Int. J. Appl. Math. Comput. Sci., 2024, Vol. 34, No. 3, 361–373
DOI: 10.61822/amcs-2024-0026

OBSERVER–BASED SLIDING–MODE FAULT–TOLERANT CONSISTENT CONTROL FOR HYBRID EVENT–TRIGGERED MULTI–AGENT SYSTEMS

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An observer-based hybrid event-triggered sliding mode fault-tolerant consistent control strategy is proposed for actuator faults in nonlinear second-order leader–follower multi-agent systems. A fault observer is designed to obtain the velocity and additive fault of the agents at the current moment. In order to save network resources and avoid the proliferation of actuator fault information, a hybrid event-triggered mechanism is given based on the actuator fault output from the fault observer. Then, a sliding mode fault-tolerant control strategy is investigated based on the speed and hybrid event-triggered mechanism of the fault observer output and combined with a linear sliding mode surface. As a result, the multi-agent system can still realize state consistency when there is an actuator fault. Conditions under which the consistent error of the multi-agent system is bounded are given. Finally, the effectiveness of the designed fault observer, sliding mode fault-tolerant controller, and hybrid event-triggered mechanism is verified by simulation in a leader–follower multi-agent system connected by a directed graph.

Keywords: multi-agent systems, fault-tolerant control, fault observers, sliding mode control, hybrid event-triggered mechanisms.

1. Introduction

In recent years, due to the flexibility and structural diversity of multi-agent systems, they have been widely used in aerospace, military, and industrial fields (Dong *et al.*, 2018; Darvishpoor *et al.*, 2020; Zhai *et al.*, 2023). At the same time, various control problems for multi-agent systems have become a research hotspot. Among them, consistent control is the basic problem of multi-agent systems. In addition, containment control (Wang *et al.*, 2014), formation control (González *et al.*, 2022) and fault-tolerant control are also the investigation focus.

Because of aging equipment, communication errors, etc., various faults may occur in multi-agent systems in practice. These faults will affect the state of some agents, and most of the inter-agent communication relies on the state information of neighbor agents. If the faults are not handled or isolated in a timely manner, then the fault information will be quickly passed on to other agents. This may eventually lead to system paralysis or destruction (Domyshev and Sidorov, 2022; Pham *et al.*, 2020; Khalili *et al.*, 2018; Xu *et al.*, 2018).

Solving the fault-tolerant control problem for multi-agent systems has become an important research direction in multi-agent system control (Chen *et al.*, 2019; Wang *et al.*, 2022; Salmanpour *et al.*, 2023; Liu *et al.*, 2021; Yu *et al.*, 2019). A class of multi-agent models containing only additive actuator faults was studied by Chen *et al.* (2019). Adaptive rates are used to estimate an upper bound for additive faults, and a fault tolerant controller is designed based on this fault upper bound. For a class of fuzzy multi-agent systems, distributed fault-tolerant consistent control strategies were designed by Wang *et al.* (2022). A class of second-order multi-agent systems with actuator faults is discussed by Salmanpour *et al.* (2023). A navigator state observer is constructed for estimating the state of the leader and a fault-tolerant controller with finite time convergence

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is designed.

A multi-agent system containing multiple leaders and followers is investigated by Chen *et al*. (2023). A neural network based adaptive observer is designed to estimate the unmeasurable states of the system. And a fault-tolerant control is designed based on the measured values. Also, for a class of nonlinear multi-agent systems with multiple leaders and followers, a finite-time fuzzy fault-tolerant controller was designed by Liu *et al*. (2021). For a class of multi-agent systems with fixed topology and switching topology with incipient actuator faults, a novel distributed fault-tolerant consistency tracking controller was proposed by Yu *et al*. (2019). However, most of the current results do not use observers to obtain faults in multi-agent systems. There is not enough research in fault diagnostics. Faults are not well utilized in the design of fault-tolerant controllers.

In the sliding mode control method, the system error is constructed into a hyperplane in a certain way, and then the control law is designed to make the system error converge to the corresponding equilibrium point along the designed hyperplane. Currently, this control method has been widely studied because it is easier to be designed and has good robustness (Yorgancioğlu and Redif, 2019; Li *et al*., 2020b; Dong *et al*., 2019). Because multi-agent systems usually encounter various nonlinear disturbances in practical applications, the sliding mode control method is also an important control strategy for multi-agent systems (Chen *et al*., 2020; Parsa and Akbarzadeh-T, 2020; Li *et al*., 2020a; Khoygani *et al*., 2021).

The second-order consistent problem for a class of hybrid multi-agent systems with unknown disturbances is considered by Chen *et al*. (2020). Based on the equivalent approximation law and the state information among the agents, a sliding mode control protocol was proposed. A class of high-order uncertain stochastic multi-agent systems is studied by Parsa and Akbarzadeh-T (2020). A new distributed fuzzy sliding mode controller is designed by combining consistency and tracking error. A distributed recursive linear sliding mode control scheme is proposed for a class of high-order nonlinear multi-agent systems by using backstepping by Li *et al*. (2020a). An observer-based sliding mode consistent control algorithm is proposed for a class of multi-agent omnidirectional wheeled robots by Khoygani *et al*. (2021). A class of uncertain nonlinear multi-agent systems is studied by Li *et al*. (2022a). Based on the principle of sliding mode control and the reinforcement learning technique, a novel sliding mode control design method was proposed.

In practical applications, multi-agent systems often suffer from faults. Despite the good robust performance of sliding mode fault-tolerant control, the advantages of sliding mode control have not been fully utilized to solve the fault-tolerance problem of multi-agent systems in most of the existing studies. In addition, due to modeling errors, environmental noise, and other reasons, multi-agent systems often exhibit nonlinear characteristics. Therefore, when establishing a multi-agent system model, incorporating nonlinear functions and uncertainty disturbances can make the system more general. The designed controller also has better robustness (Li *et al*., 2020c; Menon and Edwards, 2013; Yu *et al*., 2023; Siavash *et al*., 2019; Peng *et al*., 2021).

A second-order multi-agent system with fading channels is considered and a sliding mode controller is designed by Li *et al*. (2022b). An Euler–Lagrange multi-agent system with random disturbances is discussed by Siavash *et al*. (2019). A consistent controller is designed for actuator failures occurring in this system. A high-order multi-agent system with external disturbances and uncertainties is considered by Peng *et al*. (2021), and a neural network based distributed coherent controller is presented.

Unlike traditional periodic sampling mechanisms, the system's network resources can be conserved by using an event-triggered mechanism to limit the number of samples taken by the system (Aranda-Escolastico *et al*., 2020; Åarzén, 1999; Garcia and Antsaklis, 2012; Cheng *et al*., 2016). The event-triggered mechanism was first proposed by Åarzén (1999) and has since then been applied in digital control theory to optimize network resources. Experimental results show that the number of measurements required for a double-tank system can be greatly reduced by using an event-triggered mechanism. A class of fuzzy Markovian jump systems is considered by Cheng *et al*. (2016), and a switching strategy based on an event-triggered mechanism is proposed to improve the efficiency of transmission per sample.

In the multi-agent system, the control information obtained by each agent depends on its neighbor agents (Gong *et al*., 2023; Zhang *et al*., 2019; Yang *et al*., 2019; Hu *et al*., 2020; Li *et al*., 2019). When the system contains a large number of agents, each instance of communication will be accompanied by a large amount of information transfer, not all of which is useful. In order to reduce network communication resources, event-triggered mechanisms are introduced into multi-agent systems (Ma *et al*., 2016; Yao *et al*., 2022; Xu *et al*., 2022; Huang *et al*., 2022). A fixed-threshold event-triggered mechanism is introduced for a class of multi-agent systems containing sensor saturation faults by Ma *et al*. (2016). A class of nonlinear multi-agent systems containing unknown perturbations is studied by Yao *et al*. (2022). An adaptive event-triggered mechanism is introduced to adaptively adjust the threshold of event trigger to further reduce the trigger frequency. A dynamic event-triggered mechanism is introduced for a class of multi-agent systems containing saturated inputs by Xu *et al*. (2022). The trigger threshold contains an exponential function with time as the variable.

For a class of distributed multi-agent systems, a dynamic event-triggered mechanism is designed for the system state and an auxiliary variable in the system by Huang *et al*. (2022). However, there is limited research that extensively addresses the issue of fault-tolerant control in multi-agent systems using event-triggered mechanisms. Furthermore, the integration of fault diagnostic information into event-triggered mechanisms is still in its early stages of development.

