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DIAGNOSTIC INFERENCE BASED ON PREDICTED EFFECTIVE TRAJECTORY

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The technical and operational state of a object is characterized by so called, describing quantity set. Those quantities among others are random functions of time. Each of the describing quantities has an upper and a lower constraint. Using an object in a time interval ΔT guarantees the achievement of a reliable effect (product). The effect value is a random function of the initial describing quantity values, describing a quantity process of realization of changes in the ΔT time interval, constraint function realizations, random failure distribution and also the object operation time ΔT.

1. Introduction

The technical and operational states of an object can be characterized by quantities of finite (in overall) physical quantity number called describing quantities. Taking advantage of the knowledge in the prognostic process about the changes of the describing quantity values in the past instants and the previously used conditions one predicts those quantity values in instants and prospective conditions in the prognostic process.

The changes of describing quantity process modelling problem and the prediction of their course are the subject of many works by authors specializing in mathematical and control theory. The present paper shall not deal with this problem. The most important prognostic process element is the interference of an object state based with up–to–date and prognostic courses of the describing quantities.

From the diagnostic point of view one should clearly distinguish the instantaneous ability and the task ability of the object. The instantaneous ability (e.g. operating ability) for task realization exists when the object efficiency (instantaneous productivity) satisfies the user's requirements. An engine real power is an example of the efficiency.

The object using effect depends, first of all on the object using properties and also on using conditions and object steering stimulations. Therefore, in predicting the expected effect as the result of realizing prospecting tasks we should take into account the mentioned qualities.

The notion of an effect can be understood as any object operating effect; we can, therefore, interpret the effect e.g. in technical, economic, operational safety

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categories. As an example of an effect is the work done by an engine in the task realization time interval.

The diagnostic reality interpretation requires to distinguish:

the technical object state: describing quantity values' set of structural, material and technical object properties,

the operating object state: quantity values' set describing the operating process occurring in the object,

failure: an event which consists of object transition from the ability to the unserviceable state,

stable nonoperating state: a state as a result of a failure and hold constant (does not disappear without therapeutic operations),

falling off nonoperating state: most frequently caused by disturbances following a cause ceasing.

Failure modes are, as a rule, different from nonoperating modes. The failures can be classified in, at least, three categories:

Ageing failures. Therefore are characterized commonly by an analogous change of an object technical or operating state describing quantities and by hereby induced analogous efficiency change to a level exceeding one of the earlier accepted values. The describing quantities may be, in this case, expressed by a polynomial, and its coefficients are random values (with determinable distribution parameters).

Random independent failures. They are characterized commonly by a jump in an object technical or operating describing quantities change and caused thereafter a jump efficiency change to a level exceeding one of the earlier accepted values. Pathogenesis of those failures is not known. We know only their some statistical properties (e.g. failure rate).

Random dependent failures. They are characterized, similar to the random independent failures, by a jump in describing quantity change. Their pathogenesis is known at least in the range of statistical dependence of the individual describing values the object operating conditions (within also of the object steering excitations) and also of the object operating time.

Ageing failures can be put in the comparatively easy predicted failure category. Independent random failures, obvious regards to, we should classify into the unpredictable failure category. Dependent random failures may be classified as hard to predicted failure category.

2. Assumptions

1) The technical describing quantity set is known.
2) One of describing quantity measurement and their course of prediction are taken in accordance with the well known rules.
3) The efficiency is characterized in any instance by the object operating and technical states.

4) The object utility effect is, in mathematical understanding, an object efficiency integral in the considered time interval of the task realization.
5) The measure of the object task ability is the expected effect value in the considered time interval.
6) The independent random failures intensity is constant.
7) Within the object independent and dependent random failures there are also possible ageing failures.
8) The dependent failures intensity is an object utility time function and also one of the describing quantity values.
9) The utility effect is created only in the object which keeps its instantaneous ability in the full task realization time interval ΔT.
10) The efficiency dependence of object describing quantities on object state and utility conditions are well known.
11) The object efficiency values and constraint functions are known.

3. The Measure of Object Ability

The object *instantaneous ability in the time instant* t for utility conditions Ω is the efficiency

$$W(t) = W(L(t), \Omega(t)) \tag{1}$$

where

$$L(t) = \{l_i(t)\}, \qquad i = 1, 2, ..., I \tag{2}$$

is object state describing quantities set in the time instant t.

