Int. J. Appl. Math. Comput. Sci., 2020, Vol. 30, No. 3, 501–515
DOI: 10.34768/amcs-2020-0037

FRACTIONAL ORDER TUBE MODEL REFERENCE ADAPTIVE CONTROL FOR A CLASS OF FRACTIONAL ORDER LINEAR SYSTEMS

HANANE BALASKA[a], SAMIR LADACI[b,c,*], ABDELBAKI DJOUAMBI[a],
HORST SCHULTE[d], BACHIR BOUROUBA[e]

[a] LGEA Laboratory, Department of Electrical Engineering
University of Oum El-Bouaghi
Oum El-Bouaghi 04000, Algeria
e-mail: ha_balaska@yahoo.com, djouambi_abdelbaki@yahoo.fr

[b] Department of Electronics, Electrical Engineering and Automation
National Polytechnic School of Constantine
N.V. Ali Mendjli, Constantine 25100, Algeria

[c] SP-LAB Laboratory, Department of Electronics
Mentouri University of Constantine 1
Route de Ain Elbey, Constantine 25000, Algeria
e-mail: samir_ladaci@yahoo.fr

[d] School of Engineering—Energy and Information
University of Applied Sciences (HTW) Berlin
Wilhelminenhofstrasse 75A, 12459 Berlin, Germany
e-mail: schulte@htw-berlin.de

[e] Department of Electrical Engineering
Setif-1 University
BP: El Bez, Stif 19000, Algeria
e-mail: bourouba_b@yahoo.fr

We introduce a novel fractional order adaptive control design based on the tube model reference adaptive control (TMRAC) scheme for a class of fractional order linear systems. By considering an adaptive state feedback control configuration, the main idea is to replace the classical reference model with a single predetermined trajectory by a fractional order performance tube guidance model allowing a set of admissible trajectories. Besides, an optimization problem is formulated to compute an on-line correction control signal within specified bounds in order to update the system performance while minimizing a control cost criterion. The asymptotic stability of the closed loop fractional order control system is demonstrated using an extension of the Lyapunov direct method. The dynamical performance of the fractional order tube model reference adaptive control (FOTMRAC) is compared with the standard fractional order model reference adaptive control (FOMRAC) strategy, and the simulation results show the effectiveness of the proposed control method.

Keywords: fractional order linear system, model reference adaptive control, fractional adaptive control, optimization, performance tube, fractional order TMRAC.

1. Introduction

Fractional calculus is a topic involving a large scientific research community in various fields of science and engineering applications. Indeed, fractional order differential equations have proven to be more appropriate for the modeling of many physical systems, with a growing number of examples in the research literature. Their applications concern as well renewable energy systems (Neçaibia *et al.*, 2015), long transmission lines (Clarke *et al.*, 2004), robotics (Angel and

*Corresponding author

Viola, 2018), economics (Dadras and Momeni, 2010), electromechanical systems (Cheng *et al.*, 2002), biological systems (Ahmad and Abdel-Jabbar, 2006), cardiac behavior (Goldberger *et al.*, 1985), modeling and identification (Benchellel *et al.*, 2007), and so on.

In the meantime, differential operators of fractional order are becoming more and more prominent in control theory where the system to be controlled and/or the regulator are governed by fractional differential equations (Ladaci and Bensafia, 2016; He *et al.*, 2019). Fractional order operators allow more freedom in designing controllers and setting parameters. A major advantage provided by the resulting controls is an improved robustness compared with classical regulators, as demonstrated in the CRONE (Commande Robuste d'Ordre Non Entier) approach introduced by Oustaloup (1991), the fractional order $\mathrm{PI}^{\lambda}\mathrm{D}^{\mu}$ control with different adjustment strategies (Bettayeb *et al.*, 2017; Neçaibia and Ladaci, 2014; Mondal *et al.*, 2020), the fractional order robust control based on the ideal Bode transfer function (Djouambi *et al.*, 2008), the fractional order high-gain adaptive control (Charef *et al.*, 2013), fractional order dynamic soft variable structure control (Kamal and Bandyopadhyay, 2015) and the model-free fractional order adaptive control (Yakoub *et al.*, 2015).

Special attention has been paid to the fractional order adaptive control structures based on model reference adaptive control (MRAC) in the recent literature (Ladaci and Charef, 2012; Bourouba *et al.*, 2019). Indeed, since the first works of Vinagre *et al.* (2002) as well as Ladaci and Charef (2006), many innovative research studies were developed around this promising adaptive control configuration and its applications.

Various FOMRAC control strategies were developed in the literature; one can cite the MIT adaptive control law based on the fractional order integral (Ladaci and Charef, 2003), the robust feed-forward based FOMRAC (Ladaci *et al.*, 2009) and the composite FOMRAC control (Wei *et al.*, 2016). Also, numerous applications of FOMRAC have been realized, such as electronic systems (Aguila-Camacho and Duarte-Mermoud, 2013), tank level supervision (Balaska *et al.*, 2018), cruise control of an electrical vehicle (Balaska *et al.*, 2019), a multi-source renewable energy system (Djebbri *et al.*, 2020), etc. Some theoretical results on the stability of the FOMRAC control have consolidated this growing technique (Aguila-Camacho *et al.*, 2014; Duarte-Mermoud *et al.*, 2015).

Recently, a new concept of MRAC with performance tube was developed by Mirkin *et al.* (2011). It is called the tube based model reference adaptive control (TMRAC), where the control objective is not only to guaranty the closed loop stability, robustness and asymptotical tracking, but also to optimize the control cost with respect to some criterion. An application of this novel adaptive control scheme was performed for the water level of a cylindrical tank system (Chittillapilly and Hepsiba, 2015).

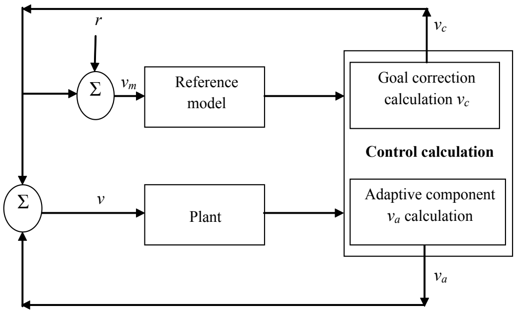

Fig. 1. Performance tube based MRAC configuration.

Fractional order MRAC with the performance tube constraint. The concept of tube reference model based adaptive control with real time input cost adaptation (TMRAC) was proposed and developed by Mirkin *et al.*, (2011; 2012) along with Mirkin and Gutman (2013). In contrast to the standard adaptive control design approach in which the desired performance of the closed loop control system is set using a guidance model with a predefined unique trajectory, in TMRAC control the reference is not imposed as a unique trajectory but as set of acceptable trajectories called tube reference model.

In the control principle detailed by Mirkin and Gutman (2013), a goal correction control is used to define the input signal for the guidance model which can evolve within a chosen range as represented in Fig. 1. By tuning this input signal with respect to the authorized interval, an acceptable set of orientation trajectories are induced. The feedback control signal is thus improved with the goal correction control. The resulting TMRAC strategy has two control aims: the first is the asymptotical stability and tracking, while the second is to satisfy an additional performance objective which can include decreasing the necessary control energy by varying the goal correction control within the permitted range.

In this paper, we propose a novel fractional order tube model reference adaptive control (FOTMRAC) approach to control a class of fractional order linear systems. Using a state feedback control configuration, we set two objectives: the first is to make the system states asymptotically track the states of a stable fractional order "guidance" model and the second goal is to reduce the control energy cost (Sajewski, 2017).

Compared with the literature (Mirkin and Gutman, 2013; Chittillapilly and Hepsiba, 2015), the main contribution is the generalization of the performance tube technique from integer order systems to a larger class of plants involving fractional order derivatives of the states with an improvement of the performance indexes. Many preceding works have proven that fractional order MRAC is advantageously robust against additive noise and disturbances (Ladaci *et al.*, 2009). The challenging problem in this paper is to investigate the fractional order control behavior against model parameter uncertainties, within a fixed set of constraints.

The FOTMRAC design procedure can be summarized in two major stages. In the first one we compute a control law that guarantees the closed loop stability of the fractional order guidance model for any reference input, and the asymptotic tracking of the allowed orientation trajectory. In the second stage, we formulate an optimization problem in order to calculate an on line correction signal in order to minimize the control energy cost. Knowing this step is only feasible after succeeding in the first design stage.

The paper is organized as follows. In Section 2 we describe some necessary preliminaries and present some theoretical concepts on fractional order systems. The proposed FOTMRAC control design is exposed in Section 3, whereas Section 4 outlines the main results of this work and performs the stability analysis of the proposed adaptive control approach. Section 5 is dedicated to numerical simulation examples in order to illustrate the FOTMRAC efficiency. Finally, concluding remarks are given in Section 6.