Several studies have been conducted on fault-tolerant consistent control for multi-agent systems. In the aforementioned works, the research on fault-tolerant consistent control of multi-agent systems under an event-triggered mechanism was mainly focused on the design of an event-triggered mechanism, state observers, and fault-tolerant controllers. However, there is limited research on attenuating the diffusion of fault information and conserving system network resources through event-triggered mechanisms in multi-agent systems. If the number of communications between agents is further reduced in the event of a failure in a multi-agent system, the transmission of failure information will be attenuated. Meanwhile, there are not many studies that adopt sliding mode controllers in order to solve the fault tolerance problem of multi-agent systems.

Based on the above analysis, the present paper makes the following main contributions:

1. In order to obtain an estimate of the faults for the design of an active fault-tolerant controller, a fault observer is designed, the speed of the agent and the degree of actuator failure are estimated, and the necessary conditions for the convergence of the observation error are given.

2. To attenuate the spread of fault information and waste less communication resources, a new hybrid an event-triggered mechanism is designed by combining fault observations and event-triggered mechanisms. By adjusting the event triggering interval in real time according to the fault estimation value, the number of agent communications is further reduced, and the network resources are saved while the spreading of fault information of the multi-agent system is attenuated.

3. Combining a fault observer and a hybrid event-triggered threshold, a hybrid event-triggered sliding mode fault-tolerant controller is designed. The consistency of the multi-agent system in the event of an actuator failure is ensured and the conditions for the convergence of the system consistency error are given.

2. Problem preparation and statement

A leader–follower multi-agent system consisting of one leader and N followers is considered in this paper. The communication topology structure can be described by $G = (v, E, \Upsilon)$, where $v = \{0, 1, \ldots, N\}$ denotes the set of nodes of the graph G, $E \subset v \times v$ denotes the set of edges and $\Upsilon = (\mathrm{a}_{ij})_{N\times N}$ denotes the adjacency matrix of the graph G. If the i-th follower agent can receive information from the j-th follower agent, then $a_{ij} = 1$, otherwise $a_{ij} = 0$. Define

$$d_i = \sum_{\substack{j=1\\ i\neq j}}^{N} a_{ij}$$

as the degree of incidence of node i. For the whole system, is $D = \mathrm{diag}\{d_1, \ldots, d_N\} \in \mathbb{R}^{N\times N}$. The Laplace matrix of the graph G is $L = D - \Upsilon$. If there exists a node in the graph G that can pass information to other nodes, then the graph G is said to be a spanning tree containing a node with that node as the root node.

Lemma 1. (Hao and Yang, 2013) *For a full rank matrix Λ and a diagonal matrices $\ell = \mathrm{diag}(\ell_1, \ldots, \ell_m)$, where the variable elements of the diagonal matrices ℓ are bounded and satisfy $0 < \underline{\ell} \leq \ell_i \leq \overline{\ell} \leq 1$, there exists a positive constant ς such that $\Lambda\ell\Lambda^T \geq \varsigma\Lambda\Lambda^T$.*

Lemma 2. (Rayleigh–Ritz theorem) *For a symmetric matrix $W \in \mathbb{R}^{n\times n}$ and vector $\vartheta \in \mathbb{R}^{n\times 1}$, we have*

$$\lambda_{\min}\{W\}\,\vartheta^T\vartheta \leq \vartheta^T W\vartheta \leq \lambda_{\max}\{W\}\,\vartheta^T\vartheta,$$

where $\lambda_{\min}\{W\}$ and $\lambda_{\max}\{W\}$ denote the minimum and maximum eigenvalues of the matrix W, respectively.

The leader–follower multi-agent system considered in this paper has one leader agent and N follower agents. The mathematical model of the leader agent can be described as follows:

$$\begin{cases} \dot{y}_0(t) = g_0(t), \\ \dot{g}_0(t) = u_0(t), \end{cases} \tag{1}$$

where $y_0(t)$ is the position of the leader, $g_0(t)$ is the speed of the leader, and $u_0(t)$ denotes the control input of the leader.

The mathematical model of the i-th follower can be described as follows:

$$\begin{cases} \dot{y}_i(t) = g_i(t), \\ \dot{g}_i(t) = u_i^F(t) + \kappa_i(t), \end{cases} \tag{2}$$

where $i = 1, 2, \ldots, N$, $y_i(t)$ is the follower position, $g_i(t)$ is the follower velocity, $u_i^F(t)$ denotes the follower fault input, and $\kappa_i(t)$ denotes the nonlinear disturbance.

In practice, the forms of actuator faults are often complex and diverse, and actuator faults can be described by the coexistence of multiplicative and additive faults, which are mathematically modeled as

$$u_i^F(t) = \sigma_i u_i(t) + \eta_i(t), \tag{3}$$

where $u_i(t)$ is the actuator control input of the i-th agent, σ_i is the multiplicative fault coefficient of the actuator, and $\eta_i(t)$ denotes the time-varying additive fault coefficient of the actuator, and different types of actuator faults occur in the system when σ_i and $\eta_i(t)$ take different values. When $0 < \sigma_i < 1$ and $\eta_i = 0$, a partial failure fault of the actuator has occurred. When $\sigma_i = 0$ and $\eta_i \neq 0$, a stuck-at fault of the actuator has occurred. When $\sigma_i = 0$ and $\eta_i = 0$, an interrupted fault of the actuator has occurred. When $\sigma_i = 1$ and $\eta_i \neq 0$, an offset fault of the actuator has occurred.

Define the consistent error of the i-th agent as

$$\begin{cases} e_{yi}(t) = \sum\limits_{j=1}^{N} a_{ij}(y_i(t) - y_j(t)) + b_i(y_i(t) - y_0(t)), \\ e_{gi}(t) = \sum\limits_{j=1}^{N} a_{ij}(g_i(t) - g_j(t)) + b_i(g_i(t) - g_0(t)). \end{cases} \tag{4}$$

Let $\tilde{y}(t) = y_i(t) - 1 \otimes y_0(t)$ and $\tilde{g}(t) = g_i(t) - 1 \otimes g_0(t)$. Equation (4) can be expressed as

$$\begin{cases} e_y(t) = ((L + B) \otimes I_m)\tilde{y}(t) = \overline{L}\tilde{y}(t), \\ e_g(t) = ((L + B) \otimes I_m)\tilde{g}(t) = \overline{L}\tilde{g}(t), \end{cases} \tag{5}$$

which yields the corresponding equations for the time derivatives

$$\begin{cases} \dot{e}_y(t) = e_g(t), \\ \dot{e}_g(t) = \overline{L}\dot{\tilde{g}}(t) \\ \qquad\; = \overline{L}(\sigma_i u_i(t) + \eta_i(t) + \kappa_i(t) - 1_n \otimes, u_0(t)), \end{cases} \tag{6}$$

The objective of this paper is to design a fault-tolerant controller such that the consistent errors $e_y(t)$ and $e_g(t)$ of the multi-agent systems (1) and (2) with the fault model (3) satisfy the following relationship:

$$\begin{cases} \lim\limits_{t\to\infty} e_y(t) \le h_1, \\ \lim\limits_{t\to\infty} e_g(t) \le h_2, \end{cases} \tag{7}$$

where h_1 and h_2 are positive constants or bounded variables.

3. Main design and analysis

In practical multi-agent failure systems, the agent speed is generally not easy to be obtained directly. At the same time, fault diagnosis of the agent is usually required to obtain better fault-tolerant control. Drawing on Yang *et al.* (2022), in this paper, a fault observer is designed for obtaining an estimate of the follower's speed and the additive fault. Unlike the approach of Yang *et al.* (2022), the fault observer designed in this paper is applicable to actuator fault types where multiplicative and additive faults coexist, which is more versatile. The specific design is as follows:

$$\begin{cases} \alpha = p_1 \tanh^{\frac{1}{2}} \theta_{1i}(t) + \hat{\eta}_i(t) + q_1\theta_{1i}(t), \\ \dot{\hat{g}}_i(t) = \alpha + \sigma_i u_i(t) + \kappa_i(t), \\ \dot{\hat{\eta}}_i(t) = p_2 \operatorname{sgn}(\alpha - \hat{\eta}_i(t)) + q_2(\alpha - \hat{\eta}_i(t)), \end{cases} \tag{8}$$

where α is the intermediate variable of the observer and it indirectly indicates the magnitude of the additive fault estimate, $\theta_{1i}(t)$ and $\theta_{2i}(t)$ denote the observation error, defined as

$$\begin{cases} \theta_{1i}(t) = g_i(t) - \hat{g}_i(t), \\ \theta_{2i}(t) = \eta_i(t) - \hat{\eta}_i(t), \end{cases} \tag{9}$$

$\tanh(\cdot)$ denotes the hyperbolic tangent function, p_1, p_2, q_1 and q_2 are positive constants to be designed, where p_1 and q_1 are used to appropriately amplify the effect of the feedback value of the speed error on the intermediate variable, and p_2 and q_2 denote the magnitude of the feedback from the fault estimation error to the fault estimate.