Let us assume, that every describing quantity can be expressed by a polynomial in the form of

$$l_i(t) = l_{i,0} + k_{i,1}t + k_{i,2}t^2 + ... + k_{i,v_i}t^{v_i}, \quad i = 1, 2, ..., I \tag{3}$$

where $k_{i,1}, k_{i,2}, ..., k_{i,v_i}$ are polynomial coefficients of the describing quantity l_i, $l_{i,0}$ is describing quantity initial value l_i in the time instant t_0.

The object *instantaneous ability criterion* has a form of inequality

$$W_{\min}(t) \leq W(t) \leq W_{\max}(t) \tag{4}$$

where

$$W_{\min}(t) = W(L_{\min}(t), \Omega(t)) \tag{5}$$

is minimal acceptable object efficiency in the time instant t for conditions Ω;

$$W_{\max}(t) = W(L_{\max}(t), \Omega(t)) \tag{6}$$

is maximal acceptable object efficiency in the time instant t for conditions Ω.

The describing quantity values, at which the object efficiency achieves the accepted value (minimal or maximal one) creates constraint hyperfaces:

$$L_{\min}(t) = \{l_{i\min}(t)\}, \qquad i = 1, 2, ..., I \tag{7}$$

$$L_{\max}(t) = \{l_{i\max}(t)\}, \qquad i = 1, 2, ..., I \tag{8}$$

whereby, analogous with (3), we accept that

$$l_{i\min}(t) = l_{i\min,0} + k_{i_d,1}t + k_{i_d,2}t^2 + ... + k_{i_d,v_{i_d}}t^{v_{i_d}}, \quad i = 1, 2, ..., I \tag{9}$$

$$l_{i\max}(t) = l_{i\max,0} + k_{i_g,1}t + k_{i_g,2}t^2 + ... + k_{i_g,v_{i_g}}t^{v_{i_g}}, \quad i = 1, 2, ..., I \tag{10}$$

Usually it is desired that the instatenteneous ability criterion is fulfilled in the entire time interval, in which the utility task is realized, i.e.:

$$\bigwedge_{t \in \Delta T} (W_{\min}(t) \le W(t) \le W_{\max}(t)) \tag{11}$$

where $\Delta T = [t_0, t_0 + T]$ is task realization time interval.

The interval measure of object ability (in the time interval ΔT) is the object utility effect in this interval. In accordance with the assumptions this effect is an efficiency integral

$$F(\Delta T) = \int_{\Delta T} W(t)\,\mathrm{d}t \tag{12}$$

Let us denote

$$\mu = \sum_{i=1}^{i=I} v_i \qquad \mu_d = \sum_{i=1}^{i=I} v_{i_d} \qquad \mu_g = \sum_{i=1}^{i=I} v_{i_g} \tag{13}$$

and assume that:

- all polynomial coefficients (3) (total) create a multidimensional random variable with a distribution
$$f_k = f(k_1, k_2, ..., k_\mu)$$
- all polynomial coefficients (9) (total) create a multidimensional random variable with a distribution
$$f_{k_d} = f(k_{1_d}, k_{2_d}, ..., k_{\mu_d})$$
- all polynomial coefficients (10) (total) create a multidimensional random variable with a distribution
$$f_{k_g} = f(k_{1_g}, k_{2_g}, ..., k_{\mu_g})$$

In accordance with the accepted ability criterion (4), the object is considered instantaneous unoperational when the efficiency adopted values are taken outside the accepted interval. In this connection the object may be not always utilized in the entire time interval ΔT. When an ageing or a random failure comes in this interval, the object quits the task realization beginning from the instant $t < t_0 + T$, and the effect obtained after this time instant is lost (in accordance with the assumptions).

The effect created by the object in the time interval ΔT is, as can be seen, a random functional:

$$F(\Delta T) = F(\Delta T;\ L(t_0), L_{\min}(t_0), L_{\max}(t_0), K, K_d, K_g, \Omega(t)) \tag{14}$$

where:

$$K = \{k_j\} \qquad j = 1, 2, ..., \mu$$
$$K_d = \{k_r\} \qquad r = 1, 2, ..., \mu_d$$
$$K_g = \{k_s\} \qquad s = 1, 2, ..., \mu_g$$

Even with deterministic sets of initial values $L(t_0)$, $L_{\min}(t_0)$ and $L_{\max}(t_0)$ and also with deterministic time interval ΔT, the polynomial coefficients (3), (9) and (10) are random variables and only their distributions f_k, f_{k_d} and f_{k_g} are known. Apart from this the time of random failure is also a random variable.

