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MULTIDIMENSIONAL SPECTRAL FACTORIZATION THROUGH THE REDUCTION METHOD OF MULTIDIMENSIONAL POLYNOMIAL FACTORIZATION

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In this paper, the (unsolved) Spectral Factorization problem, in m dimensions, is considered. A pure mathematical solution is attempted through the m-D polynomial factorization method of the Reduction. The necessary and sufficient condition is proved. The proposed method, which is also automated via a suitable computer code, is illustrated by a two-dimensional example.

1. Introduction

Multidimensional (m-D) Systems have recently attracted attention of many researchers and practitioners. The reasons are their increasing mathematical interest and their extensive technical applications (digital filter design, image processing, computer-aided tomography, design of passive sonar arrays, seismic data processing, underwater acoustics, etc.). Linear and Shift Invariant (LSI) m-D systems can be described by partial difference/differential equations, m-D transfer functions (that are ratios of m-D polynomials) and appropriate state-space models. The characteristic polynomial of all these models are polynomials in m variables which are called multivariable or multidimensional (m-D) polynomials.

So, factorization of m-D polynomials is among the primary processes in the m-D systems field, since it helps in performing simpler realizations (Galkowski, 1994), simpler stability tests and simpler controllers. However, factorizing an m-D polynomial is not a simple task since most of the available 1-D theorems and techniques are not applicable to the m-D case. Up to now, several methods have been proposed for the m-D polynomial factorization. In (Mastorakis *et al.*, 1994), a new powerful method for factorizing an m-D polynomial is presented. According to this method, we separate the variables $z_1, \ldots, z_m$ of the m-D polynomial into two sets of m_1 and $m-m_1$ variables ($m_1 < m$). Let us denote the complex vectors of the m, m_1, $m-m_1$ variables by z, $\bar{z}$, z', respectively. The constant 1 is with the vector z'. The given $f(z)$ polynomial is written as a sum of three terms. These terms are polynomials of z, $\bar{z}$, z' respectively. So, we write $f(z) = u(\bar{z}) + l(z') + w(z)$. Since the constant 1 has been included in z' the constant term of $f(z)$ is included in $l(z')$. Then, always following the method, possible factors of $f(z)$ are the polynomials $q_l(z') - r \cdot s_k(\bar{z})$ with $q_l(z')$, $s_k(\bar{z})$ the factors of $l(z')$ and $u(\bar{z})$ respectively and r is a constant. We

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check the validity of two theorems (see Appendix) and the constant r is simultaneously evaluated. This method factorizes a wide class of m-D polynomials actually by factorizing two other polynomials of less $(m_1, m - m_1)$ variables. In the case of 2-D polynomials, the factorization of two 1-D polynomials is needed which is always possible (numerically). This method is called in (Mastorakis *et al.*, 1994) as the method of Reduction.

One of the most significant and popular problems in the signal processing is the problem of the *spectral factorization* (Dudgeon, 1975; Dudgeon and Mersereau, 1984; Ekstrom and Woods, 1976a; 1976b; Ekstrom and Twogood, 1977a; 1977b; Orfanidis, 1990; Pistor, 1974). In a 1-D case, this problem is stated as follows: *Given* a 1-D polynomial $b(z)$, *Find* a "stable" 1-D polynomial $f(z)$ such that

$$f(z)f(z^{-1}) = b(z)b(z^{-1}) \tag{1}$$

The polynomial $f(z)$ is said to be stable if it corresponds to a stable LSI system. In other words, the polynomial $f(z)$ is stable if $f(z) \neq 0$ for $|z| \leq 1$. The 1-D spectral factorization problem always has a solution (Dudgeon and Mersereau, 1984; Orfanidis, 1990).

In the m-D case, the 1-D solution cannot be extended in a straightforward manner. The reason is that not ever m-D polynomial is factorized (Mastorakis and Theodorou, 1993). Several different practical approaches and engineering methods, however, have been adopted for the solution of the problem (Dudgeon, 1975; Ekstrom and Woods, 1976a; 1976b; Ekstrom and Twogood, 1977a; 1977b; Pistor, 1974) but a mathematical solution does not exist.

