

SCHEDULING PROBLEMS WITH A COMMON DUE WINDOW ASSIGNMENT: A SURVEY

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In this article a survey of studies on scheduling problems with a common due window assignment and earliness/tardiness penalty functions is presented. A due window is a generalization of the classical due date and describes a time interval in which a job should be finished. If a job is completed before or after the due window, it incurs an earliness or a tardiness penalty, respectively. In this survey we separately analyse the classical models with job-independent and job-dependent earliness/tardiness penalty functions and some other more complicated models. We describe the computational complexity of the problems and the main features of the approaches developed to solve them. Particular attention is paid to practical applications of the analysed models. As turns out, some complicated models combining classical scheduling problems with, e.g., learning and aging effects have no reasonable practical justification in the literature.

Keywords: scheduling, due window assignment, earliness/tardiness, practical example.

1. Introduction

Decision making is a very complex and widely applicable process. It is often discussed in terms of various technical applications, starting from vehicle control (Gáspár *et al.*, 2012) to resource allocation in computational grids (Kołodziej and Xhafa, 2011). In manufacturing systems one of the most crucial decisions that has to be made is determining the production schedule. The international competitive pressure resulting from the globalisation process and the everchanging customer demand forced companies to reevaluate their strategies. Only enterprises that are flexible and responsive in serving customers are able to succeed on the global market.

In order to obtain higher quality products and services, many industries have made substantial progress in shortening their execution lead times through the adoption of the Just-In-Time (JIT) philosophy (Gunn, 1992). As with any inventory system, the JIT system's main objective is to ensure that required material is available exactly when it is needed, while maintaining the inventory at the lowest possible level and eliminating such waste components like overproduction, waiting time, transportation, unnecessary processing, inventory and defective products (Kramer and Lee, 1993). Therefore, it is important that management systems and scheduling

theory contribute to the realization of JIT in a manufacturing environment.

Usually, JIT scheduling models assume the existence of job due dates and discourage early as well as tardy jobs. If a job is completed before its due date, it implies an earliness penalty such as holding cost. On the other hand, job finished after its due date implies tardiness costs such as late charges, express delivery, or loss of sales. For earlier surveys of problems with due date, see those by Baker and Scudder (1990), Gordon *et al.* (2002), as well as Lauff and Werner (2004). Good customer service means finishing jobs (or orders) as close as possible to their due dates. In practice, a “fuzzy” due date can be observed, as a compromise between the customers' expectations and the production capabilities of the manufacturers. The supply contract between the supplier and the customer often specifies a time interval in which the job is to be finished. The time interval is called the due window of a job, and the left end and the right end of the window are called the starting time and the finishing time of the due window, respectively. If the job is completed within the due window, then it is considered to be on time and will not incur any penalty. If a job is finished earlier than the starting time of its due window, then it has to be stored as an inventory, which results in an earliness penalty. On the other hand, if a job is finished later than the

finishing time of its due window, then it incurs a tardiness penalty as stated in the contract. It is clear that a wide due window increases the suppliers' production and delivery flexibility. However, a large due window and delayed job completion reduce the suppliers' competitiveness and customer service level.

In the scientific literature there can be found many examples of practical problems that can be considered a scheduling problem with a due window assignment (Koulamas, 1996; Kramer and Lee, 1993; Lee, 1991; Wu and Lai, 2007). For instance, consider a manufacturer with a limited storage area that produces on demand, e.g., located in a city centre. Such a manufacturer needs to rent a storage area nearby, which is very expensive, or transport the produced goods to other warehouses outside the city centre. The lease of a remote storage area is much cheaper than a city centre lease but implies additional transportation costs. In order to reduce the costs of transporting the produced goods to the customer, the majority of the order is transported to the client using a chosen (most effective) means of mass transit, e.g., train or truck. Until the delivery is made, the products are stored in the warehouses. The goods that have been produced after the delivery of the main part of the order have to be transported to the customer using other (more expensive) means of transport, e.g., a courier.

In the case of a large order, the manufacturer has to prepare a proper production schedule, decide for how long to rent the additional, much more expensive, storage area close to the factory, and at which point to start the delivery of the main part of the order to the customer in order to minimize the costs of transport and storage. This situation can be transformed into a scheduling problem with a common due window assignment as follows. The end of the common due window is the moment of the delivery of the main part of the order to the client, whereas the due window size is the lease time of the warehouse close to the factory. The costs of renting and transporting goods produced before the lease of the warehouse to other remote storage area are the earliness penalties, while the costs of the transport of the products manufactured after the main delivery had been made (using other, more expensive means of transport) are the tardiness penalties (Winczaszek, 2006).

In the paper we survey all scheduling problems with a common due window assignment that appeared in the scientific literature. Apart from the classical scheduling problems with a common due window assignment, we also consider more complicated models that combine the above mentioned classical ones with changeable job processing times, maintenance activities, deteriorating, learning and aging effects. We concentrate mainly on the computational complexity results of the problems considered and the proposed solution algorithms.

