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OPTIMAL ROUTING AND FRAME DISTRIBUTION ASSIGNMENT PROBLEM IN THE INTEGRATED SERVICES DIGITAL NETWORK*

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In the paper, the optimal routing and frames distribution assignment problem in Integrated Services Digital Network (ISDN) is considered. The problem consists of simultaneously finding the routes for both voice and data traffic and the distribution of frames between voice and data traffics which minimize the total average delay per packet such that the traffic requirements are satisfied. The problem presented here is the nonlinear mixed–integer programming problem. The paper is divided to present the exact algorithm for the above formulated problem. Since this problem is NP–complete then a branch and bound method is used to construct this optimal algorithm.

1. Introduction

The concept of the ISDN is a single network that provides universal facilities among many different types of users. An efficient ISDN uses a common transmission system for all types of traffic and allows a standard access to the network for all the different types of terminal equipment that it must serve. Existing but noncompatible terminals are converted to support this standard access by the use of terminal adapters, whereas terminals designed for the ISDN automatically conform to it (Inose, 1979; Williams and Leon–Garcia, 1984).

The ISDN offers a single network for providing all communication services in order to achieve the economy of sharing. This economy motivates the general idea of an integrated services network. Integration avoids the need for many overlaying networks, which complicate network management and make the introduction and evaluation of services inflexible.

Integration within the network can have different meanings, depending on the part of network being considered (Hui, 1989; Little and Ghafoor, 1990). First, the easiest to achieve, integrated access involves the sharing, among services from an end user, of a single interface to a single transmission link connecting the end–user to a larger network. These links comprise the local access network within the larger network. Second, integrated transport involves the flexible sharing, among

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services from many users, of high–capacity transmission links beyond the local access network. Integrated transport avoids the segregation of different traffic types and media onto different transmission links. The integration of media on the same link may facilitate easier interactions between media within the network (Schurmann and Holzmann–Kaiser, 1990). Third, integrated switching involves switching multirate, multimedia services within a single switching machine, in particular a single interconnection network. An integrated switch would avoid the necessity of putting in an interconnection network whenever a new service of distinct traffic characteristics is introduced. An integrated switching network must flexibly meet the delay and bandwidth requirements of each service.

The advantages of the ISDN to both administration and users are numerous. One Integrated Service Digital Network is much more cost effective than several different networks. One network, using common equipment for all types of traffic, uses the available network resources more efficiently and additionally represents an overall reduction in maintenance costs. It also achieves the desirable goal of universal multi–mode communication capabilities, and offers significantly enhanced quality of transmission compared with the performance of present–day analog networks. The higher transmission rates offered by the ISDN are particularly attractive for data services (Fujioka *et. al.*, 1991; Iffland *et. al.*, 1989; Stephenson and McGaw, 1989).

When proposed, the concept of Pulse Code Modulation (PCM) was acknowledged world–wide to be revolutionary in the sense that two separately developed functions of telecommunications, transmission and switching, were integrated. In this system, speech signals are coded and time–division–multiplexed at the points of origin, and demultiplexed and decoded only at the points of destination. This prevents the quality of the speech signal from being degraded, no matter how many links and nodes are involved in its path. This arrangement also drastically cuts down the cost of the network by avoiding modulation and demodulation at each link and by replacing the inefficient space–division–switching at each node (Fujioka *et. al.*, 1991; Inose, 1979).

It is digital technology that makes the integration of transmission and switching possible (Hui, 1989; Iffland *et. al.*, 1989). The benefits of digital technology are multifold. First, digital circuitry provides a drastic reduction in hardware costs, extremely high reliability and maintenability, and substantial savings in space, material and energy. Secondly, digital techniques enhance transmission capability. Extremely broadband transmission media such an optical fibers and millimeter waveguides are available. Data of various speeds can be transmitted much more efficiently by digital transmission as compared with its analog counterpart. Speech, data and visual signals for the customers, as well as control and supervisory signals for setting up and releasing connections through the network, can be transmitted by digital techniques in an unified stream. Thirdly, digital techniques permit sophisticated signal processing and efficient signal handling. Coding of analog signals permits digital processing to reduce redundancy and provides opportunities for more sophisticated processing such as feature extraction. Digital information can be temporarily stored without distortion in digital memories. The store permits more efficient use of network facilities and provides a variety of buffering benefits such a refreshing and speed conversion. Above all, digital technology provides better interfacing with computers and terminals (Stephenson and McGaw, 1989; Williams and Leon–Garcia, 1984).