In this paper, the m-D Spectral Factorization problem is stated and a mathematical solution is presented by using the main results of the m-D factorization algorithm presented in (Mastorakis *et al.*, 1994) and briefly discussed above. An example is also included.

2. Multidimensional Spectral Factorization

2.1. Statement of the Problem

In m-D case, the problem is stated as follows:

Let the m-D polynomial

$$b(z_1, \ldots, z_m) = \sum_{i_1=0}^{N_{b,1}} \cdots \sum_{i_m=0}^{N_{b,m}} a_b(i_1, \ldots, i_m) z_1^{i_1} \ldots z_m^{i_m} \tag{2}$$

where $N_{b,1}, \ldots, N_{b,m}$ are the degrees of $b(z_1, \ldots, z_m)$ with respect to $z_1, \ldots, z_m$.

Find a "stable" m-D polynomial $f(z_1, \ldots, z_m)$

$$f(z_1, \ldots, z_m) = \sum_{i_1=0}^{N_{f,1}} \cdots \sum_{i_m=0}^{N_{f,m}} a_f(i_1, \ldots, i_m) z_1^{i_1} \ldots z_m^{i_m} \tag{3}$$

where $N_{f,1}, \ldots, N_{f,m}$ are the degrees of $f(z_1, \ldots, z_m)$ with respect to $z_1, \ldots, z_m$. such that

$$f(z_1, \ldots, z_m) f(z_1^{-1}, \ldots, z_m^{-1}) = b(z_1, \ldots, z_m) b(z_1^{-1}, \ldots, z_m^{-1}) \tag{4}$$

A polynomial $f(z_1, \ldots, z_m)$ such that the transfer function $1/f(z_1, \ldots, z_m)$ corresponds to a stable m-D system is meant by the term a "stable" polynomial. Obviously, in the case where $b(z_1, \ldots, z_m)$ is already a stable polynomial, the obvious solution

$$f(z_1, \ldots, z_m) = b(z_1, \ldots, z_m) \tag{5}$$

there exists. Now, the question is: For what other polynomials $b(z_1, \ldots, z_m)$ eqn. (4) has a solution. Before giving a complete answer to this question, some very important results of the m-D systems stability theory should be presented. More specifically the following theorems have been already proved.

Theorem 1. (Shanks *et al.*, 1972) *The m-D polynomial $f(z_1, \ldots, z_m)$ is stable if and only if*

$$f(z_1, \ldots, z_m) \neq 0 \quad \text{for} \quad |z_1| \leq 1 \quad \text{and} \ \ldots \ \text{and} \quad |z_m| \leq 1$$

Theorem 2 is a more applicable Theorem than Theorem 1.

Theorem 2. (Anderson and Jurry, 1974) *The m-D polynomial $f(z_1, \ldots, z_m)$ is stable if and only if*

$$1) \quad f(z_1, 0, \ldots, 0) \neq 0 \quad \text{for} \quad |z_1| \leq 1$$

and

$$2) \quad f(z_1, z_2, \ldots, 0) \neq 0 \quad \text{for} \quad |z_1| = 1 \quad \text{and} \quad |z_2| \leq 1$$

and

$$\vdots$$

and

$$m) \quad f(z_1, z_2, \ldots, z_m) \neq 0 \quad \text{for} \ |z_1| = 1 \ \text{and} \ \ldots \ \text{and} \ |z_{m-1}| = 1 \ \text{and} \ |z_m| \leq 1$$

To test the various conditions of the above theorems several tests have been proposed. These test are easier to be applied than the above theorems.

Schur-Cohn test, Table test, Zeheb-Walach test, Bose and Basu test, etc are some of the popular tests that are used to check the conditions of Theorems 1 and 2. For a detailed discussion see (Tzafestas, 1986).

In the present paper, one definition is still given.