The next section is devoted to the commonly used

notation of the problems in question. In Sections 3 and 4 we present the results obtained for the classical models with job-independent and job-dependent earliness/tardiness penalty functions, respectively. Section 5 concerns some other scheduling problems with a common due window assignment combined with mathematically more complicated models of job processing times. Some conclusions are given in Section 7.

2. Problem notation

This section presents the problem notation which is used in the article. We investigate problems of scheduling n jobs (J_1, J_2, \dots, J_n) on m parallel machines (M_1, M_2, \dots, M_m) to minimize a given criterion. It is assumed that a machine can process at most one job at a time, and all jobs are ready for processing at time $t = 0$. Each job is characterized by its processing time p_j and a given set of weights, e.g., $\alpha_j, \beta_j, \gamma, \delta, \theta$.

Let us define the following:

- e_j : the beginning of the due window (earliest due date) of job J_j ;
- d_j : the end of the due window (latest due date) of job J_j ;
- $D_j = d_j - e_j$: the size of the due window;
- S_j : the starting time of job J_j ;
- C_j : the completion time of job J_j ;
- $E_j = \max\{e_j - C_j, 0\}$: the earliness of job J_j ;
- $T_j = \max\{C_j - d_j, 0\}$: the tardiness of job J_j ;
- $\langle e, d \rangle$: the common due window, i.e., $e_j = e$ and $d_j = d$ for each job J_j , where e and d are to be assigned in an optimization process. The beginning and end of the due window are usually real numbers, unless stated otherwise.

The above notation appears in the second and the third field of the three-field notation introduced by Graham *et al.* (1979). The machine environment is described in the first field of this notation.

3. Classical models with job-independent earliness/tardiness penalty functions

In this section we analyze the problems with the common due window assignment and job-independent earliness and tardiness penalty functions. We will start with the sum type criteria.

3.1. Sum type criteria. The problem in which both the beginning and the size of the due window are decision variable,

$$1|\langle e, e + D \rangle| \sum (\alpha E_j + \beta T_j), \quad (1)$$

was investigated by Kramer and Lee (1993) as well as Weng and Ventura (1996). The authors constructed $O(n \log n)$ time optimal algorithms to solve the problem.

Observe that the penalty function of the above mentioned scheduling problem does not take into account the size of the due window nor its location. Adding the due window size and location penalties makes the problems more realistic. The following problems:

$$1|\langle e, d \rangle| \sum (\alpha E_j + \beta T_j) + \gamma(d - e) + \delta e, \quad (2)$$

$$1|\langle e, d \rangle| \sum (\alpha E_j + \beta T_j) + \gamma(d - e) \quad (3)$$

were studied by Liman *et al.* (1998) as well as Janiak and Marek (2009; 2004), respectively. Optimal $O(n \log n)$ time algorithms were constructed for these problems.

The problem in a parallel identical machine environment was considered by Janiak and Winczaszek (2005a):

$$P|\langle e, d \rangle| \sum (\alpha E_j + \beta T_j) + \gamma(d - e). \quad (4)$$

It was demonstrated that the problem is strongly NP-hard and an optimal algorithm based on dynamic programming method with time complexity $O(mn^m(\sum p_j)^{m-1})$ was provided.

Even with unit earliness/tardiness penalty weights the problem still remains strongly NP-hard. Janiak *et al.* (2007) along with Janiak and Marek (2003) investigated the following problem:

$$P|\langle e, d \rangle| \sum (E_j + T_j) + \gamma(d - e). \quad (5)$$

The authors showed that it is equivalent to the problem $P||C_{\max}$, in the sense that an optimal solution of one of them can be derived from an optimal solution of the other one in polynomial time. Hence, the analysed problem is strongly NP-hard, since $P||C_{\max}$ is strongly NP-hard (Garey and Johnson, 1978). A heuristic algorithm with the worst case ratio equal to $\frac{4}{3} - \frac{1}{3m}$ and an optimal algorithm based on dynamic programming method with computational complexity $O(n(\sum p_j)^m)$ can be provided.

Janiak and Winczaszek (2003) investigated a problem in which the beginning and the size of the due window are decision variables but the due window size is constrained from above and below:

$$1|\langle e, d \rangle, D_{\min} \leq d - e \leq D_{\max}| \alpha \sum E_j + \beta \sum T_j + \theta e + f_W(d - e), \quad (6)$$

where f_W is an arbitrary concave increasing function. The time complexity of the algorithm provided $O(n \log n + \log p_{\max})$. It can be noticed that if $D_{\max} \geq \sum p_j$ and $D_{\min} = 0$, then in fact the due window size can be freely established and, if $D_{\min} = D_{\max}$, then in fact due window size is given in advance. Thus, this problem is a generalization of the single machine problems described earlier in this section.

A more general problem with non-linear earliness/tardiness penalty functions was studied by Janiak and Winczaszek (2004):

$$1|\langle e, d \rangle, D_{\min} \leq d - e \leq D_{\max}| \sum f_E(E_j) + \sum f_T(T_j) + f_D(e) + f_W(d - e), \quad (7)$$

where f_E , f_T and f_W are arbitrary nondecreasing functions with $f_E(0) = f_T(0) = f_W(0) = 0$. It was shown that the problem is NP-hard and the authors proposed a pseudopolynomial time $O(n(\sum p_j - D_{\min})^2)$ solution algorithm.