To illustrate the effectiveness of the designed fault observer (8), consider the following result:

Theorem 1. *The fault observer (8) that satisfies the conditions*

$$\begin{cases} p_1 > 4, \\ -p_1^3 + p_1^2(2p_2 - 2q_1 + 2q_2 - \frac{1}{4}q_1^2) \\ +p_1(2q_1(2p_2 + q_2) - 8p_2 - \bar{\eta}^2) \\ -16p_2q_1 + 4\bar{\eta}^2 - 4q_2^2 > 0, \end{cases} \tag{10}$$

where $\overline{\eta}$ is the upper bound of $\eta(t)$, can observe the velocity $g(t)$ and additive faults $\eta(t)$ of the leader–follower multi-agent system (1) and (2) with the actuator faults (3), and the observation error will eventually converge to zero.

Proof. Take the derivative of the observation error,

$$\begin{aligned} \dot{\theta}_{1i}(t) &= \dot{g}_i(t) - \dot{\hat{g}}_i(t) \\ &= \sigma_i u_i(t) + \eta_i(t) + \kappa_i(t) - \alpha - \sigma_i u_i(t) - \kappa_i(t) \\ &= \eta_i(t) - \alpha \\ &= \eta_i(t) - p_1 \tanh^{\frac{1}{2}} \theta_{1i}(t) - \hat{\eta}_i(t) - q_1\theta_{1i}(t) \\ &= \theta_{2i} - p_1 \tanh^{\frac{1}{2}} \theta_{1i}(t) - q_1\theta_{1i}(t), \end{aligned}$$

$$\begin{aligned} \dot{\theta}_{2i}(t) &= \dot{\eta}_i(t) - \dot{\hat{\eta}}_i(t) \\ &= \dot{\eta}_i(t) - p_2 \operatorname{sgn}(\alpha - \hat{\eta}_i(t)) - q_2(\alpha - \hat{\eta}_i(t)). \end{aligned}$$

On account of

$$\begin{aligned}\theta_{2i}(t)-\dot{\theta}_{1i}(t)&=\eta_i(t)-\hat{\eta}_i(t)-(\eta_i(t)-\alpha)\\&=\alpha-\hat{\eta}_i(t),\end{aligned}$$

we have

$$\begin{aligned}\dot{\theta}_{2i}(t)=\dot{\eta}_i(t)&-p_2\operatorname{sgn}(\theta_{2i}(t)-\dot{\theta}_{1i}(t))\\&-q_2(\theta_{2i}(t)-\dot{\theta}_{1i}(t)).\end{aligned}$$

Let

$$\xi_i^T(t)=\begin{bmatrix}|\theta_{1i}(t)|^{\frac{1}{2}} & \theta_{2i}(t)\end{bmatrix},$$

$$J=\begin{bmatrix}p_1^2+4p_2 & -p_1\\ -p_1 & 2\end{bmatrix}.$$

Define the Lyapunov function

$$V_1=\xi_i^TJ\xi_i.$$

The time derivative of $\xi_i^T(t)$ is

$$\begin{aligned}\dot{\xi}_i^T(t)&=|\theta_{1i}(t)|^{-\frac{1}{2}}\left\{\begin{bmatrix}-\frac{p_1+q_1}{2} & \frac{1}{2}\\ -(p_2+q_2) & 0\end{bmatrix}\xi_i(t)\right.\\&\quad\left.+|\theta_{1i}(t)|^{\frac{1}{2}}\begin{bmatrix}0\\1\end{bmatrix}\dot{\eta}_i(t)\right\}.\end{aligned}$$

Let $w_i(t)=|\theta_{1i}(t)|^{\frac{1}{2}}\,\dot{\eta}_i(t)$,

$$A=\begin{bmatrix}-\frac{p_1+q_1}{2} & \frac{1}{2}\\ -(p_2+q_2) & 0\end{bmatrix},\quad B=\begin{bmatrix}0\\1\end{bmatrix}.$$

Then we get

$$\dot{\xi}_i^T(t)=\left|\theta_{1i}(t)\right|^{-\frac{1}{2}}\left(A\xi_i(t)+Bw_i(t)\right).$$

The time derivative of V_1 is

$$\begin{aligned}\dot{V}_1&=\left|\theta_{1i}(t)\right|^{-\frac{1}{2}}\begin{bmatrix}\xi_i(t)\\w_i(t)\end{bmatrix}^T\begin{bmatrix}A^TP+PA & PB\\ B^TP & 0\end{bmatrix}\begin{bmatrix}\xi_i(t)\\w_i(t)\end{bmatrix}\\&\le\left|\theta_{1i}(t)\right|^{-\frac{1}{2}}\begin{bmatrix}\xi_i(t)\\w_i(t)\end{bmatrix}^T\begin{bmatrix}A^TP+PA & PB\\ B^TP & 0\end{bmatrix}\begin{bmatrix}\xi_i(t)\\w_i(t)\end{bmatrix}\\&\quad+\left|\theta_{1i}(t)\right|\overline{\eta}^2-\left(w_i(t)\right)^2\\&\le\frac{1}{2}\left|\theta_{1i}(t)\right|^{-\frac{1}{2}}\xi_i^T(t)\big(A^TP+PA+PBB^TP\\&\quad+\overline{\eta}^2M\big)\xi_i(t)\\&\le-\frac{1}{2}\left|\theta_{1i}(t)\right|^{-\frac{1}{2}}\xi_i^T(t)Q\xi_i(t),\end{aligned}$$

where

$$M=\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}^T,\quad Q=\begin{bmatrix}A_1 & A_2\\A_2 & A_3\end{bmatrix},$$

and we define $A_1={p_1}^3+{p_1}^2(p_1-1)+2p_1(p_2-q_2)+4p_2q_1-\overline{\eta}^2$, $A_2=-{p_1}^2+p_1(2-\frac{1}{2}p_1)+2q_2$ and $A_3=p_1-4$.

When p_1, p_2, q_1 and q_2 satisfy the condition (10), Q is a positive definite matrix and the time derivative of the Lyapunov function V_1 satisfies

$$\dot{V}_1\le-\frac{1}{2}|\theta_{1i}(t)|^{-\frac{1}{2}}\xi_i^T(t)Q\xi_i(t).$$

Lemma 3 now implies

$$\dot{V}_1\le-\frac{1}{2}\left|\theta_{1i}(t)\right|^{-\frac{1}{2}}\lambda_{\max}\left\{Q\right\}\left\|\xi_i(t)\right\|^2<0.$$

It follows from the Lyapunov stability that the observer error eventually converges to zero. The proof is complete. ■

Remark 1. From Eqn. (10), the coefficients p_1, p_2, q_1 and q_2 of the fault observer can be chosen in many different combinations due to the presence of higher order nonlinear inequalities. However, the selection experience can be drawn from a large number of experiments. The larger the value of p_1, the higher the observation accuracy and the lower the chattering of the system error, but the number of communications will increase. The larger the value of p_2, the faster the convergence of the observer, but the more pronounced the chattering will be. The larger the value of q_1, the faster the convergence of the faults, and it has little effect on the number of communications. The larger the value of q_2, the faster the fault observation error converges, but q_2 is too large and tends to cause instability in the observations.

In order to conserve the communication resources of the multi-agent system in executing the fault-tolerant control algorithm and to attenuate the spreading of fault information among the agents, a hybrid event-triggered mechanism is designed by introducing the fault estimation value into the triggering threshold. When the system meets the trigger conditions, it is sampled and the input of the controller is updated at the same time. Define the k-th sampling moment of the i-th agent as t_k^i, and the system state and velocity at this time as $y(t_k^i)$ and $g(t_k^i)$, respectively. Define the sampling error at moment t as

$$\begin{cases}\delta_i^y=y_i(t_k^i)-y_i(t),\\ \delta_i^g=g_i(t_k^i)-g_i(t).\end{cases}\tag{11}$$

The hybrid event-triggered mechanism is designed as

$$\begin{aligned}t_{k+1}^i\triangleq\min\big\{t>t_k^i\mid\\ \|\delta_i^{\mathrm{y}}\|+\|\delta_i^{\mathrm{g}}\|\ge\varpi+\|\hat{\eta}_i(t)\|\big\},\end{aligned}\tag{12}$$

where t_{k+1}^i denotes the $(k+1)$-th sampling moment of the i-th agent, ϖ is the fixed threshold of the hybrid event-triggered mechanism, and $\hat{\eta}_i(t)$ is the observed value of the additive fault of the i-th agent obtained by the fault observer (8). The larger the additive fault, the longer the sampling interval. This design can effectively suppress the spread of fault information.

Based on the position error $e_{yi}(t)$ and velocity error $e_{gi}(t)$ of the i-th follower agent, consider the following linear sliding mode variables:

$$s_i(t) = k_1 e_{yi}(t) + e_{gi}(t), \tag{13}$$

where $k_1 = \|\bar{L}\|$.

