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UNIVERSAL ADAPTIVE TRACKING CONTROLLER FOR RIGID MANIPULATORS

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The problem of (semiglobal) joint space trajectory tracking of rigid manipulators is considered. A new exponentially stable control algorithm is introduced. This algorithm requires only the knowledge of the manipulator dynamics along a desired trajectory. A set of n local universal adaptive controllers joined to each robot arm plays the role of generators of the 'correction of control'. The proposed universal adaptive tracking algorithm (UATA) has the following advantages: (i) a balanced gain distribution in the feedback-loop, (ii) small overshoots and (iii) a fast transient response.

1. Introduction

The aim of this paper is to propose an alternative approach to the tracking control problem of rigid manipulators. Earlier solutions to this problem have been known for many years (see e.g. Sadegh and Horowitz, 1990; Slotine and Li, 1988; Wen and Bayard, 1988). There exist quite many tracking algorithms based on various levels of knowledge of robot dynamics. In (Qu and Dorsey, 1991) it has been shown that a control algorithm using the classical PD controller without information about the robot model yields uniform ultimate boundedness (also known as practical stability) of the tracking error, i.e. the error tends in a finite time to a bounded region around zero. The basic disadvantage of this algorithm is a trade-off between the knowledge of robot dynamics and large gains of the controller. Continuously, growing up requirements regarding the quality of robot control imply a search for new control algorithms. It appears that at least partial knowledge of manipulator dynamics is necessary to preserve asymptotic (exponential) stability of the control algorithm. The knowledge of the robot model along a desired trajectory is often used in tracking control algorithms (Sadegh and Horowitz, 1990; Wen and Bayard, 1988).

In the mid 1980s a new approach to adaptive control for linear systems was developed which is called universal adaptive control (Mareels, 1984; Martensson, 1985; Morse, 1983; Nussbaum, 1983). The basic idea of universal adaptive control is to achieve control objectives for an unknown system using only output data without explicit parameter identification. Universal adaptive control has been prospering in linear systems for some time. Recently, great efforts have been put forth to extend this approach to nonlinear systems (Allgoewer and Ilchmann, 1995; Ilchmann, 1993).

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In this paper, we propose to solve a tracking control problem for rigid manipulators using universal adaptive control (Mazur, 1993; 1996). Due to good properties of the universal adaptive control, we expect that the obtained universal adaptive tracking algorithm will exhibit significant advantages: a balanced gain distribution in the feedback-loop, small overshoots, and a fast transient response.

We have shown that this algorithm is semiglobally exponentially stable. The control algorithm presented here consists of two parts: (i) a reference control, which needs knowledge of the robot model along a desired trajectory, (ii) a correction term, which is a collection of local universal adaptive controllers defined by Byrnes and Willems (1984). Since each of the local controllers resembles a dynamic PD controller, the introduced control algorithm may be considered as a dynamic version of the Wen and Bayard algorithm (Wen and Bayard, 1988). We have proved stability of the new algorithm using the Lyapunov function technique. Our Lyapunov function is a modified function derived by Wen and Bayard (1988). We have proved the exponential stability similarly to the procedure presented by Sadegh and Horowitz (1990).

A simulation study has been made for the IRb-6 manipulator limited to the first three degrees of freedom. The new algorithm is compared with the Wen and Bayard algorithm. The proposed universal adaptive tracking algorithm offers significant improvements of the quality of control.

The paper is organized as follows. In Section 2, the problem formulation and main result (Theorem 1) are presented. Section 3 is devoted to the proof of the main result. In Section 4 a simulation study is presented and the universal adaptive tracking control for the IRb-6 manipulator is developed. Section 5 contains some conclusions.

2. Problem Statement and Main Result

In this paper, we consider the problem of joint space trajectory tracking of rigid manipulators. We will present a control law which preserves the convergence of a real manipulator's trajectory to a desired trajectory. Throughout this paper we will consider the standard model of an n-link rigid manipulator (Spong and Vidyasagar, 1989) given by the following equation:

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) = u \tag{1}$$

where $x \in \mathbb{R}^n$ represents the joint positions, $\dot{x} \in \mathbb{R}^n$ stands for the velocities of the joints, and $u \in \mathbb{R}^n$ is a vector of control inputs (torques). $M(x)$ is a positive-definite $n \times n$ inertia matrix, the matrix $C(x,\dot{x})$ of size $n \times n$ is defined via Christoffel's symbols and represents the Coriolis and centripetal torques, and $G(x) \in \mathbb{R}^n$ is a vector of gravitational torques.

We define a universal adaptive tracking control law for the robot model (1) as

$$u = M(x_d)\ddot{x}_d + C(x_d,\dot{x}_d)\dot{x}_d + G(x_d) - kP_1e_1 - kP_2e_2 \tag{2}$$

where P_1, P_2 are positive constants and e_1, e_2 represent tracking errors of positions and velocities, respectively, defined as follows:

$$e_1(t) = (e_{11}, \dots, e_{1i}, \dots, e_{1n})^T = x(t) - x_d(t), \qquad e_1 \in \mathbb{R}^n \tag{3}$$

$$e_2(t) = (e_{21}, \dots, e_{2i}, \dots, e_{2n})^T = \dot{x}(t) - \dot{x}_d(t) = \dot{e}_1, \qquad e_2 \in \mathbb{R}^n \tag{4}$$

The matrix k is diagonal and describes the gains of the local universal adaptive controllers:

$$k(t) = \mathrm{diag}\Big\{k_i(t)\Big\}, \qquad i = 1, \dots, n \tag{5}$$

where k_i is the gain of the local dynamic universal adaptive controller assigned to the i-th manipulator joint. Each local universal adaptive controller is in the form proposed by Byrnes and Willems (1984):

$$\dot{k}_i(t) = (P_1 e_{1i} + P_2 e_{2i})^2 = P_1^2 e_{1i}^2 + P_2^2 e_{2i}^2 + 2P_1 P_2 e_{1i} e_{2i}, \quad i = 1, \dots, n \tag{6}$$

In what follows, we will assume $k_i(0) > 0$. It is easy to see that eqn. (2) consists of two parts. The first part, which should ensure the robot motion along a desired trajectory, is actually the reference signal, and the second part which can be regarded as a correction, is in fact the universal adaptive tracking controller:

$$u = u_d + u_{BW}, \qquad u_{BW} = -kP_1 e_1 - kP_2 e_2 \tag{7}$$

Now we are ready to present the main result of this paper in the form of the following theorem:

Theorem 1. *Consider the model of the manipulator dynamics given by eqn. (1) in a closed loop with the control law (2). Let $k(t)$ be a diagonal matrix of local universal adaptive controller gains $k_i(t)$*

$$k_i(t) = k_i(0) + \int_0^t \dot{k}_i(t)\,\mathrm{d}t, \quad k_i(0) > 0$$

where $\dot{k}_i(t)$ is given by (6), and let $x_d(t)$ be a desired trajectory along which the manipulator moves. Then, for any bounded desired trajectory and for any initial conditions $x(0), \dot{x}(0)$, there exist $k(0), P_1, P_2$ such that the position tracking error $e_1(t)$ and the velocity tracking error $e_2(t)$ defined, respectively, as

$$e_1(t) = x(t) - x_d(t), \quad e_1(t) \in \mathbb{R}^n$$

$$e_2(t) = \dot{x}(t) - \dot{x}_d(t), \quad e_2(t) \in \mathbb{R}^n$$

tend exponentially to zero while all the local universal controller gains $k_i(t)$ remain bounded, i.e.

$$\lim_{t\to\infty} k_i(t) \ \textit{exists and is finite}$$

for all $i = 1, \dots, n$, where i is the the number of robot links.

A block diagram of the control system considered in Theorem 1 is shown in Fig. 1. The proof of Theorem 1 is given in the next section.

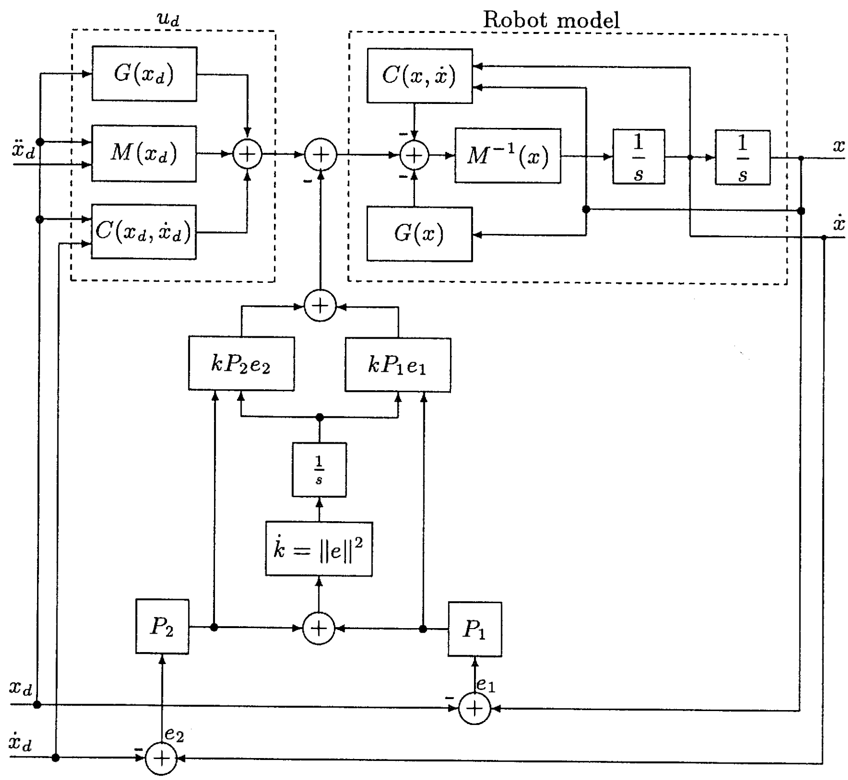


Fig. 1. Block diagram of the universal adaptive tracking algorithm (the manipulator and controller in a closed feedback loop).

3. Proof of the Main Result

The proof is based on the Lyapunov theory and falls naturally into four parts. First, we propose a Lyapunov function candidate and prove that it is positive definite. Then, we present an important lemma which gives us the necessary estimate of the time derivative of the Lyapunov function. Next, we compute the time derivative of the Lyapunov function along the trajectories of the closed-loop system. Finally, an exponential stability is proved for the presented algorithm applied to a robot manipulator.

3.1. Positive Definiteness of the Lyapunov Function

Our proof of the exponential convergence of tracking errors e_1 and e_2 is based on a standard Lyapunov technique along with a lemma by Wen and Bayard (Wen and Bayard, 1988) exploited to estimate the time derivative of the Lyapunov function.

We will use the following Lyapunov function that is a modification of the function used in (Wen and Bayard, 1988):

$$V(t, e_1, e_2, k) = \frac{1}{2} e_2^T M(x) e_2 + \frac{1}{2} e_1^T k P_1 e_1 + \varepsilon e_1^T M(x) e_2 + \frac{1}{2} \varepsilon e_1^T k P_2 e_1 \qquad (8)$$

where $k = k(t)$, $x = x_d + e_1$ and ε is a small constant. We will show that there exists a $\varepsilon > 0$ such that V is positive definite in e_1 and e_2. For this purpose, we underbound all terms of V and show that this underbounding is positive, if ε is chosen properly.

First, some notation is introduced: $\lambda_m(M(x))$ is the minimum eigenvalue of the matrix $M(x)$ and $\lambda_M(M(x))$ stands for the maximum eigenvalue of the matrix $M(x)$, both computed for a fixed x. Now, a well-known property of quadratic forms yields

$$\frac{1}{2} e_2^T M(x) e_2 \geq \frac{1}{2} e_2^T \lambda_m(M(x)) e_2 = \frac{1}{2} \lambda_m(M(x)) \|e_2\|^2 \qquad (9)$$

Similarly, the second and fourth terms in the Lyapunov function can be underbounded respectively by

$$\frac{1}{2} e_1^T k P_1 e_1 \geq \frac{1}{2} \lambda_m(k P_1) \|e_1\|^2 \qquad (10)$$

$$\frac{1}{2} \varepsilon e_1^T k P_2 e_1 \geq \frac{1}{2} \varepsilon \lambda_m(k P_2) \|e_1\|^2 \qquad (11)$$

From the definition of the inner product we derive the following expression:

$$\varepsilon e_1^T M(x) e_2 = \varepsilon \|e_1\| \, \|M(x) e_2\| \cos \angle(e_1, M e_2) \geq -\varepsilon \|e_1\| \, \|M(x) e_2\| \qquad (12)$$

From the definition of the norm of a vector we get

$$\|M e_2\| = \sqrt{e_2^T M^T M e_2} = \sqrt{e_2^T M^2 e_2} \leq \sqrt{\lambda_M^2(M) \|e_2\|^2} = \lambda_M(M) \|e_2\| \qquad (13)$$