Janiak and Winczaszek (2006) studied further the parallel machine version of the problem

$$P|\langle e, d \rangle, D_{\min} \leq d - e \leq D_{\max}| \sum f_E(E_j) + \sum f_T(T_j) + f_D(e) + f_W(d - e), \quad (8)$$

where f_E , f_T and f_W are arbitrary nondecreasing functions with $f_E(0) = f_T(0) = f_W(0) = 0$. It was proven that the problem is strongly NP-hard and an optimal algorithm based on the dynamic programming method with time complexity $O(n(\sum p_j)^{3m-1})$ was provided.

Wu and Wang (1999) analysed a problem in which the due window size is constrained from above and a flowtime penalty is also taken into account:

$$1|\langle e, d \rangle, d - e \leq \min p_j| \sum (\alpha E_j + \beta T_j + \theta C_j) + \delta d. \quad (9)$$

It was shown that the problem is solvable in $O(n \log n)$ time.

Mosheiov and Sarig (2008) extended a classical single machine due window assignment problem to the case of position-dependent processing times:

$$1|\langle e, d \rangle, p_{ij}| \sum (\alpha E_j + \beta T_j) + \gamma(d - e) + \delta e. \quad (10)$$

The authors did not restrict the processing time function to be either monotonically increasing (to reflect deterioration) or monotonically decreasing (to reflect learning). They provided an $O(n^3)$ time optimal algorithm.

Mosheiov and Oron (2004) addressed a common due-window assignment problem on parallel identical machines with unit processing time jobs,

$$Pm|\langle e, d \rangle| \sum_{i=1}^m \sum_{j=1}^{n_i} \alpha E_{ij} + \beta T_{ij} + \gamma e + \delta(d - e). \quad (11)$$

The authors showed that the number of candidate values for the optimal due-window starting time and for the optimal due-window completion time is bounded by 2. It was also proven that the starting time of the first job on each of the machines is either 0 or 1, thus allowing introduction of a fairly simple, constant-time solution for the problem.

3.2. Min-max type criteria. So far we have discussed only the scheduling problems with minimization of the sum of earliness/tardiness penalties and the due window size and location penalties. Now, we will survey a group of scheduling problems with a common due window assignment concerning the min-max criterion.

Janiak *et al.* (2007) showed that the single machine problem

$$1|\langle e, d \rangle | \max\{\alpha \max_j E_j, \beta \max_j T_j, \gamma(d - e)\} \quad (12)$$

is polynomially solvable. They constructed an $O(n)$ time optimal algorithm to solve the problem.

Mosheiov and Sarig (2009a) studied a similar problem:

$$1|\langle e, d \rangle | \max\{\alpha \max_j E_j, \beta \max_j T_j, \gamma e, \delta(d - e)\}. \quad (13)$$

They provided an $O(n)$ time solution algorithm.

Janiak *et al.* (2007) investigated the following parallel machine problem with the min-max criterion:

$$P|\langle e, d \rangle | \max\{\alpha \max_j E_j, \beta \max_j T_j, \gamma(d - e)\}. \quad (14)$$

The authors showed that it is equivalent to the problem $P|\langle e, d \rangle | \sum(E_j + T_j) + \gamma(d - e)$, in the sense that an optimal solution of one of them can be derived from an optimal solution of the other one in polynomial time. Therefore, the analysed problem is strongly NP-hard, since $P|\langle e, d \rangle | \sum(E_j + T_j) + \gamma(d - e)$ is strongly NP-hard, as we have observed earlier. Each algorithm for the problem $P|\langle e, d \rangle | \sum(E_j + T_j) + \gamma(d - e)$ can be adapted to solve the problem $P|\langle e, d \rangle | \max\{\alpha \max_j E_j, \beta \max_j T_j, \gamma(d - e)\}$.

Mosheiov (2001) considered the following problem:

$$P|\langle e, d \rangle | \max\{\alpha \max_j E_j, \beta \max_j T_j, \gamma e, \delta(d - e)\}. \quad (15)$$

He proposed an $O(\max\{nm, n \log n\})$ time asymptotically optimal approximation algorithm. If the job processing times are independent and identically distributed random variables with bounded support, then the absolute error of the solution obtained by this algorithm approaches zero. Moreover, for the special case of the problem with unit processing times of jobs, this algorithm delivers an optimal solution.

A single machine version of the above problem

$$1|\langle e, d \rangle | \max\{\alpha \max_j E_j, \beta \max_j T_j, \gamma e, \delta(d - e)\}, \quad (16)$$

was proven to be polynomially solvable by Mosheiov and Sarig (2009a), who provided an $O(n)$ time optimal algorithm.

4. Classical models with job-dependent earliness/tardiness penalty functions

The due window assignment problems with job-dependent penalty functions are much more general and, therefore, might imply wider practical applications. However, they also are much more complex.

