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OPTIMAL SENSORS LOCATION FOR PARAMETER IDENTIFICATION OF DISTRIBUTED SYSTEMS*

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A computational method is proposed for optimal sensors location in parameter identification of two–dimensional parabolic distributed systems with noise–contaminated outputs. After a short survey, the concepts of optimality are discussed, followed by a derivation of the Fisher information matrix for systems with spatially varying parameters to be identified. The determinant of the information matrix and the sensitivity criterion are taken as measures of identification accuracy. Numerical examples illustrate the proposed approach.

1. Introduction

The development of parameter identification methods that are more effective and reliable has drawn researchers' attention for almost thirty years now, especially in distributed parameter systems (DPS), i.e. systems that are characterized mathematically by partial differential equations (PDE's) (see Kubrusly, 1977; Polis, 1982; Polis and Goodson, 1976; for survey papers). In spite of the fact that distributed models involve using very sophisticated mathematical methods, they make possible to describe the process more accurately and to implement more effective control strategies. Examples of applications areas abound. They include structural analysis and design where vibrations and dynamic behaviour are central, acoustics problems and design in the basic process industries, where heat and mass transfer and chemical reactions are important. They also concern geophysical analysis, including underground water and oil exploration, air and water pollution propagation and nuclear energetics (see Preprints of 5th IFAC Symp., 1989).

Although it is well known that estimation accuracy of DPS parameters depends significantly on the choice of experimental conditions, there have been very few contributions to the optimum experimental design for that systems. The problem is to choose the experimental conditions subject to constraints such that the expected accuracy is maximized in some sense. It should be noticed that the problem considered is essentially different from the optimal measurement problem for state estimation (see Kubrusly and Malebranche, 1985) since in the first one the state depends non–linearly on unknown parameters.

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Since it is generally impossible to measure the system state over the entire spatial domain, the problem of where to locate measurement devices becomes an important one if we want to reduce the cost of instrumentation and increase the efficiency of the identifiers.

There are three possible strategies of taking measurements:

- location of a given number of sensors (Kubrusly and Malebranche, 1988),
- using movable sensors (Carotenuto *et al.*, 1987),
- scanning, i.e. using only some sensors of all the stationary located ones at the moments when the measurements can be taken (Nakano and Sagara, 1988; Korbicz, 1991).

The above mentioned techniques involve the following problems, respectively:

- how to choose the minimal number of the sensors and their location,
- how to choose their optimal trajectories and velocities,
- how to choose optimal location of the sensors at given moments.

These topics have been widely studied by many authors for state estimation. However, only a few papers have appeared specifically on the optimal sensors location problem for DPS identification.

The existing methods can be divided into three main groups:

- methods leading to state estimation,
- methods leading to optimum experimental design,
- methods leading to random fields analysis.

This classification is shown in Figure 1.

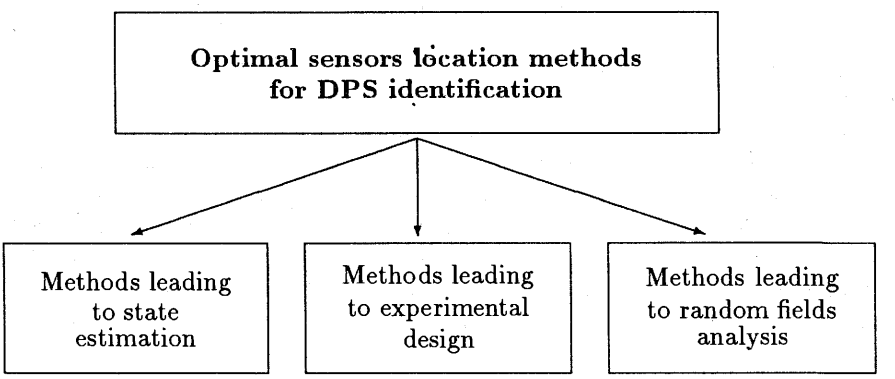


Fig. 1. Classification of the existing methods for sensors location.

The methods belonging to the first group seem to be flexible enough since their use involves relatively few constraints and the optimal sensors location problem for state estimation has already been explored widely (Korbicz and Zgurowski, 1991; El Jai and Pritchard, 1988; Kubrusly and Malebranche, 1985). However, the essential problem is that after augmentation of the state space for estimating parameters jointly with the state our task becomes very non–linear. Because of

that a sequence of linearizations at suitable trajectories is performed in (Malebranche, 1988) and a special suboptimal filtering algorithm is used in (Korbicz and Zgurowski, 1991). After transforming the parameter identification problem into the state estimation, one of the algorithms presented by Kubrusly and Malebranche (1985) can be applied. Nevertheless, practical implementation of many of them is restricted because the matrix Riccati equations for the filter error covariance matrix should be solved. Therefore, an upper bound for the filter covariance is used by Malebranche (1988) and the suboptimal filtering is implemented in (Korbicz and Zgurowski, 1991). In those works the sensors location problem is considered as an optimization one so that the trace of the covariance error matrix is minimized with respect to the dynamic constraints in the form of the Kalman filter equations.