After multiplying (13) by the term $-\varepsilon\|e_1\|$ we derive the inequality:

$$-\varepsilon \|e_1\| \, \|M e_2\| \geq -\varepsilon \|e_1\| \, \|e_2\| \lambda_M(M) \qquad (14)$$

From (12) and (14) we get the following evaluation:

$$\varepsilon e_1^T M(x) e_2 \geq -\varepsilon \lambda_M(M(x)) \|e_1\| \, \|e_2\| \qquad (15)$$

Obviously, we have

$$\lambda_m(k(t) P_1) = P_1 \lambda_m(k(t))$$

so eventually the above inequalities result in the following estimate:

$$V(t, e_1, e_2, k) \geq \frac{1}{2} \lambda_m(M(x)) \|e_2\|^2 + \frac{1}{2} P_1 \lambda_m(k(t)) \|e_1\|^2 + \frac{1}{2} \varepsilon P_2 \lambda_m(k(t)) \|e_1\|^2 - \varepsilon \|e_1\| \, \|e_2\| \lambda_M(M(x)) \qquad (16)$$

Observe that we can rewrite relation (16) as

$$V(t,e_1,e_2,k) \;\geq\; \frac{1}{2}\lambda_m(M(x))\left(\|e_2\| - \varepsilon\frac{\lambda_M(M)}{\lambda_m(M)}\|e_1\|\right)^2 + \frac{1}{2}\left[P_1\lambda_m(k(t)) + \varepsilon P_2\lambda_m(k(t)) - \varepsilon^2\frac{\lambda_M^2(M(x))}{\lambda_m(M(x))}\right]\|e_1\|^2 \quad (17)$$

Clearly, since the gains of local universal adaptive controllers $k_i(t)$ do not decrease as $t \to \infty$, the following condition is sufficient for the positive definiteness of V:

$$P_1\lambda_m(k(0)) + \varepsilon P_2\lambda_m(k(0)) - \varepsilon^2\Lambda \geq 0 \quad (18)$$

where

$$\Lambda = \max_x \frac{\lambda_M^2(M(x))}{\lambda_m(M(x))} > 0$$

We can compute ε for which inequality (18) holds. Since we have assumed $\varepsilon > 0$, it is immediate that V is positive definite provided that

$$0 < \varepsilon \leq \frac{P_2\lambda_m(k(0)) + \sqrt{P_2^2\lambda_m^2(k(0)) + 4\Lambda P_1\lambda_m(k(0))}}{2\Lambda} \quad (19)$$

3.2. Wen-Bayard Lemma

In this section, a stability lemma is presented that plays an important role in further considerations. This lemma has been proved in (Wen and Bayard, 1988).

Lemma 1. *For a dynamical system*

$$\dot{x}_i = f_i(x_1,\dots,x_N,t), \qquad x_i \in \mathbb{R}^{n_i}, \quad t \geq 0, \quad i = 1,\dots,N$$

let f_i be locally Lipschitz with respect to $x_1,\dots,x_N$, uniformly in t on bounded intervals and continuous in t for $t \geq 0$.

Let us suppose that a function $V : \mathbb{R}^{(n_1+n_2+\dots n_N)} \times (\mathbb{R}_+ \cup \{0\}) \to (\mathbb{R}_+ \cup \{0\})$ is given such that

$$V(x_1,\dots,x_N,t) = \sum_{i,j=1}^{N} x_i^T P_{ij}(x_1,\dots,x_N,t)x_j \quad (20)$$

where, for each $i = 1,\dots,N$, there exists $\xi_i > 0$ such that

$$\xi_i\|x_i\|^2 \leq V(x_1,\dots,x_N,t) \quad (21)$$

and the following inequality holds:

$$\dot{V}(x_1,\dots,x_N,t) \leq -\sum_{i\in I_1}\left(\alpha_i - \sum_{j\in I_{2_i}}\gamma_{ij}\|x_j(t)\|^{k_{ij}}\right)\|x_i(t)\|^2 \quad (22)$$

where $\alpha_i, \gamma_{ij}, k_{ij} > 0, I_{2_i} \subset I_1 \subset \{1, \ldots, N\}$.

Let $V_0 = V(x_1(0), \ldots, x_N(0), 0)$. *If for each* $i \in I_1$

$$\alpha_i > \sum_{j \in I_{2_i}} \gamma_{ij} \left(\frac{V_0}{\xi_j} \right)^{k_{ij}/2} \tag{23}$$

then for all

$$\lambda_i \in \left(0, \alpha_i - \sum_{j \in I_{2_i}} \gamma_{ij} \left(\frac{V_0}{\xi_j} \right)^{k_{ij}/2} \right) \tag{24}$$

the following inequality holds:

$$\dot{V}(x_1, \ldots, x_N, t) \leq -\sum_{i \in I_1} \lambda_i \|x_i\|^2, \quad \forall t \geq 0 \tag{25}$$

In the remainder of this section, we shall show that the Lyapunov function (8) satisfies the assumptions of Lemma 1, and hence inequality (25) holds. Next, we find a number $\xi_1 > 0$ which satisfies the condition

$$V(e_1, e_2, k, t) \geq \xi_1 \|e_1\|^2 \tag{26}$$

From (17) we obtain

$$\begin{aligned} V(e_1, e_2, k, t) &\geq \frac{1}{2} \lambda_m(M(x)) \left(\|e_2\| - \varepsilon \frac{\lambda_M(M(x))}{\lambda_m(M(x))} \|e_1\| \right)^2 \\ &\quad + \frac{1}{2} \left\{ (P_1 + \varepsilon P_2) \lambda_m(k(t)) - \varepsilon^2 \frac{\lambda_M^2(M(x))}{\lambda_m(M(x))} \right\} \|e_1\|^2 \\ &\geq \frac{1}{2} \left\{ (P_1 + \varepsilon P_2) \lambda_m(k(t)) - \varepsilon^2 \frac{\lambda_M^2(M(x))}{\lambda_m(M(x))} \right\} \|e_1\|^2 \\ &= \xi_1' \|e_1\|^2 \\ &\geq \xi_1 \|e_1\|^2 \end{aligned}$$

The term ξ_1' is time-varying and depends on the position x, therefore we take

$$\xi_1 = \frac{1}{2} \left\{ \lambda_m(k(0)) (P_1 + \varepsilon P_2) - \varepsilon^2 \Lambda \right\} \tag{27}$$