Azizoglu and Webster (1997) considered an NP-hard job scheduling problem on a single machine with an unrestricted due window to minimize the total weighted earliness and tardiness cost:

$$1|\langle e, e + D \rangle | \sum(\alpha_j E_j + \beta_j T_j). \quad (17)$$

This model is closely related to the one studied by Kramer and Lee (1993). However, allowing the penalty rates to be job-dependent not only enabled modeling new problems, where the relative importance of jobs with respect to avoiding early or tardy completion can vary, but also affected the problem's complexity. The problem is NP-hard even for the case of an unrestricted common due instance (Hall and Posner, 1991). Kramer and Lee (1993) presented properties that help characterize the form of optimal schedules and defined an efficient method for calculating a lower bound on the optimum cost. The properties and lower bounding method were incorporated into a two-step branch-and-bound algorithm. Computational test results indicate that the algorithm solves efficiently problems with up to 25 jobs.

Limana and Ramaswamy (1994) analysed a single machine scheduling problem to minimize the sum of weighted earliness and a weighted number of tardy jobs given a delivery window:

$$1|\langle e, e + D \rangle | \sum \alpha_j E_j + \beta_j U_j. \quad (18)$$

The problem was proven to be NP-hard. The authors presented a dynamic programming based solution algorithm ($O(n^2 D \sum p_j)$).

Yeung *et al.* (2001a) studied a single machine scheduling problem to minimize the weighted number of early and tardy jobs with a common due window. As in previously presented studies, the window size was a given parameter, but the window location was a decision variable and the objective of the problem was to find a schedule that minimizes the weighted number of early and tardy jobs and the due window location penalty:

$$1|\langle e, e + D \rangle | \sum(\alpha_j U_j + \beta_j V_j) + \gamma e. \quad (19)$$

The authors showed that the problem is NP-hard in the ordinary sense only and developed a dynamic programming based pseudo-polynomial $O(n^2 p_{\max}(D + 1))$ algorithm. They also provided polynomial time algorithms for two special cases:

- an $O(n \log n)$ time algorithm, if $\alpha_j = \alpha$ and $\beta_j = \beta$:

$$1|\langle e, e + D \rangle | \sum (\alpha U_j + \beta V_j) + \gamma e, \quad (20)$$

- and an $O(n)$ time algorithm for the unit job processing times case:

$$1|\langle e, e + D \rangle, p_j = 1 | \sum (\alpha_j U_j + \beta_j V_j) + \gamma e. \quad (21)$$

Yeung *et al.* (2001b) analysed several single-machine non-preemptive scheduling problems to minimize the sum of weighted earliness-tardiness, a weighted number of early and tardy jobs, common due window location, and flowtime penalties. The authors assumed that the due window location has a tolerance (\hat{d}_0) and the window size is a given parameter. All of the general problems studied in this paper were proved to be NP-hard in the ordinary sense, and pseudo-polynomial time algorithms were developed:

- an $O(n^3(D + 1)p_{\max}^2(\sum p_j + 1)^2)$ time, dynamic programming based algorithm, under the assumption that the ratios of the job processing times to the earliness-tardiness weights are agreeable:

$$1|\langle e, e + D \rangle, p_i/\alpha'_i \geq p_k/\alpha'_k \\ \Leftrightarrow p_i/\beta'_i \geq p_k/\beta'_k | \sum (\alpha'_j E_j + \beta'_j T_j + \alpha_j U_j + \beta_j V_j) \\ + \gamma \max\{0, d - \hat{d}_0\}, \quad (22)$$

- an $O(n^3(D + 1)(\sum p_j + 1)p_{\max}^2(\sum p_j - D + 1)^2)$ algorithm for the problem, in which the flowtime is included in the objective function:

$$1|\langle e, e + D \rangle | \sum (\alpha_j U_j + \beta_j V_j + \theta_j C_j) \\ + \gamma \max\{0, d - \hat{d}_0\}, \quad (23)$$

- an $O(n^2(D + 1)(\sum p_j + 1)(\sum p_j - p_{\min} + 1)^2 p_{\max})$ algorithm, if zero tolerance ($\hat{d}_0 = 0$) for the due window is assumed:

$$1|\langle e, e + D \rangle | \sum (\alpha_j U_j + \beta_j V_j + \theta_j C_j) + \gamma d, \quad (24)$$

- and a polynomial $O(n \log n)$ time algorithm, for a job-independent penalty ratios:

$$1|\langle e, e + D \rangle | \sum (\alpha' E_j + \beta' T_j + \alpha U_j + \beta V_j) + \gamma d. \quad (25)$$

Janiak and Winczaszek (2005b) proved NP-hardness of the following problem:

$$1|\langle e, d \rangle, D_{\min} \leq d - e \leq D_{\max} | \\ \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e), \quad (26)$$

where the due window size is bounded to be within the interval $[D_{\min}, D_{\max}]$, and investigated the following special cases:

- under the assumption that the ratios of the job processing times to the earliness/tardiness weights are agreeable, the problem

$$1|\langle e, d \rangle, D_{\min} \leq d - e \leq D_{\max}, \\ p_i/\alpha_i < p_j/\alpha_j \Leftrightarrow p_i/\beta_i < p_j/\beta_j | \\ \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e) \quad (27)$$

remains NP-hard and is solvable in pseudopolynomial time $O(n(\sum p_j)^2)$;

