state-space equations, each preceded by nonlinear blocks, representing interconnected Hammerstein systems (Kamoun and Kamoun, 2016):

$$\left\{\begin{aligned} X_1(k+1) &= \begin{bmatrix} 0 & 1 \\ -0.38 & -1.06 \end{bmatrix} X_1(k) \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\tanh(0.4 \times u_1(k)) + f(k)) \\ &\quad + \begin{bmatrix} 0.15 & 0.24 \\ 0.34 & 0.09 \end{bmatrix} X_{12}(k), \\ M_1 &= 4 \times 10^{-3}, \\ X_2(k+1) &= \begin{bmatrix} 0 & 1 \\ -0.26 & 0.88 \end{bmatrix} X_2(k) \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\tanh(0.4 \times u_2(k)) + f(k)) \\ &\quad + \begin{bmatrix} 0.08 & 0.32 \\ 0.17 & 0.37 \end{bmatrix} X_{21}(k), \\ M_1 &= 3 \times 10^{-3}, \\ f(k) &= 0.001 \times \sin(\tfrac{2k\pi}{100}) \qquad \text{for} \ \ 50 \le k \le 80, \end{aligned}\right. \tag{40}$$

where the initial values are defined as $X(1) = [0.1, 0.5]^T$, $X(2) = [0.91, 0.95]^T$, the switching vector is given by $G_1 = [0, 0.15]$ and $G_2 = [0, 0.5]$. The control parameters are chosen as $\phi_1 = 0.5$ and $\phi_2 = 0.81$.

The simulation results using the DSMC and ETSMC methods are presented in Figs. 2–7. Figure 2 displays the evolution of control signals and the sliding surface for the first subsystem, while Fig. 3 shows the same for the second one. This analysis is further supported by the state evolution depicted in Figs. 4 and 5. The simulation results indicate that the ETSMC method utilizes only 34% for Subsystem 1 and 31% for Subsystem 2 from the total sampled dataset, showcasing a significant reduction in communication resource consumption. This highlights the capability of the proposed ETSMC approach to effectively halve the usage of communication resources.

Additionally, to enhance the assessment of this method, another simulation example will be presented in the subsequent section.

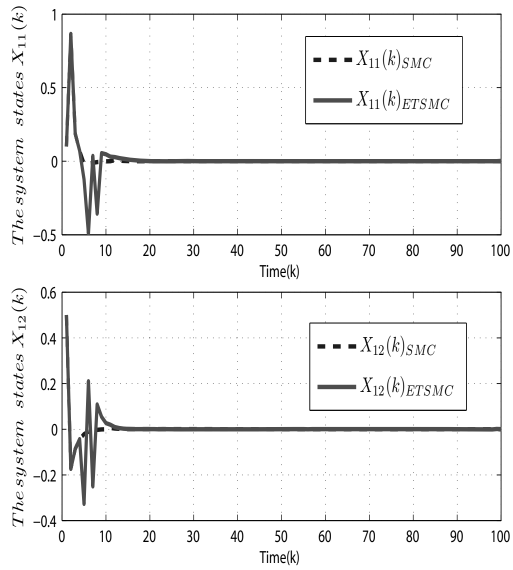

Fig. 3. Evolution of the system states $X_{11}(k)$ and $X_{12}(k)$ (Example 1).

7.2. Example 2. The system to be controlled is described as follows:

$$\left\{\begin{aligned} X_1(k+1) &= \begin{bmatrix} 0 & 1 \\ -0.32 & 1.2 \end{bmatrix} X_1(k) \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\sin(0.2 \times u_1(k)) + f(k)) \\ &\quad + \begin{bmatrix} 0.1 & 0.4 \\ 0.2 & -0.1 \end{bmatrix} X_{12}(k), \\ M_1 &= 4 \times 10^{-3}, \\ X_2(k+1) &= \begin{bmatrix} 0 & 1 \\ -0.2 & -0.92 \end{bmatrix} X_2(k) \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\tanh(0.4 \times u_2(k)) + f(k)) \\ &\quad + \begin{bmatrix} 0.2 & 0.1 \\ -0.3 & 0.1 \end{bmatrix} X_{21}(k), \\ M_1 &= 4 \times 10^{-3}, \\ f(k) &= 0.001 \times \sin(\tfrac{2k\pi}{100}) \qquad \text{for} \ \ 50 \le k \le 80, \end{aligned}\right. \tag{41}$$