Note that the condition $\xi_1 > 0$ results from eqns. (18)–(19). Next, we will find a number $\xi_2 > 0$ for which the following condition holds:

$$V(e_1, e_2, k, t) \geq \xi_2 \|e_2\|^2 \tag{28}$$

To compute ξ_2, we rewrite (16) as

$$\begin{aligned} V(e_1,e_2,k,t) &\geq \frac{1}{2}\|e_1\|^2\alpha + \frac{1}{2}\lambda_m(M)\|e_2\|^2 - \varepsilon\lambda_M(M)\|e_1\|\,\|e_2\| \\ &= \frac{1}{2}\alpha\left(\|e_1\| - \frac{\varepsilon\lambda_M(M)}{\alpha}\|e_2\|\right)^2 + \frac{1}{2}\lambda_m(M)\left(1 - \frac{\varepsilon^2\lambda_M^2(M)}{\alpha\lambda_m(M)}\right)\|e_2\|^2 \\ &\geq \frac{1}{2}\lambda_m(M)\left[1 - \frac{\varepsilon^2\lambda_M^2(M)}{\alpha\lambda_m(M)}\right]\|e_2\|^2 \\ &\geq \frac{1}{2}\lambda_m(M)\left[1 - \frac{\varepsilon^2\Lambda}{\lambda_m(k(0))(P_1+\varepsilon P_2)}\right]\|e_2\|^2 \\ &\geq \frac{\Delta}{\lambda_m(k(0))(P_1+\varepsilon P_2)}\xi_1\|e_2\|^2 = \xi_2\|e_2\|^2 \end{aligned} \tag{29}$$

where the symbols α i Δ are respectively defined as

$$\alpha = \lambda_m(k)(P_1+\varepsilon P_2), \qquad \Delta = \min_x \lambda_m(M) \tag{30}$$

Consequently,

$$\xi_2 = \frac{\Delta\xi_1}{\lambda_m(k(0))(P_1+\varepsilon P_2)} \tag{31}$$

The numbers $\xi_1, \xi_2 > 0$ can be chosen in such a way that the Lyapunov function defined by (8) satisfies assumption (21) of Lemma 1. To use Lemma 1, we have to show that assumption (22) holds for V as well.

3.3. Time Derivative of the Lyapunov Function

In this section, we will find an overbound for the time derivative of V along solutions of the closed-loop system obtained by substituting (2) into (1)

$$\begin{aligned} &M(x)\dot{e}_2 = -\Delta M\ddot{x}_d - C(x,\dot{x})e_2 - \Delta C\dot{x}_d - \Delta G - kP_1e_1 - kP_2e_2 \\ &\dot{k} = \mathrm{diag}\left\{P_1^2e_{1i}^2 + P_2^2e_{2i}^2 + 2P_1P_2e_{1i}e_{2i}\right\} \\ &\dot{e}_1 = e_2 \\ &x = x_d + e_1 \\ &\dot{x} = \dot{x}_d + e_2 \end{aligned} \tag{32}$$

where

$$\Delta M = M(x) - M(x_d), \quad \Delta C = C(x,\dot{x}) - C(x_d,\dot{x}_d), \quad \Delta G = G(x) - G(x_d)$$

The time derivative of V defined by (8) is equal to

$$\begin{aligned} \dot{V} \;=\; & e_2^TM\dot{e}_2 + \frac{1}{2}e_2^T\dot{M}e_2 + e_1^TkP_1e_2 + \frac{1}{2}e_1^T\dot{k}P_1e_1 + \varepsilon e_2^TMe_2 \\ & + \varepsilon e_1^T\dot{M}e_2 + \varepsilon e_1^TM\dot{e}_2 + \varepsilon e_1^TkP_2e_2 + \frac{1}{2}\varepsilon e_1^T\dot{k}P_2e_1 \end{aligned} \tag{33}$$

After substitution of (32) into (33), we compute $\dot{V}$ along the solutions of the closed-loop robot system:

$$\begin{aligned}\dot{V} &= e_2^T(-\Delta M\ddot{x}_d - Ce_2 - \Delta C\dot{x}_d - \Delta G - kP_1e_1 - kP_2e_2) + \frac{1}{2}e_2^T\dot{M}e_2 \\ &\quad + e_1^TkP_1e_2 + \frac{1}{2}e_1^T\dot{k}P_1e_1 + \varepsilon e_2^TMe_2 + \varepsilon e_1^T\dot{M}e_2 + \varepsilon e_1^TkP_2e_2 \\ &\quad + \varepsilon e_1^T(-\Delta M\ddot{x}_d - Ce_2 - \Delta C\dot{x}_d - \Delta G - kP_1e_1 - kP_2e_2) \\ &\quad + \frac{1}{2}\varepsilon e_1^T\dot{k}P_2e_1 \end{aligned} \tag{34}$$

By using the identities

$$\dot{M} = C + C^T, \qquad \frac{1}{2}e_2^T\dot{M}e_2 = e_2^TCe_2$$

(34) reduces to

$$\begin{aligned}\dot{V} &= -e_2^T\Delta M\ddot{x}_d - e_2^T\Delta C\dot{x}_d - e_2^T\Delta G - e_2^TkP_2e_2 + \frac{1}{2}e_1^T\dot{k}P_1e_1 \\ &\quad + \varepsilon e_2^TMe_2 + \varepsilon e_1^TC^Te_2 - \varepsilon e_1^T\Delta M\ddot{x}_d - \varepsilon e_1^T\Delta C\dot{x}_d - \varepsilon e_1^T\Delta G \\ &\quad - \varepsilon e_1^TkP_1e_1 + \frac{1}{2}\varepsilon e_1^T\dot{k}P_2e_1 \end{aligned} \tag{35}$$