Digital technology permits integration of information in various forms. Such analog information as speech and visual signals are converted into digital and thereby combined with data for transmission, switching, processing and retrieval. Digital technology further provides an opportunity for the integration of telecommunication and broadcasting.

Requirements for switched digital communications systems indicate an increasing diversity of traffic characteristics, such as for voice and data. The motivations for considering integrated voice and data traffic in a shared network environment include: the advent of new voice related applications with the technology now existing to support them, and the desire to plan for and design future integrated for reasons of economy and flexibility (Lassers, 1989).

Data traffic is bursty in nature and real–time delivery is not of primary importance. There is no constraint that a data message must be delivered within a certain time. However, the delay must be reasonable. In data transmission subsystems strict error control and recovery procedures are required.

Voice traffic contains calls which last for a few minutes and for nearly 60 percent of the conversation time, the channel remains idle. Voice traffic can be modeled as having alternating talkspurts and silences with the generation of voice packets at a constant rate during talkspurts and no packet generation during silences. To preserve the integrity of a conversation, voice packets must be delivered within some time bound (typically 150–200 ms). Conversation is inherently robust, and as a result, speech can be reconstructed at the destination with acceptable quality if the loss of voice packets is less than some specified fraction (typically 1 percent) (Suda and Bradley, 1989).

It is known that by transmitting a series of impulses carrying the instantaneous amplitudes of the analog signal obtained by sampling with the period $1/2f_0$, and at the receiving end, by filtering the samples by means of an ideal low–pass filter with the cut–off frequency f_0, the original analog signal can be completely recovered (sampling theorem). At each voice user, a continuous voice analog signal is digitized by a coder (for instance, a typical PCM encoder produces one 8 bit word every 125 μs).

The multiplexing of a number of speech signals into a digital stream may be understand with reference to the configuration and the frame structure of a PCM terminal equipment for 24 speech channels. Each of the 24 speech signals is sampled every 125 μs. Since the sampling pulses assigned to the speech signals are 5.18 μs apart from each other, the 24 PAM pulses obtained by the sampling are aligned every 5.18 μs at the common bus. A common encoder converts each of the PAM pulses into a 8–bit PCM word in a 5.18 μs slot. At the end of the 24th time slot, the timing circuit inserts another pulse which is used for framing.

Thus 193 pulses are aligned in a frame of 125 μs. Twelve successive frames are combined to form a multiframe of 1.5 ms. For the purpose of identifying frames and multiframes, the framing pulses in the odd–numbered frames assume binary "1" and "0" alternately and those in the even–numbered frames assume the binary pattern "001110" successively. The last significant digit, i.e., the 8th bit in each time slot, is stolen every six frames to transmit signals (Fujioka *et. al.*, 1991).

Integration of voice and data (bulk or interactive) is provided by hybrid–switched multiplexing scheme. This scheme assures for both circuit– and packet–switching in the same transmission link by the use of special time–division multiplexing format. Transmission consist of fixed–duration frames that have two compartments: one part dedicated to voice traffic and the other part dedicated to data traffic. The boundary between the two parts is said to be "movable" when data traffic is allowed to use any frame capacity temporarily unused by voice traffic. The state of the system is given by the number of voice calls and the number of data packets in the system, and the performance is specified by the voice call blocking probability and the mean packet delay (Kasprzak, 1989).

There are several reasons why a hybrid arrangement is considered. Circuit–switching is well suited for satisfying all classes of traffic. Various studies comparing circuit– and packet–switching have concluded that circuit–switching is more cost–effective for traffic characterized by long messages, and packet–switching is more cost–effective for short messages. The exact message size at which the crossover occurs is a function of the various models used for calculating switching and transmission costs (Ross and Mowafi, 1982).

In the paper, the optimal routing and frames distribution assignment problem is considered. The problem consists of simultaneously finding the routes for both voice and data traffic and the distribution of frames between voice and data traffics which minimize the total average delay per packet such that the traffic requirements are satisfied.

2. Model Description

We assume that there are two classes of traffic in the considered network: packet–switched traffic (data) and circuit–switched traffic (voice). The assumption follows from the analysis of services available in the ISDN; the mentioned classes are representative for non real time and real time applications.

The properties of the ISDN can be taken into consideration in the theoretical model. The model of the considered system is the network consisting of a set of arcs, a set of nodes and capacity function. In the model arcs correspond to the channels of the ISDN network; the directed and the undirected arcs correspond to the unidirectional and to the half–duplex channels, respectively.