2. Preliminaries

Fractional order differential equations are widely addressed in the mathematical and control literature (Podlubny, 1999). Their main drawback is generally the lack of exact solutions leading to the use of numerical and approximation techniques for their analysis and implementation. For the purpose of the proposed fractional order adaptive control design, we will present the Grünwald—Letnikov and Caputo derivative definitions with a numerical approximation algorithm for the Grünwald–Letnikov integral and derivative to be used later in our simulations. Also, we will recall some important results that are useful for the stability analysis of the proposed fractional order control system (Aguila-Camacho *et al.*, 2014; Alikhanov, 2013), and the fractional order extension of the Lyapunov theorem (Li and Podlubny, 2010; Kaczorek, 2019).

2.1. Caputo and Grünwald–Letnikov definitions.

The Caputo definition of the derivative of fractional order α has the following form (Kaczorek and Borawski, 2016):

$$ {}_0^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \left[\int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} \mathrm{d}\tau \right] \quad (1) $$

where α is a real number such that $n-1 \leq \alpha \leq n$ and $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y}\, \mathrm{d}y$ is the Gamma function.

The Grünwald–Letnikov derivative is defined as

$$ {}_0^{GL} D_t^\alpha f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{r=0}^{t/h} (-1)^r \binom{\alpha}{r} f(t-rh) \quad (2) $$

where the binomial coefficients ($r > 0$) are given by

$$ \binom{\alpha}{0} = 1, \quad \binom{\alpha}{r} = \frac{\alpha(\alpha-1)\cdots(\alpha-r+1)}{r!} \quad (3) $$

with $r \in \mathbb{N}$ and h denotes the sampling period.

2.2. Grünwald–Letnikov numerical approximation.

For a causal function $f(t)$, with the time operator $t = kh$, the fractional order derivative is (Balaska *et al.*, 2019)

$$ D_t^\alpha f(t) = h^{-\alpha} \sum_{r=0}^{t/h} (-1)^r \binom{\alpha}{r} f(kh-rh) \quad (4) $$

The fractional order integral is given by (Podlubny, 1999)

$$ \begin{aligned} I_t^\alpha f(t) &= D_t^{-\alpha} f(kh) \\ &\approx h^\alpha \sum_{r=0}^{t/h} (-1)^r \binom{-\alpha}{r} f(kh-rh). \end{aligned} \quad (5) $$

Besides, we have the following important property of the Caputo derivative of fractional order (Aguila-Camacho *et al.*, 2014).

Lemma 1. *Let $\xi(t) \in \mathbb{R}^n$ be a continuous and derivable function. Then for any time instant $t \geq t_0$*

$$ \frac{1}{2} {}_{t_0}^C D_t^\alpha \left(\xi^T(t)\xi(t) \right) \leq \xi^T(t) {}_{t_0}^C D_t^\alpha \xi(t) \quad (6) $$

2.3. Stability of fractional order systems.

Consider the fractional order nonlinear time varying systems of the form

$$ {}_{t_0}^C D_t^\alpha \xi(t) = f(\xi, t), \quad (7) $$

where t represents time and $\alpha \in (0,1)$. Here the definition (1) is used. Then we have the following important result for the stability analysis of the system (7) using the direct Lyapunov theorem (Alikhanov, 2010).

Definition 1. A continuous function $\gamma : [0,t) \to [0,\infty)$ is said to be a *class-K function* if it is strictly increasing and $\gamma(0) = 0$ (Duarte-Mermoud *et al.*, 2015).

Theorem 1. *Consider a non autonomous fractional order system (7), and let $\xi = 0$ be its equilibrium point. Assume the existence of a Lyapunov function $V(t, \xi(t))$ and class-K functions γ_i $(i = 1, 2, 3)$ that satisfy*

$$ \gamma_1(\|\xi\|) \leq V(t, \xi(t)) \leq \gamma_2(\|\xi\|), \quad (8) $$

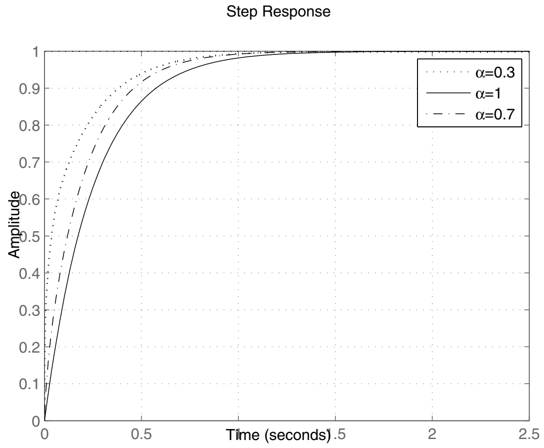

Fig. 2. Comparison of step responses for a first order-like system.

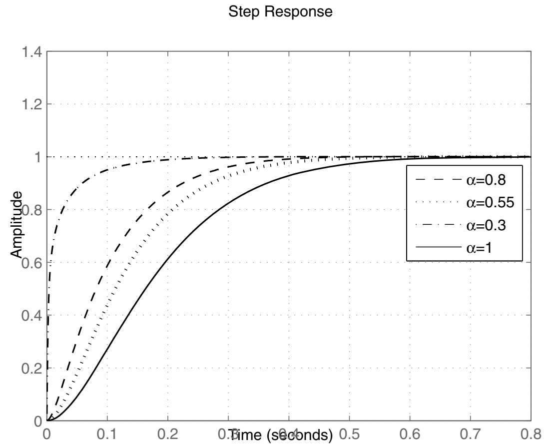

Fig. 3. Comparison of step responses for a second order-like system.

$$ {}_{t_0}^{C}D_t^{\beta} V(t, \xi(t)) \leq -\gamma_3 (\|\xi\|) \tag{9} $$

with $\beta \in (0, 1)$*. Then the system (7) is asymptotically stable.*

Proof. See the proof by Muñoz-Vázquez *et al.* (2019). ■

2.4. Performance of fractional order systems. Advantageous properties of fractional order models compared with standard integer order ones have been pointed out by many previous studies as the best reference models for control applications (Ladaci and Charef, 2006; Mondal and Biswas, 2011). Indeed, fractional order systems offer better quality, in terms of time response and transition dynamic stability, in addition to their intrinsic robustness action against disturbances and noise (Ladaci *et al.*, 2006).

In order to illustrate this fact, consider two examples of systems with respectively first order and second order-like models given by

$$ H_1(s) = \frac{1}{\left(1 + \dfrac{s}{4}\right)^{\alpha}}, \tag{10} $$

$$ H_2(s) = \frac{1}{\left(\dfrac{s^2}{\omega_n^2} + 2\xi\dfrac{s}{\omega_n} + 1\right)^{\alpha}} \tag{11} $$

with $\omega_n = 10$ rad/s, $\xi = 0.95$.

The step responses of systems $H_1(s)$ and $H_2(s)$ for the integer case ($\alpha = 1$) and the fractional order values of (α) are given in Figs. 2 and 3, respectively. They show the gain in the rise time in both the cases.

3. Problem statement

We consider the plant represented by the fractional order linear state model

$$ D^{\alpha}\xi(t) = A\xi(t) + Bv(t), \tag{12} $$

where $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times l}$ are unknown constant parameter matrices, α is a real number such that $0 < \alpha < 1$, $\xi(t) \in \mathbb{R}^n$ and $v(t) \in \mathbb{R}^l$ are the system state and the input signal, respectively. We assume that the plant state variables are measured.

To simplify the notation, we write D^{α} for the fractional derivative of order α instead of ${}_{t_0}^{C}D_t^{\alpha}$.

The first objective of the proposed control approach is to determine the control signal $v(t)$ such that $\xi(t)$ tracks asymptotically the desired fractional order reference model specified by

$$ D^{\alpha}\xi_m(t) = A_m\xi_m(t) + B_m v_m(t) \tag{13} $$

with

$$ v_m(t) = r(t) + v_c(t) \tag{14} $$

where $A_m \in \mathbb{R}^{n\times n}$ is a stable matrix, $B_m \in \mathbb{R}^{n\times l}$, $\xi_m \in \mathbb{R}^n$ and $v_m \in \mathbb{R}^l$ are the reference model state and the reference model input signal, respectively.