Now, using the properties of the inner product and square forms, we can overbound the right-hand side of eqn. (35) as follows:

$$-e_2^T\Delta M\ddot{x}_d \le \|e_2\|\,\|\Delta M\ddot{x}_d\| \tag{36}$$

$$-e_2^T\Delta C\dot{x}_d \le \|e_2\|\,\|\Delta C\dot{x}_d\| \tag{37}$$

$$-e_2^T\Delta G \le \|e_2\|\,\|\Delta G\| \tag{38}$$

$$-e_2^TkP_2e_2 \le -\|e_2\|^2\lambda_m(kP_2) \tag{39}$$

$$\frac{1}{2}e_1^T\dot{k}P_1e_1 \le \frac{1}{2}\|e_1\|^2\lambda_M(\dot{k}P_1) \tag{40}$$

$$\varepsilon e_2^TMe_2 \le \varepsilon\|e_2\|^2\lambda_M(M) \tag{41}$$

$$\varepsilon e_1^TC^Te_2 \le \varepsilon\|Ce_1\|\,\|e_2\| \tag{42}$$

$$-\varepsilon e_1^T\Delta M\ddot{x}_d \le \varepsilon\|e_1\|\,\|\Delta M\ddot{x}_d\| \tag{43}$$

$$-\varepsilon e_1^T\Delta C\dot{x}_d \le \varepsilon\|e_1\|\,\|\Delta C\dot{x}_d\| \tag{44}$$

$$-\varepsilon e_1^T \Delta G \ \leq \ \varepsilon \|e_1\| \, \|\Delta G\| \tag{45}$$

$$-\varepsilon e_1^T k P_1 e_1 \ \leq \ -\varepsilon \|e_1\|^2 \lambda_m(kP_1) \tag{46}$$

$$\frac{1}{2}\varepsilon e_1^T \dot{k} P_2 e_1 \ \leq \ \frac{1}{2}\varepsilon \|e_1\|^2 \lambda_M(\dot{k}P_2) \tag{47}$$

To find more exact estimates of the right-hand sides of inequalities (36)–(47) we will assume that for all $t \geq 0$ the desired trajectory and its first and second time derivatives $x_d(t), \dot{x}_d(t), \ddot{x}_d(t)$ are bounded. By applying the mean-value theorem we obtain, already known from (Sadegh and Horowitz, 1990), the evaluations of the terms, in which the elements of the robot dynamics are present:

$$\|\Delta M \ddot{x}_d\| \leq a_1 \|e_1\| \tag{48}$$

$$\|\Delta C \dot{x}_d\| \leq a_2 \|e_1\| + a_3 \|e_2\|, \qquad a_1, a_2 \geq 0 \tag{49}$$

$$\|\Delta G\| \leq a_4 \|e_1\| \tag{50}$$

$$\|C(x, \dot{x}) e_1\| \leq a_5 \|e_1\| + a_6 \|e_1\| \, \|e_2\| \tag{51}$$

where $a_i, i = 1, \ldots, 6$ are positive constants.

Before we start the evaluation of the sign of the time derivative of V, we have to estimate the right-hand sides of expressions (40) and (47). A plausible evaluation of (40) is as follows:

$$\frac{1}{2} e_1^T \dot{k} P_1 e_1 \leq \frac{1}{2} \|e_1\|^2 \lambda_M(\dot{k}P_1) \leq \frac{1}{2} \|e_1\|^2 P_1 \sum_{i=1}^{n} \dot{k}_i$$

After substitution of expression (6), which describes the dynamics of the universal adaptive controller, into the above inequality, we compute

$$\begin{aligned}
\frac{1}{2} e_1^T \dot{k} P_1 e_1 \ &\leq \ \frac{1}{2} \|e_1\|^2 P_1 \sum_{i=1}^{n} \dot{k}_i \\
&= \ \frac{1}{2} \|e_1\|^2 P_1 \sum_{i=1}^{n} (P_1^2 e_{1i}^2 + P_2^2 e_{2i}^2 + 2 P_1 P_2 e_{1i} e_{2i}) \\
&\leq \ \frac{1}{2} \|e_1\|^2 P_1 \sum_{i=1}^{n} (P_1^2 e_{1i}^2 + P_2^2 e_{2i}^2 + 2 P_1 P_2 |e_{1i}| \, |e_{2i}|) \\
&= \ \frac{1}{2} \|e_1\|^2 P_1 \Big(\sum_{i=1}^{n} P_1^2 e_{1i}^2 + \sum_{i=1}^{n} P_2^2 e_{2i}^2 + \sum_{i=1}^{n} 2 P_1 P_2 |e_{1i}| \, |e_{2i}| \Big)
\end{aligned}$$

Now, using the relation

$$2ab \leq a^2 + b^2$$

we conclude that

$$\begin{aligned}\frac{1}{2}e_1^T\dot{k}P_1e_1 &\leq \frac{1}{2}\|e_1\|^2P_1\Big(P_1^2\sum_{i=1}^{n}e_{1i}^2+P_2^2\sum_{i=1}^{n}e_{2i}^2+\sum_{i=1}^{n}(P_1^2e_{1i}^2+P_2^2e_{2i}^2)\Big)\\ &= \frac{1}{2}\|e_1\|^2P_1\Big(2P_1^2\|e_1\|^2+2P_2^2\|e_2\|^2\Big)\\ &= P_1^3\|e_1\|^4+P_1P_2^2\|e_1\|^2\|e_2\|^2 \end{aligned} \tag{52}$$

Similarly, we can evaluate expression (47)

$$\begin{aligned}\frac{1}{2}\varepsilon e_1^T\dot{k}P_2e_1 &\leq \frac{1}{2}\varepsilon\|e_1\|^2P_2\lambda_M(\dot{k})\\ &= \frac{1}{2}\varepsilon\|e_1\|^2P_2\sum_{i=1}^{n}\dot{k}_i\\ &\leq \varepsilon\|e_1\|^2P_2\Big(P_1^2\|e_1\|^2+P_2^2\|e_2\|^2\Big)\\ &= \varepsilon P_1^2P_2\|e_1\|^4+\varepsilon P_2^3\|e_1\|^2\|e_2\|^2 \end{aligned} \tag{53}$$

We apply the above inequalities to get an evaluation of $\dot{V}$. Thus $\dot{V}$ can be over-bounded by

$$\begin{aligned}\dot{V} \leq\ & -\|e_2\|^2\Big\{P_2\lambda_m(k)-a_3-\varepsilon\lambda_M(M)\Big\}\\ & -\|e_1\|^2\varepsilon\Big\{P_1\lambda_m(k)-a_1-a_2-a_4\Big\}\\ & +\|e_1\|\,\|e_2\|\Big\{a_1+a_2+a_4+\varepsilon a_5+\varepsilon a_3\Big\}\\ & +\|e_1\|^4\left\{P_1^3+\varepsilon P_1^2P_2\right\}\\ & +\|e_1\|^2\|e_2\|^2\left\{P_1P_2^2+\varepsilon P_2^3\right\}\\ & +\varepsilon a_6\|e_1\|\,\|e_2\|^2 \end{aligned}$$