A capacity of each arc is equal to the capacity of the channel which corresponds to this arc. The capacity of each channel and the length of the transmission frame in this channel are interdependent.

The channel's capacity in the network supported different classes of traffic, expressed in number of slots, is partitioned into two compartments: circuit–

switched and packet–switched. Transmission of the considered classes, i.e., data and voice traffic in the ISDN correspond to the flows in the proposed model. They consist of multicommodity flow and non–bifurcated flow. The multicommodity flow and the non–bifurcated flow represent the transmission of the packet–switched traffic (data) and the circuit–switched traffic (voice), respectively.

For the multicommodity flow, a commodity consists of an ensemble of packets generated in the same node and transmitted to the same destination. For the non–bifurcated flow, a commodity consists of the ensemble of calls having the same source and the same destination (Batycki and Kasprzak, 1983).

Let the connected and directed graph $G =< N, L >$ represent the structure of the network $S; N$ denotes the set of nodes and L denotes the set of arcs. By definition, the multicommodity flow in the network S is the family of functions

$$f^k : L \rightarrow R^+ \cup \{0\}$$

which assign values $f^k(x,y)$ $(k = 1,2,...,q)$ to each arc $< x,y >\in L$. The value $f^k(x,y)$ is called a flow of commodity k on arc $< x,y >$. The arc flows must satisfy the following constraints.

i) For the conservation of the flow at each node, commodity by commodity, we have

$$\sum_{y\in A(x)} f^k(x,y) - \sum_{y\in B(x)} f^k(y,x) = \begin{cases} r_k & \text{for } x \in s_k \\ -r_k & \text{for } x = u_k \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

for every $x \in N$, $k = 1,2,...,q$.

ii) For nonnegativity of the flows, we have

$$f^k(x,y) \geq 0 \text{ for every } < x,y >\in L, \ k = 1,2,...,q \tag{2}$$

where $A(x) = \{y : y \in N \text{ and } < x,y >\in L\}$,
$B(x) = \{y : y \in N \text{ and } < y,x >\in L\}$,
r_k is the flow value to be send from the source s_k to the destination u_k (value of the commodity k), s_k is the source of the commodity k, u_k is the destination of the commodity k, q is the number of all commodities.

A multicommodity flow is defined to be non–bifurcated if each commodity flows along one route only (Fratta *et. al.*, 1973). There are two well–known criteria applied to evaluate the flow in the model (Williams and Leon–Garcia, 1984). The non–bifurcated flow and the multicommodity flow are evaluated using the blocking probability and the total average delay per packet, respectively.

The problem formulated and considered in this paper may be applied for design of the ISDN using the following assumptions:

- the topology of the ISDN and channels capacities are given,
- ISDN allows to transmit all data and voice calls due to traffic requirements among nodes given by a number of voice calls and a number of data packets in the time unit,

- there are not a lost voice calls in the ISDN.

Then the total average delay per packet may be only used as the quality coefficient for traffic flows in the ISDN. The choice of routes for voice calls (non–bifurcated flow in the considered model) has influence on the total average delay. It is because the increase of a capacity of circuit–switched compartment in the frame decreases – in the same time – the capacity available for the packet–switched traffic; the total average delay depends on the length of the packet–switched compartments.

Assuming the Kleinrock's model (Klenrock, 1964), i.e.,

- the external arrivals are Poissonian,
- the packet length distribution is exponential,
- the nodal storage is infinite,
- there is no node delay and no lost traffic,
- the interarrival and transmission times on each transmission channel are independent,

then the total average delay per packet is expressed as follows

$$T = \frac{1}{\gamma}\sum_{i=1}^{p}\frac{f_i}{c_i - f_i}$$

where f_i is the average bit rate on the channel i, c_i is the capacity of the channel i, γ is the total packet arrival rate from external sources, p is the number of all channels.

The routing and frames' distribution assignment problem in the ISDN is formulated as follows:

given: network topology,
channel capacities,
traffic requirements (for data and voice traffic),
minimize: total average delay per packet,
over: multicommodity flow,
non–bifurcated flow,
subject to: multicommodity flow and non–bifurcated flow satisfy the constraints (1) and (2),
arcs capacities constraints (total flow in each arc is not greater then their capacity).

The problem presented here is the nonlinear mixed–integer programming problem. The paper is divided to present the exact algorithm for the above formulated problem. Since this problem is NP–complete then a branch and bound method is used to construct this optimal algorithm.