Here $r(t) \in \mathbb{R}$ is the reference input signal, and $v_c(t) \in \mathbb{R}$ is the goal correction control signal varying in the following interval:

$$ v_c(t) \in \left[v_c^-(t), v_c^+(t)\right]. \tag{15} $$

Many practical processes can be modeled in the form (12). In fact, fractional order models have proven to be more accurate in representing many physical phenomena than the classical "integer-order" methods. In (Stiassnie, 1979) and (Bagley, 1983), viscoelastic models are formulated by fractional order differential equations. Another typical fractional order system is the voltage-current relation of a semi-infinite lossy transmission line (Wang, 1987) and diffusion of the heat through a semi-infinite solid, where heat flow is equal to the half-derivative of the temperature (Podlubny, 1999). An automotive application of the fractional-order modeling of high-pressure fluid-dynamic flows is presented by Lino *et al*. (2015). Movahhed *et al*. (2016) proposed a fractional state space model for a fractional DC/DC Buck converter. Also a fractional order model is used to represent electrically coupled neuron systems by Moaddy *et al*. (2012) and electrical circuits by Kaczorek (2011). System identification for thermal dynamics of buildings uses fractional order models in the work of Chen *et al*. (2016). Even in biomedicine and biology engineering, a fractional order model is proposed to model immune cells influenced by cancer cells (Ucar *et al*., 2019) and dynamics of HIV infection (Rihan, 2013) (see a more complete review by Ionescu *et al*. (2017)).

Besides, by (14) we define the fractional order reference model input signal, which can be changed within specified limits, and like that, we do not have unique reference trajectory but a set of admissible desired trajectories which constitute the performance tube reference model. In many applications, such as industrial process or flight control, it is not necessary to follow exactly a single reference trajectory, but some specified deviation from it is permitted (Duarte-Mermoud *et al*., 2015).

Then, a second objective is to find the correction signal satisfying the condition (15) that minimizes the cost function,

$$J(v_c) = \int_0^t v^2(\tau)\,\mathrm{d}\tau \tag{16}$$

In order to be able to guarantee the first goal of the proposed fractional order adaptive control strategy, we assume that there exist a constant vector $\theta_\xi^\star \in \mathbb{R}^n$ and a nonzero constant scalar $\theta_m^\star$ such that the following conditions are satisfied:

$$A + B\theta_\xi^{\star T} = A_m, \tag{17}$$

$$B\theta_m^\star = B_m. \tag{18}$$

We assume that the structure of A and B is known and the pair (A, B) is controllable. We also assume that θ_m^* is positive. This last hypothesis is not restrictive because, first, it is usually used in similar designs (Ioannou and Sun, 1996) and we have the ability to choose A_m and B_m in a way that guaranties the existence of $\theta_\xi^\star \in \mathbb{R}^n$ and $\theta_m^\star$. We note that if we consider arbitrary matrices A, B, A_m, B_m, there may be no $\theta_\xi^\star \in \mathbb{R}^n$ and scalar $\theta_m^\star$ satisfying (17) and (18), and, therefore, the control law may not have sufficient structural flexibility to realize the control objective.

In case the structure of A and B is known, we can design the matrices A_m and B_m so that the matching conditions (17) and (18) have a solution for some $\theta_\xi^\star$ and $\theta_m^\star$. In this study, we assume their existence which means that the controller has sufficient structural flexibility to realize the control objective.

The proposed control $v(t)$ is the sum of two signals: an adaptive signal $v_a(t)$, and a goal correction control signal $v_c(t)$,

$$v(t) = v_a(t) + v_c(t) \tag{19}$$

Defining the tracking error $e(t) = \xi(t) - \xi_m(t)$ for any $v(t)$ such that

$$\begin{aligned} D^\alpha e(t) &= D^\alpha \xi(t) - D^\alpha \xi_m(t) \\ &= A\xi(t) + Bv(t) - A_m\xi_m(t) + B_m v_m(t) \\ &= \left(A_m - B\theta_\xi^{\star T}\right)\xi(t) + B\left(v_a(t) + v_c(t)\right) \\ &\quad - A_m\xi_m(t) - B_m\left(r(t) + v_c(t)\right) \\ &= A_m e(t) + B\left[v_a(t) - \theta_\xi^{\star T}\Psi(t) - \theta_c^\star v_c(t)\right] \end{aligned} \tag{20}$$

with

$$\begin{cases} \Psi(t) = [\xi(t)\ r(t)]^T, \\ \theta^\star = [\theta_\xi^{\star T}\ \theta_m^\star]^T, \\ \theta_c^\star = \theta_m^\star - 1, \end{cases} \tag{21}$$

$$v_a(t) = \theta^T\Psi(t) + \theta_c v_c(t), \tag{22}$$

$$\tilde\theta(t) = \theta(t) - \theta^\star, \quad \tilde\theta_c(t) = \theta_c(t) - \theta_c^\star \tag{23}$$

which results in the fractional order error equation

$$D^\alpha e(t) = A_m e(t) + B\tilde\theta^T(t)\Psi(t) + B\tilde\theta_c(t)v_c(t). \tag{24}$$

Following the results of Mirkin and Gutman (2013), we are able to propose the adaptive control of equations (19) and (22), with the following gain adaptation laws:

$$\begin{cases} D^\alpha\theta(t) = -\Lambda_1\Phi(t) - \Lambda_p D^\alpha\Phi(t), \\ \Phi(t) = \Omega(t)\Psi(t), \\ \Omega(t) = e^T(t)PB_m, \\ D^\alpha\theta_c(t) = -\Lambda_c\Omega(t)v_c(t) + \theta_{c0}(t), \end{cases} \tag{25}$$

where

$$\theta_{c0}(t) = \begin{cases} 0 & \text{if} \quad |\theta_c + 1| \ge \epsilon, \\ \Lambda_c\Omega(t)v_c(t) & \text{otherwise} \end{cases} \tag{26}$$

is the projection function, $\Lambda_1 = \Lambda_1^T > 0$, $\Lambda_p = \Lambda_p^T > 0$ and $\Lambda_c > 0$ are the design parameters, ϵ is a known lower bound to $|\theta_c^\star + 1| = |\theta_m^\star| = \theta_m^\star$, P is the solution of the Lyapunov equation

$$A_m^T P + PA_m + Q = 0, \quad Q = Q^T > 0. \tag{27}$$

In the next section, we shall prove that the proposed control law guarantees the asymptotic stability in the sense of Lyapunov of the closed loop system for any choice of $v_c \in [v_c^-, v_c^+]$, and thus, the first control goal is achieved independently from the second one.

The objective for the proposed FOTMRAC control scheme it to design an updating mechanism for the goal correction control $v_c(t) \in [v_c^-(t), v_c^+(t)]$ aiming to minimize the control energy cost evaluated by the optimization criterion (16).

From (16) and (19) we get $J(v_c) = (v_a(t) + v_c(t))^2$. Thus, for any bounded $\theta^T(t)$, $\Psi(t)$ and $\theta_c(t)$, the optimization criterion (16) leads to the following solution:

$$v_c^{\text{opt}}(t) = \begin{cases} -v_a(t) & \text{if } v_a \in [v_c^-, v_c^+], \\ \arg\min(J(v_c^-), J(v_c^+)) & \text{otherwise} \end{cases} \tag{28}$$

where $J(v_c^-$ and $J(v_c^+)$ are the values of $J(v_c) = (v_a(t)+v_c(t))^2$ when $v_c = v_c^-$ and $v_c = v_c^+$, respectively.

From the given controller of Eqns. (19), (25), and (28), it results that the two control goals are realized and obviously, the first stage problem solution is indispensable to deal with the second design step.

Remark 1. Note that the solution of the optimization problem (16) in case when $v_a \in [v_c^-, v_c^+]$ has the form $-(1 + \theta_c(t))^{-1}\theta^T(t)\Psi(t)$, so the adaptive gain $\theta_c(t)$ has a point of singularity. In order to prevent $\theta_c(t)$ from taking the value -1, we suggest the parameter projection algorithm for updating this gain (Ioannou and Sun, 1996).

4. Main results

In order to be able to perform the stability analysis of the proposed performance tube based adaptive control scheme for the discussed class of fractional order systems represented by equation (12) for any bounded choice of $v_c(t) \in [v_c^-(t), v_c^+(t)]$, we need to recall the following lemma (Duarte-Mermoud *et al.*, 2015).

Lemma 2. *Consider a continuous and differentiable function $\xi(t) \in \mathbb{R}^n$, with $Q = Q^T \geq 0 \in \mathbb{R}^{n\times n}$. For any time instant $t \geq t_0$, we have*

$$\frac{1}{2}{}_{t_0}^{C}D_t^\alpha \xi^T(t)Q\xi(t) \leq \xi(t)Q{}_{t_0}^{C}D_t^\alpha \xi(t), \quad \forall\alpha \in (0,1). \tag{29}$$

Proof. We assume that there exist a matrix $P \in \mathbb{R}^{n\times n}$ and a vector $y(t) \in \mathbb{R}^n$ such that

$$Q = P^T P, \quad P\xi(t) = y(t). \tag{30}$$

Thus

$$\frac{1}{2}{}_{t_0}^{C}D_t^\alpha \xi^T(t)P^T P\xi(t) = \frac{1}{2}{}_{t_0}^{C}D_t^\alpha y^T(t)y(t). \tag{31}$$

From (31) and Lemma 1, we get $\forall\alpha \in (0,1)$,

$$\frac{1}{2}{}_{t_0}^{C}D_t^\alpha \xi^T(t)Q\xi(t) \leq y^T(t){}_{t_0}^{C}D_t^\alpha y(t), \tag{32}$$

$$\frac{1}{2}{}_{t_0}^{C}D_t^\alpha \xi^T(t)Q\xi(t) \leq \xi^T(t)P^T P{}_{t_0}^{C}D_t^\alpha \xi(t). \tag{33}$$

Consequently,

$$\frac{1}{2}{}_{t_0}^{C}D_t^\alpha \xi^T(t)Q\xi(t) \leq \xi^T(t)Q{}_{t_0}^{C}D_t^\alpha \xi(t). \tag{34}$$

■

Now we present the main result of this paper concerning the Lyapunov stability of the proposed FOTMRAC control strategy.