To simplify further developments, we propose to introduce the following notations:

$$\frac{1}{2}\Big\{P_2\lambda_m(k(t))-a_3-\varepsilon\lambda_M(M(x))\Big\}\geq A=\frac{1}{2}\Big(P_2\lambda_m(k(0))-a_3-\varepsilon\Lambda_1\Big)$$

$$P_1\lambda_m(k(t))-a_1-a_2-a_4\geq B=P_1\lambda_m(k(0))-a_1-a_2-a_4$$

$$T=a_1+a_2+a_4+\varepsilon a_5+\varepsilon a_3$$

$$E=P_1^3+\varepsilon P_1^2P_2$$

$$D=P_1P_2^2+\varepsilon P_2^3$$

$$S=\varepsilon a_6$$

where

$$\Lambda_1 = \max_x \lambda_M(M)$$

Now let us transform $\dot{V}$ to the form (22) required by Lemma 1

$$\begin{aligned} \dot{V} &\leq -2A\|e_2\|^2 - \varepsilon B\|e_1\|^2 + T\|e_1\|\,\|e_2\| \\ &\quad + E\|e_1\|^4 + D\|e_1\|^2\|e_2\|^2 + S\|e_1\|\,\|e_2\|^2 \\ &= -A\left[\|e_2\| - \frac{T}{2A}\|e_1\|\right]^2 - \left[\varepsilon B - \frac{T^2}{4A}\right]\|e_1\|^2 - A\|e_2\|^2 + E\|e_1\|^4 \\ &\quad + D\|e_1\|^2\|e_2\|^2 + S\|e_1\|\,\|e_2\|^2 \end{aligned}$$

Note that for $A > 0$ the first term is always negative, so that we can write

$$\dot{V} \leq -\left[\varepsilon B - \frac{T^2}{4A}\right]\|e_1\|^2 - A\|e_2\|^2 + E\|e_1\|^4 + D\|e_1\|^2\|e_2\|^2 + S\|e_1\|\,\|e_2\|^2$$

Let $B_1 = \varepsilon B - \frac{T^2}{4A}$. Then

$$\dot{V} \leq -\Big(B_1 - E\|e_1\|^2 - D\|e_2\|\Big)\|e_1\|^2 - \Big(A - S\|e_1\|\Big)\|e_2\|^2$$

so $\dot{V}$ satisfies assumption (22) of Lemma 1.

Let $V_0 = V(e_1(0), e_2(0), k(0), 0)$. If the following conditions hold:

$$B_1 > E\frac{V_0}{\xi_1} + D\sqrt{\frac{V_0}{\xi_2}} \tag{54}$$

$$A > S\sqrt{\frac{V_0}{\xi_1}} \tag{55}$$

then, for each λ_i satisfying

$$0 < \lambda_1 < B_1 - E\frac{V_0}{\xi_1} - D\sqrt{\frac{V_0}{\xi_2}} \tag{56}$$

$$0 < \lambda_2 < A - S\sqrt{\frac{V_0}{\xi_1}} \tag{57}$$

the following estimate holds:

$$\dot{V} \leq -\sum_{i=1}^{2}\lambda_i\|e_i\|^2, \quad \forall t \geq 0 \tag{58}$$

3.4. Exponential Stability of the Tracking Control System

In this subsection, we will show that the Lyapunov function satisfying (58) implies exponential stability of the universal adaptive tracking algorithm, if only the gains of the local dynamic tracking controllers remain bounded. Expression (58) will also be used to prove that the gains of tracking controllers $k_{i\infty}$ tend to finite limits.

To prove the exponential closed-loop stability (i.e. the exponential convergence of errors e_1 and e_2 to zero) we need to show that V decreases exponentially to zero. First, we overbound all the terms of the Lyapunov function V given by (8):

$$\frac{1}{2}e_2^T M(x)e_2 \leq \frac{1}{2}\|e_2\|^2\lambda_M(M(x))$$

$$\frac{1}{2}e_1^T kP_1 e_1 \leq \frac{1}{2}\|e_1\|^2\lambda_M(kP_1) \leq \frac{1}{2}\|e_1\|^2 P_1\lambda_M(k(t))$$

$$\frac{1}{2}\varepsilon e_1^T kP_2 e_1 \leq \frac{1}{2}\varepsilon\|e_1\|^2\lambda_M(kP_2) \leq \frac{1}{2}\varepsilon\|e_1\|^2 P_2\lambda_M(k(t))$$

$$\varepsilon e_1^T M(x)e_2 = \varepsilon\|e_1\|\,\|M(x)e_2\|\cos\angle(e_1, Me_2) \leq \varepsilon\|e_1\|\,\|e_2\|\lambda_M(M(x))$$

where

$$\|M(x)e_2\| = \sqrt{e_2^T M^T M e_2} = \sqrt{e_2^T M^2 e_2} \leq \|e_2\|\lambda_M(M(x))$$

Using again the relation

$$2ab \leq a^2 + b^2$$

we get

$$\begin{aligned}\varepsilon\lambda_M(M(x))\|e_1\|\,\|e_2\| &\leq \left(\frac{\varepsilon}{2}\|e_1\|\right)^2 + \left(\lambda_M(M(x))\|e_2\|\right)^2 \\ &= \frac{\varepsilon^2}{4}\|e_1\|^2 + \lambda_M^2(M(x))\|e_2\|^2\end{aligned}$$

Finally, an upper bound for V can be established as

$$\begin{aligned}V &\leq \frac{1}{2}\lambda_M(M(x))\|e_2\|^2 + \frac{1}{2}P_1\lambda_M(k(t))\|e_1\|^2 + \frac{\varepsilon^2}{4}\|e_1\|^2 \\ &\quad + \lambda_M^2(M(x))\|e_2\|^2 + \frac{1}{2}\varepsilon P_2\lambda_M(k(t))\|e_1\|^2\end{aligned}$$

and after completing terms

$$\begin{aligned}V &\leq \|e_1\|^2\left(\frac{1}{2}P_1\lambda_M(k(t)) + \frac{1}{2}\varepsilon P_2\lambda_M(k(t)) + \frac{\varepsilon^2}{4}\right) \\ &\quad + \|e_2\|^2\left(\frac{1}{2}\lambda_M(M(x)) + \lambda_M^2(M(x))\right) = \gamma_1\|e_1\|^2 + \gamma_2\|e_2\|^2\end{aligned}$$

where

$$\gamma_1 = \max_x \left(\frac{1}{2} P_1 \lambda_M(k(t)) + \frac{1}{2}\varepsilon P_2 \lambda_M(k(t)) + \frac{\varepsilon^2}{4} \right)$$

$$\gamma_2 = \max_x \left(\frac{1}{2} \lambda_M(M(x)) + \lambda_M^2(M(x)) \right)$$

This overbounding of the Lyapunov function is valid if γ_1 is bounded $\forall t \geq 0$. To prove that $\gamma_1 < \infty$, it will be shown that all local dynamic universal adaptive controller gains satisfy $k_i(t) < \infty$ for each $t \geq 0$.