The second class of the methods is closely related to classical optimum experimental design. System description in the form of deterministic PDE's with noisy observations is assumed. The optimal sensors location criteria are essentially the same: maximization of a scalar measure of the Fisher information matrix associated with the parameters to be identified. It is worth noting that the inverse of this information matrix is a solution of the Riccati equation, related to the state estimation problem, for the case of no input noise. In (Quereshi, 1980) two simple examples were given. In order to avoid computational difficulties sinusoidal excitations were assumed and the position of a single sensor optimized. The optimal design depends on unknown parameters observed in the above examples that is typical for experimental design in dynamic systems. This requires the necessity of certain *a priori* knowledge, such as nominal parameter values which can be obtained by a preliminary experiment or by physical analysis. In (Rafajłowicz, 1981) the information matrix was associated to the system eigenvalues rather than to the system parameters. Conditions for optimality of the experiment design were derived after some simplifications (among others the infinite observation time and restricted form of the inputs were assumed). Under the optimal experimental conditions one can observe sensors clusterization, i.e. sensors are assembled at some points of the spatial domain. This is because of the assumption that the measurement noises are produced independently by sensors placed very close to each other and the sensors dimensions are neglible. In other works (Rafajłowicz, 1984; Rafajłowicz and Myszka, 1986) much attention is focused on the problems of optimum input signal choice for DPS identification and in (Rafajłowicz, 1986b) an approach to the optimum choice of moving sensors trajectories is proposed.

The third group of the methods is very interesting for applications. This approach is based on random fields theory. Since DPS are described by PDE's, direct use of that theory is impossible. The work (Kazimierczyk, 1989) is typical for that group. Initial description for some mechanical system subjected to the action of a random load is reduced to two characteristics of a random field (the mean and the covariance function). Those characteristics contain unknown parameters of the initial description. The very important fact is that the qualitatively proper results of the design require the more terms in the Fourier series approximation of the re-

sponse of the system the more sensors positions are being optimized. For optimum sensors placement in random fields analysis the theory of optimum experimental design turns out to be very effective (Brimkulov *et al.*, 1986).

The use of the above mentioned methods is restricted because they involve both computational (the methods belonging to the first group) and realizing difficulties (the methods of the other groups). Apart from that the simulated results presented in the optimal sensors location literature have generally been developed for DPS with one–dimensional spatial domain and constant coefficients. Some generalizations are still expected in this connection.

In this paper, the relationship between the experimental conditions and the achievable accuracy in parameter estimation for parabolic systems is studied. The numerical solution is proposed based upon an optimization of the criterion that uses the Fisher information matrix. That approach enables us to consider two–dimensional systems with spatially–varying parameters and to study problems when no analytical solution exists.

2. Problem Statement

Distributed parameter systems are described in most cases by linear or non– linear partial differential equations. Assume that the system under consideration can be described by the following equation

$$\frac{\partial \boldsymbol{X}(t,\boldsymbol{z})}{\partial t} = \boldsymbol{f}\left(t,\boldsymbol{z},\boldsymbol{X},\frac{\partial \boldsymbol{X}}{\partial z_1},\ldots,\frac{\partial \boldsymbol{X}}{\partial z_3},\frac{\partial^2 \boldsymbol{X}}{\partial z_1^2},\ldots,\frac{\partial^2 \boldsymbol{X}}{\partial z_3^2},\boldsymbol{U},\boldsymbol{\Theta}\right) \tag{1}$$

$$\boldsymbol{z} \in \Omega, \quad t \in (0,t_f]$$

where $\boldsymbol{f}$ is a known vector function continuously differentiable with respect to z and t and twice continuously differentiable with respect to the remaining parameters, $\boldsymbol{z} = [z_1,\ldots,z_3]^T$ – the spatial coordinate vector, $\boldsymbol{X}(t,\boldsymbol{z})$ – the state vector, $\boldsymbol{U}(t,\boldsymbol{z})$ – the input vector, $\boldsymbol{\Theta}$ – the unknown system parameter vector, Ω – the open spatial domain with smooth boundary $\partial\Omega$, $\overline{\Omega} = \Omega \cup \partial\Omega$.
The initial and boundary conditions for (1) are given, respectively, by

$$\boldsymbol{X}(0,\boldsymbol{z}) = \boldsymbol{X}_0(\boldsymbol{z}), \qquad \boldsymbol{z} \in \Omega \tag{2}$$

and

$$\boldsymbol{g}(t,\boldsymbol{z},\boldsymbol{X},\frac{\partial \boldsymbol{X}}{\partial \boldsymbol{z}}) = 0, \qquad \boldsymbol{z} \in \partial\Omega \tag{3}$$

where $\boldsymbol{g}$ is some known function.