Theorem 2. *Consider the class of fractional order systems given by (12), and the designed reference model given by (13), with the reference input (14) and the goal correction control satisfying (15). The FOTMRAC control law given by (19) and (22) with the gain adaptation laws (25) and (28) guarantees that all the closed-loop signals are bounded, the tracking error tends asymptotically to zero, and also minimizes the control cost defined by (16).*

Proof. Define the Lyapunov candidate function

$$V = e^T(t)Pe(t) + \frac{1}{\theta_m^\star}\left[\tilde{\Phi}^T(t)\Lambda_1^{-1}\tilde{\Phi}(t) + \Lambda_1^{-1}\tilde{\theta}_c^2(t)\right], \tag{35}$$

where

$$\tilde{\Phi}(t) = \tilde{\theta} + \Lambda_p\Phi(t). \tag{36}$$

Then

$$\begin{aligned} D^\alpha V = {} & D^\alpha\left[e^T(t)Pe(t)\right] \\ & + \frac{1}{\theta_m^\star}\Big(D^\alpha\left[\tilde{\Psi}^T(t)\Lambda_1^{-1}\tilde{\Psi}(t)\right] \\ & + \Lambda_1^{-1}D^\alpha\left[\tilde{\theta}_c^2(t)\right]\Big). \end{aligned} \tag{37}$$

Applying Lemma 2 for each fractional order derivative term, we get

$$\begin{aligned} & D^\alpha\left[e^T(t)Pe(t)\right] \\ & \leq 2e^T(t)PD^\alpha e(t) \\ & \leq 2\Big[e^T(t)PA_m e(t) \\ & \quad + \frac{1}{\theta_m^\star}\left[\tilde{\theta}^T(t)\Psi(t) + \tilde{\theta}_c(t)\Omega(t)v_c(t)\right]\Big], \end{aligned} \tag{38}$$

$$D^\alpha\left[\tilde{\Psi}^T(t)\Lambda_1^{-1}\tilde{\Psi}(t)\right] \leq 2\tilde{\Psi}^T(t)\Lambda_1^{-1}D^\alpha\tilde{\Psi}(t). \tag{39}$$

But

$$D^\alpha \tilde{\Psi}(t) = D^\alpha \theta(t) + \Lambda_p D^\alpha \Psi(t) = -\Gamma_1 \Psi(t), \tag{40}$$

which yields

$$\begin{aligned} D^\alpha &\left[\tilde{\Psi}^T(t)\Lambda_1^{-1}\Psi(t)\right] \\ &\leq 2\left[\tilde{\theta}^T(t) + \Psi^T(t)\Lambda_p\right]\Lambda_1^{-1}\left[-\Lambda_1\Psi(t)\right] \\ &\leq -2\left[\tilde{\theta}^T(t) + \Psi^T(t)\Lambda_p\right]\Psi(t), \end{aligned} \tag{41}$$

$$\begin{aligned} D^\alpha \left(\tilde{\theta}_c^2(t)\right) &\leq 2\tilde{\theta}_c(t) D^\alpha \tilde{\theta}_c(t) \\ &\leq 2\tilde{\theta}_c(t)\left[-\Lambda_c \Omega(t) v_c(t) + \theta_{c0}(t)\right]. \end{aligned} \tag{42}$$

From (38), (41), and (42), we get

$$\begin{aligned} D^\alpha V \leq{}& 2e^T(t) P A_m e(t) \\ &+ \frac{2}{\theta_m^\star}\left[\tilde{\theta}^T(t)\Psi(t) + \tilde{\theta}_c(t)\Omega(t)v_c(t)\right] \\ &+ \frac{2}{\theta_m^\star}\left[-\tilde{\theta}^T(t)\Psi(t) - \Psi^T(t)\Lambda_p\,\Psi(t)\right] \\ &+ \frac{2\Lambda_c^{-1}}{\theta_m^\star}\left[-\Lambda_c\tilde{\theta}_c(t)\Omega(t)v_c(t) + \tilde{\theta}_c(t)\theta_{c0}(t)\right], \end{aligned} \tag{43}$$

$$\begin{aligned} D^\alpha V \leq{}& 2e^T(t) P A_m e(t) \\ &- \frac{2}{\theta_m^\star}\Psi^T(t)\Lambda_p\,\Psi(t) + \frac{2}{\theta_m^\star \Lambda_c}\tilde{\theta}_c(t)\theta_{c0}(t). \end{aligned} \tag{44}$$

Since

$$\begin{aligned} 2e^T(t) P A_m e(t) &= e^T(t) P A_m e(t) \\ &\quad + e^T(t) A_m^T P e(t) \\ &= e^T(t)\left[A_m^T P + P A_m\right] e(t) \\ &= -e^T(t) Q e(t), \end{aligned} \tag{45}$$

we have

$$\begin{aligned} D^\alpha V \leq{}& -e^T(t) Q e(t) \\ &- \frac{2}{\theta_m^\star}\Psi^T(t)\Lambda_p\,\Psi(t) + \frac{2}{\theta_m^\star \Lambda_c}\tilde{\theta}_c(t)\theta_{c0}(t) \end{aligned} \tag{46}$$

and the last term satisfies $\tilde{\theta}_c(t)\theta_{c0}(t) \leq 0$ (Ioannou and Sun, 1996).

From (46), we have $D^\alpha V \leq 0$, so that the Lyapunov stability of the proposed control scheme is proved for any $v_c(t) \in [v_c^-(t), v_c^+(t)]$. Consequently, the first goal stage is achieved. It is clear that choosing $v_c^{\text{opt}}(t)$ satisfying (28) allows minimizing the control cost defined by (16). Thus, the two control goals are achieved; we guarantee the stability of the closed loop, the tracking convergence and also the minimization of the control cost.

The two control goals are achieved separately, but the solution of the first stage is necessary for the solution of the second one. ■

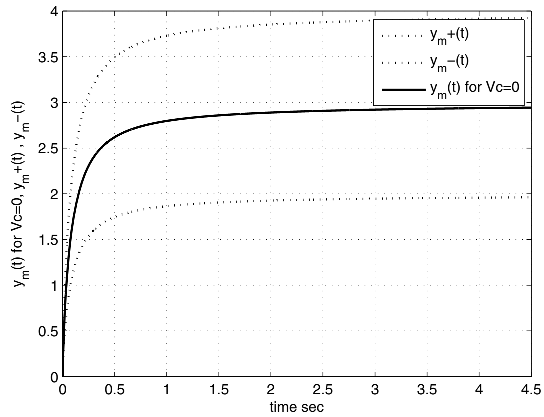

Fig. 4. Reference model response for $v_c(t) = 0$ (solid line) and the performance tube (dotted line) for Example 1.

5. Simulation examples

In this section, we propose two numerical simulation examples to illustrate the efficiency of the proposed FOTMRAC adaptive control strategy. As discussed before, the proposed fractional order models can represent a wide range of practical processes.

5.1. Example 1.

Let the unstable fractional order scalar plant be modeled as

$$D^\alpha y(t) = a y(t) + b v(t), \quad y(0) = y_0. \tag{47}$$

Assume that the desired reference model is

$$\begin{aligned} D^\alpha y_m(t) &= a_m y_m(t) + b_m v_m(t), \\ v_m(t) &= r(t) + v_c(t), \end{aligned} \tag{48}$$

where $a = 1$, $b = 0.5$, $a_m = -5$, $b_m = 5$, $v_c^- = -1$, $v_c^+ = +1$, $\alpha = 0.75$ and $r(t)$ is a step signal with amplitude $+3$.

By the choice of a_m and b_m we describe the performance of the reference model. In this numerical example, we choose a_m as a stable pole, and b_m in order to have a unitary gain.

Since the reference signal $r(t)$ is a step of $+3$ in amplitude, we choose the range of $v_c(t)$ in $[-1, 1]$, and thus, we define the admissible range for the reference trajectories. The variables $y_m^+(t)$ and $y_m^-(t)$ in Fig. 4, represent the reference model response for the inputs $v(t) = r(t) + v_c^+$ and $v(t) = r(t) + v_c^-$, respectively.