Definition 1. The polynomial $f(z_1, \ldots, z_m)$ is *perfectly unstable* if and only if the polynomial $z_1^{N_{f,1}} \ldots z_m^{N_{f,m}} \cdot f(z_1^{-1}, \ldots, z_m^{-1})$ is stable. The numbers $N_{f,1}, \ldots, N_{f,m}$ have been defined as the degree of $f(z_1, \ldots, z_m)$ with respect to $z_1, \ldots, z_m$.

The proof of the following Lemma is achieved by using the conditions of Theorems 1 and 2.

Lemma

1. *The product of two or more than two stable polynomials is a stable polynomial.*
2. *The product of two or more than two perfectly unstable polynomials is a perfectly unstable polynomial.*

2.2. Solution of the m-D Spectral Factorization Problem

In this paragraph, the m-D spectral factorization problem is faced as follows.

In order to find a stable polynomial $f(z_1, \ldots, z_m)$ as a solution to eqn. (4), the m-D polynomial factorization method, presented in (Mastorakis *et al.*, 1994) and briefly discussed above, is used. Using this method the polynomial $b(z_1, \ldots, z_m)$ is factorized (if it is possible).

Suppose that the following factorization is obtained.

$$b(z_1, ..., z_m) = b_1(z_1, ..., z_m) \ldots b_k(z_1, ..., z_m)u_1(z_1, ..., z_m) \ldots u_l(z_1, ..., z_m) \quad (6)$$

where

$$b_i(z_1, \ldots, z_m) = \sum_{i_1=0}^{N_{b_i,1}} \cdots \sum_{i_m=0}^{N_{b_i,m}} a_{b_i}(i_1, \ldots, i_m) z_1^{i_1} \ldots z_m^{i_m} \qquad i = 1, \ldots, k \quad (7)$$

are stable factors, while

$$u_i(z_1, \ldots, z_m) = \sum_{i_1=0}^{N_{u_i,1}} \cdots \sum_{i_m=0}^{N_{u_i,m}} a_{u_i}(i_1, \ldots, i_m) z_1^{i_1} \ldots z_m^{i_m} \qquad i = 1, \ldots, l \quad (8)$$

are perfectly unstable ones.

Let us define

$$f(z_1, \ldots, z_m) = \prod_{j=1}^{k} b_j(z_1, \ldots, z_m) \prod_{j=1}^{l} z_1^{N_{u_j,1}} \ldots z_m^{N_{u_j,m}} \cdot u_j(z_1^{-1}, \ldots, z_m^{-1}) \quad (9)$$

Therefore

$$f(z_1^{-1}, \ldots, z_m^{-1}) = \prod_{j=1}^{k} b_j(z_1^{-1}, \ldots, z_m^{-1}) \prod_{j=1}^{l} z_1^{-N_{u_j,1}} \ldots z_m^{-N_{u_j,m}} \cdot u_j(z_1, \ldots, z_m) \quad (10)$$

Obviously, $f(z_1, \ldots, z_m)$ is a stable polynomial (Lemma) and it satisfies eqn. (4). Now, we are ready to formulate the following theorem.

m-D Spectral Factorization Theorem. *The problem of finding a stable polynomial $f(z_1, \ldots, z_m)$ satisfying eqn. (4), has a solution* if and only if *the polynomial $b(z_1, \ldots, z_m)$ is factorized in a product only of stable and perfectly unstable factors (Eqn. (6)). The solution, in this case, is given by eqn. (9).*

Proof. Sufficient: It has been already been proved analytically before the statement of the above theorem. Necessary: Suppose that there exists a stable polynomial

$f(z_1, \ldots, z_m)$ such that eqn. (4) holds. If this polynomial is factorized, all of its factors should be stable factors. Therefore

$$f(z_1, \ldots, z_m) = \prod_{j=1}^{\nu} f_j(z_1, \ldots, z_m) \tag{11}$$

where

$$f_j(z_1, \ldots, z_m) = \prod_{i_1=0}^{N_{f_i,1}} \cdots \prod_{i_m=0}^{N_{f_i,m}} a_{f_j}(i_1, \ldots, i_m) z_1^{i_1} \ldots z_m^{i_m} \tag{12}$$