where the initial values are defined as $X(1) = [0.1, 0.5]^T$, $X(2) = [0.1, 0.5]^T$, and the switching vector is given by $G_1 = [0.01, 0.2]$ and $G_2 = [0, 0.01]$. The control parameters are chosen as $\phi_1 = 0.42$ and $\phi_2 = 0.651$.

Remark 2. The function $\sin(\cdot)$ is noninvertible over

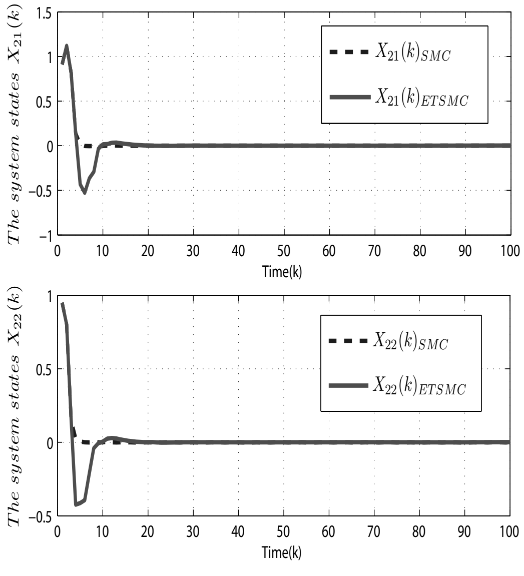

Fig. 4. Evolution of the system states $X_{21}(k)$ and $X_{22}(k)$ (Example 1).

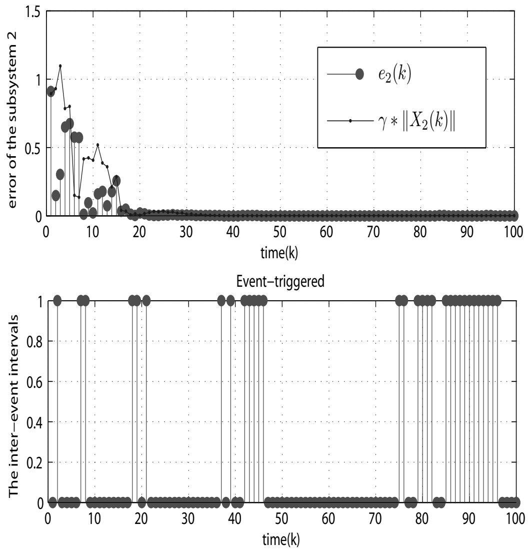

Fig. 6. Release intervals and instants of the ETM of Subsystem 2 (Example 1).

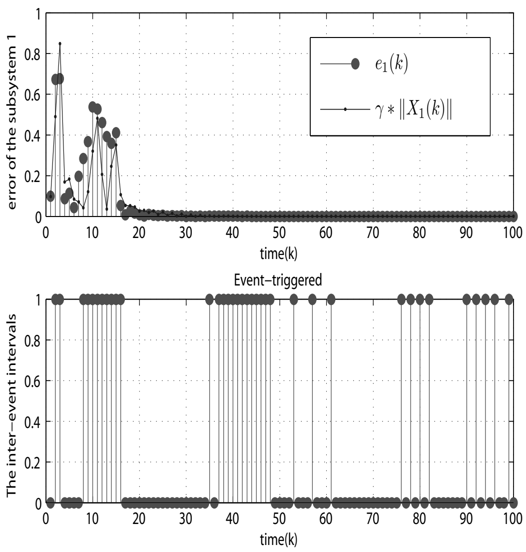

Fig. 5. Release intervals and instants of the ETM of Subsystem 1 (Example 1).