We assume that measurements of the state $\boldsymbol{X}(t,\boldsymbol{z})$ are made continuously in time at N fixed points $\{\boldsymbol{z}^j\}_{j=1}^N \subset \Omega_{ad} \subseteq \Omega$ and the observation system is non–inertial. Then the observation equations are algebraic and given by

$$\boldsymbol{Y}_j(t) = \boldsymbol{h}_j(t, \boldsymbol{X}(t, \boldsymbol{z}^j)) + \varepsilon(t, \boldsymbol{z}^j), \qquad t \in [0, t_f] \tag{4}$$

$$j = 1, ..., N$$

Here Ω_{ad} is the part of $\overline{\Omega}$ where the measurements can be made, $\boldsymbol{h}_j$ – some scalar function that describes the j–th sensor and $\varepsilon(t, \boldsymbol{z}^j)$ denotes the measurement noise (usually it is assumed that the noise is gaussian, spatial uncorrelated and white). The observation scheme is shown in Figure 2.

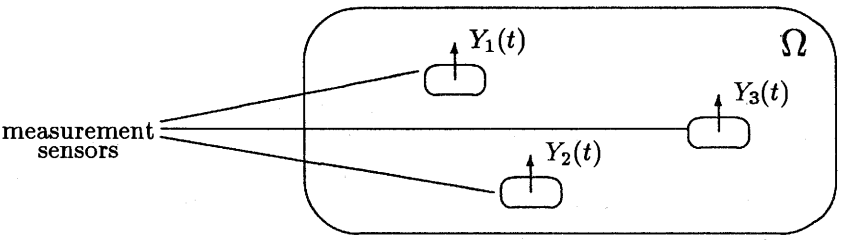


Fig. 2. The measurement process.

We formulate the parameter identification problem as follows: given the model (1) and the outcomes of the measurements $\boldsymbol{Y}$ at the points $\{\boldsymbol{z}^j\}_{j=1}^N$ and the input $\boldsymbol{U}$, it is necessary to estimate the parameter vector $\boldsymbol{\Theta}$ which minimizes the least–squares performance criterion, i.e. to find $\widehat{\boldsymbol{\Theta}}$ such that

$$\widehat{\boldsymbol{\Theta}} = \arg \min_{\boldsymbol{\Theta} \in \Omega_{ad}} \sum_{j=1}^{N} \int_0^{t_f} \|\boldsymbol{Y}_j(t) - \boldsymbol{h}_j(t, \widehat{\boldsymbol{X}}(t, \boldsymbol{z}^j; \boldsymbol{\Theta}))\|^2 dt \tag{5}$$

where $\widehat{\boldsymbol{X}}(t, \boldsymbol{z}; \boldsymbol{\Theta})$ denotes the solution of the equation (1) for the given parameter $\boldsymbol{\Theta}$ and $\|\cdot\|$ – the euclidean norm. When the measurements are taken under noisy condition we are faced with new problems, namely where to locate the sensors and what input signals should be used in order to achieve the best accuracy of the estimates of unknown parameters in some sense.

There are two widely used estimation accuracy criteria for solving the above–mentioned problems:

i) maximizing or minimizing some scalar measure based on so called the average Fisher information matrix that is defined as follows (Quereshi *et al.*, 1980)

$$\overline{\boldsymbol{M}}(\boldsymbol{z}^1, ..., \boldsymbol{z}^N) \triangleq \frac{1}{t_f} \sum_{j=1}^{N} \int_0^{t_f} \frac{\partial \boldsymbol{X}(t, \boldsymbol{z}^j)^T}{\partial \boldsymbol{\Theta}} \frac{\partial \boldsymbol{h}_j(t, \boldsymbol{X}(t, \boldsymbol{z}^j))^T}{\partial \boldsymbol{X}} \sum^{-1} \times$$

$$\times \frac{\partial \boldsymbol{h}_j(t, \boldsymbol{X}(t, \boldsymbol{z}^j))}{\partial \boldsymbol{X}} \frac{\partial \boldsymbol{X}(t, \boldsymbol{z}^j)}{\partial \boldsymbol{\Theta}} dt \tag{6}$$

where $\sum = \text{diag}\{\text{var}\,\varepsilon(t,z^j), j = 1,...,N\}$; in most cases the determinant $\det \overline{M}$ is maximized (the D–optimality criterion) or the trace of the inverse $\text{trace}\,\overline{M}^{-1}$ is minimized (the A–optimality criterion); this approach has come down from optimum experimental design for lumped parameter systems (LPS);

ii) minimizing the mean square error (Korbicz and Zgurowski, 1991; Malebranche, 1988)

$$J(\Theta(z^1,...,z^N)) \triangleq E\left[\sum_{j=1}^{N}\int_0^{t_f}\|Y_j(t)-h_j(t,\widehat{X}(t,z^j;\Theta(z^1,...,z^N))\|^2 dt\right] \tag{7}$$

In the further considerations we are going to restrict our attention to the problem of sensors location only by assuming that the input signal is already given.

We could also refer to other equivalent forms of DPS descriptions to be met in literature. Thus, in research papers that are more theoretical in character, advanced functional analysis is used (theory of strongly continouous semigroups of linear operators) (Banks and Kunisch, 1989; Curtain and Pritchard, 1978; Kubrusly and Malebranche, 1985; Omatu and Seinfeld, 1989). In some applications it is convenient to take advantage of the notion of Green function (Rafajłowicz, 1983; 1984; 1986b). On the other hand its use is restricted to relatively simple systems.