Let

$$\begin{aligned} s(t) &= \left[s_1^T(t), s_2^T(t), \cdots, s_N^T(t)\right]^T, \\ e_y(t) &= \left[e_{y1}^T(t), e_{y2}^T(t), \cdots, e_{yN}^T(t)\right]^T, \\ e_g(t) &= \left[e_{g1}^T(t), e_{g2}^T(t), \cdots, e_{gN}^T(t)\right]^T. \end{aligned}$$

Then the sliding mold surface can be expressed as

$$s(t) = k_1 e_y(t) + e_g(t). \tag{14}$$

Taking the derivative, we get

$$\begin{aligned} \dot{s}(t) &= k_1 \dot{e}_y(t) + \dot{e}_g(t) \\ &= k_1 e_g(t) + \overline{L}(U^F(t) + K(t) - 1_n \otimes u_0(t)), \end{aligned} \tag{15}$$

where $K(t) = [\kappa_1(t), \kappa_2(t), \cdots, \kappa_N(t)]^T$, $U^F(t) = [{u_1}^F(t), {u_2}^F(t), \cdots, {u_N}^F(t)]^T$.

For the multi-agent systems (1), (2), the fault model (3) and the sliding mold surface (14), the following sliding mold fault-tolerant controller (16) based on the hybrid event-triggered mechanism (12) is designed:

$$\begin{aligned} U(t_k) = \varsigma^{-1} \{&1_N \otimes u_0(t) - k_1 e_y(t_k) - e_g(t_k) \\ &-(\hat{\eta}(t) + \bar{K}) \operatorname{sign}(s(t))\}, \end{aligned} \tag{16}$$

where $U(t_k) = [u_1(t_k), u_2(t_k), \ldots, u_N(t_k)]^T$, ς is a positive constant to be designed and $\bar{K}$ is upper bound of $K(t)$.

For the designed sliding-mode fault-tolerant controller (16), we get the following result:

Theorem 2. *The state error of a leader–follower multi-agent systems (1) and (2) with actuator faults (3) can be driven to the sliding-mode area*

$$\Theta = \{y_i(t) \in \mathbb{R}^N, g_i(t) \in \mathbb{R}^N : \|s_i(t)\| \le \chi\}, \tag{17}$$

where $\chi = 2N \|\overline{L}\| (\varpi + \|\hat{\eta}(t)\|)$, under the effect of a sliding-mode fault-tolerant controller (16).

Proof. Define the Lyapunov function

$$V_2 = \frac{1}{2} s^T s. \tag{18}$$

Taking the time derivative, we have

$$\begin{aligned} \dot{V}_2 &= s^T \dot{s} \\ &= s^T(\dot{e}_g(t_k) + k_1 \dot{e}_y(t_k)) \\ &= s^T(\overline{L}\sigma U(t_k) + \overline{L}(\eta(t) + K(t) - 1_n \otimes u_0(t)) \\ &\quad + k_2 e_g(t_k)) \\ &= s^T(\overline{L}\sigma\varsigma^{-1}\{1_n \otimes u_0(t) - k_1 e_y(t_k) - e_g(t_k) \\ &\quad - (\hat{\eta}(t_k) + \overline{K}) \operatorname{sign}(s(t_k))\} + \overline{L}(\eta(t) + K(t) \\ &\quad - 1_n \otimes u_0(t)) + k_1 e_g(t_k)) \\ &= s^T(\varsigma^{-1}\overline{L}\sigma\overline{L}^{-1}\overline{L}\{1_n \otimes u_0(t) - k_1 e_y(t_k) \\ &\quad - e_g(t_k) - (\hat{\eta}(t_k) + \overline{K}) \operatorname{sign}(s(t_k))\} + \overline{L}(\eta(t) \\ &\quad + K(t) - 1_n \otimes u_0(t)) + k_1 e_g(t_\kappa)), \end{aligned} \tag{19}$$

where $\sigma = \operatorname{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_N\}$, and from Lemma 1, $\bar{L}\sigma\bar{L}^{-1} \ge \varsigma\bar{L}\bar{L}^{-1}$.

Therefore, (19) can be simplified as

$$\begin{aligned} \dot{V}_2 &\le s^T(\bar{L}\{1_n \otimes u_0(t) - k_1 e_y(t_k) - e_g(t_k) \\ &\quad -(\hat{\eta}(t_k) + \bar{K}) \operatorname{sign}(s(t_k))\} \\ &\quad + \bar{L}(\eta(t_k) + K(t) - 1_n \otimes u_0(t)) + k_1 e_g(t_k)) \\ &\le s^T(\bar{L}\{\eta(t_k) + K(t) - k_1 e_y(t_k) - e_g(t_k) \\ &\quad -(\hat{\eta}(t_k) + \bar{K}) \operatorname{sign}(s(t_k))\} + k_1 e_g(t_k)) \\ &\le \|s\| (\|\bar{L}\{\eta(t_k) + K(t) - k_1 e_y(t_k) - e_g(t_k) \\ &\quad -(\hat{\eta}(t_k) + \bar{K}) \operatorname{sign}(s(t_k))\}\| + k_1 e_g(t_k)). \end{aligned} \tag{20}$$

Due to $k_1 = \|\bar{L}\|$, Eqn. (20) can be simplified to

$$\begin{aligned} \dot{V}_2 &\le \|s\| (\|\bar{L}\{\eta(t_k) + K(t) - k_1 e_y(t_k) - e_g(t_k) \\ &\quad -(\hat{\eta}(t_k) + \bar{K}) \operatorname{sign}(s(t_k))\}\| + \|\bar{L}\| e_g(t_k)) \\ &\le \|s\| \|\bar{L}\{\eta(t_k) + K(t) - k_1 e_y(t_k) \\ &\quad -(\hat{\eta}(t_k) + \bar{K}) \operatorname{sign}(s(t_k))\}\|. \end{aligned} \tag{21}$$

The hybrid event-triggered mechanism has been added and two cases need to be discussed. The first one is $\operatorname{sign}(s(t)) = \operatorname{sign}(s(t_k))$, and the other case is $\operatorname{sign}(s(t)) \ne \operatorname{sign}(s(t_k))$.

Combining Theorem 1, if $\operatorname{sign}(s(t)) = \operatorname{sign}(s(t_k))$, due to $K(t) \le \bar{K}$, Eqn. (21) can be simplified to:

$$\begin{aligned} \dot{V}_2 &\le \|s\| \|\bar{L}\| (-k_1 e_y(t_k)) \\ &\le -\|s\| \|\bar{L}\|^2 \|e_y(t_k)\| \le 0. \end{aligned}$$

From the Lyapunov stability, the system error of the faulty multi-agent systems (1) and (2) can reach the sliding mode surface (14) under the action of the controller (16).

Case 2. From the linear sliding mold surface (14), one can obtain

$$\begin{aligned} s(t) &= k_1 e_y(t) + e_g(t) \\ &= \overline{L}(k_1 \tilde{y}_i(t) + \tilde{g}_i(t)) \\ &= \overline{L}(k_1(\tilde{g}(t_k^i) - \delta^y) + (\tilde{g}(t_k^i) - \delta^g)) \\ &= s(t_k^i) - \overline{L}(k_1 \delta^y + \delta^g). \end{aligned}$$

As $k_1 = \left\|\bar{L}\right\| \geq 0$, we deduce that

$$\begin{aligned} \|s(t)| &\leq \left\|s(t_k^i)\right\| + \left\|\overline{L}\right\| (\|\delta^y\| + \|\delta^g\|) \\ &\leq \left\|s(t_k^i)\right\| + \left\|\overline{L}\right\| (\varpi + \|\hat{\eta}(t)\|) \\ &\leq (2N-1) \left\|\overline{L}\right\| (\varpi + \|\hat{\eta}(t)\|) \\ &\quad + \left\|\overline{L}\right\| (\varpi + \|\hat{\eta}(t)\|) \\ &\leq 2N \left\|\overline{L}\right\| (\varpi + \|\hat{\eta}(t)\|) = \chi, \end{aligned}$$

where $\chi = 2N \left\|\overline{L}\right\| (\varpi + \|\hat{\eta}(t)\|)$. Therefore, the current state of the multi-agent system, whether in the sliding phase or after reaching the sliding mold surface, the consistent error of the faulty multi-agent systems (1) and (2) is driven to the sliding mold area (17) by the sliding mold fault-tolerant controller (16). ■

Theorem 3. *The leader-follower multi-agent system (1) and (2) with actuator faults (3) can make the system position and velocity errors converge into the following bounded region with the sliding mode fault-tolerant controller (16):*

$$\begin{cases} \|e_y(t)\| \leq \dfrac{\|\beta\|\,\chi}{k_1}, \\ \|e_g(t)\| \leq 2\,\|\beta\|\,\chi. \end{cases} \tag{22}$$