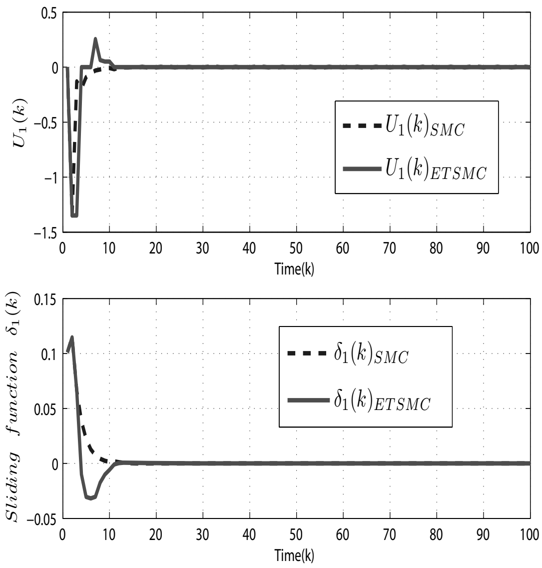

Fig. 7. Evolution of the control signal $U_1(k)$ and the sliding surface $\delta_1(k)$ (Example 2).

its entire range due to its periodic nature, as it repeats its values every 2π. However, within a single cycle, such as from $-\pi/2$ to $\pi/2$, $\sin(\cdot)$ is invertible. This implies that each output value in that range corresponds to one input value.

Example 2 illustrates a scenario where the input signal range in $\sin(0.2u_1(k))$ falls within the invertible part of $\sin(\cdot)$. Therefore, the input signal is confined to a range that permits the nonlinearity to be invertible, ensuring control efficiency is not negatively impacted by a noninvertible nonlinearity.

The simulation results for discrete event-triggered SMC are presented in Figs. 8–13. They emphasize that the ETSMC technique utilizes only 13% of the total sampled data for Subsystem 1 and 60% for Subsystem 2, resulting in a substantial reduction in communication resource usage.

7.3. Discussion. In the discussion section, we will investigate the proposed controller's effectiveness by

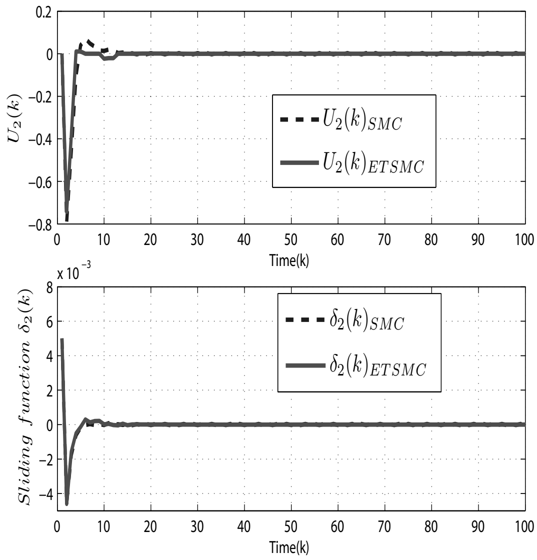

Fig. 8. Evolution of the control signal $U_2(k)$ and the sliding surface $\delta_2(k)$ (Example 2).

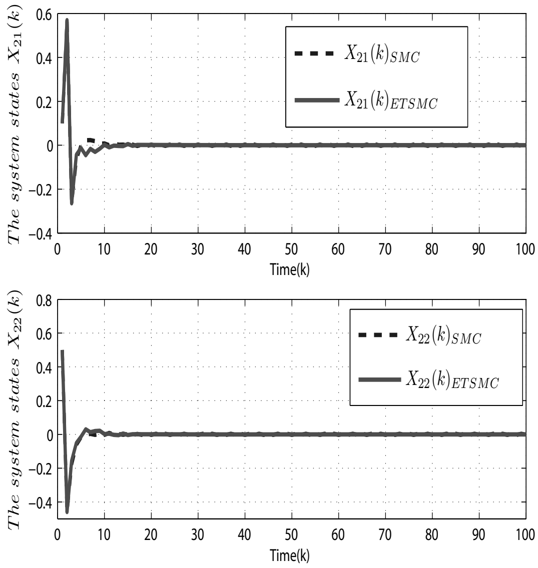

Fig. 10. Evolution of the system states $X_{21}(k)$ and $X_{22}(k)$ (Example 2).