Regarding the random character of the effect it is desirable to utilize the expected value.

It has the form:

$$\overline{F}(\Delta T) = \overline{F}(\Delta T;\ L(t_0), L_{\min}(t_0), L_{\max}(t_0), f_k, f_{k_d}, f_{k_g}, g(\Theta)$$
$$= \int_T^{\infty} g(\Theta)\, \mathrm{d}\Theta \int\!\!\int_{\chi} \cdots \int f_k f_{k_d} f_{k_g} F(\Delta T)\, \mathrm{d}k_1 ... \mathrm{d}k_{\mu}\, \mathrm{d}k_{1_d} ... \mathrm{d}k_{\mu_d}\, \mathrm{d}k_{1_g} ... \mathrm{d}k_{\mu_g} \tag{15}$$

where:

$$g(\Theta) = \big(\lambda_1 + \lambda_2(T)\big) \exp\Big(-\big(\lambda_1 T + \int_0^T \lambda_2(t)\, \mathrm{d}t\big)\Big) \tag{16}$$

is time distribution Θ until a random failure occurs.

In the upper expression we denote:

$\lambda_1 =$ const is independent failure intensity, $\lambda_2(t)$ is dependent failure intensity, and $\chi = \chi(\Delta T;\ L(t_0), L_{\min}(t_0), L_{\max}(t_0))$ is $(\mu + \mu_d + \mu_g)$–dimensional area in the polynomial coefficients space (3), (9) and (10) for which the inequality (4) is fulfilled in the time interval ΔT.

4. Effect Trajectories

The effect expected value as a function of task realization time interval ΔT, for various object categories, allows to determine the effect trajectories, i.e. the effect expected changes of graphic images as a function of time.

Let us distinguish some effect trajectories.

1) The object effect trajectory:
   - of a random unfailuring, i.e. an object with failure intensities: $\lambda_1 = 0$ and $\lambda_2(t) = 0$;
   - with efficiency equal to the expected value, maximal accepted, i.e.

$$W(t) = \overline{W}_{\max}(t) \quad \text{for} \quad t \geq t_0$$

It is the ideal object trajectory, a theoretical one. Let us denote it as the *optimum trajectory*. It is described by an expression:

$$\overline{F}(\Delta T) = \overline{F}_{\mathrm{opt}}(\Delta T) = \int_{\Delta T} \overline{W}_{\mathrm{max}}(t)\,\mathrm{d}t \tag{17}$$

An example of such a trajectory is the line $\overline{F}_{\mathrm{pes}}(\Delta T)$ shown in Figure 1.

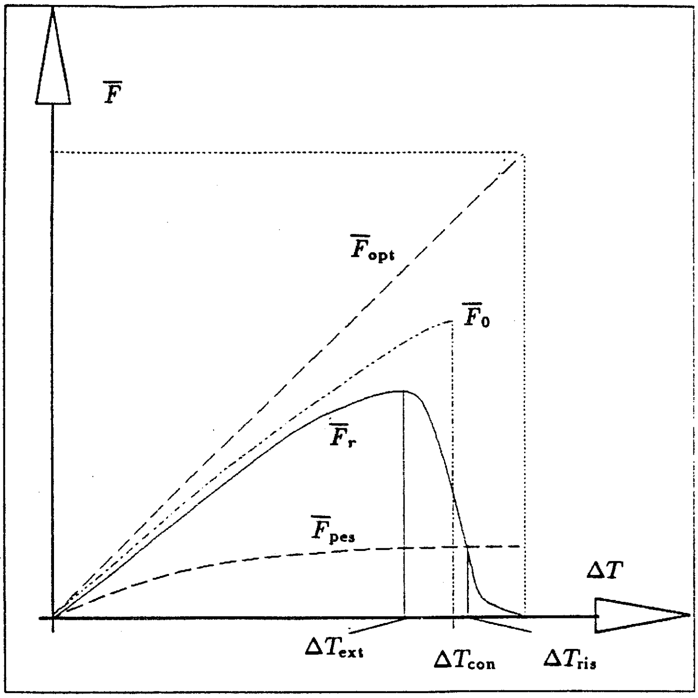

Fig. 1. The illustration of expected effect changes illustration as a function of utility task realization time interval for:
- a theoretical object, $\overline{F}_{\mathrm{opt}}$;
- a real object, predictable, $\overline{F}_0$;
- a real object, actual, $\overline{F}_r$;
- a real object, the worst acceptable, $\overline{F}_{\mathrm{pes}}$.