So

$$f(z_1^{-1}, \ldots, z_m^{-1}) = \prod_{j=1}^{\nu} f_j(z_1^{-1}, \ldots, z_m^{-1}) \tag{13}$$

Then $b(z_1, \ldots, z_m)$ can be derived by a proper recombination of the factors of the numerator of the product $f(z_1, \ldots, z_m) \cdot f(z_1^{-1}, \ldots, z_m^{-1})$. Therefore

$$b(z_1, \ldots, z_m) = \prod_{j \in S} f_j(z_1, \ldots, z_m) \prod_{j \in S'} z_1^{N_{f_j,1}} \ldots z_m^{N_{f_j,m}} f_j(z_1^{-1}, \ldots, z_m^{-1}) \tag{14}$$

where $S \subseteq \left\{1, 2, \ldots, \nu\right\}$, $S' = \left\{1, 2, \ldots, \nu\right\} - S$.

Hence, $b(z_1, \ldots, z_m)$ consists of stable and perfectly unstable factors. Also, eqn. (4) holds (as one can see after a simple algebraic manipulation).

Example. Given

$$b(z_1, z_2) = 10z_1z_2^2 - z_2^2 + 10z_1^2z_2 + 34z_1z_2 - z_2 + 5z_1^2 + 17z_1 + 6 \tag{15}$$

find a stable polynomial $f(z_1, z_2)$ such that

$$f(z_1, z_2)f(z_1^{-1}, z_2^{-1}) = b(z_1, z_2)b(z_1^{-1}, z_2^{-1}) \tag{16}$$

The first step is the factorization of $b(z_1, z_2)$ using the method which is presented in (Mastorakis *et al.*, 1994), as well as, without details, in the Introduction of the present paper.

To this end, we write

$$b(z_1, z_2) = u(z_1) + l(z_2) + w(z_1, z_2) \tag{17}$$

where

$$u(z_1) = 5z_1^2 + 17z_1 \tag{18}$$

$$l(z_2) = -z_2^2 - z_2 + 6 \tag{19}$$

$$w(z_1, z_2) = 10z_1^2z_2^2 + 10z_1^2z_2 + 34z_1z_2 \tag{20}$$

The factors of $u(z_1)$ are: $1, z_1, 5z_1+17, (5z_1+17)z_1$ while the factors of $l(z_2)$ are: $1, 2-z_2, 3+z_2, (2-z_2)(3+z_2)$.

A linear combination of two factors one of $u(z_1)$ and one of $l(z_1)$ to be a factor of $b(z_1, z_2)$ is possible. Let $q_l(z_1)$ and $s_k(z_2)$ be these factors, respectively. A possible factor of the polynomial $b(z_1, z_2)$ is the polynomial $q_l(z_1) - r \cdot s_k(z_2)$ where r is a (complex) constant. Generally, for every selection of $q_l(z_1)$ and $s_k(z_2)$, we check the validity of Theorems A or B (see Appendix). After some trials, one can find the proper polynomials: $q_l(z_1) = z_1$, $s_k(z_2) = 3 + z_2$. By checking these theorems, r is found equal to 1. Therefore $z_1 + z_2 + 3$ is a factor of $b(z_1, z_2)$ and by carrying out the division $b(z_1, z_2) : (z_1 + z_2 + 3)$ the quotient $5z_1 - z_2 + 10z_1z_2 + 2$ is found. Therefore

$$b(z_1, z_2) = (z_1 + z_2 + 3)(5z_1 - z_2 + 10z_1z_2 + 2) \tag{21}$$

The next step is the study of the factors of $b(z_1, z_2)$ with respect to the stability. The factor z_1+z_2+3 is a stable factor since $z_1+3 \neq 0$ for $|z_1| \leq 1$ and $z_1+z_2+3 \neq 0$ for $|z_1| = 1$ and $|z_2| \leq 1$. This is true because if this is not the case $z_2 = -z_1 - 3$, therefore $|z_2| = |z_1 + 3| \geq |-1+3| = 2 > 1$.