These variables define the tube of the admissible reference trajectories. The reference model response for $v_c(t) = 0$ is represented in Fig. 4 (the solid line).

The fractional order tube model reference adaptive controller of (19) and (22) with the gain adaptation

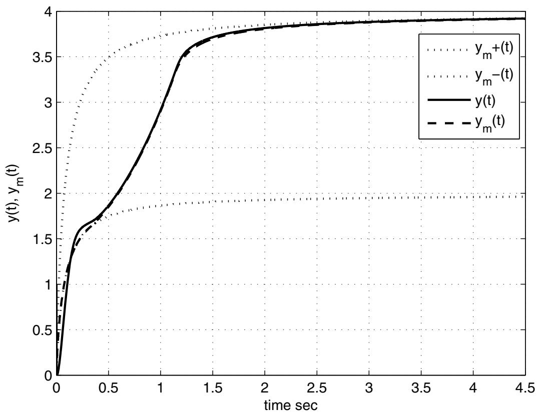

Fig. 5. Controlled system output (solid line) and the reference trajectory (dotted line) within the performance tube (dashed line).

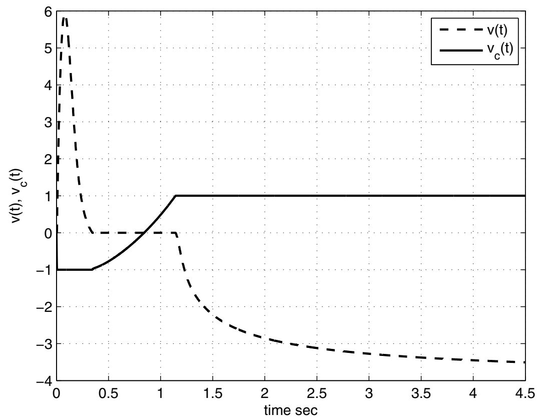

Fig. 6. FOTMRAC control signal (solid line) and the goal correction control signal (dashed line).

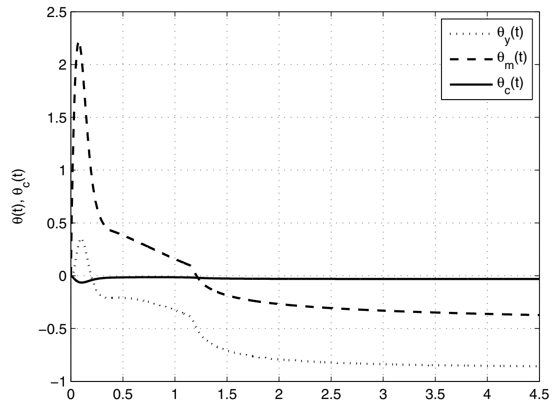

Fig. 7. Adaptive gain vectors.

laws (25) and the optimization problem solution (28) are applied to the plant (47) with the controller parameters

$$\Lambda_1 = 9, \quad \Lambda_p = 0.6, \quad P = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/5 \end{bmatrix}. \qquad (49)$$

P is the solution of the Lyapunov equation (27). Besides, in order to compare the performance of the proposed FOTMRAC strategy with the FOMRAC scheme (i.e., $v_c = 0$ and $\theta_c = 0$ in (19) and (22)), we adjust the value of the reference model input in order to have the same steady state trajectory reaching.

For the needs of numerical simulation, we use the Gründwald–Letnikov approximation of the derivative and integrator operators (4) and (5), respectively.

Our experimental setting for the proposed control application includes two cases with two different values for the reference model inputs $r(t) = +3$ and $r(t) = -3$. First we take the reference model input $r(t) = +3$. The simulation results for the obtained closed loop control system output and the guidance trajectory within the performance tube are presented in Fig. 5.

The FOTMRAC control signal, the goal correction control signal and the adaptive gain vectors are respectively represented in Figs. 6 and 7.

The closed loop control system outputs for both the cases of fractional order adaptive control (FOMRAC) with optimization and without optimization are shown in Fig. 8, whereas the absolute control signals are illustrated in Fig. 9.

By taking the reference model input $r(t) = -3$, we obtain the results summarized in Figs. 10–14, illustrating, respectively, the controlled system output and the reference trajectory within the performance tube (Fig. 10), the FOTMRAC control signal and the goal correction control signal (Fig. 11), the adaptive gain vectors (Fig. 12), the system outputs for the cases of fractional order adaptive control with and without optimization (Fig. 13), and the absolute control signals for the same cases (Fig. 14).

From these simulation results, it is obvious that the correction control signal is updated at each instant of time, and varies within the preset limits in order to reduce the energy control criterion. It implies also that the reference trajectory is updated in real time and remains in the predefined model reference tube, whereas the system output tracks perfectly the updated trajectory.

Figures 9 and 14 show the absolute value of the control signals $|v(t)|$ for the cases of fractional order adaptive control with and without optimization, respectively. We notice that the control signal cost is clearly lower with optimization, and thus, we have the same steady response space with a minimum control effort.

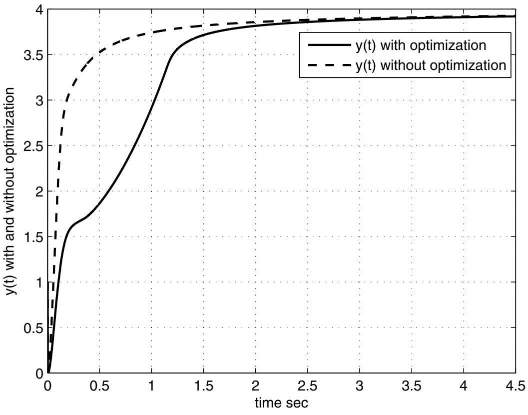

Fig. 8. System outputs for case of fractional order adaptive control with and without optimization.

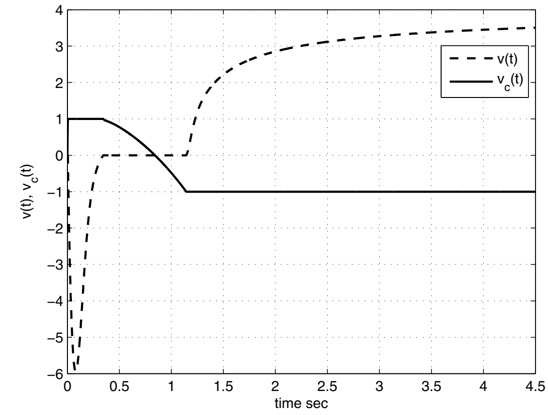

Fig. 11. FOTMRAC control signal and the goal correction control signal.

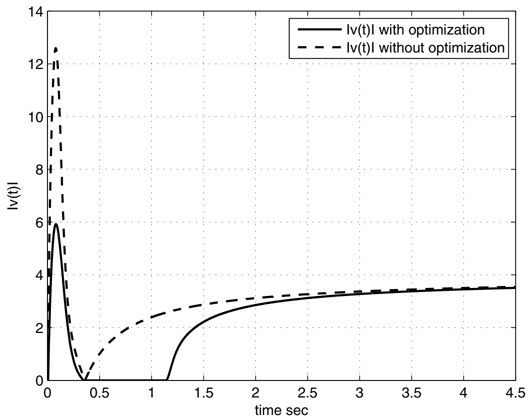

Fig. 9. Absolute control signals for case of fractional order adaptive control with and without optimization.

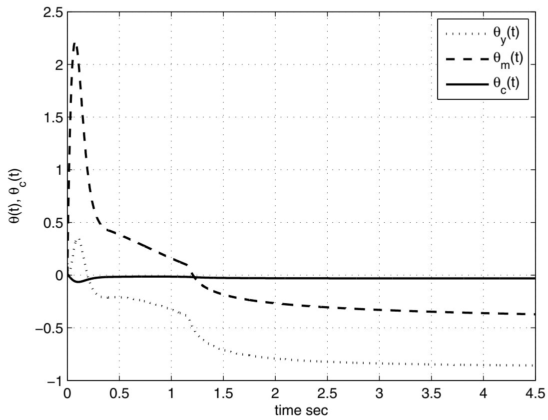

Fig. 12. Adaptive gain vectors.

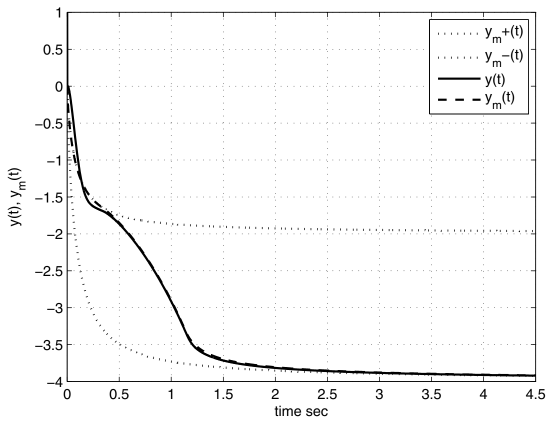

Fig. 10. Controlled system output and the reference trajectory within the performance tube.