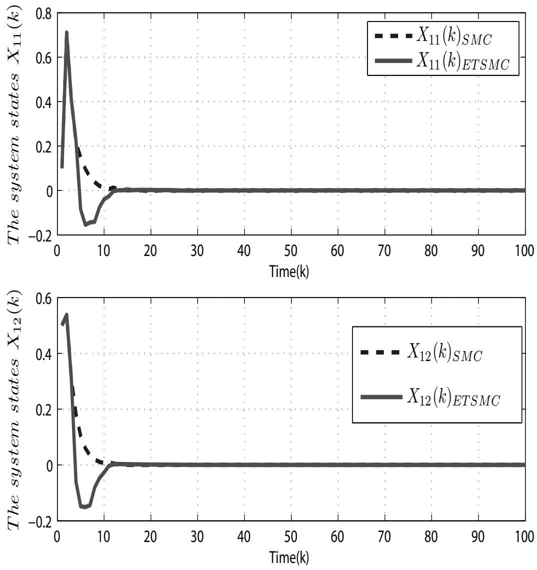

Fig. 9. Evolution of the system states $X_{11}(k)$ and $X_{12}(k)$ (Example 2).

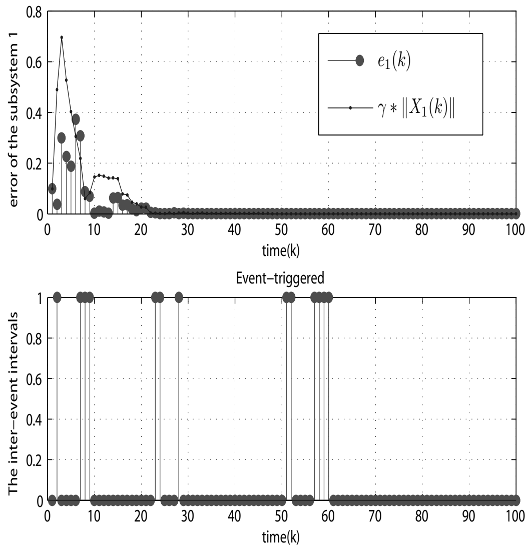

Fig. 11. Release intervals and instants of the ETM of Subsystem 1 (Example 2).

examining its behavior during transition periods and its robustness against external disturbances. We will also measure its performance using the RMSD index and evaluate the consistency of the ETSMC approach by adjusting the event-triggering threshold.

Reaching phase analysis. The reaching phase ($0 < k < 20$ in this case) is crucial in sliding mode control because it represents a critical period of transition where the system's state is directed toward a predefined switching surface. This phase is essential for ensuring that the system reaches the sliding mode accurately and efficiently. During this time, the system is most sensitive to external influences and disturbances. Optimizing this phase through methods such as linear matrix inequalities (LMI) in future work can lead to faster convergence and improved overall performance

Counteracting external disturbances. In the discussion of the proposed method's performance, the ability

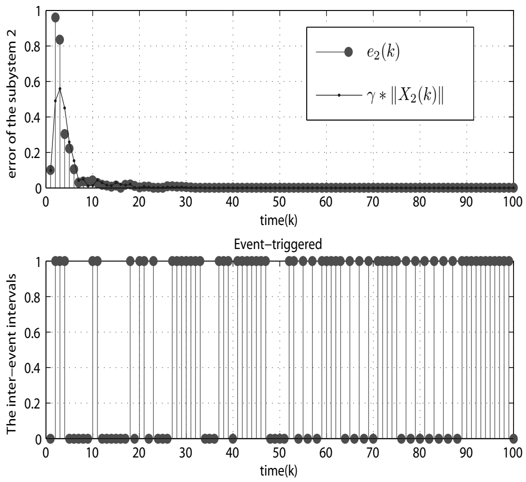

Fig. 12. Release intervals and instants of the ETM of Subsystem 2 (Example 2).