Proof. From Theorem 2, the sliding mode surface (14) can be rewritten as

$$s(t) = e_g(t) + k_1 e_y(t) = \beta\chi, \tag{23}$$

where $\beta = [\beta_1, \beta_2, \ldots, \beta_N]$, $\beta_i \in (-1, 1)$, which yields

$$e_g(t) = \beta\chi - k_1 e_y(t) = \dot{e}_y(t). \tag{24}$$

Taking the Lyapunov function

$$V_3 = \frac{1}{2} e_y^T(t) e_y(t),$$

we can compute its time derivative

$$\begin{aligned} \dot{V}_3 &= \left(\beta\chi - k_1 e_y(t)\right)^T e_y(t) \\ &= \beta^T \chi e_y(t) - k_1 e_y^T(t) e_y(t) \\ &\leq \beta^T \chi e_y(t) - k_1 \left\|e_y(t)\right\|^2 \\ &\leq k_1 \Big(\left\|e_y(t)\right\| - \frac{\|\beta\|\chi}{k_1} \Big) \left\|e_y(t)\right\|. \end{aligned}$$

Since $k_1 = \left\|\overline{L}\right\| \geq 0$, the position error of the system can remain bounded when the system error enters the sliding mode area (17), that is,

$$\|e_y(t)\| \leq \frac{\|\beta\|\,\chi}{k_1}. \tag{25}$$

From Eqns. (24) and (25), we get

$$\begin{aligned} \|e_g(t)\| &\leq \|\beta\|\,\chi + k_1 \left\|e_y(t)\right\| \\ &\leq \|\beta\|\,\chi + k_1 \Big(\frac{\chi \left\|\beta\right\|}{k_1} \Big) \\ &\leq 2\|\beta\|\chi. \end{aligned} \tag{26}$$

Therefore, the velocity error of the system can remain bounded, that is, $\|e_g(t)\| \leq 2\,\|\beta\|\,\chi$. ■

Finally, it is explored whether the designed hybrid event-triggered mechanism will exhibit Zeno behavior. By the hybrid event-triggered mechanism (12),

$$\|\delta^y\| + \|\delta^g\| \geq \varpi + \|\hat{\eta}(t)\|.$$

Let $\delta = \|\delta^y\| + \|\delta^g\|$. Since $\delta^y = y(t) - y(t_k)$ and $\delta^g = g(t) - g(t_\kappa)$, we have

$$\begin{aligned} \dot{\delta} &\leq \|\dot{y}(t)\| + \|\dot{g}(t)\| \\ &= \|g(t)\| + \|\sigma U(t) + \eta(t) + K(t)\| \\ &\leq \|g(t)\| + \left\|-\varsigma^{-1}\sigma\left(1_n \otimes u_0(t) - e_g(t) - k_2 e_y(t)\right.\right. \\ &\quad \left.\left. -(\eta(t) + \bar{K})\operatorname{sign}(s(t_k))\right)\right\| + \left(\bar{\eta} + \bar{K}\right). \end{aligned} \tag{27}$$

By using the $\tanh(\cdot)$ function approximating the $\operatorname{sign}(\cdot)$ function, we have

$$\operatorname{sign}(s(t)) \approx \tanh(\gamma \otimes I_m s(t_k)), \tag{28}$$

where γ is a diagonal matrix.

Since $-1 \leq \tanh(\cdot) \leq 1$, we have

$$\|\tanh(\gamma \otimes I_m s(t_k))\| \leq \|I_m N\| = 1. \tag{29}$$

Therefore, (27) can be simplified as

$$\begin{aligned} \dot{\delta} &\leq \|g(t)\| + \left\|-\varsigma^{-1}\sigma(1_n \otimes u_0(t) - e_g(t) - k_1 e_y(t))\right\| \\ &\quad + \left\|-\varsigma^{-1}\sigma(\eta(t) + \bar{K})\operatorname{sign}(s(t_k)))\right\| + \left(\bar{\eta} + \bar{K}\right) \\ &\leq \|g(t)\| + \varsigma^{-1}\left|\sigma\right| \|e_g(t) + k_1 e_y(t) + u_0(t)\| \\ &\quad + \varsigma^{-1}\left|\sigma\right| \left(\bar{\eta} + \bar{K}\right) + \left(\bar{\eta} + \bar{K}\right) \\ &\leq \|g(t)\| + \varsigma^{-1}\left|\sigma\right| \|e_g(t) + k_1 e_y(t) + u_0(t)\| \\ &\quad + \left(\varsigma^{-1}\left|\sigma\right| + 1\right)\left(\bar{\eta} + \bar{K}\right), \end{aligned} \tag{30}$$

where $\overline{K}$ denotes an upper bound of the nonlinear disturbance $K(t)$, and $t \in (t_k, t_{k+1})$. We have

$$\delta(t) - \delta(t_k) \leq \int_{t_k}^{t} \rho(\tau)\,\mathrm{d}\tau,$$

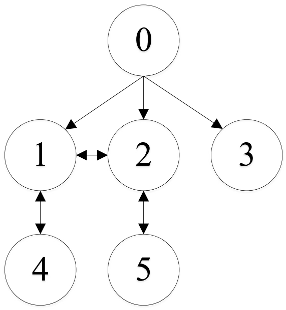

Fig. 1. Communication topology.

where

$$\rho(\tau) = \|g(\tau)\| + \varsigma^{-1} |\sigma| \|e_g(\tau) + k_1 e_y(\tau) + u_0(\tau)\| + (\varsigma^{-1} |\sigma| + 1) \left(\overline{\eta} + \overline{K}\right).$$

Because of $\delta(t_k) = 0$, we have

$$\begin{aligned}\delta(t) &\le (t - t_k) \left(\|g(t)\| + \varsigma^{-1} |\sigma| \|e_g(t) + k_1 e_y(t) + u_0(t)\| + (\varsigma^{-1} |\sigma| + 1) \left(\bar{\eta} + \bar{K}\right)\right) \\ &\le (t - t_k)\, \rho(t).\end{aligned} \tag{31}$$

When $e_y(t)$ and $e_g(t)$ satisfy the trigger condition at moment t_{k+1}, there exists a positive constant μ such that $\delta(t_{k+1}) > \mu$. Then Eqn. (31) can be transformed into

$$\mu \le \delta(t_{k+1}) \le (t_{k+1} - t_\kappa)\rho(t). \tag{32}$$

Since $\rho(t) > 0$, it follows that

$$t_{k+1} - t_k \ge \frac{\mu}{\rho(t)} > 0. \tag{33}$$

Therefore, the designed hybrid event-triggered mechanism does not exhibit Zeno behavior.

4. Simulation study

Next, a simulation study is conducted to verify the validity of the control strategy proposed in this paper.

Consider a multi-agent system consisting of one leader and five followers, whose communication topology is shown in Fig. 1. Where the leader index is 0, the follower index is i $(i = 1, 2, \ldots, 5)$, and the weight values of the edges are all 1. From Fig. 1, the Laplace matrix L and the connection matrix B can be expressed as follows:

$$L = \begin{bmatrix} 1 & -1 & 0 & -1 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix},$$

$$B = \mathrm{diag}(1, 1, 1, 0, 0).$$

In the simulation, the control input of the leader agent is $u_0(t) = 0.5\sin(0.2t)$, and the multi-agent system failure input is $u^F(t) = \sigma u(t) + \eta(t)$, where the multiplicative fault failure degree σ can be expressed as $\sigma = \begin{bmatrix}\sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \sigma_5\end{bmatrix}^T$. Since the multiplicative failure is difficult to be determined in practical applications, for the rigor of the simulation and to verify the robustness of the controller, σ_i is selected as a random number in a certain region, and the selected ranges are $\sigma_1 \in [0.5, 0.6]$, $\sigma_2 \in [0.3, 0.5]$, $\sigma_3 \in [0.6, 0.75]$, $\sigma_4 \in [0.5, 0.55]$, $\sigma_1 \in [0.5, 0.65]$. Combined with Lemma 1, the range of values of the normal quantities in Eqn. (19) can be obtained: $0 < \varsigma \le 1.03$, and $\varsigma = 0.1$ is selected for better control performance. The additive fault $\eta(t)$ can be expressed as

$$\eta(t) = \begin{bmatrix} 0.2\sin(0.6\pi t) \\ 0.1 + 0.1\cos(0.5\pi t) \\ 0.2(1 - e^{-0.1t}) \\ 0.2\sin(0.5\pi t)\cos(0.3\pi t) \\ 0 \end{bmatrix}.$$