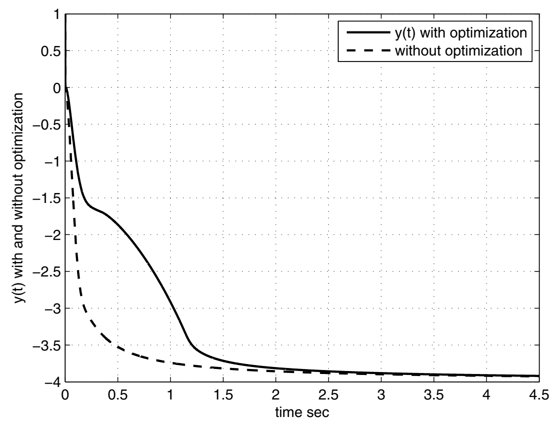

Fig. 13. System outputs for the case of fractional order adaptive control with and without optimization.

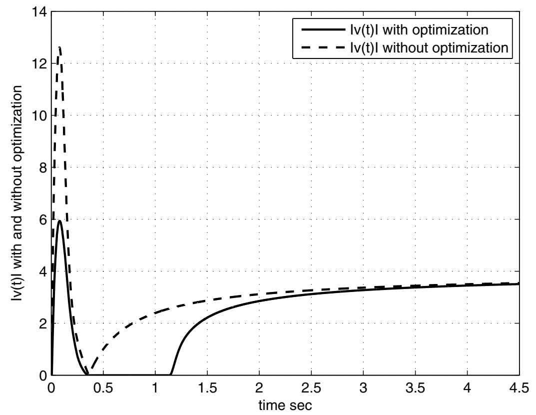

Fig. 14. Absolute control signals for the case of fractional order adaptive control with and without optimization.

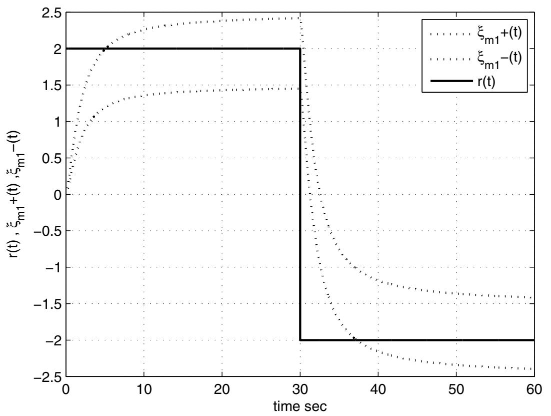

Fig. 15. Model reference tube and the reference input signal for Example 2.

Table 1. Comparison of control cost evaluations for Example 1.

Cost	FOTMRAC strategy (with optimization)	FOMRAC strategy (without optimization)
$J = \sum_k v^2(k)$	35171	54100

To evaluate the quadratic criterion J given in (16), we compare for the two cases (with and without optimization) the control cost function given in Table 1.

We observe that the control cost of the proposed method is much lower (about 35%) than in the case of the standard model reference adaptive control.

5.2. Example 2. Let us now consider the unstable SIMO system defined by

$$\begin{bmatrix} D^\alpha \xi_1(t) \\ D^\alpha \xi_2(t) \end{bmatrix} = A \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} + B\, v(t), \quad (50)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},$$

and the order $\alpha = 0.8$.

Our objective is to design a state feedback control law $v(t)$ according to our proposed FOTMRAC strategy, in order to control the first state variable $\xi_1(t)$. For that purpose we choose the following stable reference model:

$$\begin{bmatrix} D^\alpha \xi_m 1(t) \\ D^\alpha \xi_m 2(t) \end{bmatrix} = A_m \begin{bmatrix} \xi_m 1(t) \\ \xi_m 2(t) \end{bmatrix} + B_m\, v_m(t), \quad (51)$$

where

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

and

$$v_m(t) = r(t) + v_c(t).$$

The controller parameter values are chosen as follows:

$$P = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

$$\Lambda_p = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}, \quad \Lambda_c = 2. \quad (52)$$

The reference input signal $r(t)$ is represented in Fig. 15. The variables $\xi_{m1}^{+}(t)$ and $\xi_{m1}^{-}(t)$ in Fig. 15 represent the first state variable of the reference model for the input $v(t) = r(t) + v_c^{+}$ and $v(t) = r(t) + v_c^{-}$, respectively, where $v_c^{-} = -0.5$, $v_c^{+} = +0.5$. These variables define the tube of the admissible reference trajectories.

Figure 16 shows the controlled output system, tracking perfectly the model reference trajectory within the reference tube. This reference trajectory is updated by the adaptive control law such that the control effort is minimized.

The control signal and the goal correction control signal are shown in Fig. 17, whereas the adaptive gain vectors are represented in Fig. 18.

In order to compare the performance of the proposed FOTMRAC strategy with the FOMRAC scheme (i.e., $v_c(t) = 0$ and $\theta_c = 0$ in (19) and (22)), we adjust the value of the reference model input in order to have the same steady state trajectory reaching. The controlled output system and the absolute value of the control signals for the

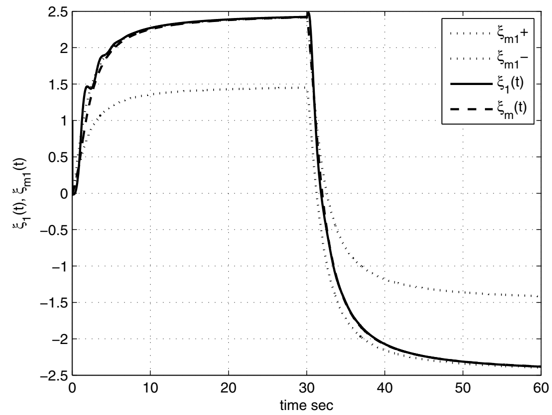

Fig. 16. Controlled output system and the reference trajectory within the performance tube.

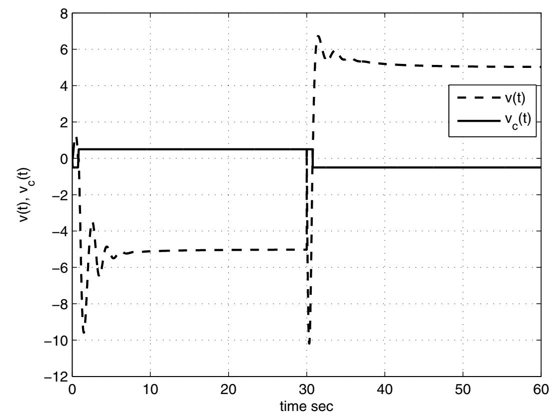

Fig. 17. FOTMRAC control signal and the goal correction control signal.

Table 2. Comparison of control cost evaluations for Example 2.

Cost	FOTMRAC strategy (with optimization)	FOMRAC strategy (without optimization)
$J = \sum_k v^2(k)$	162190	165500

scheme of adaptive control with and without optimization are respectively illustrated in Figs. 19 and 20.

For the same steady state response, we remark that the proposed FOTMRAC controller finds the reference trajectory within the performance tube so that the control effort is minimized.

From Fig. 20, it is obvious that the control cost with FOTMRAC strategy is lower than the control cost applying a simple FOMRAC control without optimization. We notice that the gain in control cost compared with the classical scheme is realized in the transitory phase, because in the steady state phase the curves of the proposed and the standard control converge to the same value. From the instant 0 [s] to 10 [s], the gain equals 5.75%. Notice that this improvement in control cost could be increased by augmenting the interval $[v_c^-, v_c^+]$. For example, if we take the interval $[-1.5, +1.5]$, the gain in command cost becomes 13.98%.

Also to evaluate the criterion J, we present a comparative evaluation of the control cost function in the two cases (with and without optimization) in Table 2.

In regard of the obtained simulation results, we can say that our proposed FOTMRAC strategy gives a better control performance since all the closed loop signals are bounded, the plant response tracks the reference trajectory within the admissible range (the performance tube) with a minimum control effort. Even if this performance quantification is not linearly related to the fractional order, we can always find a result improvement using a fractional order tube MRAC control.

6. Conclusion

In this article, a novel fractional order tube model reference adaptive control (FOTMRAC) has been developed for a class of fractional order linear systems. Based on the TMRAC control scheme proposed by Mirkin and Gutman (2013), the proposed adaptive control scheme generalizes the performance tube technique to the class of arbitrary order systems.

Using an adaptive state feedback configuration, this approach has two main objectives: the first one is to ensure the stability of the resulting fractional order closed loop system and the asymptotic tracking of the guidance trajectory within the performance tube. The second objective is the minimization of a control cost. The stability analysis has been performed in Theorem 2 using an extended version of the Lyapunov theorem to fractional order systems.