3. Optimal Sensors Location for Processes Described by Parabolic PDE's

Let us study first a simple example of a single sensor placement by using the optimum experimental design methods.

Example 1. Consider the scalar, one–dimensional heat conduction system governed by

$$\frac{\partial X(t,z)}{\partial t} = \mu\frac{\partial^2 X(t,z)}{\partial z^2} \qquad z\in(0,1),\ t\in(0,t_f] \tag{8}$$

$$\text{B.C.}\quad X(t,0) = X(t,1) = 0 \qquad t\in(0,t_f]$$

$$\text{I.C.}\quad X(0,z) = \sin\pi z \qquad z\in[0,1]$$

where $\mu > 0$. In this case the point measurement is available such that

$$Y(t) = X(t,z^1) + \varepsilon(t,z^1) \tag{9}$$

where ε denotes the measurement noise (assumed gaussian, spatial uncorrelated and white). The noise is assumed stationary with variance σ^2. The diffusivity μ is assumed to be unknown. It is necessary to calculate the optimal sensor position for estimating μ.

This system has a unique solution continuous in z expressed by $X(t,z) = \sin\pi z\cdot\exp(-\mu\pi^2 t)$. The average information matrix (here it is a scalar) takes the following form

$$\overline{M}(z)=\frac{1}{\sigma^2 t_f}\int_0^{t_f}\left(\frac{\partial X}{\partial \mu}\right)^2 dt=\frac{1}{\sigma^2 t_f}\int_0^{t_f}\left[-\pi^2 t\sin \pi z\exp(-\mu\pi^2 t)\right]^2 dt=$$

$$\sin^2\pi z\left[\frac{1}{\sigma^2 t_f}\pi^4\int_0^{t_f} t^2\exp(-2\mu\pi^2 t)dt\right]=\text{const}\cdot\sin^2\pi z,\quad \text{const}>0$$

Hence, $\det\overline{M}(z)$ attains its maximum at $z=0.5$ and this is the optimal sensor location.

One can ask about a greater number of sensors to be located. If there are N sensors, the performance index is expressed by

$$\det\overline{M}(z^1,...,z^N)=\frac{1}{\sigma^2 t_f}\sum_{j=1}^{N}\int_0^{t_f}\left(\frac{\partial X(t,z^j)}{\partial\mu}\right)^2 dt=\text{const}\sum_{j=1}^{N}\sin^2\pi z^j$$

and attains its maximum iff $z^1=z^2=...=z^N=0.5$, but that means locating all the sensors at one point!

That phenomenon is more general. If there is only one parameter θ to be identified and noisy system state is directly observed, then $\det\overline{M}$ can be expressed in the following form

$$\det\overline{M}(z^1,...,z^N)=\sum_{j=1}^{N}r(z^j) \tag{10}$$

where

$$r(z)=\frac{1}{\sigma^2 t_f}\int_0^{t_f}\left(\frac{\partial X(t,z)}{\partial\mu}\right)^2 dt$$

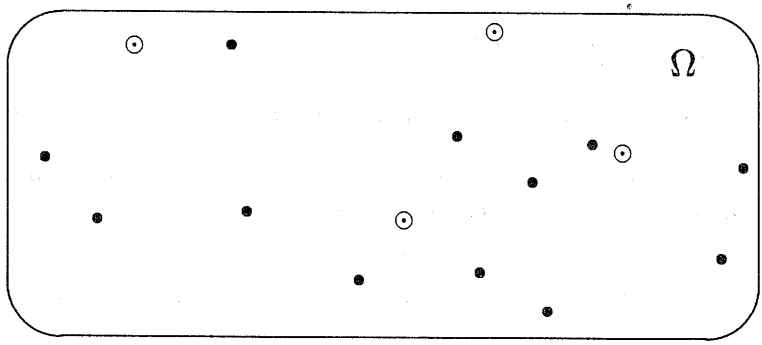
If the function $r(z)$ has only one global maximum at the point $z=z_0$, then it is evident that $\overline{M}(z^1,...,z^N)$ attains its global maximum iff $z^1=z^2=...=z^N=z_0$. But this occurence is only apparently strange. It results from the assumption that measurements made by one sensor have no influence on measurements made by the other one and that spatial dimensions of the sensors can be neglected.

The following approach is most often used in order to avoid the possibility that several of the points might be clustered in a small region. Let us assume that there should be located N sensors. It is useful to prescribe *a priori* a set of possible locations $\boldsymbol{z}^i$, $i=1,2,...,M$, where $M>N$. Then we seek the best set of N locations from among the M possible. It is shown in Figure 3.

This formulation in no way restricts the generality of the problem, since M can in principle be chosen as large as desired (if the system is solved numerically, the maximum value of M is the number of grid points in the domain Ω).

It should be noticed that in most cases the optimum experimental conditions depend on the unknown parameters and on the form of the input signal. The implication is simply that the design should be based on nominal parameters values

which can be obtained by a preliminary experiment or by physical analysis. The problems of optimum input signal choice were discussed by Rafajłowicz (1986a).