Then $\overline{\eta} = 0.2$, and the nonlinear disturbance is expressed as

$$K(t) = \begin{bmatrix} \kappa_1(t) \\ \kappa_2(t) \\ \kappa_3(t) \\ \kappa_4(t) \\ \kappa_5(t) \end{bmatrix} = \begin{bmatrix} 0.5(\cos(y_1(t)) + \cos(g_1(t))) \\ 0.5(\cos(y_2(t)) + \cos(g_2(t))) \\ 0.5(\cos(y_3(t)) + \cos(g_3(t))) \\ 0.5(\cos(y_4(t)) + \cos(g_4(t))) \\ 0.5(\cos(y_5(t)) + \cos(g_5(t))) \end{bmatrix}.$$

Then $\bar{K} = 0.5$, and the fixed threshold in the hybrid event-triggered mechanism is selected as $\varpi = 0.2$. The initial state value of the leader agent is $y_0(0) = 5$, $g_0(0) = 1$. The initial state values of the follower agents are $y_1(0) = 5$, $y_2(0) = 6$, $y_3(0) = -5$, $y_4(0) = -1$, $y_5(0) = 0$, $g_1(0) = 1.5$, $g_2(0) = 1.2$, $g_3(0) = 2$, $g_4(0) = 0.5$ and $g_5(0) = 0$, respectively. To satisfy Eqn. (10), the parameters of the observer are chosen as $p_1 = 5$, $p_2 = 15$, $q_1 = 50$ and $q_2 = 25$. It can be verified that $p_1 > 4$ and $-p_1^3 + p_1^2(2p_2 - 2q_1 + 2q_2 - \frac{1}{4}q_1^2) + p_1(2q_1(2p_2 + q_2) - 8p_2\overline{\eta}^2) - 16p_2q_1 + 4\overline{\eta}^2 - 4{q_2}^2 = 6649.96 > 0$, so the values of p_1, p_2, q_1 and q_2 satisfy Eqn. (10).

The speed observations output by the fault observer, the fault observations, the speed observation error of the fault observer on the system, and the observation error of the fault observer on the additive fault of the actuator are presented in Figs. 2–5, respectively.

As can be seen in Fig. 4, due to the addition of the hybrid event-triggered mechanism and the selection of the linear sliding mode surface, the fault observer produces chattering when observing the agent speed. However, the chattering amplitude is not large, and the highest chattering value is 0.147 at 3.82 s. As shown in Fig. 5, the chattering of the observer for additive faults is not obvious. Meanwhile, the convergence of the fault

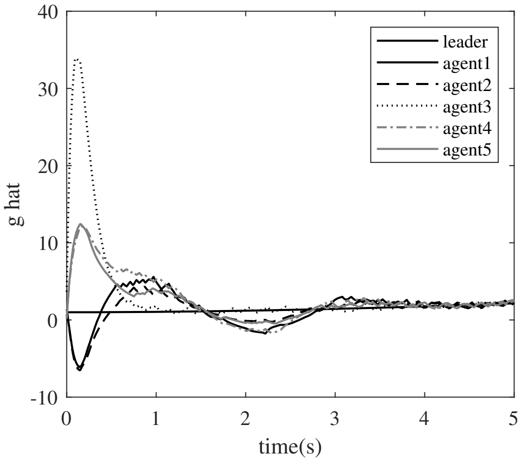

Fig. 2. Speed observation curves.

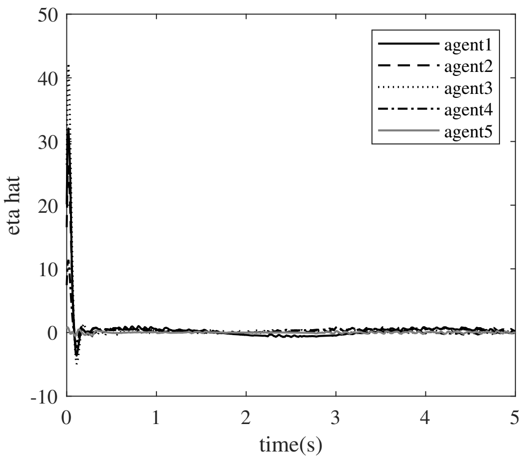

Fig. 3. Fault observation curves.

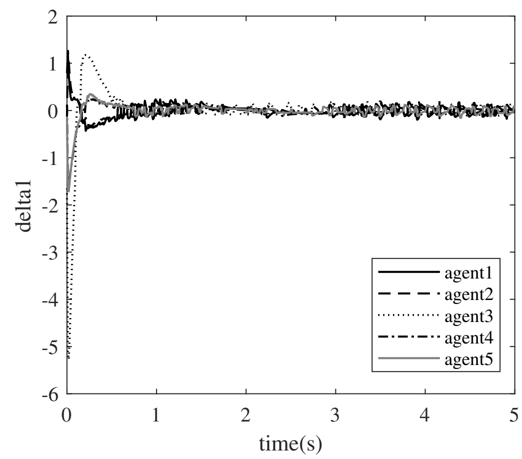

Fig. 4. Observation error curves of the speed.

observer is fast, and the observation errors of the system speed and additive faults converge to near 0 in about 0.76 s.

From the experiment, we have $\|s_i(t)\| = 201.51$ and $\chi = 2N\left\|\overline{L}\right\|(\varpi + \|\eta(t)\|) = 207.64$, so that Theorem 2 holds and the system error will finally converge into the sliding mode region (17). Also from the experiment $\|e_y(t)\| = 98.65$, $\|e_g(t)\| = 350.73$, $k_1 = \left\|\overline{L}\right\| = 3.9563$. Taking $\beta = \begin{bmatrix}0.9 & 0.9 & 0.9 & 0.9 & 0.9\end{bmatrix}$, we have $\|\beta\|\chi/k_1 = 105.49$ and $2\|\beta\|\chi = 834.71$, so that Theorem 3 holds and the position error and velocity error of the system can converge into the bounded region (22). The curves of the positions for the follower agents are shown in Fig. 6. The position errors between the agents are presented in Fig. 7. The speed errors between the agents are given in Fig. 8. Due to the use of a linear sliding mode surface in the controller, some degree of system state chattering occurs, but has little effect on the system state and speed convergence.

As can be seen in Fig. 7, the position error of the follower agents changes faster and converges to near 0 after 1.37 s, while the peak of chattering occurs at 7.66 s with a peak of 0.871, after which the chattering phenomenon tends to decrease. As can be seen in Fig. 8, the speed error of the follower agents converges to the vicinity of 0 after 1.36 s, and the peak of chattering occurs at 4.19 s and has a peak value of 1.5, after which the chattering phenomenon shows a decreasing trend.

Next, the fault-tolerant controller proposed in this paper and the one set forth by Li (2013) will be used for comparison. The position error and velocity error of the output of the fault-tolerant control protocol proposed by Li (2013) are represented in Figs. 9 and 10, respectively. The comparison shows that the fault-tolerant control protocol proposed in this paper converges faster in the event of a fault. At the same time, the control objective is realized by the fault-tolerant controller proposed in this paper while the network resources are saved due to the event-triggering mechanism being used.

The system sampling scenario under the hybrid event-triggered mechanism is illustrated in Fig. 11. It can be seen from the figure that the hybrid event-triggered mechanism adopted can well reduce the number of samples and reduce the waste of system network resources. If periodic sampling is used and the sampling period is set to 0.01 s, then the numbers of state updates for the multi-agent system will be in the range of 500 in 5 s. As shown in Fig. 11, the number of state updates from Agent 1 to Agent 5 under the hybrid event-triggered mechanism is 43, 34, 72, 46 and 56, respectively. Therefore, the number of updates to the state of the agents is significantly reduced, which greatly saves the network resources of the multi-agent system. The sampling of the event-triggered mechanism without fault estimates is presented by Fig. 12, where the number of

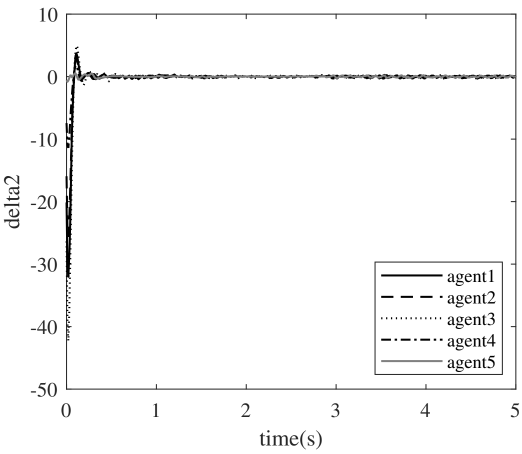

Fig. 5. Observation error curves for additive faults.