Two illustrative examples have been presented to demonstrate the effectiveness and the accuracy of the proposed method. Applying the FOTMRAC control scheme has given satisfactory performance for the two plants considered, namely:

- The asymptotic stability is guaranteed in both the cases despite the fact that the model parameters are unknown.
- The systems' outputs lay in the reference model tube domain.
- The control energy cost is minimized.
- The performance indexes are much better than those of the classical TMRAC control scheme.

Future research will concern the extension of this adaptive control approach to fractional order nonlinear and MIMO systems.

References

Aguila-Camacho, N. and Duarte-Mermoud, M.A. (2013). Fractional adaptive control for an automatic voltage regulator, *ISA Transactions* **52**(6): 807–815.

Aguila-Camacho, N., Duarte-Mermoud, M. and Gallegos, J. (2014). Lyapunov functions for fractional order systems, *Communications in Nonlinear Science and Numerical Simulation* **19**(9): 2951–2957.

Ahmad, W. and Abdel-Jabbar, N. (2006). Modeling and simulation of a fractional order bioreactor system, *Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and Its Applications, Porto, Portugal*, pp. 1–5.

Alikhanov, A.A. (2010). On the stability and convergence of nonlocal difference schemes, *Differential Equations* **46**(7): 949–961.

Alikhanov, A.A. (2013). Stability and convergence of difference schemes approximating a two-parameter nonlocal boundary value problem, *Differential Equations* **49**(7): 796–806.

Angel, L. and Viola, J. (2018). Fractional order PID for tracking control of a parallel robotic manipulator type delta, *ISA Transactions* **79**: 172–188.

Bagley, R. (1983). Fractional calculus – a different approach to the analysis of viscoelastically damped structures, *AIAA Journal* **21**(5): 741–748.

Balaska, H., Ladaci, S., Schulte, H. and Djouambi, A. (2019). Adaptive cruise control system for an electric vehicle using a fractional order model reference adaptive strategy, *IFAC-PapersOnLine* **52**(13): 194–199.

Balaska, H., Ladaci, S. and Zennir, Y. (2018). Conical tank level supervision using a fractional order model reference adaptive control strategy, *Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotics, ICINCO 2018, Porto, Portugal*, pp. 224–231.

Benchellel, A., Poinot, T. and Trigeassou, J.-C. (2007). Modelling and identification of diffusive systems using fractional models, *in* J. Sabatier *et al.* (Eds), *Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering*, Springer, Berlin, pp. 213–225.

Bettayeb, M., Mansouri, R., Al-Saggaf, U. and Mehedi, I. (2017). Smith predictor based fractional-order-filter PID controllers design for long time delay systems, *Asian Journal of Control* **19**(2): 1–12.

Bourouba, B., Schulte, H. and Ladaci, S. (2019). A novel MRAC-based fractional adaptive control design for a class of fractional order linear systems, *Proceedings of the 8th International Conference on Systems and Control (ICSC 2019), Marrakesh, Morocco*, pp. 303–308.

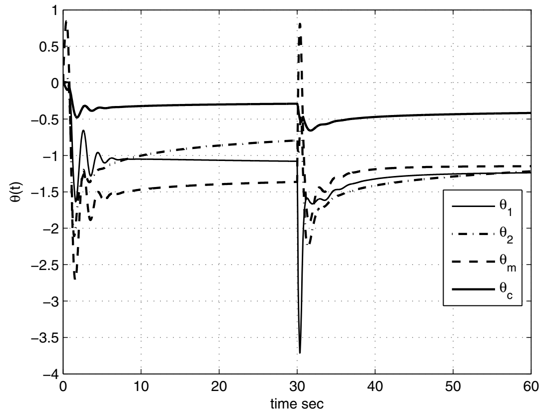

Fig. 18. Adaptive gain vectors.

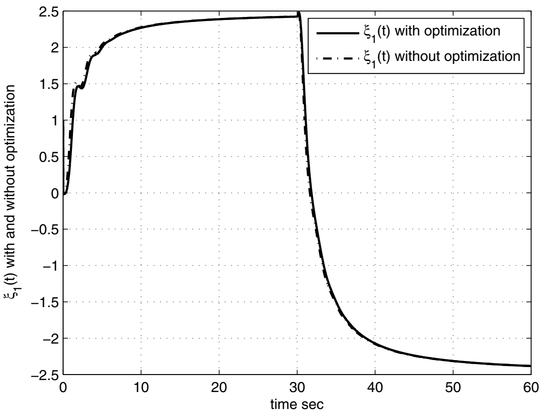

Fig. 19. System output for the cases of fractional order control with and without optimization.

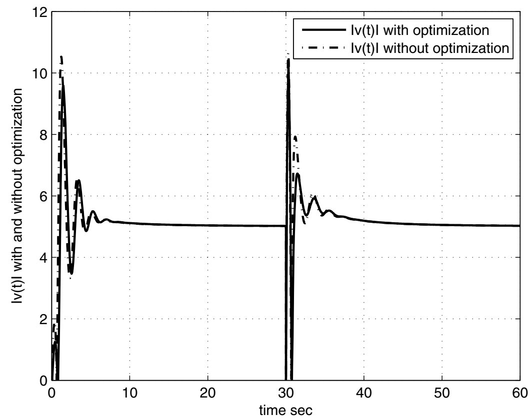

Fig. 20. Absolute control signals for the cases of fractional order control with and without optimization.

Charef, A., Assabaa, M., Ladaci, S. and Loiseau, J. (2013). Fractional order adaptive controller for stabilized systems via high-gain feedback, *IET Control Theory and Applications* **7**(6): 822–828.

Chen, L., Basua, B. and McCabe, D. (2016). Fractional order models for system identification of thermal dynamics of buildings, *Energy and Buildings* **133**: 381–388.

Cheng, C., Chen, C. and Chiu, G.-C. (2002). Predictive control with enhanced robustness for precision positioning in frictional environment, *IEEE/ASME Transactions On Mechatronics* **7**(3): 385–392.

Chittillapilly, R. and Hepsiba, D. (2015). Tube model reference adaptive control for a cylindrical tank system, *Proceedings of the International Conference on Innovations in Information, Embedded and Communication Systems (ICIIECS), Coimbatore, India*, pp. 1–4.

Clarke, T., Narahari-Achar, B. and Hanneken, J. (2004). Mittag-Leffler functions and transmission lines, *Journal of Molecular Liquids* **114**(1): 159–163.

Dadras, S. and Momeni, H. (2010). Control of a fractional-order economical system VIA sliding mode, *Physica A* **389**(12): 2434–2442.

Djebbri, S., Ladaci, S., Metatla, A. and Balaska, H. (2020). Fractional-order model reference adaptive control of a multi-source renewable energy system with coupled DC/DC converters power compensation, *Energy Systems* **11**(2): 1–41 .

Djouambi, A., Charef, A. and Voda, B. (2008). Fractional order robust control based on Bode's ideal transfer function, *RS-JESA, Fractional order systems* **42**: 999–1014.

Duarte-Mermoud, M., Aguila-Camacho, N., Gallegos, J. and Castro-Linares, R. (2015). Using general quadratic Lyapunov functions to prove Lyapunov uniform stability for fractional order systems, *Communications in Nonlinear Science and Numerical Simulation* **22**(1): 650–659.

Goldberger, A., Bhargava, V., West, B. and Mandell, A. (1985). On the mechanism of cardiac electrical stability, *Biophysics Journal* **48**(3): 525–528.

He, B.-B., Zhou, H.-C., Kou, C.-H. and Chen, Y.-Q. (2019). Stabilization of uncertain fractional order system with time-varying delay using BMI approach, *Asian Journal of Control*, DOI: 10.1002/asjc.2193.

Ioannou, P. and Sun, J. (1996). *Robust Adaptive Control*, Prentice-Hall, Englewood Cliffs, NJ.

Ionescu, C., Lopes, A., Copot, D., Machado, J. and Bates, J. (2017). The role of fractional calculus in modelling biological phenomena: A review, *Communications in Nonlinear Science and Numerical Simulation* **51**: 141–159.

Kaczorek, T. (2011). Singular fractional linear systems and electrical circuits, *International Journal of Applied Mathematics and Computer Science* **21**(2): 379–384, DOI: 10.2478/v10006-011-0028-8.

Kaczorek, T. (2019). Absolute stability of a class of fractional positive nonlinear systems, *International Journal of Applied Mathematics and Computer Science* **29**(1): 93–98, DOI: 10.2478/amcs-2019-0007.

Kaczorek, T. and Borawski, K. (2016). Fractional descriptor continuous–time linear systems described by the Caputo–Fabrizio derivative, *International Journal of Applied Mathematics and Computer Science* **26**(3): 533–541, DOI: 10.1515/amcs-2016-0037.

Kamal, S. and Bandyopadhyay, B. (2015). High performance regulator for fractional order systems: A soft variable structure control approach, *Asian Journal of Control* **17**(5): 1–5.