• – points where the sensor can be located
⊙ – points where the sensor is located after optimization

Fig. 3. Avoiding clusterization.

4. Computation of Information Matrix

Let us assume that the process under consideration is described by the following parabolic differential equation

$$\frac{\partial X(t,\boldsymbol{z})}{\partial t} = \nabla_{\boldsymbol{z}}(\mu(\boldsymbol{z})\nabla_{\boldsymbol{z}}X(t,\boldsymbol{z})) + U(t,\boldsymbol{z}) \tag{11}$$

$$\text{B.C.} \quad X(t,\boldsymbol{z}) = 0, \qquad \boldsymbol{z} \in \partial\Omega, \quad t \in (0,t_f]$$

$$\text{I.C.} \quad X(0,\boldsymbol{z}) = X_0(\boldsymbol{z}), \qquad \boldsymbol{z} \in \Omega$$

Within the framework of the finite element or the finite difference approximation (see for example, Twizell, 1984) , after discretizing the whole space domain, solutions to (10) are governed by a system of first–order differential equations in time of the form

$$\boldsymbol{A}(\mu)\widetilde{\boldsymbol{X}}(t) + \boldsymbol{B}\frac{d}{dt}\widetilde{\boldsymbol{X}}(t) = \widetilde{\boldsymbol{U}}(t) \tag{12}$$

$$\text{I.C.} \quad \widetilde{\boldsymbol{X}}(0) = \widetilde{\boldsymbol{X}}_0$$

where $\widetilde{\boldsymbol{X}}(t)$ denotes the nodal approximation solution vector, $\widetilde{\boldsymbol{U}}(t)$ – the nodal excitation vector, and $\boldsymbol{A}(\mu)$ and $\boldsymbol{B}$ are the global coefficient matrices. $\boldsymbol{A}(\mu)$ contains coefficients depending on the unknown parameters, whereas the coefficients of $\boldsymbol{B}$ are constant.

Assume that the function μ of the spatial coordinates can be approximated by the following expression to any desired accuracy by choosing a sufficiently large number of terms n_p

$$\mu = \mu(c) = \sum_{\nu=1}^{n_p} c_\nu \phi_\nu(z), \; c \in \mathbb{R}^{n_p} \tag{13}$$

For given n_p, the approximating functions $\phi_\nu(z)$, $\nu = 1, ..., n_p$ are known and linearly independent and the coefficients c_ν, $\nu = 1, ..., n_p$ are to be identified. By differentiating both sides of (12) with respect to c_ν we obtain the following system of the sensitivity equations

$$\boldsymbol{A}(\mu)\widetilde{\boldsymbol{X}}_\nu(t) + \boldsymbol{B}\frac{d}{dt}\widetilde{\boldsymbol{X}}_\nu(t) = -\boldsymbol{D}_\nu\widetilde{\boldsymbol{X}}(t) \tag{14}$$

$$\text{I.C.} \quad \widetilde{\boldsymbol{X}}_\nu(0) = \mathrm{O}$$

where

$$\widetilde{\boldsymbol{X}}_\nu = \left.\frac{\partial\widetilde{\boldsymbol{X}}}{\partial c_\nu}\right|_{\mu=\mu(c)} \qquad \boldsymbol{D}_\nu = \left.\frac{\partial \boldsymbol{A}}{\partial c_\nu}\right|_{\mu=\mu(c)}$$

The above equations can be used in order to obtain the sensitivity functions $\partial\widetilde{\boldsymbol{X}}/\partial c_\nu$, $\nu = 1, ..., n_p$ that are necessary to calculate the information matrix (see (6)).

5. Measurement Optimization Problem

The general algorithm can be expressed as follows

(i) solve the system (12) in order to obtain $\widetilde{\boldsymbol{X}}$,

(ii) solve the system (14) for $\nu = 1, ..., n_p$,

(iii) choose the best (in some sense) set of locations z^j, $j = 1, 2, ..., N$ from among all the points of the spatial grid.

Direct minimization of trace $\overline{\boldsymbol{M}}^{-1}$ or maximization of $\det \overline{\boldsymbol{M}}$ can involve very time–consuming computational efforts if there are more than one parameter to be identified. Hence, an approximate solution can be achieved indirectly by maximizing the trace of the information matrix (Nakamori 1980; Ghosh 1989). This approach leads to increased sensitivity of the outputs with respect to the unknown parameters. In this sense we call trace $\overline{\boldsymbol{M}}$ *the sensitivity criterion.*

The sensors location problem can now be formulated as the following linear 0–1 integer programming problem

$$\underset{\delta_i \in \{0,1\}}{\text{maximize}} \quad J = \sum_{i=1}^{M} \delta_i \, \text{trace} \, \overline{\boldsymbol{M}}_i \tag{15}$$

subject to the constraint

$$\sum_{i=1}^{M} \delta_i = N$$

where

$$\overline{M}_i = \frac{1}{t_f}\int_0^{t_f} \nabla_{\mathbf{c}} X(t, \boldsymbol{z}^i) \nabla_{\mathbf{c}}^T X(t, \boldsymbol{z}^i) dt$$

and $\boldsymbol{z}^i$ denotes a possible sensor location (a grid point), $i = 1, ..., M$.