- even without the due window size restriction, the problem

$$1|\langle e, d \rangle, p_i/\alpha_i < p_j/\alpha_j \Leftrightarrow p_i/\beta_i < p_j/\beta_j | \\ \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e) \quad (28)$$

still remains NP-hard, although there exists a fully polynomial-time approximation scheme with time complexity $O(n^2 \log^3(\max\{n, 1/\epsilon, p_j, \alpha_j, \beta_j, \gamma\})/\epsilon^3)$;

- the special case with unit processing times of jobs:

$$1|\langle e, d \rangle, D_{\min} \leq d - e \leq D_{\max}, p_j = 1 | \\ \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e) \quad (29)$$

is polynomially solvable in $O(n^3)$ time.

Mosheiov and Sarig (2011) showed that the parallel machine problem with unit job processing times:

$$P|\langle e, d \rangle, p_j = 1 | \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e) \quad (30)$$

is polynomially solvable in $O(n^4)$ time.

Mosheiov and Sarig (2010) studied the following problems:

$$1|\langle e, d \rangle | \max_j \{\alpha_j E_j + \beta_j T_j + \gamma e + \delta(d - e)\}, \quad (31)$$

$$Pm|\langle e, d \rangle | \max_j \{\alpha_j E_j + \beta_j T_j + \gamma e + \delta(d - e)\}, \quad (32)$$

$$Fm|\langle e, d \rangle | \max_j \{\alpha_j E_j + \beta_j T_j + \gamma e + \delta(d - e)\}. \quad (33)$$

They showed that for a given job sequence the optimal due window size and location can be found in polynomial time by solving linear programs.

5. Other models

In this section we analyse the models of problems with a common due window assignment extended to such additional complicated situations as changeable job processing times, deteriorating jobs, learning and aging effects, maintenance activities, and so on.

The learning effect is a phenomenon of improving production efficiency during a long time learning process of workers. The aging effect is a process opposite to the learning one, i.e., the workers get tired during the long time production process and their efficiency decreases. The concept of the learning effect was introduced to scheduling theory by Meilijson and Tamir (1984) as well as Dondeti and Mohanty (1998).

Liman et al. (1997) examined a problem with a common due window assignment and changeable job processing times:

$$1|\langle e, d \rangle; p_j = p'_j - x_j | \sum (\alpha E_j + \beta T_j + G_j x_j) + \delta e + \gamma(d - e), \quad (34)$$

where p_j is the normal processing time, p'_j is minimum processing time, while x_j is the amount of reduction in processing time ($0 \leq x_j \leq p_j - p'_j$) and $G_j x_j$ is the cost of shortening the job. The problem considered is solvable in $O(n^3)$ by transformation to the assignment problem.

Wang and Wang (2011) considered a single machine deteriorating jobs scheduling problem with a common due window assignment and the learning effect:

$$1|\langle e, d \rangle, p_j = (a_j + bt)r^\alpha | \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e). \quad (35)$$

The current processing time of job J_j scheduled at position r in a sequence is given by $p_j = (a_j + bt)r^\alpha$, where $t > 0$ is the starting time for job J_j , $a \leq 0$ is the learning index and b is the deterioration rate, which is common for all jobs. An $O(n \log n)$ time optimal solution algorithm was proposed.

Yang (2010) studied a very similar problem with both start-time dependent learning and position dependent aging effects:

$$1|\langle e, d \rangle, p_{jr} = (p_j - at)r^b | \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e), \quad (36)$$

where $p_{jr} = (p_j - at)r^b$ is the current processing time of job J_j started at time t and scheduled in the r -th position in a sequence, $a > 0$ is a common learning rate and $b \geq 0$ is the aging factor. In this model the job processing time consists of two parts. One part is a decreasing start-time dependent function and the other part is an increasing position-dependent function. An $O(n \log n)$ time optimal algorithm was provided.

Cheng et al. (2010) examined another common due window assignment scheduling problem,

$$1|\langle e, d \rangle, p_j = a_j + bS_j | \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e), \quad (37)$$

in which the job processing times follow a linear time-dependent deterioration model. Thus, the current processing time of job J_j is given by $p_j = a_j + bS_j$, where a_j is the normal processing time, $b > 0$ is a common deteriorating factor and S_j is the starting time of job J_j . They showed that the problem is polynomially solvable by an $O(n \log n)$ time algorithm.

Mosheiov and Sarig (2009b) introduced some maintenance activities to scheduling problems with a common due window assignment. A maintenance activity is an optional activity that requires a fixed time interval during which the production is stopped, as the machine is turned off. However, after the activity, the machine becomes more efficient, which is reflected in the new shortened job processing times. Mosheiov and Sarig (2009b) studied a problem

$$1|\langle e, d \rangle, ma | \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e), \quad (38)$$

in which the scheduler has an option to schedule a maintenance activity (*ma*) that lasts a constant t units of time. During this activity no production is performed. The processing time of job J_j is denoted by p_j if the job is processed prior to the maintenance activity, and $\lambda_j p_j$ ($0 < \lambda_j \leq 1$) if it is scheduled after. λ_j is the *modifying rate* of job J_j . This model reflects a maintenance break in a production environment during which the machine is stopped and no production is performed. They provided an $O(n^4)$ time solution algorithm.