Ladaci, S. and Bensafia, Y. (2016). Indirect fractional order pole assignment based adaptive control, *Engineering Science and Technology, an International Journal* **19**(1): 518–530.

Ladaci, S. and Charef, A. (2003). MIT adaptive rule with fractional integration, *Proceedings of the CESA 2003 IMACS Multiconference on Computational Engineering in Systems Applications, Lille, France*, pp. 1–6.

Ladaci, S. and Charef, A. (2006). On fractional adaptive control, *Nonlinear Dynamics* **43**(4): 365–378.

Ladaci, S. and Charef, A. (2012). Fractional order adaptive control systems: A survey, *in* E. Mitchell and S. Murray (Eds), *Classification and Application of Fractals*, Nova Science Publishers, Inc., New York, NY, pp. 261–275.

Ladaci, S., Charef, A. and Loiseau, J.J. (2009). Robust fractional adaptive control based on the strictly positive realness condition, *International Journal of Applied Mathematics and Computer Science* **19**(1): 69–76, DOI: 10.2478/v10006-009-0006-6.

Ladaci, S., Loiseau, J. and Charef, A. (2006). Using fractional order filter in adaptive control of noisy plants, *Proceedings of the International Conference on Advances in Mechanical Engineering And Mechanics, ICAMEM 2006, Hammamet, Tunisia*, pp. 1–6.

Li, Y. and Podlubny, I. (2010). Stability of fractional-order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag-Leffler stability, *Computers & Mathematics with Applications* **59**(5): 1810–1821.

Lino, P., Maione, G. and Saponaro, F. (2015). Fractional-order modeling of high-pressure fluid-dynamic flows: An automotive application, *IFAC-PapersOnLine* **48**(1): 382–387, DOI: 10.1016/j.ifacol.2015.05.093.

Mirkin, B. and Gutman, P.-O. (2013). Tube model reference adaptive control, *Automatica* **49**(4): 1012–1018.

Mirkin, B., Gutman, P.-O. and Shtessel, Y. (2012). Coordinated decentralized sliding mode MRAC with control cost optimization for a class of nonlinear systems, *Journal of the Franklin Institute* **349**(4): 1364–1379.

Mirkin, B., Gutman, P.-O. and Sjoberg, J. (2011). Output-feedback MRAC with reference model tolerance of nonlinearly perturbed delayed plants, *18th IFAC World Congress, Milan, Italy*, pp. 6751–6756.

Moaddy, K., Radwan, A.G., Salama, K., Momani, S. and Hashim, I. (2012). The fractional-order modeling and synchronization of electrically coupled neuron systems, *Computers & Mathematics with Applications* **64**(10): 3329–3339.

Mondal, D. and Biswas, K. (2011). Performance study of fractional order integrator using single-component fractional order element, *IET Circuits Devices & Systems* **5**(4): 334–342, DOI: 10.1049/iet-cds.2010.0366.

Mondal, R., Chakraborty, A., Dey, J. and Halder, S. (2020). Optimal fractional order PID controller for stabilization of cart-inverted pendulum system: Experimental results, *Asian Journal of Control* **22**(3): 1345–1359.

Movahhed, A., Shandiz, H. and Sani, S. (2016). Comparison of fractional order modelling and integer order modelling of fractional order buck converter in continuous condition mode operation, *Advances in Electrical and Electronic Engineering: Power Engineering and Electrical Engineering* **14**(5): 531–542.

Muñoz-Vázquez, A., Parra-Vega, V., Sánchez-Orta, A. and Martínez-Reyes, F. (2019). Robust Mittag-Leffler stabilisation of fractional-order systems, *Asian Journal of Control*, DOI: 10.1002/asjc.2195.

Neçaibia, A. and Ladaci, S. (2014). Self-tuning fractional order $PI^{\lambda}D^{\mu}$ controller based on extremum seeking approach, *International Journal of Automation and Control* **8**(2): 99–121.

Neçaibia, A., Ladaci, S., Charef, A. and Loiseau, J. (2015). Fractional order extremum seeking approach for maximum power point tracking of photo-voltaic panels, *Frontiers in Energy* **9**(1): 43–53.

Oustaloup, A. (1991). *La commande CRONE*, Hermès, Paris.

Podlubny, I. (1999). *Fractional Differential Equations*, Academic Press, San Diego, CA.

Rihan, F. (2013). Numerical modeling of fractional-order biological systems, *Abstract and Applied Analysis* **2013**(816803): 1–11, DOI: 10.1155/2013/816803.

Sajewski, L. (2017). Minimum energy control of descriptor fractional discrete-time linear systems with two different fractional orders, *International Journal of Applied Mathematics and Computer Science* **27**(1): 33–41, DOI: 10.1515/amcs-2017-0003.

Stiassnie, M. (1979). On the application of fractional calculus for the formulation of viscoelastic models, *Applied Mathematical Modelling* **3**(4): 300—-302.

Ucar, E., Ozdemir, N. and Altun, E. (2019). Fractional order model of immune cells influenced by cancer cells, *Mathematical Modelling of Natural Phenomena* **14**(308): 1–12.

Vinagre, B., Petras, I., Podlubny, I. and Chen, Y. (2002). Using fractional order adjustment rules and fractional order reference models in model-reference adaptive control, *Nonlinear Dynamics* **29**(1–4): 269–279.

Wang, J. (1987). Realizations of generalized Warburg impedance with RC ladder networks and transmission lines, *Journal of The Electrochemical Society* **134**(8): 1915—1920.

Wei, Y., Sun, Z., Hu, Y. and Wang, Y. (2016). On fractional order composite model reference adaptive control, *International Journal of Systems Science* **47**(11): 2521–2531.

Yakoub, Z., Amairi, M., Chetoui, M., Saidi, B. and Aoun, M. (2015). Model-free adaptive fractional order control of stable linear time-varying systems, *ISA Transactions* **67**: 193–207.

Hanane Balaska received her automatics engineering degree in 2004 and her MSc degree in advanced automatics in 2008, both from the Department of Electrical Engineering, University of Skikda, Algeria. From 2007 to 2012, she worked as an industrial instrumentation engineer at the Sonatrach refinery, Skikda. She joined the University of Skikda in 2012, where she has been teaching until today. She is currently pursuing her PhD on fractional robust adaptive control in the Department of Electrical Engineering, University of Larbi Ben M'hidi, Oum El Bouaghi, Algeria. Her research interests include fractional order adaptive control, fault diagnosis and fault tolerant control.

Samir Ladaci was born in 1971. He obtained his state engineering degree in automatics from the Polytechnic School of Algiers in 1995 and his MSc degree in industrial automation from Annaba University, Algeria, in 1999. He received his PhD and *Habilitation Universitaire* from the Department of Electronics, Mentouri University of Constantine, Algeria, in 2007 and 2009, respectively. Currently he is employed as a full professor at the National Polytechnic School of Constantine. He is the author or a co-author of a book and more than 150 other publications, including over 40 journal articles. His research interests include fractional order systems and control, adaptive control, robust control, chaotic systems, identification.

Abdelbaki Djouambi is currently a full professor at the Department of Electrical Engineering, University of Oum El Bouaghi, Algeria. He received his PhD degree from the Department of Electronics, University of Constantine, Algeria, in 2008 and his HDR degree from the Department of Electronics, University of Batna, Algeria, in 2011. His current research interest is fractional order systems identification and robust control.

Horst Schulte received his diploma degree in electrical engineering from TU Berlin and his PhD degree in control engineering from the University Kassel (Germany). He joined the Bosch Group in 2005, where he worked in R&D projects in the field of modeling, optimization and advanced control of actuators, power systems and drive trains. Since 2009, he has been a full professor at the University of Applied Sciences HTW Berlin. His research interests include nonlinear controller and observer design with Takagi–Sugeno (TS), LPV and sliding-mode techniques, robust control system design, active fault-tolerant control (FTC) system design with industrial applications. He is the author of more than 145 scientific publications, including international journal papers, book chapters, patents, and conference papers. Prof. Schulte is a European Advanced Control and Diagnosis (EACD) steering committee member and a member of IFAC TC 6.4 on Fault Detection, Supervision & Safety of Technical Processes as well as IFAC TC 7.1 on Automotive Control.

Bachir Bourouba received his state engineering degree in automatic control from the Ferhat Abbes University of Setif 1, Algeria, in 2000, and his MSc degree at the same university in 2005. He obtained his DSc degree from the Department of Electronics, Mentouri Brothers University of Constantine, Algeria, in 2018. Since 2008, he has been an assistant professor at the Department of Electrical Engineering at the Ferhat Abbes University of Setif 1. His research interests focus on fractional systems, optimal control, chaotic system and fractional adaptive intelligent control.

Received: 20 January 2020
Revised: 23 March 2020
Re-revised: 24 April 2020
Accepted: 29 April 2020