6. Numerical Examples

To illustrate the application of the proposed algorithm, the two-dimensional diffusion process is considered. It is described by the following equation

$$\frac{\partial X(t, z_1, z_2)}{\partial t} = \frac{\partial}{\partial z_1}\left(\mu(z_1, z_2)\frac{\partial X(t, z_1, z_2)}{\partial z_1}\right) + \\ + \frac{\partial}{\partial z_2}\left(\mu(z_1, z_2)\frac{\partial X(t, z_1, z_2)}{\partial z_2}\right) + U(t, z_1, z_2) \tag{16}$$

$$(z_1, z_2) \in \Omega = (0, 2) \times (0, 2), \quad t \in (0, 0.5]$$

with the appropriate boundary and initial conditions.

The observation process is represented by

$$Y_j(t) = X(t, z_1^j, z_2^j) + \varepsilon(t, z_1^j, z_2^j), \ j = 1, ..., N \tag{17}$$

where N is the number of sensors. This means that the state is measured directly at each observation point. Furthermore it is assumed that the measurement errors are gaussian, spatial uncorrelated and white with respect to time and space.

The state and sensitivity equations were solved by the finite element method. The sampling interval and coordinate divisions were $\Delta t = 0.05$ and $\Delta z_1 = \Delta z_2 = 0.2$, respectively.

The objective is to estimate the diffusion coefficient $\mu(z_1, z_2)$ based on the measurements as accurately as possible over the period of observations. In order to avoid clusterization only the nodes of the spatial grid are allowed for sensors location. Using the above mentioned algorithms, two numerical experiments were performed.

Variant 1. Let us assume the following form for the diffusion coefficient to be identified

$$\mu(z_1, z_2) = 1 + 2z_1 + 2z_2$$

Thus (see (13)), $\phi_1(z_1, z_2) = 1$, $\phi_2(z_1, z_2) = z_1$, $\phi_3(z_1, z_2) = z_2$ and $c = [1\ 2\ 2]^T$. Let us also assume that $U(t, z_1, z_2) = 0$ and

I.C.	$X(0, z_1, z_2) = 0,$	$(z_1, z_2) \in \Omega$
B.C.	$X(t, z_1, z_2) = 100,$	$(z_1, z_2) \in \partial\Omega, \quad t \in (0, 0.5]$

Figures 4 and 5 show the surface plot and the contour map for the sensitivity criterion trace $\overline{M}$ for one sensor. It attains its maximum at the point (0.8, 0.8). This means that sensors should be placed as close to that point as possible. For example, if there are three sensors, they would be located at the points (0.8, 0.6), (0.8, 0.8) and (0.6, 0.8). It is noteworthy that the initial and boundary conditions and the form of the diffusion coefficient assume one axis of symmetry for that system (it is a line $z_1 - z_2 = 0$) and that axis remains the same if we consider the contour lines for the sensitivity criterion (see Figure 5).

Variant 2. Another simulation experiment was performed for an assumed diffusion coefficient

$$\mu(z_1, z_2) = 1$$

and

$$\text{I.C.} \quad X(0, z_1, z_2) = 50, \qquad (z_1, z_2) \in \Omega$$

$$\text{B.C.} \quad X(t, z_1, z_2) = \begin{cases} 200, & (z_1, z_2) \in (0,1) \times \{0\}, \qquad t \in (0, 0.5] \\ 0, & (z_1, z_2) \in \partial\Omega \backslash (0,1) \times \{0\} \qquad t \in (0, 0.5] \end{cases}$$

and

$$U(t, z_1, z_2) = 10 \exp(z_1 \cdot z_2)$$

The input signal plots are presented in Figures 6 and 7. The appropriate results (for the D–optimality criterion) are shown in Figures 8 and 9. In such a case it is difficult to guess where the sensor should be placed: closer to the maximum of the input signal or to the boundary perturbation. The results suggest that the best choice is to locate the sensor at the point (1.0, 0.4).

7. Concluding Remarks

The relatively simple numerical method to locate sensors for parameter identification has been given. It enables us to consider two– or three–dimensional spatial domains. This approach can also be applied for identification of the unknown spatially–varying parameters. By using it we are able to solve problems when any analytical solution does not exist.