The factor $5z_1 - z_2 + 10z_1z_2 + 2$ is unstable since $5z_1 + 2 = 0$ for $z_1 = -2/5$ ($|-2/5| \leq 1$). Furthermore, the perfect instability of this factor is examined. Thus, the stability of the polynomial $z_1z_2(5z_1^{-1} - z_2^{-1} + 10z_1^{-1}z_2^{-1} + 2)$ is studied. This is rewritten as $5z_2 - z_1 + 10 + 2z_1z_2$. One observes that $-z_1 + 10 \neq 0$ for $|z_1| \leq 1$ and for $|z_1| = 1$, $|z_2| \leq 1$ we have $5z_2 - z_1 + 10 + 2z_1z_2 \neq 0$ because if this is not the case, then

$$z_2 = \frac{-10 + z_1}{5 + 2z_1} \Longrightarrow |z_2| = \frac{|10 - z_1|}{|5 + 2z_1|} > \frac{9}{|5 + 2z_1|} > \frac{9}{7} > 1$$

Therefore $5z_1 - z_2 + 10z_1z_2 + 2$ is perfectly unstable. Since $b(z_1, z_2)$ is a product of stable and perfectly unstable factors, the m-D Spectral Factorization problem has a solution. The solution is the stable polynomial.

$$f(z_1, z_2) = (z_1 + z_2 + 3)(5z_2 - z_1 + 10 + 2z_1z_2) \tag{22}$$

or

$$f(z_1, z_2) = 2z_1z_2^2 + 2z_1^2z_2 + 5z_2^2 + 10z_1z_2 - z_1^2 + 25z_2 + 7z_1 + 30 \tag{23}$$

This is a stable polynomial that satisfies eqn. (16).

Remark 1. The stability of the polynomial factors that discussed above can also be checked by using the various tests like the Schur-Cohn test, the Table test, etc.

3. Conclusion

The m-D spectral factorization problem has a *solution* if and only if the given polynomial can be written as a product of stable and perfectly unstable factors. The factorization of the given m-D polynomial is achieved by the method of reduction.

The good algorithmic form of the above method permits also the computer implementation. This is left for the readers.

Appendix

First, we give some indispensable definitions. We start with the extention of the known definitions of the 1-D case.

Definition 1. A multidimensional polynomial is *prime*, if and only if it has only trivial divisors, i.e. itself and the zero degree polynomial (scalars).

Definition 2. Two multidimensional polynomials are called *coprime* if and only if they have only trivial common divisors.

Definition 3. A multidimensional polynomial is called *composite* if and only if it is not prime.

The multidimensional polynomials divisibility theory is not a simple extension of the 1-D one, because the basic 1-D theorems do not hold in the m-D case. However, several conclusions from the 1-D polynomials theory also hold in the m-D polynomials divisibility theory. The following fundamental principles do not hold for multidimensional polynomials.

1. $\forall$ couple f, g, $\exists\, \pi, \upsilon$ which are unique such that: $f = g\pi$ or $f = g\pi + \upsilon$ and degree $(\upsilon) <$ degree $(g)\ \forall\, z_1, \ldots, z_m$.
2. If f, g are coprime polynomials, there exists a, b (multidimensional polynomials in $z_1, \ldots z_m$) such that $1 = af + bg$.

Two very important theorems are now presented. These theorems are proved in a different way than the 1-D case, because the above principles 1 and 2 do not hold.