3. Problem Formulation

Consider an ISDN consisting of n nodes and p channels. Let the unigraph $G =< N, L >$ represent the topology of the ISDN; N is the set of nodes and

$L =< w^1, w^2, ..., w^p >$ is the set of arcs which correspond to the channels of the ISDN.

Let c_0^i denote the capacity of the channel i. Let a capacity function c be defined as follows

$$c: \; c(w^i) = c_0^i \text{ for every } w^i \in L$$

Then, we obtain the network $S =< G; c >$ which is the model of the considered network.

Let there be q commodities in the network S. The commodities are ordered in such a way that the commodities numbered from 1 to e correspond to data or to voice calls to be send using circuit–switched technology (non–bifurcated flow). The commodities from $e+1$ to q correspond to data transferred using packet–switched technology (multicommodity flow).

Let $\{\pi_i^k, \; k = 1, 2, ..., l_i\}$ be a set of all possible routes from source to destination for commodity i, where $i = 1, 2, ..., e$; the commodity i may be transferred from its source to destination along exactly one route from this set. Each route π_i^k $(k = 1, 2, ..., l_i)$ for commodity $i (1 \leq i \leq e)$ has a previously designed capacity ρ_i. It means that in all the arcs belonging to the route discussed, a part of their capacity, equal to ρ_i, is reserved solely for the flow of commodity i $(1 \leq i \leq e)$. We assume further that for each commodity i $(1 \leq i \leq e)$ the value ρ_i is attributed arbitrarily. Obviously for every $i = 1, 2, ..., e$, $\rho_i \geq r_i$ where r_i is the value of commodity i.

Each route π_i^k $(1 \leq i \leq e)$ is associated with the binary variable z_i^k, and

$$z_i^k = \begin{cases} 1, & \text{if the commodity } i \text{ flows along the route } \pi_i^k \\ 0, & \text{otherwise.} \end{cases}$$

Since each commodity $i\,(1 \leq i \leq e)$ must be flown along one route only, then

$$\sum_{k=1}^{l_i} z_i^k = 1 \qquad \text{for every } i = 1, 2, ..., e \tag{3}$$

Let Y_r be a permutation of values of all variables z_i^k $(k = 1, 2, ..., l_i, \; i = 1, 2, ..., e)$ for which the condition (3) is satisfied and let $\widehat{Y}_r$ be the set of all variables which in Y_r are equal to one. Then each set $\widehat{Y}_r$ determines a unique non–bifurcated flow (i.e., the routes for commodities from 1 to e). Let $\mathbb{R}$ be the family of all permutation Y_r.

Let U_j^r be the sum of capacities of routes π_i^k such that the binary variables z_i^k belong to the set $\widehat{Y}_r$, containing the arc w^j, for commodities $i = 1, 2, ..., e$. Then

$$U_j^r = \sum_{i=1}^{e} \sum_{k=1}^{l_i} \rho_i z_i^k u_i^{kj} \qquad \text{for } j = 1, 2, ..., p$$

where

$$u_i^{kj} = \begin{cases} 1, & \text{if the arc } w^j \text{ is on the route } \pi_i^k \\ 0, & \text{otherwise} \end{cases}$$

Then, U_j^r and $c(w^j) - U_r^j$ are the parts of capacity of arc w^j designed for non–bifurcated and multicommodity flows, respectively. Thus, the value U_j^r defines the distribution of the capacity of arc w^j between two classes of traffic in the ISDN. It follows from the above considerations that each permutation Y_r determines the unique values U_j^r, $j = 1, 2, ..., p$.

Let $T(Y_r)$ be the minimal average delay per packet in the ISDN in which the routes for commodities from 1 to e, i.e., circuit–switched traffic are defined by value of variables z_i^k from the permutation Y_r. Using the Kleinrock's results (Klenrock, 1964; Kleinrock, 1976), $T(Y_r)$ can be obtained solving the following multicommodity flow problem in the network S (Grzech and Kasprzak 1991)

$$T(Y_r) = \min_{\underline{f}} \frac{1}{\gamma} \sum_{k=1}^{p} \frac{f(w^k)}{c(w^k) - U_k^r - f(w^k)} \tag{4}$$

subject to

$$\boldsymbol{f} \in \mathbb{F}_w \tag{5}$$

$$f(w^k) \leq c(w^k) - U_k^r \qquad \text{for every } w^k \in L \tag{6}$$

where $\boldsymbol{f}$ is the column vector of total flows of commodities from $e+1$ to q on all arcs in the network S, i.e., $\boldsymbol{f} = \left[f(w^1), f(w^2), ..., f(w^p)\right]^T$, $f(w^k) = \sum_{i=e+1}^{q} f^i(w^k)$ is the total flow of commodities numbered from $(e+1)$ to q on the arc w^k, $f^i(w^k)$ is the flow of the commodity i $(e+1 \leq i \leq q)$ on the arc w^k, $\mathbb{F}_w$ is the set of vectors $\boldsymbol{f}$ which correspond to a multicommodity flow.