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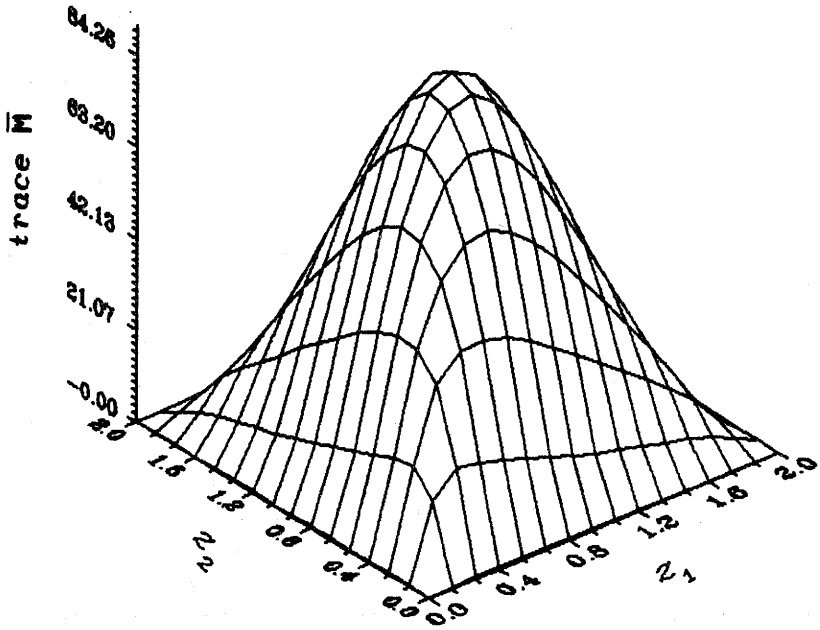

Fig. 4. The sensitivity criterion for Variant 1.

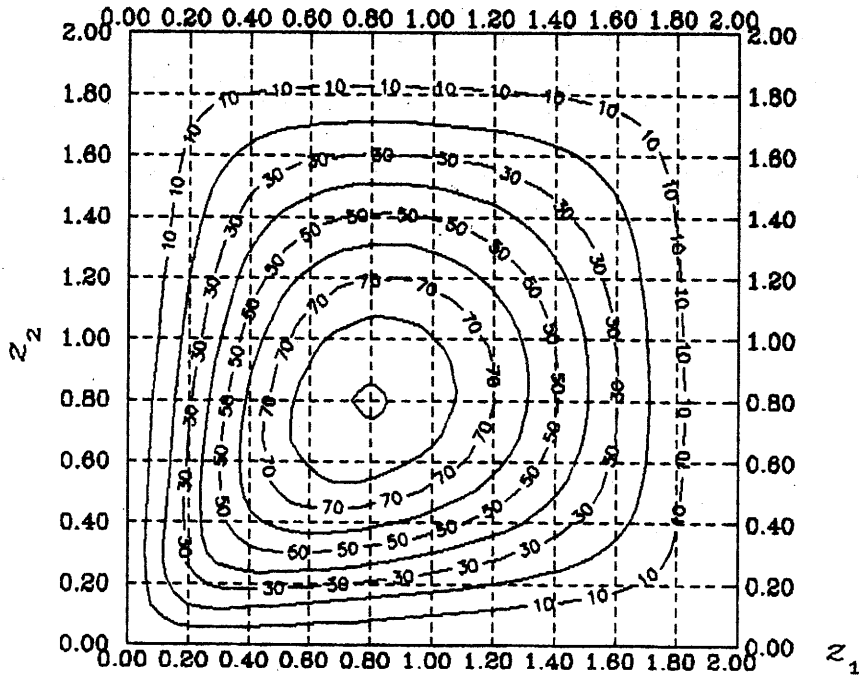

Fig. 5. The contour lines for Variant 1.

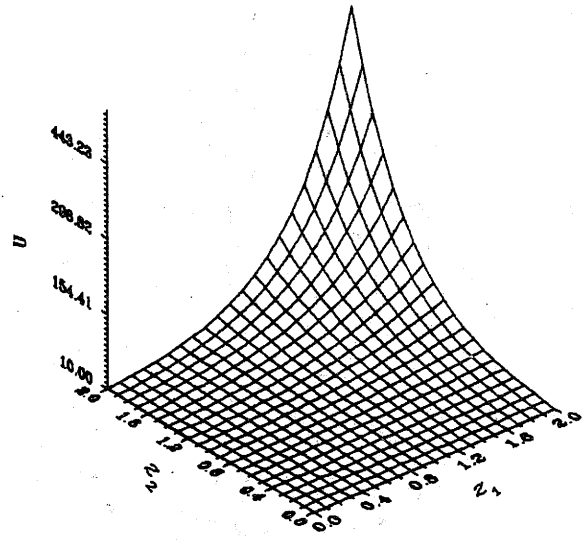

Fig. 6. The surface plot for the input signal (Variant 2).