The gain of the i-th controller is equal to

$$\begin{aligned} k_i(t) &= k_i(0) + \int_0^t (P_1 e_{1i} + P_2 e_{2i})^2 \, \mathrm{d}t \\ &\leq k_i(0) + \int_0^t (2P_1^2 e_{1i}^2 + 2P_2^2 e_{2i}^2) \, \mathrm{d}t \\ &\leq k_i(0) + \int_0^t \left(2P_1^2 \Big(\sum_{i=1}^n e_{1i}^2 \Big) + 2P_2^2 \Big(\sum_{i=1}^n e_{2i}^2 \Big) \right) \mathrm{d}t \\ &\leq k_i(0) + 2P_1^2 \int_0^t \|e_1\|^2 \, \mathrm{d}t + 2P_2^2 \int_0^t \|e_2\|^2 \, \mathrm{d}t \end{aligned} \quad (59)$$

so we have to show that both integrals on the right-hand side of (59) are bounded. But we know that for the time derivative of the Lyapunov function V inequality (58) holds, so integrating both sides of (58) we get

$$V(t) - V(0) \leq -\lambda_1 \int_0^t \|e_1\|^2 \, \mathrm{d}t - \lambda_2 \int_0^t \|e_2\|^2 \, \mathrm{d}t \quad (60)$$

Clearly, (60) implies

$$0 \leq V(t) \leq V(0) - \lambda_1 \int_0^t \|e_1\|^2 \, \mathrm{d}t - \lambda_2 \int_0^t \|e_2\|^2 \, \mathrm{d}t \quad (61)$$

which means that both integrals in (61) are bounded, and hence $k_i(t) < \infty$.

We have proved that all the gains of the local universal adaptive tracking controllers have bounded limits. To get a time-invariant overbounding of the Lyapunov function, it is sufficient to choose

$$\gamma_1 = \frac{1}{2} P_1 \sum_{i=1}^n k_i(\infty) + \frac{1}{2}\varepsilon P_2 \sum_{i=1}^n k_i(\infty) + \frac{\varepsilon^2}{4}$$

Since $k_i(\infty)$ is not explicitly known, we use (60) to obtain an effective estimate for γ_1. From (61) we deduce

$$\lambda_1 \int_0^t \|e_1\|^2 \, \mathrm{d}t + \lambda_2 \int_0^t \|e_2\|^2 \, \mathrm{d}t \leq V(0) \quad (62)$$

Hence

$$\int_0^t \|e_1\|^2 \, dt \le \frac{V(0)}{\lambda_1}, \qquad \int_0^t \|e_2\|^2 \, dt \le \frac{V(0)}{\lambda_2} \tag{63}$$

Let us substitute now (63) into (59) to evaluate $k_i(\infty)$ and then $k_i(\infty)$ into γ_1:

$$\gamma_1 = \frac{1}{2}(P_1 + \varepsilon P_2) \sum_{i=1}^{n} k_i(0) + (P_1 + \varepsilon P_2) \left(\frac{P_1^2}{\lambda_1} + \frac{P_2^2}{\lambda_2} \right) V(0) + \frac{\varepsilon^2}{4} \tag{64}$$

By defining $\gamma = \max(\gamma_1, \gamma_2)$, where $\gamma_1, \gamma_2 > 0$, we obtain

$$V \le \gamma \Big(\|e_1\|^2 + \|e_2\|^2 \Big) \tag{65}$$

Inequality (58) can be rewritten as

$$\dot{V} \le -\lambda_1 \|e_1\|^2 - \lambda_2 \|e_2\|^2 \le -\lambda \Big(\|e_1\|^2 + \|e_2\|^2 \Big) \tag{66}$$

with $\lambda = \min(\lambda_1, \lambda_2)$. After multiplying the inequality (65) by $-\lambda/\gamma$ we obtain

$$-\lambda \Big(\|e_1\|^2 + \|e_2\|^2 \Big) \le -\frac{\lambda}{\gamma} V = -\gamma_0 V, \qquad \gamma_0 = \frac{\lambda}{\gamma}$$

which, along with (66), yields

$$\dot{V} \le -\gamma_0 V \le 0 \tag{67}$$

i.e.

$$V(e_1, e_2, k, t) \le V\Big(e_1(0), e_2(0), k(0), 0\Big) e^{-\gamma_0 t} = V_0 e^{-\gamma_0 t} \tag{68}$$

Expression (68) implies that the Lyapunov function V decreases exponentially. Using this property of V, we can easily show that the errors e_1 and e_2 converge exponentially to zero. Namely, by combining (26) and (68), we obtain the following relation:

$$\xi_1 \|e_1\|^2 \le V_0 e^{-\gamma_0 t}$$

Thus

$$\|e_1\| \le \sqrt{\frac{V_0}{\xi_1}} e^{-\frac{1}{2}\gamma_0 t} \tag{69}$$

Analogously, we combine (28) and (68) to get

$$\|e_2\| \le \sqrt{\frac{V_0}{\xi_2}} e^{-\frac{1}{2}\gamma_0 t} \tag{70}$$

Expressions (69) and (70) establish the exponential convergence of e_1 and e_2 to zero. It is also of interest to examine the domain of attraction of the presented control algorithm. The stability we are able to guarantee is called the semiglobal stability (Loria and Ortega, 1995). This kind of stability means that for any initial conditions and any bounded desired trajectory it is always possible to find controller gains to ensure the exponential convergence of errors to zero. Thus, with a suitable choice of the controller gains, the domain of attraction can be enlarged arbitrarily. The universal adaptive tracking algorithm given by Theorem 1 is stable, if inequalities (54) and (55) hold. The term $V(0, k(0), e_1(0), e_2(0))$ on the right-hand sides of these inequalities depends on the initial values of errors, which means that it depends on the initial position $x(0)$ and velocity $\dot{x}(0)$ of the joints. On the other hand, conditions (54) and (55) depend on initial controller gains $k_i(0)$. It is easy to show that the above inequalities are satisfied for a sufficiently large $k_i(0)$ for fixed initial conditions of the joint trajectory. Conditions (54) and (55) also hold, if $k_i(0)$ are small and P_1 and P_2 are sufficiently large. It is important to remember that P_2 cannot be too large (P_1 should be greater than aP_2^2), otherwise ξ_1 decreases to zero, so Lemma 1 does not hold any more.