The duration of the maintenance activity does not have to be constant. Cheng et al. (2010) introduced the following model of the scheduling problem:

$$1|\langle e, d \rangle, ma, p_j = a_j + bS_j | \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e), \quad (39)$$

in which the job processing times follow a linear time-dependent deterioration model. Thus, the current processing time of job J_j (p_j) is given by $p_j = a_j + bS_j$, where a_j is the normal processing time, $b > 0$ is a common deteriorating factor and S_j is the starting time for job J_j . As before, a maintenance activity is allowed to improve production efficiency. But unlike in the previous model, the maintenance activity does not introduce a modifying rate λ_j . At most one maintenance activity is allowed throughout the planning horizon. Maintenance can be performed immediately after the processing of any completed job. However, the position and starting time of the maintenance activity in the production

schedule are unknown in advance. It is assumed that the machine will revert to its initial conditions after the maintenance activity and machine deterioration start anew. The duration of the maintenance activity is a linear function of its starting time and is given by $f(t) = \mu + \sigma t$, where $\mu > 0$ is the basic maintenance time, $\sigma \geq 0$ is a maintenance factor, and t is the starting time of the maintenance activity. The later the production manager schedules the maintenance break, the worse the condition of the machine and the longer it takes to restore it to its initial state. They proposed an $O(n^2 \log n)$ optimal time algorithm.

Yang *et al.* (2010) also studied the same deteriorating maintenance but with a different job processing time model:

$$1|(e, d), ma, p_{jr} = p_j r^{a_j}| \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e), \quad (40)$$

where job J_j has a normal processing time p_{jr} and a job-dependent aging factor $a_j > 0$. If job J_j is scheduled in the r -th position in a sequence, then its current processing time equals $p_{jr} = p_j r^{a_j}$. The maintenance activity (*ma*) is defined as above. An $O(n^4)$ time optimal solution was provided.

Yang (2010) investigated a due window scheduling problem with joint start-time dependent learning and position dependent aging effects under the deteriorating maintenance activity:

$$1|(e, d), ma, p_{jr} = (p_j - at)r^b| \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e), \quad (41)$$

where $p_{jr} = (p_j - at)r^b$ is the current processing time of job J_j started at time t and scheduled in the r -th position in a sequence, $a > 0$ is a common learning rate and $b \geq 0$ is the aging factor. In this model, each job processing time consists of two parts. One part is a decreasing start-time dependent function and the other part is an increasing position-dependent function. The maintenance activity (*ma*) is defined as in the work of Mosheiov and Sarig (2009b). An $O(n^2 \log n)$ time optimal solution algorithm was provided.

Zhao and Tang (2010) studied a single machine due window assignment problem with deteriorating jobs and a rate-modifying activity:

$$1|(e, d), rm, p_j^A = p_j + bS_j| \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e). \quad (42)$$

In this problem the current processing time of a job (p_j^A) is a linear function of its starting time (S_j) and the deterioration rates ($b > 0$) are identical for all jobs. The rate-modifying activity (*rm*) takes t units of time, during

which no job can be executed by the machine. The normal processing time of job J_j is denoted by p_j if the job is processed prior to the rate-modifying activity, and $\lambda_j p_j$ if it is scheduled afterwards, where $\lambda_j > 0$ is the *modifying rate*. If the normal processing time of job J_j is p_j ($\lambda_j p_j$) the current processing time of job J_j , if processed from time S_j , is given by $p_j^A = p_j + bS_j$ ($p_j^A = \lambda_j p_j + bS_j$). The authors provided an $O(n^4)$ time solution algorithm.

6. Discussion and recommendations

Recently, there has been a lot of studies concerning scheduling with a common due window assignment as well as learning and aging effects. The concept of the learning effect (without any scheduling processes) was first observed during the process of plane production in 1936 and was described by Wright (1936). Obviously, there are several practice examples of the dependency of operation processing times on the learning (or aging) effect but without any scheduling process. Biskup (1999) introduced into scheduling theory a dependency of job processing times on the position of the job in the sequence (i.e., on the number of already processed jobs). The concept of the learning effect was introduced to scheduling theory by Meilijson and Tamir (1984) as well as Dondeti and Mohanty (1998). However, they did not give any reasonable real-life practical example for the model combining the scheduling process with the learning one. Next, several authors (Wang and Xia, 2005; Wu, 2005; Chen *et al.*, 2006; Wang, 2006) considered various scheduling problems with the learning effect even in multimachine flow shop environments without any real-life practical justification.

In this survey, we must emphasize that a combination of scheduling problems and learning or aging effects in one model has no reasonable real-life applications in the literature and that we see no sense in continuing further research considering these scheduling problems from practical (computer engineering, automatic control, technical and economical) point of view, unless such a reasonable real-life example is presented. For a detailed survey of scheduling problems with learning and aging effects, see the work of Janiak *et al.* (2011), where particular attention was paid to practical applications of the mentioned models.