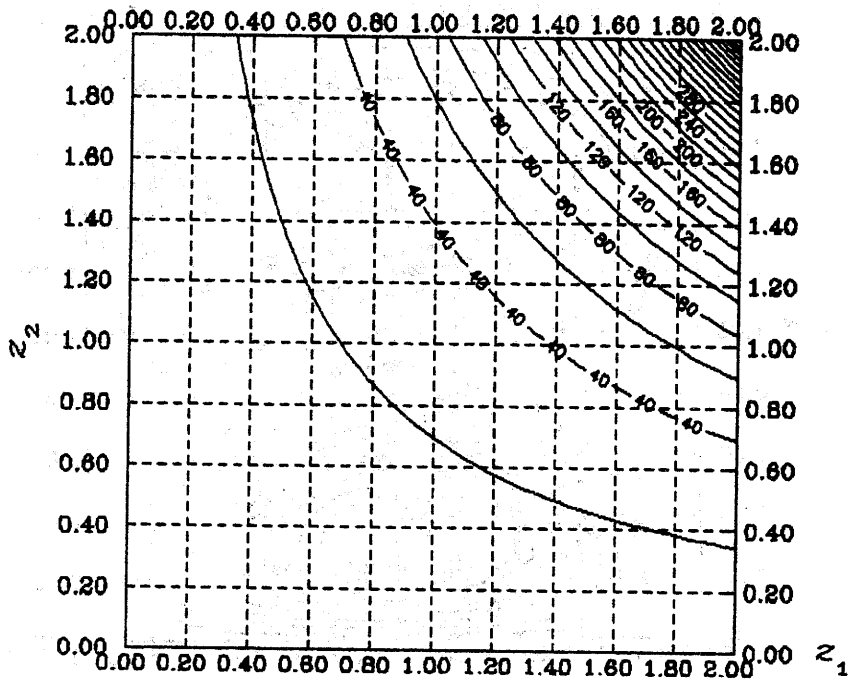

Fig. 7. The contour map for the input signal (Variant 2).

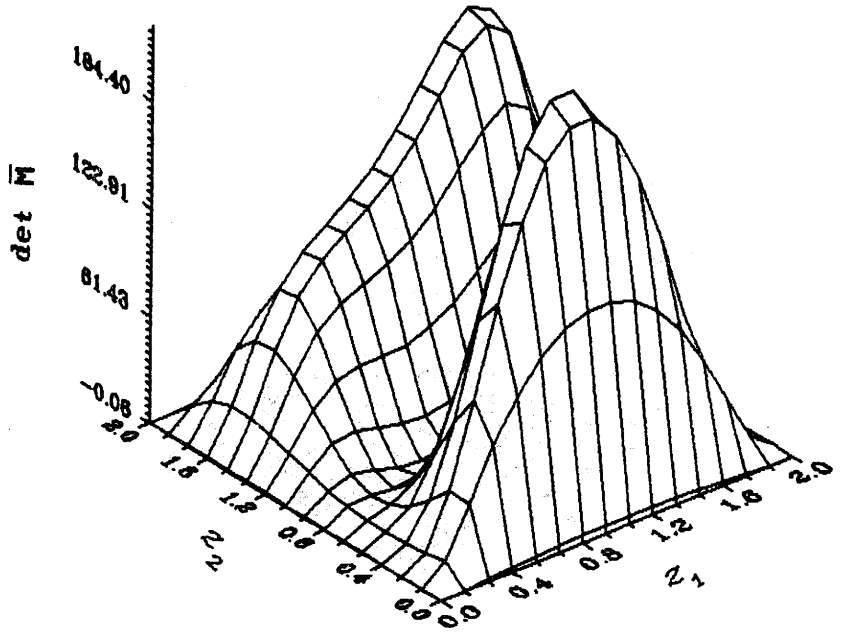

Fig. 8. The determinant of the information matrix for Variant 2.

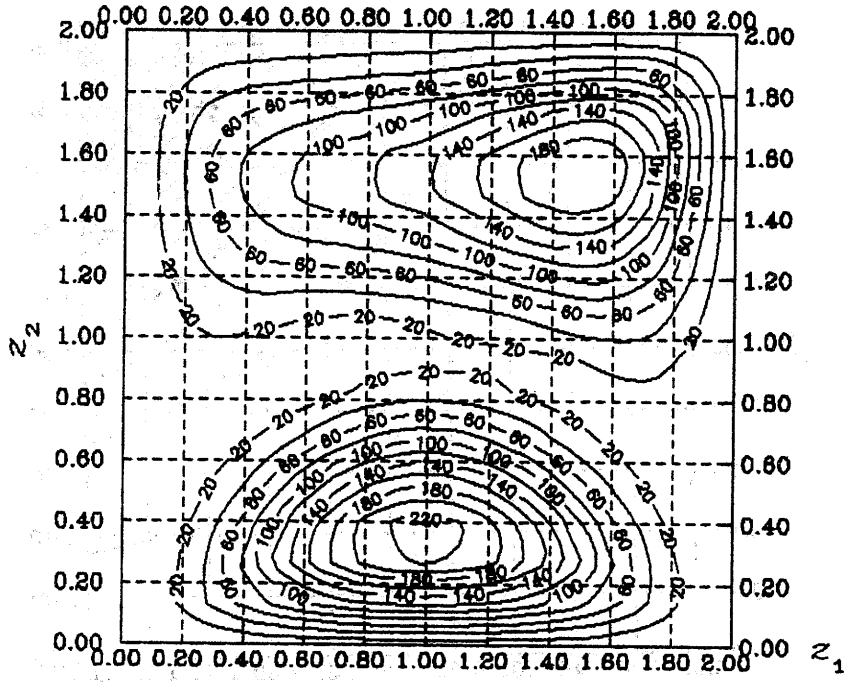

Fig. 9. The contour lines for Variant 2.