The problem (4)–(6) can be solved using the FD method (Fratta *et. al.*, 1973). Then, the optimal flows and distribution of arcs capacities assignment problem in the model S can be formulated as follows

$$\min_{Y_r} T(Y_r) \tag{7}$$

subject to

$$Y_r \in \mathbb{R} \tag{8}$$

$$U_k^r \leq c(w^k) \text{ for every } w^k \in L \tag{9}$$

4. Calculation Scheme

The problem (7)–(9) is NP–complete (Batycki and Kasprzak, 1983). Then, a branch and bound method (Balas, 1967; Kasprzak, 1989) is used to construct the exact algorithm for solving this problem.

Let $Y_1 \in \mathbb{R}$ be the initial permutation. Starting with the permutation Y_1 we generate a sequence of permutation Y_r. Each permutation Y_s is obtained from a permutation Y_r by substitution $z_i^j = 0$, $z_i^k = 1$ (where $z_i^j \in \widehat{Y}_r$) which satisfies the condition (3); in the permutation Y_r we have $z_i^k = 0$, $z_i^j = 1$, and in the permutation Y_s we have $z_i^j = 0$, $z_i^k = 1$, while other values of variables in both permutations Y_s and Y_r are not changed. The generating process can be presented in the form of solution tree H. Each node in H corresponds to a permutation Y_r. Each arc in the solution tree H represents a pair of permutations (Y_r, Y_s) such that Y_s is obtained from Y_r by substitution $z_i^j = 0$, $z_i^k = 1$. We say that Y_s is the successor of Y_r if there is a path in the solution tree H between Y_r and Y_s. The initial permutation Y_1 is the root in H.

The generation of a new permutation involves the choice of a certain pair of variables from Y_r for substitution. This operation is called the choice operation. The choice of such variables is based on local optimization criterion. For each permutation Y_r from the sequence we perform a testing operation (i.e., lower bound) to check the possibility of the generation of the permutation Y_s with less total average delay per packet than that already found. If such Y_s does not exist, we abandon the considered permutation Y_r and all its successors and next backtrack to the predecessor Y_p from which Y_r was generated.

If the new permutation Y_s is generated from permutation Y_r by substitution $z_i^k = 1$, $z_i^j = 0$ ($z_i^j \in \widehat{Y}_r$), then we constantly fix the value of the variable z_i^k. It means that the values of all variables z_i^l ($l = 1, ..., l_i$) can not be changed in every possible successors of the permutation Y_s. If we backtrack in the solution tree H from the permutation Y_s to the permutation Y_r by the variable z_i^j ($z_i^j \in \widehat{Y}_s$ and $z_i^j \notin Y_r$) we momentarily fix this variable, then $z_i^j = 0$. If we backtrack $(l_i - 1)$-th times by z_i^j, we constantly fix the value of z_i^j ($z_i^j = 1$). So, for each permutation Y_r we constantly fix a set $F_r \subset \widehat{Y}_r$ and momentarily fix a set F_r^t. The variables in the set F_r which also belong to $\widehat{Y}_1$ are constantly fixed and represent the path from the root to Y_r in H. Each variable z_i^k in F_r, which not belongs to $\widehat{Y}_1$, is constantly fixed and represents $l_i - 1$ variables $z_i^1, z_i^2, ..., z_i^{k-1}, z_i^{k+1}, ..., z_i^{l_i}$ which have been abandoned during the backtracking process. The values of all variables in F_r are equal to one, and the values of all variables in F_r^t are equal to zero.

5. Testing Operation

The basic task of this operation is to compute the lower bound LB_r of the total average delay per packet for the permutation Y_r, and for every possible successor Y_s generated from Y_r.

The part of the capacity of each arc from the set L designed for the non–bifurcated flow in the network S can be described as the sum of two variables

$$U_k^r = U_k^{ru} + U_k^{rw},$$

where

$U_k^{ru} = \sum_{i,l:z_i^l \in F_r} \rho_i u_i^{lk}$ is the sum of capacities of routes containing the arc w^k, for which the variables z_i^l are equal to one and belong to F_r,

$U_k^{rw} = \sum_{i,l:z_i^l \in \widehat{Y}_r - F_r} \rho_i u_i^{lk}$ is the sum of capacities of routes containing the arc w^k, for which the variables z_i^l are equal to one in the permutation Y_r and do not belong to F_r.