2) The object effect trajectory:

– random failuring with maximal and constant failure intensity:

$$\lambda_{\mathrm{max}} = \lambda_1 + \lambda_2\mathrm{max} = \mathrm{const}$$

where:

$$\lambda_{2\mathrm{max}} = \max_{t \in \Delta T}\{\lambda_2(t)\}$$

– with efficiency equal to the expected minimal value of the accepted efficiency, it is

$$W(t) = \overline{W}_{\min}(t) \quad \text{for} \quad t \geq t_0$$

This trajectory will be denoted pessimal. It is given by an expression:

$$\overline{F}_{\text{pes}}(\Delta T) = R(\Delta T;\ \lambda_{\max}) \int_{\Delta T} \overline{W}_{\min}(t)\,\mathrm{d}t \tag{18}$$

An example of such a trajectory is the curve $\overline{F}_{\text{pes}}(\Delta T)$, shown in Figure 1.

3) The object effect trajectory. In the object there occurs independent random failures, no dependent random failures are present and the ageing changes are determined and well known.

It is a trajectory which enables an exact and fully trustworthy prediction of ageing changes. Such case, except theoretical, is an ideal case from the prognostic point of view.

The effect trajectory is here described by an expression:

$$\overline{F}_0(\Delta T) = \begin{cases} \exp(-\lambda_1 \Delta T) \int_{\Delta T} W(t)\,\mathrm{d}t & \Leftarrow \Delta T \leq \Delta T_{\text{con}} \\ 0 & \Leftarrow \Delta T > \Delta T_{\text{con}} \end{cases} \tag{19}$$

where T_{con} is time to the determined ageing failure, $\Delta T_{\text{con}} = [0, T_{\text{con}}]$.

An example of such a trajectory is the curve $\overline{F}_0(\Delta T)$ shown in Figure 1.

4) The effect trajectory of an object subject to random independent ageing failures.

The random failures of the statistical parameters and also of the describing quantity changes are well known.

Such a trajectory is an image of an effect creation through a real object. It is determined by the expression (15).

An example of such a trajectory is the curve $\overline{F}_r(\Delta T)$ in Figure 1.

Let us note, that for each real effect trajectory and each task realization time interval $\Delta T \leq \Delta T_{\text{con}}$ we have:

– efficiency

$$W_{\min}(t) \leq W(t) \leq W_{\max}(t) \quad \text{for} \quad t \in \Delta T$$

– the summary intensity of independent and dependent random failures

$$\lambda_1 \leq \lambda(t) \leq \lambda_{\max} \quad \text{for} \quad t \in \Delta T$$

where $\lambda(t) = \lambda_1 + \lambda_2(t)$ is summary failure intensity.

Therefore, each real effect trajectory lies:

– below the optimal trajectory, i.e.:

$$\overline{F}_r(\Delta T) \leq \overline{F}_{\mathrm{opt}}(\Delta T)$$

– above the pessimal trajectory in a certain interval T_{ris}, i.e.:

$$\overline{F}_r(\Delta T) \geq \overline{F}_{\mathrm{pes}}(\Delta T) \quad \Leftarrow 0 < \Delta T \leq \Delta T_{\mathrm{ris}}$$

It means that the optimal and pessimal trajectories determine a real and accepted field, regarding the object task ability, of effective trajectories.

Notice, that the trajectory $\overline{F}_r(\Delta T)$ aims at trajectory $\overline{F}_0(\Delta T)$, when:

- the describing quantity polynomial coefficient distributions aims at to single-point distributions,
- the intensity of random independent failures aim at zero,
- the random dependent failure intensity aims at zero or the intensity function aims at the unitary jump in an instant, when the efficiency achieves the boundary value (i.e. for $T = T_{\mathrm{con}}$),
- the efficiency function aims at maximal efectivity.

5. Task Ability Criteria

Criterion No 1. (absolute).