A separate survey of scheduling problems with maintenance activity with particular attention paid to practical applications of such models should be provided.

7. Conclusion

In this article we have presented the state of the art of scheduling problems with a common due window assignment and earliness/tardiness penalty functions. We have reviewed the results concerning the classical models

Table 1. Computational complexity and solution algorithm results for scheduling problems with a common due window assignment and job-independent earliness/tardiness penalty functions.

No.	Problem	Complexity	Algorithms
1	$1 \langle e, e + D \rangle \sum (\alpha E_j + \beta T_j)$	P	$O(n \log n)$ (Kramer and Lee, 1993; Weng and Ventura, 1996)
2	$1 \langle e, d \rangle \sum (\alpha E_j + \beta T_j) + \gamma(d - e) + \delta e$	P	$O(n \log n)$ (Liman et al., 1998)
3	$1 \langle e, d \rangle \sum (\alpha E_j + \beta T_j) + \gamma(d - e)$	P	$O(n \log n)$ (Janiak et al., 2009; Janiak and Marek, 2004)
4	$P \langle e, d \rangle \sum (\alpha E_j + \beta T_j) + \gamma(d - e)$	s. NP-hard	$O(mn^m (\sum p_j)^{m-1})$ (Janiak and Winczaszek, 2005a)
5	$P \langle e, d \rangle \sum (E_j + T_j) + \gamma(d - e)$	s. NP-hard	$O(n(\sum p_j)^m)$, $\frac{4}{3} - \frac{1}{3m}$ (Janiak et al., 2007; 2009)
6	$1 \langle e, d \rangle, d - e \leq \min p_j \sum (\alpha E_j + \beta T_j + \theta C_j) + \delta d$	P	$O(n \log n)$ (Wu and Wang, 1999)
7	$1 \langle e, d \rangle, D_{\min} \leq d - e \leq D_{\max} \alpha \sum E_j + \beta \sum T_j + \theta e + f_W(d - e)$	P	$O(n \log n + \log p_{\max})$ (Janiak and Winczaszek, 2003)
8	$1 \langle e, d \rangle, D_{\min} \leq d - e \leq D_{\max} \sum f_E(E_j) + \sum f_T(T_j) + f_D(e) + f_W(d - e)$	NP-hard	$O(n(\sum p_j - D_{\min})^2)$ (Janiak and Winczaszek, 2004)
9	$P \langle e, d \rangle, D_{\min} \leq d - e \leq D_{\max} \sum f_E(E_j) + \sum f_T(T_j) + f_D(e) + f_W(d - e)$	s. NP-hard	$O(n(\sum p_j)^{3m-1})$ (Janiak and Winczaszek, 2006)
10	$1 \langle e, d \rangle, p_{ij} \sum (\alpha E_j + \beta T_j) + \gamma(d - e) + \delta e$	P	$O(n^3)$ (Mosheiov and Sarig, 2008)
11	$P \langle e, d \rangle \max \{ \alpha \max_j E_j, \beta \max_j T_j, \gamma(d - e) \}$	s. NP-hard	Janiak et al. (2007)
12	$1 \langle e, d \rangle \max \{ \alpha \max_j E_j, \beta \max_j T_j, \gamma(d - e) \}$	P	$O(n)$ (Janiak et al., 2007)
13	$P \langle e, d \rangle \max \{ \alpha \max_j E_j, \beta \max_j T_j, \gamma e, \delta(d - e) \}$	s. NP-hard	heuristic (Mosheiov, 2001)
14	$1 \langle e, d \rangle \max \{ \alpha \max_j E_j, \beta \max_j T_j, \gamma e, \delta(d - e) \}$	P	$O(n)$ (Mosheiov and Sarig, 2009a)
15	$Pm \langle e, d \rangle \sum_{i=1}^m \sum_{j=1}^{n_i} \alpha E_{ij} + \beta T_{ij} + \gamma e + \delta(d - e)$	P	(Mosheiov and Oron, 2004)
16	$1 \langle e, e + D \rangle \sum (\alpha U_j + \beta V_j) + \gamma e$	P	$O(n \log n)$ (Yeung et al., 2001a)
17	$1 \langle e, e + D \rangle \sum (\alpha' E_j + \beta' T_j + \alpha U_j + \beta V_j) + \gamma d$	P	$O(n \log n)$ (Yeung et al., 2001b)

of problems with job-independent and job-dependent earliness/tardiness penalties. We have also analysed the research findings related to other mathematically complicated models. Unfortunately, it was stated that some of the last models, e.g., with learning and aging effects have no reasonable practical justification in the literature. Then, there is no sense to continue research considering these scheduling problems from practical (computer engineering, automatic control, technical and economical) point of view, unless such a reasonable real-life example appears.

The concise presentation of the computational complexity and solution algorithm results obtained for the problems presented in this survey are given in Tables 1–3 (P, NP-hard and s. NP-hard denote class polynomially solvable, NP-hard and strongly NP-hard problems, respectively).

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Table 2. Computational complexity and solution algorithm results for scheduling problems with a common due window assignment and job-dependent earliness/tardiness penalty functions.