We have, that $U_r^s \geq U_k^{ru}$ for any successor Y_s generated from the permutation Y_r, since in all successors of Y_r the values of variables from the set F_r remain unchanged.

The lower bound is obtained by continous relaxation of the constraint (1) which is resulted from the assumption that the variables U_k^{rw} $(k = 1, 2, ..., p)$ are independent. It is easy to observe, that the value of function (4) decreases if the values U_k^{rw}, $k = 1, 2, ..., p$ decrease. Let UL_r be the lower bound of the sum of values of all variables U_k^{rw} for the permutation Y_r and all its successors. The lower bound UL_r may be calculated using the procedure similar to that one presented in (Kasprzak, 1989), then

$$UL_r = \sum_{i:z_i^k \in \widehat{Y}_r - F_r} \rho_i \beta_i$$

where β_i is the number of arcs belonging to the route having minimum number of arcs, for commodity i

$$\beta_i = \min_{k:z_i^k \in B_i^r} \beta_i^k$$

$\beta_i^k = \sum_{l=1}^{p} u_i^{kl}$ is the number of arcs belonging to the route π_i^k

$$B_i^r = \left\{ z_i^1, z_i^2, ..., z_i^{l_i} \right\} - F_r^t$$

Then, the lower bound LB_r of the total average delay per packet can be obtained solving the following problem

$$\min_{f, U_k^{rw}, k=1,2,...,p} \frac{1}{\gamma} \sum_{k=1}^{p} \frac{f(w^k)}{c(w^k) - U_k^{ru} - U_k^{rw} - f(w^k)} \tag{10}$$

subject to

$$\boldsymbol{f} \in \mathbb{F}_w \tag{11}$$

$$f(w^k) \leq c(w^k) - U_k^{ru} - U_k^{rw} \quad \text{for every} \quad w^k \in L \tag{12}$$

$$\sum_{k=1}^{p} U_k^{rw} \geq UL_r \tag{13}$$

The minimization of the problem (10)–(13) can be carried out first on the variables U_k^{rw}, $k = 1, 2, ..., p$, keeping $\boldsymbol{f}$ fixed, and then on $\boldsymbol{f}$. The problem (10)–(13) can be solved over the variables U_k^{rw} $(k = 1, 2, ..., p)$ using the Kuhn–Tucker optimality conditions

$$\frac{1}{\gamma} \frac{f(w^k)}{(c(w^k) - U_k^{ru} - U_k^{rw} - f(w^k))^2} - \lambda + \lambda_k = 0, \qquad k = 1, 2, ..., p$$

$$\left(UL_r - \sum_{k=1}^{p} U_k^{rw} \right) \lambda = 0$$

$$\left(f(w^k) + U_k^{ru} + U_k^{rw} - c(w^k) \right) \lambda_k = 0, \qquad k = 1, 2, ..., p$$

It is easily seen that the Lagrange multipliers $\lambda_k = 0$, $k = 1, 2, ..., p$, because the function in (10) goes to ∞ whenever $\lambda > 0$ for some k. After some algebra and using the above Kuhn–Tucker optimality conditions, we obtain

$$U_k^{rw} = c(w^k) - U_k^{ru} - f(w^k) - \frac{\sqrt{f(w^k)} \left(\sum_{k=1}^{p} \left(c(w^i) - U_i^{ru} - f(w^i) \right) - UL_r \right)}{\sum_{k=1}^{p} \sqrt{f(w^i)}}$$

By introducing the expression of U_k^{rw} into the criterion function in (10), we have

$$T_{LB}^{r\,1} = \frac{\left(\sum_{k=1}^{p} \sqrt{f(w^k)} \right)^2}{\gamma \left(CUL_r - \sum_{k=1}^{p} f(w^k) \right)} \tag{14}$$

where

$$CUL_r = \sum_{k=1}^{p} \left(c(w^k) - U_k^{ru} \right) - UL_r$$

The function (14) is quasi–concave. This implies that there are local minima. Then, the function (14) can be bounded by convex function

$$T_{LB}^{r\,2} = \frac{\left(\sum_{k=1}^{p} \frac{f(w^k)}{\sqrt{c(w^k) - U_k^{ru}}} \right)^2}{\gamma \left(CUL_r - \sum_{k=1}^{p} f(w^k) \right)} \tag{15}$$