Theorem A. *A factor* $z_1 - p(z_2, \ldots, z_m)$ *is a factor of* $f(z_1, \ldots, z_m)$ *if and only if*

$$f\Big(p(z_2, \ldots, z_m), z_2, \ldots, z_m\Big) \equiv 0\ \forall\, z_1, \ldots, z_m$$

Proof. Necessity: $f(z_1, \ldots, z_m) = (z_1 - p(z_2, \ldots, z_m)) \cdot \pi(z_1, \ldots, z_m)$ so, if we put $z_1 = p(z_2, \ldots, z_m)$ we take $f(p(z_2, \ldots, z_m), z_2, \ldots, z_m) = 0$. Sufficiency: We take:

$$f(z_1, \ldots, z_m) = \Big(z_1 - p(z_2, \ldots, z_m)\Big) \cdot \pi(z_1, \ldots, z_m) + \upsilon(z_1, \ldots, z_m) \qquad \text{(A.1)}$$

From the theory concerning the division algorithm of the two 1-D polynomials $f(z_1, \ldots, z_m)$ and $z_1 - p(z_2, \ldots, z_m)$ (with respect to z_1) we have that π and υ are unique, and $\upsilon(z_1, \ldots, z_m)$ has a smaller degree for z_1 than $z_1 - p(z_2, \ldots, z_m)$. Therefore, $\upsilon = \upsilon(z_2, \ldots, z_m)$ (i.e. it does not contain z_1) and $\pi(z_1, \ldots, z_m)$ is a polynomial of z_1, and is in general a function in $z_2, \ldots, z_m$ because $z_2, \ldots, z_m$ were considered as parameters. Now, since $f(p(z_2, \ldots, z_m), z_2, \ldots, z_m) = 0$ one has $\upsilon(z_2, \ldots, z_m) = 0$. Therefore (A.1) will become:

$$f(z_1, \ldots, z_m) = \Big(z_1 - p(z_2, \ldots, z_m)\Big) \cdot \pi(z_1, \ldots, z_m) \qquad \text{(A.2)}$$

Let n be the degree of z_1 in the polynomial $f(z_1,\dots,z_m)$, then the polynomial $\pi(z_1,\dots,z_m)$ has the formula: $p_{n-1}z_1^{n-1}+p_{n-2}z_1^{n-2}+\dots+p_0z_1^0$ where $p_0,\dots,p_{n-1}$ are functions of $z_2,\dots,z_m$. However, because the coefficient of z_1 in the divisor is constant (one), it is clear by equating the coefficients that p_{n-1} is a polynomial in $z_2,\dots,z_m$. Taking now $p_{n-2}-p\cdot p_{n-1}=a_{n-1}(z_2,\dots,z_m)$ we find that p_{n-2} is a polynomial in $z_2,\dots,z_m$, where we denote $f(z_1,\dots,z_m)=\sum_{i=0}^{n}a_i(z_2,\dots,z_m)z_1^i$. If p_{n-k} is a polynomial in $z_2,\dots,z_m$, then p_{n-k-1} is a polynomial also, because $p_{n-k-1}=p\cdot p_{n-k}+a_{n-k}(z_2,\dots,z_m)$. Therefore $p_{n-1},\dots,p_0$ are polynomials in $z_2,\dots,z_m$. Consequently $\pi(z_1,\dots,z_m)$ is a polynomial in $z_1,z_2,\dots,z_m$. ∎

Remark 2. Theorem A can be proved as an application of the following theorem, which is a generalization of the previous theorem. The proof of Theorem B is rather extensive. In any case, it can be found in (Mastorakis *et al.*, 1994).

Theorem B. *A polynomial $h(z_1,\dots,z_m)$ is a factor of $f(z_1,\dots,z_m)$ if and only if $\forall\,(z_1,\dots,z_m)$ such that:*

$$\left.\begin{array}{l} h(z_1,\dots,z_m)=0 \\ \vdots \\ \dfrac{\partial^{t_2}h(z_1,\dots,z_m)}{\partial z_i^{t_2}}=0 \end{array}\right\}\quad \forall\, i: i=1,\dots,m \tag{A.3}$$

it follows that

$$\left.\begin{array}{l} h(z_1,\dots,z_m)=0 \\ \vdots \\ \dfrac{\partial^{t_1}h(z_1,\dots,z_m)}{\partial z_i^{t_1}}=0 \end{array}\right\}\quad \forall\, i: i=1,\dots,m \tag{A.4}$$

where $t_1\ge t_2$.

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