to counteract time-varying external disturbances is key. Discrete sliding mode control (DSMC) effectively addresses these disturbances by ensuring they the switching gain M_i is higher than the product of the system parameters $\|B_i\|$ and $\|f(k)\|$. This matching condition allows the control system to maintain the desired state trajectory by stabilizing the system dynamics on the sliding surface. The control input's capability to handle varying disturbances can be seen through the successful application of an external disturbance, $f(k) = 0.001 \times \sin(2k\pi/100)$, within the range from $k = 50$ to $k = 80$. The results demonstrate high accuracy in sliding surface convergence to zero and state convergence to desired values (for Examples 1 and 2).

Performance evaluation with the RMSD index. The efficiency of the proposed ETSMC is evaluated using the root mean square deviation (RMSD) index

$$\mathrm{RMSD}_i = \sqrt{\frac{1}{N_H}\sum_{k=1}^{N_H}(Y_i(k) - Y_{r,i}(k))^2} \quad \text{for } i = 1, 2, \tag{42}$$

where $N_H = 100$ is the simulation horizon. The desired reference is given by

$$Y_{r,i}(k) = H_i X_{r,i}(k),$$

where

$$X_{r,i}(k) = [0, 0]^T, \quad H_1 = [0, 0.01], \quad H_2 = [0, 0.001].$$

Despite the presence of disturbances, the proposed ETSMC achieves excellent tracking performance. This is demonstrated by the RMSD index values:

$$\mathrm{RMSD}_1 = 5.18 \times 10^{-4}, \ \mathrm{RMSD}_1 = 1.10 \times 10^{-4}$$

for Example 1 and

$$\mathrm{RMSD}_1 = 4.9 \times 10^{-4}, \quad \mathrm{RMSD}_1 = 6.82 \times 10^{-4}$$

for Example 2.

Table 1. Event-triggered rates with different γ_i values (Example 1).

Triggering parameter γ_1	0.01	0.5	0.85	0.95
Event-triggered rates	99%	96%	48%	34%
Triggering parameter γ_2	0.01	0.5	0.85	0.95
Event-triggered rates	100%	96%	46%	32%

Assessment of ETSMC smoothness. The smoothness of the proposed ETSMC approach is assessed by varying the event-triggering threshold. Table 1 summarizes the outcomes of 100 simulation runs with different values of the event-triggering parameter γ (threshold).

These results demonstrate that higher γ values lead to a reduced event-triggered ratio, which is the proportion of event-triggering releases out of a total of simulation steps. This implies that the ETSMC approach can reduce the need for data transmissions, thereby alleviating the communication load.

In summary, the proposed controller demonstrates notable effectiveness in handling transitions and disturbances, while the RMSD index and event-triggering threshold assessment reveal its potential for consistent and efficient performance.

8. Conclusion

This paper introduced and explored a novel paradigm of decentralized sliding mode control (DSMC) for discrete interconnected Hammerstein subsystems and addressed its limitations through the introduction of an innovative event-triggered sliding mode control (ETSMC). By employing Lyapunov theory, a comprehensive stability analysis was conducted, affirming the robustness and effectiveness of the proposed controllers. The simulations presented in Section 7 underscored the effectiveness and performance enhancement achieved by the ETSMC method over traditional DSMC approaches. The results demonstrated promising advancements in mitigating communication and computational burdens while maintaining stability and control efficacy in complex systems. This research paves the way for further advancements in control strategies, emphasizing the significance of adaptive and efficient methodologies in addressing contemporary challenges within decentralized control systems.

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