The function (15) is obtained from the function (14) multiplying each addend of sum in the numerator of (14) times

$$\sqrt{f(w^k)/(c(w^k) - U_k^{ru})}$$

Then, the lower bound of the total average delay per packet for every successors of Y_r can be obtained solving the following problem

$$\min_{f} T_{LB}^{r2} \tag{16}$$

subject to

$$f \in \mathbb{F}_w \tag{17}$$

$$f(w^k) \leq c(w^k) - U_k^{ru} \qquad \text{for every } \; w^k \in L \tag{18}$$

$$CUL_r - \sum_{k=1}^{p} f(w^k) \geq 0 \tag{19}$$

It is easily seen that the lower bound of the total average delay per packet for every possible successor generated from Y_r may be obtained solving the following problem (Kasprzak, 1989)

$$\min_{f} \left(T_{LB}^{r3} = \sum_{k=1}^{p} \frac{f(w^k)}{c(w^k) - U_k^{ru} - f(w^k)} \right)$$

subject to (17) and (18).

Let T be the function of average delay per packet in (10). For each f we have

$$T_{LB}^{r1} \leq T \; \text{ and } \; T_{LB}^{r2} \leq T$$

then

$$T \geq \alpha T_{LB}^{r1} + (1 - \alpha) T_{LB}^{r2} \; \text{ for } \; \alpha \in [0, 1]$$

Thus

$$LB_r = \min_{f} \left(\alpha T_{LB}^{r1} + (1 - \alpha) T_{LB}^{r2} \right) \tag{20}$$

subject to (17)–(19) and for $\alpha \in [0, 1]$.

It is easy to observe that the function (20) incorporates the constraints (18) and (19) as the interior penalty functions for $\alpha \in (0, 1)$. It guarantees the feasibility of the solution during the application of the FD method. Then, LB_r may be obtained solving the problem (20) subject to (17) and $\alpha \in (0, 1)$. This problem may be easily solved using the FD method (Fratta *et. al.*, 1973).

6. Choice Operation

The purpose of the choice operation is to find the variable $z_i^k \in \widehat{Y}_r$ for substitution $z_i^k = 0$, $z_i^l = 1$ and generating a successor Y_s of the permutation Y_r with the least possible value of the total average delay per packet. Let Y_s be obtained from Y_r. Then

$$
\begin{aligned}
&T(Y_s) - T(Y_r) = \\
&\sum_{k=1}^{p} \frac{f_s(w^k)}{c(w^k) - U_k^s - f_s(w^k)} - \sum_{k=1}^{p} \frac{f_r(w^k)}{c(w^k) - U_k^r - f_r(w^k)} = \\
&\sum_{k=1}^{p} \left(\frac{f_s(w^k)}{c(w^k) - U_k^s - f_s(w^k)} - \frac{f_r(w^k)}{c(w^k) - U_k^s - f_r(w^k)} \right) + \\
&\sum_{k=1}^{p} \left(\frac{f_r(w^k)}{c(w^k) - U_k^s - f_s(w^k)} - \frac{f_r(w^k)}{c(w^k) - U_k^r - f_r(w^k)} \right) = \Delta T_{sr}^f + \Delta T_{sr}^u
\end{aligned}
$$

where $f_r(w^k)$ is equal to the optimal value of $f(w^k)$ which is obtained solving the problem (4)–(6) for values of U_k^r defined by Y_r.

It follows from the above expression that ΔT_{sr}^u defines the increment of a value of the total average delay per packet in terms of U_k^r, keeping $f_r(w^k)$ $(k = 1, 2, \ldots, p)$ fixed. Also, ΔT_{sr}^f defines the increment of the value of the total average delay per packet in terms of $f_r(w^k)$, keeping U_k^s $(k = 1, 2, \ldots, p)$ fixed. We are interested in the way we do the change of routes for the same commodity i $(1 \le i \le e)$, i.e., we should indicate the way of the choice two variables $z_i^k \in \widehat{Y}_r$ and z_i^l substitution.

The best way evidently is to bound the change $T(Y_s) - T(Y_r)$. In our opinion, it is not necessary to evaluate the influence of the substitution on a change of the total average delay per packet (i.e., $T(Y_s) - T(Y_r)$), because this change may be approximated by ΔT_{sr}^u. Consequently, we will evaluate the influence of the substitution on ΔT_{sr}^u because such bounding is simpler from computation point of view.