It is possible to determine the expected effect trajectory, i.e. a trajectory

$$\overline{W}(t) = W(t;\ \overline{k}_1, \overline{k}_2, ..., \overline{k}_\mu)$$

where $\overline{k}_j;\ j = 1, 2, ..., \mu$ is the polynomial coefficient (3) expected values.

Denote: T_{exp} is the first intersection time of the efficiency trajectory expected value with one limiting trajectory function ($\overline{W}_{\mathrm{min}}(t)$ or $\overline{W}_{\mathrm{max}}(t)$), i.e.:

$$T_{\mathrm{exp}} : \overline{W}(T_{\mathrm{exp}}) = \overline{W}_{\mathrm{con}}(T_{\mathrm{exp}}) \tag{20}$$

and

$$\Delta T_{\mathrm{exp}} = [0, T_{\mathrm{exp}}]$$

The object is able to carry out a task which lasts no longer than ΔT_{exp}, that means $\Delta T \leq \Delta T_{\mathrm{exp}}$.

Criterion No 2. (absolute).

Denote: ΔT_{ris} is time interval, for which the expected effect value trajectory $\overline{F}_r$ of the real object intersects with the pessimal trajectory, i.e.:

$$\Delta T_{\mathrm{ris}} : \overline{F}_r(\Delta T_{\mathrm{ris}} = \overline{F}_{\mathrm{pes}}(\Delta T_{\mathrm{ris}}) \tag{21}$$

The object is able carry out a task which lasts no longer than ΔT_{ris}, i.e.: $\Delta T \leq \Delta T_{\mathrm{ris}}$.

Criterion No 3. (absolute).

Denote: ΔT_{ext} is time interval, for which the effect expected value trajectory $\overline{F}_r$ of the real object achieves the maximal value, i.e.:

$$\Delta T_{\text{ext}} : \left. \frac{\mathrm{d}\overline{F}_r(\Delta T)}{\mathrm{d}\Delta T} \right|_{\Delta T = \Delta T_{\text{ext}}} = 0 \tag{22}$$

It this case each task lasting no longer than ΔT_{ext}, i.e.: $\Delta T \leq \Delta T_{\text{ext}}$ shall be accepted.

The most advantageous case exists when $\Delta T = \Delta T_{\text{ext}}$.

Criterion No 4. (relative).

Denote: ΔT_{tol} is time interval, for which the effect expected value trajectory $\overline{F}_r$ of the real object intersects with the tolerance trajectory:

$$\overline{F}_{\text{tol}}(\Delta T) = C_{\text{tol}} \overline{F}_{\text{opt}}(\Delta T) \tag{23}$$

where $C_{\text{tol}} < 1$ is tolerance coefficient.

Hence:

$$\Delta T_{\text{tol}} : \overline{F}_r(\Delta T_{\text{tol}}) \overline{F}_{\text{tol}}(\Delta T_{\text{tol}}) \tag{24}$$

The object is able to realize a task which lasts no longer then ΔT_{tol}, i.e.: $\Delta T \leq \Delta T_{\text{tol}}$.

Other criteria are also possible in accordance with the users requirements.

6. Final Remarks

The effect trajectory method can be utilized when the below mentioned information is known.

1. A list of prevailed quantities describing the technical and operational object states.
2. The information about the physical nature of the efficiency and by the object produced effect (i.e. the definition of those quantities in a physical sense).
3. The information of statistical parameters of the object dependent random failures (i.e. about failures intensity) or its elements.
4. The information of dependent random failures statistical parameters and about the relationship between those parameters, object technical and operational state describing parameters.
5. The information of statistical parameters concerning changes of the quantity describing the object technical and operating states related to time and object utility conditions.
6. The information about the dependence of efficiency on the quantities describing the technical and operational object state.

7. Diagnostic information concerning the prognostic instant (i.e. for $t = 0$) which allows to determine:
   - the object ability probability in the prognoses instant,
   - the describing quantity values in the prognoses moment of time.
8. The information of antropotechnic pair (user – object) utility properties, which allows to determine:
   - the object failure results in regard to the effect (damage or preserving of the effect present in failure instant),
   - the user's type (requirement level regarding trustworthiness and prediction instant).

The effect trajectory method is effective in almost all cases when:
- the independent random failure intensity is small,
- the independent random failure intensity accepts significant values only in the near neighborhood of the ageing failure instant,
- the describing quantity polynomial coefficients distributions have a considerably converge form.

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