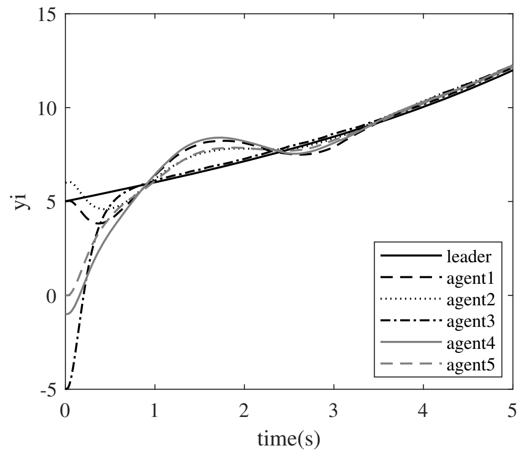

Fig. 6. Positional state of the follower agent.

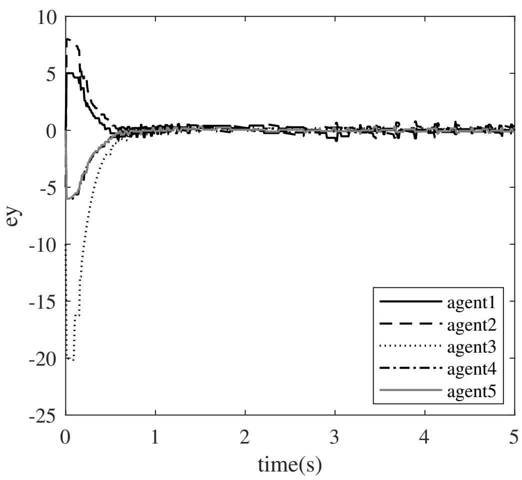

Fig. 7. Position error curves.

state updates from Agent 1 to Agent 5 within 5 seconds are 93, 90, 106, 79 and 70, respectively. It can be seen that, if the estimated value of the fault is added to the event-triggered mechanism to constitute a hybrid event-triggered mechanism, the number of sampling times of the system when the fault occurs is further reduced and the spread of the fault information is attenuated.

5. Conclusions

An observer-based hybrid event-triggered sliding mode fault-tolerant consistent control strategy was proposed for actuator faults in nonlinear second-order leader-follower multi-agent systems. The conditions for the convergence of the system consistency error were given based on Lyapunov' stability theory. A new hybrid event-triggered mechanism is designed based on the actuator fault output from the fault observer. Finally, the validity of the control strategy designed was verified by a simulation study.

Acknowledgment

This work was supported by the National Natural Science Foundation of China under the grant 62103057.

References

Åarzén, K.-E. (1999). A simple event-based PID controller, *IFAC Proceedings Volumes* **32**(2): 8687–8692.

Aranda-Escolastico, E., Guinaldo, M., Heradio, R., Chacon, J., Vargas, H., Sanchez, J. and Dormido, S. (2020). Event-based control: A bibliometric analysis of twenty years of research, *IEEE Access* **8**: 47188–47208.

Chen, L., Xue, H., Liang, H. and Zhao, M. (2023). Adaptive fuzzy fault-tolerant containment control for nonlinear multiagent systems based on modified distributed observer, *Fuzzy Sets and Systems* **464**: 108431.

Chen, S., Chen, B. and Shi, F. (2019). Distributed fault-tolerant consensus protocol for fuzzy multi-agent systems, *Circuits, Systems, and Signal Processing* **38**(2): 611–624.

Chen, S., Wang, M. and Li, Q. (2020). Second-order consensus of hybrid multi-agent systems with unknown disturbances via sliding mode control, *IEEE Access* **8**: 34973–34980.

Cheng, J., Park, J.H., Zhang, L. and Zhu, Y. (2016). An asynchronous operation approach to event-triggered control for fuzzy Markovian jump systems with general switching policies, *IEEE Transactions on Fuzzy Systems* **26**(1): 6–18.

Darvishpoor, S., Roshanian, J., Raissi, A. and Hassanalian, M. (2020). Configurations, flight mechanisms, and applications of unmanned aerial systems: A review, *Progress in Aerospace Sciences* **121**: 100694.

Domyshev, A. and Sidorov, D. (2022). Optimization of the structure of power system multi-agent control, *IFAC-PapersOnLine* **55**(9): 250–255.

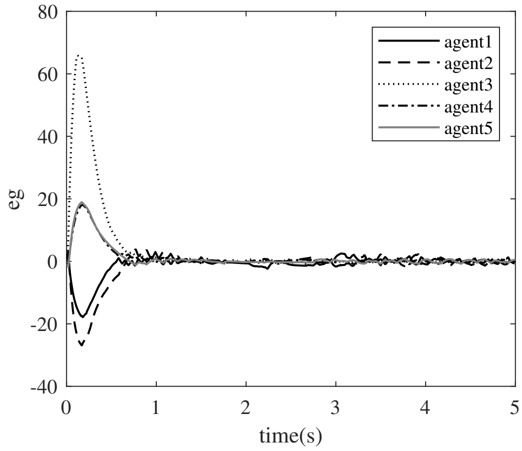

Fig. 8. Speed error curves.

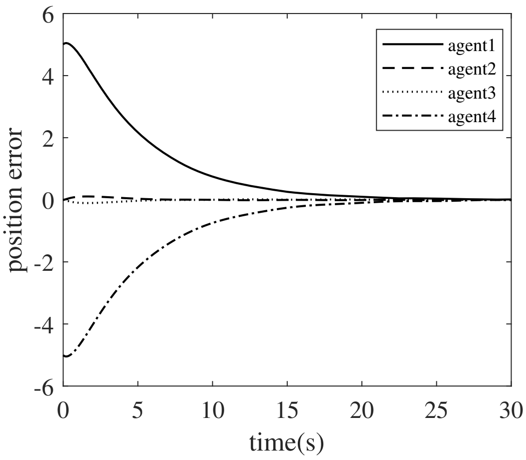

Fig. 9. Position error in the work of Li (2013).

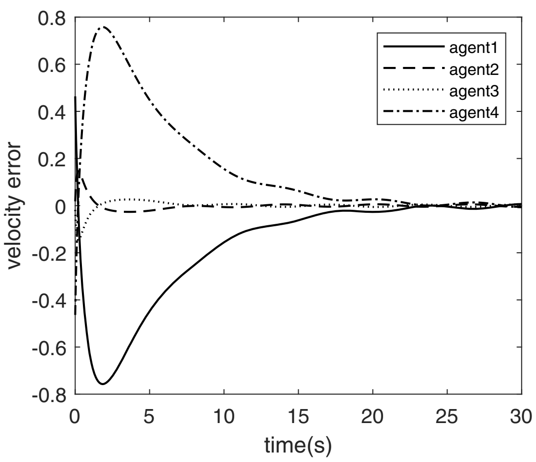

Fig. 10. Velocity error in the work of Li (2013).

Dong, L., Yu, D. and Nguang, S.K. (2019). Novel nonsingular terminal sliding mode control for multi-agent tracking systems with application to jerk circuit, *IEEE Transactions on Circuits and Systems II: Express Briefs* **67**(8): 1429–1433.

Dong, X., Hua, Y., Zhou, Y., Ren, Z. and Zhong, Y. (2018). Theory and experiment on formation-containment control of multiple multirotor unmanned aerial vehicle systems, *IEEE Transactions on Automation Science and Engineering* **16**(1): 229–240.

Garcia, E. and Antsaklis, P.J. (2012). Model-based event-triggered control for systems with quantization and time-varying network delays, *IEEE Transactions on Automatic Control* **58**(2): 422–434.

Gong, S., Zheng, M., Hu, J. and Zhang, A. (2023). Event-triggered cooperative control for high-order nonlinear multi-agent systems with finite-time consensus, *International Journal of Applied Mathematics and Computer Science* **33**(3): 439–448, DOI: 10.34768/amcs-2023-0032.

González, A., Sala, A. and Armesto, L. (2022). Decentralized multi-agent formation control with pole-region placement via cone-complementarity linearization, *International Journal of Applied Mathematics and Computer Science* **32**(3): 415–428, DOI: 10.34768/amcs-2022-0030.

Hao, L.-Y. and Yang, G.-H. (2013). Robust fault tolerant control based on sliding mode method for uncertain linear systems with quantization, *ISA Transactions* **52**(5): 600–610.

Hu, T., He, Z., Zhang, X. and Zhong, S. (2020). Leader-following consensus of fractional-order multi-agent systems based on event-triggered control, *Nonlinear Dynamics* **99**(3): 2219–2232.

Huang, B., Liu, Y., Xia, Z. and Wang, J. (2022). A bi-event-triggered multi-agent system for distributed optimization, *IEEE Transactions on Network Science and Engineering* **10**(2): 1074–1084.

Khalili, M., Zhang, X., Polycarpou, M.M., Parisini, T. and Cao, Y. (2018). Distributed adaptive fault-tolerant control of uncertain multi-agent systems, *Automatica* **87**: 142–151.