It follows from the theorem formulated and proved by Batycki and Kasprzak (1983) that if the permutation Y_s is obtained from Y_r by substitution $z_i^k = 0$, $z_i^l = 1$, where $z_i^k \in \widehat{Y}_r$, then

$$
\Delta T_{sr}^u \ge \rho_i(l(\pi_i^l) - l(\pi_i^k)) = \Delta_{kl}^i \tag{21}
$$

where

$$
l(\pi_i^k) = \sum_{a=1}^{p} u_i^{ak} \left(\left. \frac{\partial T}{\partial U_k} \right|_{U_k = U_k^r,\ f(w^k) = f_r(w^k)} \right)
$$

Let $E_r = \widehat{Y}_r - F_r$, and let

$$M_r = \left(\bigcup_{i:z_i^k \in E_r} \{z_i^l : l = 1,2,...,l_i\} \right) - E_r$$

It follows from the above considerations that the choice criterion on variables z_i^k is the expression Δ_{kl}^{ir} . We should choose such variable z_i^k from the set E_r and such variable z_i^l $(z_i^l \in M_r)$ for which the value of Δ_{kl}^{ir} is minimal.

7. Algorithm

We start with $Y_1 \in \mathrm{IR}$, $F_1 = \emptyset$, $F_1^t = \emptyset$ and $T^* = +\infty$. Let Y_r be the current permutation and let F_r and F_r^t be the current sets of variables constantly or momentarily fixed in r–th iteration of the algorithm.

Step 1 *(test step)*
If $U_k^{ru} > c(w^k)$, $k = 1,2,...,p$, then go to step 4. Otherwise compute LB_r. If $LB_r > T^*$, then go to step 4. Otherwise go to step 2.

Step 2 *(evaluation step)*
Compute $T(Y_r)$ and identify the set M_r. Next perform $M_r := M_r - F_r^t$. If $T(Y_r) < T^*$, then put $T^* = T(Y_r)$. Next go to step 3.

Step 3 *(forward step)*
If $M_r = \emptyset$, then go to step 4. Otherwise choose the variable $z_i^k \in E_r$ and $z_i^l \in M_r$ for which the value Δ_{kl}^{ir} is minimal. Next generate the permutation Y_s by substitution $z_i^k = 0$, $z_i^l = 1$, i.e., by letting $\widehat{Y}_s = (\widehat{Y}_r \cup \{z_i^l\}) - \{z_i^k\}$, $F_s = F_r \cup \{z_i^l\}$, $F_s^t = F_r^t$. Go to step 1.

Step 4 *(backtracking step)*
Backtrack to the predecessor Y_p of the permutation Y_r. If Y_r has no predecessor then the algorithm terminates. The permutation Y_* associated with the current T^* is optimal, i.e., total average delay per packet $T(Y_*)$, and U_k^*, $k = 1,2,...,p$ are optimal in the ISDN. Otherwise, drop the data for Y_r and update the data for Y_p as follows

$$M_p := M_p - \{z_i^l\}, \quad F_p^t = F_p^t \cup \{z_i^l\}$$

If the backtracking is performed for $(l_i - 1)$–th time by the variable z_i^k then perform

$$F_p = F_p \cup \{z_i^k\} \quad \text{and} \quad F_p^t := F_p^t - \left(\bigcup_{a=1}^{l_i} \{z_i^a\} \right)$$

Go to step 1.

The above exact algorithm involves to find the initial permutation $Y_1 \in \mathrm{IR}$ for which the condition (9) is satisfied and the problem (4)–(6) has solution in the network S. To find such the feasible initial permutation Y_1 we can use the heuristic algorithm similar to the one, presented by Grzech and Kasprzak (1991).

8. Conclusions

It was assumed that there are two classes of traffic in the Integrated Services Digital Network: packet– and circuit–switched traffic for data and voice, respectively. The assumption follows from the analysis of services available in the ISDN. The mentioned classes are representative for non real time and real time applications.

The properties of the ISDN has been taken into consideration in the theoretical model. The channel's capacity, expressed in the number of slots, is partitioned into two compartments which support dofferent classes of traffic. Transmission of the considered classes, i.e., data and voice traffic in the ISDN corresponds to the flows in the proposed model and consists of multicommodity flow and non–bifurcated flow.

The problem formulated here is the nonlinear mixed–integer programming problem. The exact algorithm for the presented problem has been proposed and presented. The algorithm of solving the traffic classes integration problem is based on the branch and bound method.

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