No.	Problem	Complexity	Algorithms
1	$1 \langle e, e + D \rangle \sum \alpha_j E_j + \beta_j T_j$	NP-hard	B&B (Azizoglu and Webster, 1997)
2	$1 \langle e, e + D \rangle \sum \alpha_j E_j + \beta_j U_j$	NP-hard	$O(n^2 D \sum p_j)$ (Limana and Ramaswamy, 1994)
3	$1 \langle e, e + D \rangle \sum (\alpha_j U_j + \beta_j V_j) + \gamma e$	NP-hard	$O(n^2 p_{max}(D + 1))$ (Yeung et al., 2001a)
4	$1 \langle e, e + D \rangle, p_j = 1 \sum (\alpha_j U_j + \beta_j V_j) + \gamma e$	P	$O(n)$ (Yeung et al., 2001a)
5	$1 \langle e, e + D \rangle, p_i/\alpha'_i \geq p_k/\alpha'_k \Leftrightarrow p_i/\beta'_i \geq p_k/\beta'_k \sum (\alpha'_j E_j + \beta'_j T_j + \alpha_j U_j + \beta_j V_j) + \gamma \max\{0, d - \hat{d}_0\}$	NP-hard	$O(n^3 (D+1)p_{max}^2 (\sum p_j + 1)^2)$ (Yeung et al., 2001b)
6	$1 \langle e, e + D \rangle \sum (\alpha_j U_j + \beta_j V_j + \theta_j C_j) + \gamma \max\{0, d - \hat{d}_0\}$	NP-hard	$O(n^3 (D + 1)(\sum p_j + 1)p_{max}^2 (\sum p_j - D + 1)^2)$ (Yeung et al., 2001b)
7	$1 \langle e, e + D \rangle \sum (\alpha_j U_j + \beta_j V_j + \theta_j C_j) + \gamma d$	NP-hard	$O(n^2 (D + 1)(\sum p_j + 1)(\sum p_j - p_{min} + 1)^2 p_{max})$ (Yeung et al., 2001b)
8	$P \langle e, d \rangle, D_{min} \leq d - e \leq D_{max}, p_j = 1 \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e)$	P	$O(n^4)$ (Mosheiov and Sarig, 2011)
9	$1 \langle e, d \rangle, D_{min} \leq d - e \leq D_{max} \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e)$	NP-hard	Janiak and Winczaszek (2005b)
10	$1 \langle e, d \rangle, D_{min} \leq d - e \leq D_{max}, p_i/\alpha_i < p_j/\alpha_j \Leftrightarrow p_i/\beta_i < p_j/\beta_j \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e)$	NP-hard	$O(n(\sum p_j)^2)$ (Janiak and Winczaszek, 2005b)
11	$1 \langle e, d \rangle, p_i/\alpha_i < p_j/\alpha_j \Leftrightarrow p_i/\beta_i < p_j/\beta_j \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e)$	NP-hard	$O(n^2 \log^3(\max\{n, 1/\epsilon, p_j, \alpha_j, \beta_j, \gamma\}/\epsilon^3))$ (Janiak and Winczaszek, 2005b)
12	$1 \langle e, d \rangle, D_{min} \leq d - e \leq D_{max}, p_j = 1 \sum \alpha_j E_j + \sum \beta_j T_j + \gamma(d - e)$	P	$O(n^3)$ (Janiak and Winczaszek, 2005b)

Table 3. Computational complexity and solution algorithm results for scheduling problems with a common due window assignment and some mathematically complicated models of job processing times.

No.	Problem	Complexity	Algorithms
1	$1 \langle e, d \rangle; p_j = p_j^t - x_j \sum (\alpha E_j + \beta T_j + G_j x_j) + \delta e + \gamma(d - e)$	P	$O(n^3)$ (Liman et al., 1997)
2	$1 \langle e, d \rangle, p_j = (a_j + bt)r^{\alpha} \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e)$	P	$O(n \log n)$ (Wang and Wang, 2011)
3	$1 \langle e, d \rangle, p_{jr} = (p_j - at)r^b \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e)$	P	$O(n \log n)$ (Yang, 2010)
4	$1 \langle e, d \rangle, p_j = a_j + bS_j \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e)$	P	$O(n \log n)$ (Cheng et al., 2010)
5	$1 \langle e, d \rangle, ma \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e)$	P	$O(n^4)$ (Mosheiov and Sarig, 2009b)
6	$1 \langle e, d \rangle, ma, p_{jr} = (p_j - at)r^b \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e)$	P	$O(n^2 \log n)$ (Yang, 2010)
7	$1 \langle e, d \rangle, ma, p_j = a_j + bS_j \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e)$	P	$O(n^2 \log n)$ (Cheng et al., 2010)
8	$1 \langle e, d \rangle, ma, p_{jr} = p_j r^{a_j} \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e)$	P	$O(n^4)$ (Yang et al., 2010)
9	$1 \langle e, d \rangle, rm, p_j^A = p_j + bS_j \sum (\alpha E_j + \beta T_j) + \gamma e + \delta(d - e)$	P	$O(n^4)$ (Zhao and Tang, 2010)

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