Khoygani, M. R., Ghasemi, R. and Ghayoomi, P. (2021). Robust observer-based control of nonlinear multi-omnidirectional wheeled robot systems via high order sliding-mode consensus protocol, *International Journal of Automation and Computing* **18**(5): 787–801.

Li, G., Wang, X. and Li, S. (2020a). Consensus control of higher-order Lipschitz non-linear multi-agent systems based on backstepping method, *IET Control Theory & Applications* **14**(3): 490–498.

Li, G., Wang, X. and Li, S. (2020b). Finite-time consensus algorithms of leader-follower higher-order multi-agent systems with uncertain nonlinearities, *Journal of the Franklin Institute* **357**(16): 11939–11952.

Li, J., Yuan, L., Chai, T. and Lewis, F.L. (2022a). Consensus of nonlinear multiagent systems with uncertainties using reinforcement learning based sliding mode control, *IEEE Transactions on Circuits and Systems I: Regular Papers* **70**(1): 424–434.

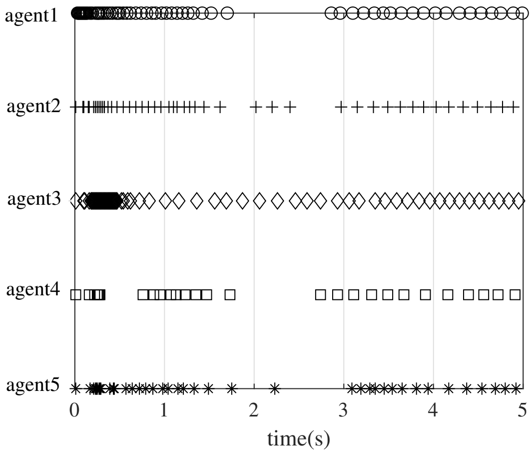

Fig. 11. Release moments and intervals for hybrid event-triggered mechanisms.

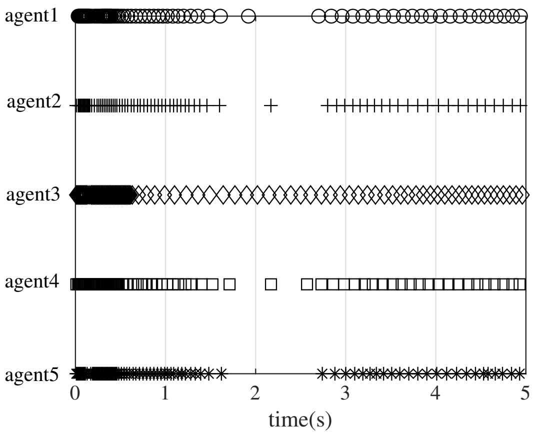

Fig. 12. Release moments and intervals for event-triggered mechanisms.

Li, J. (2013). Fault tolerant consensus of multi-agent systems with linear dynamics, *Mathematical Problems in Engineering* **2013**(1): 465–671.

Li, W., Niu, Y. and Cao, Z. (2022b). Event-triggered sliding mode control for multi-agent systems subject to channel fading, *International Journal of Systems Science* **53**(6): 1233–1244.

Li, X., Dong, X., Li, Q. and Ren, Z. (2019). Event-triggered time-varying formation control for general linear multi-agent systems, *Journal of the Franklin Institute* **356**(17): 10179–10195.

Li, Z., Yan, J., Yu, W. and Qiu, J. (2020c). Event-triggered control for a class of nonlinear multiagent systems with directed graph, *IEEE Transactions on Systems, Man, and Cybernetics: Systems* **51**(11): 6986–6993.

Liu, C., Jiang, B., Zhang, K. and Patton, R.J. (2021). Distributed fault-tolerant consensus tracking control of multi-agent systems under fixed and switching topologies, *IEEE Transactions on Circuits and Systems I: Regular Papers* **68**(4): 1646–1658.

Ma, L., Wang, Z. and Lam, H.-K. (2016). Event-triggered mean-square consensus control for time-varying stochastic multi-agent system with sensor saturations, *IEEE Transactions on Automatic Control* **62**(7): 3524–3531.

Menon, P.P. and Edwards, C. (2013). Robust fault estimation using relative information in linear multi-agent networks, *IEEE Transactions on Automatic Control* **59**(2): 477–482.

Parsa, P. and Akbarzadeh-T, M.-R. (2020). Combining consensus and tracking errors in sliding mode control of high order uncertain stochastic multi-agent systems, *2020 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Glasgow, UK*, pp. 1–8.

Peng, C., Zhang, A. and Li, J. (2021). Neuro-adaptive cooperative control for high-order nonlinear multi-agent systems with uncertainties, *International Journal of Applied Mathematics and Computer Science* **31**(4): 635–645, DOI: 10.34768/amcs-2021-0044.

Pham, T.V., Nguyen, Q.T., Messai, N. and Manamanni, N. (2020). Fault-tolerant tracking control for heterogeneous multi-agent systems, *2020 59th IEEE Conference on Decision and Control (CDC), Jeju Island, South Korea*, pp. 2696–2701.

Salmanpour, Y., Arefi, M.M., Khayatian, A. and Yin, S. (2023). Observer-based fault-tolerant finite-time control of nonlinear multiagent systems, *IEEE Transactions on Neural Networks and Learning Systems* **34**(6): 1–10.

Siavash, M., Majd, V. and Tahmasebi, M. (2019). Finite-time consensus control of Euler–Lagrange multi-agent systems in the presence of stochastic disturbances and actuator faults, *Journal of Electrical and Computer Engineering Innovations* **7**(2): 163–172.

Wang, J., Fang, Y.-M., Li, J.-X. and Li, K.-D. (2022). Fixed-time sliding mode fault-tolerant consensus control for second-order multi-agent system with actuator fault, *2022 41st Chinese Control Conference (CCC), Hefei, China*, pp. 4153–4158.

Wang, Y., Cheng, L., Hou, Z.-G., Tan, M. and Wang, M. (2014). Containment control of multi-agent systems in a noisy communication environment, *Automatica* **50**(7): 1922–1928.

Xu, C., Xu, H., Guan, Z.-H. and Ge, Y. (2022). Observer-based dynamic event-triggered semiglobal bipartite consensus of linear multi-agent systems with input saturation, *IEEE Transactions on Cybernetics* **53**(5): 3139–3152.

Xu, M., Yang, P. and Shu, Q. (2018). Consistent sliding mode fault tolerant control for second order multi-agent systems, *2018 IEEE CSAA Guidance, Navigation and Control Conference (CGNCC), Xiamen, China*, pp. 1–6.

Yang, P., Hu, X., Wang, Z. and Zhang, Z. (2022). Sliding mode fault tolerant consensus control for multi-agent systems based on super-twisting observer, *Journal of Systems Engineering and Electronics* **33**(6): 1309–1319.

Yang, X.-R., Liu, G.-P., Zhang, Z. and Lyu, J. (2019). Consensus analysis and control for rectangular singular multi-agent systems, *2019 Chinese Automation Congress (CAC), Hangzhou, China*, pp. 3632–3635.

Yao, D., Li, H. and Shi, Y. (2022). Adaptive event-triggered sliding-mode control for consensus tracking of nonlinear multiagent systems with unknown perturbations, *IEEE Transactions on Cybernetics* **53**(4): 2672–2684.

Yorgancioğlu, F. and Redif, S. (2019). Fast nonsingular terminal decoupled sliding-mode control utilizing time-varying sliding surfaces, *Turkish Journal of Electrical Engineering and Computer Sciences* **27**(3): 1922–1937.

Yu, Z., Liu, C., Wang, X. and Ren, X. (2023). Distributed output estimation error observer-based adaptive fault-tolerant consensus tracking control of multi-agent systems, *International Journal of Adaptive Control and Signal Processing* **37**(4): 1030–1048.

Yu, Z., Liu, Z., Zhang, Y., Qu, Y. and Su, C.-Y. (2019). Distributed finite-time fault-tolerant containment control for multiple unmanned aerial vehicles, *IEEE Transactions on Neural Networks and Learning Systems* **31**(6): 2077–2091.

Zhai, M., Sun, Q., Wang, R., Wang, B., Hu, J. and Zhang, H. (2023). Distributed multiagent-based event-driven fault-tolerant control of islanded microgrids, *IEEE Transactions on Cybernetics* **53**(11): 7251–7262.

Zhang, Q., Wang, J., Yang, Z. and Chen, Z. (2019). High gain feedback robust control for flocking of multi-agents system, *Transactions of the Institute of Measurement and Control* **41**(13): 3769–3776.

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Received: 26 February 2024
Revised: 8 May 2024
Re-revised: 19 June 2024
Accepted: 19 June 2024