4. Simulations

In this section, we will focus our attention on the comparison of the universal adaptive tracking algorithm defined by (2) and (6) with the algorithm proposed by Wen and Bayard (1988):

$$u = M(x_d)\ddot{x}_d + C(x_d, \dot{x}_d)\dot{x}_d + G(x_d) - K_p e_1 - K_v e_2 \tag{71}$$

Let us observe that these two algorithms differ from each other by the correction terms. In the new algorithm, the correction term comes from a set of local dynamic universal adaptive controllers. The gains of these controllers are time-varying and their dynamics depends on parameters P_1 and P_2. In the algorithm of Wen and Bayard, the correction is generated by a PD controller whose gain is constant. To ensure a stability of algorithm (71), large gains K_p and K_v are required as shown in (Wen and Bayard, 1988). To compare the quality of control of both algorithms, we use the following performance index:

$$j_i = \int_0^T e_i^2(t)\,\mathrm{d}t, \qquad i = 1, \dots, n \tag{72}$$

which is a square error computed during simulation for the i-th robot link. The final time of simulations is taken as $T = 30\,\mathrm{s}$. Simulations have been carried out with the use of the TUTSIM 0.7 software package. For simulations we have chosen the IRb-6 manipulator limited to the first three degrees of freedom (without wrist motions), described in (Gosiewski *et al.*, 1984).

4.1. Achieving a Fixed Value of the Performance Index

In this subsection, we compare the universal adaptive tracking algorithm (2), (6) with the Bayard and Wen algorithm (71) for step and periodic desired (reference) trajectories.

For all the manipulator joints the step desired trajectory has been chosen identical, i.e. $x_{1d}(t) = x_{2d}(t) = x_{3d}(t) = 1\,\mathrm{rad}$. In practice, this trajectory is realizable only for the first joint of the IRb-6 manipulator, but this is not important for the theoretical results and simulations. Also the initial conditions have been assumed the same and equal to $x_i(0) = 0$ and $\dot{x}_i(0) = 0$, $i = 1, 2, 3$.

The objective of the simulations is to examine the behaviour of the tracking errors $e_1(t)$ and to check minimal values of controller gains required to achieve a prescribed value of the performance index (i.e. $j_i = 0.1$ for a desired step trajectory) for each joint of the manipulator.

In Fig. 2, we present several graphs of the tracking errors of the Wen and Bayard algorithm while assuming a step trajectory.

In Table 1, we show values of gains K_p and K_v needed to achieve the prescribed value of the performance index $j_i = 0.1$ for each link.

Simulations have been performed for two different initial values of the universal adaptive controller gains, i.e. for small initial gains ($k_i(0) = 0.1$) and for large initial gains ($k_i(0) = 10$). It has been shown that by enlarging the initial gains one can improve the performance of the universal controller.

In Fig. 3, we present the profiles of the tracking errors appearing in the universal adaptive tracking algorithm during tracking the step trajectory for $j_i = 0.1$ and $k_i(0) = 0.1$, whereas in Fig. 5 we show the evolution of the tracking errors for $k_i(0) = 10$ and $j_i = 0.1$.

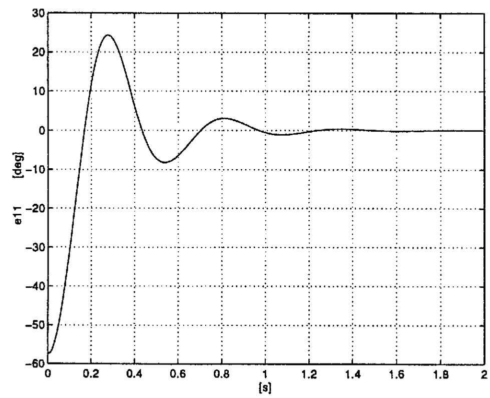
Local universal controller gains $k_i(t)$ corresponding to errors from Fig. 3 are presented in Fig. 4. The gains corresponding to errors from Fig. 5 are shown in Fig. 6.

From the behaviour of tracking errors presented in Figs. 2, 3 and 5 we can deduce that the transient response of the universal adaptive tracking algorithm (UATA) is faster, leading to much smaller overshoots for all of joints than in the Wen and Bayard algorithm. The last feature of UATA is particularly useful in obstacle avoidance.

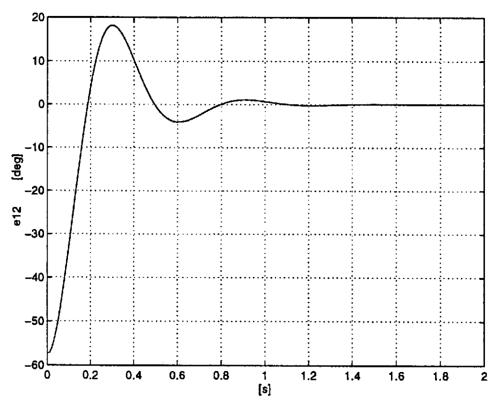
Minimal gains needed in both algorithms to obtain the prescribed value $j_i = 0.1$ of the performance index for each joint are collected in Table 1.

Tab. 1. Controller gains needed to obtain the value $j_i = 0.1$ of the performance index during tracking the step trajectory.

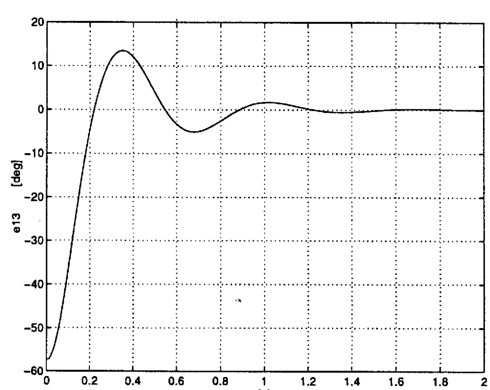
Algorithm	Gains for j_1	Gains for j_2	Gains for j_3
Wen-Bayard	$K_p = 2000$, $K_v = 100$	$K_p = 1500$, $K_v = 100$	$K_p = 1500$, $K_v = 100$
UATA, $k_i(0) = 0.1$	$k_1 P_1 = 394$, $k_1 P_2 = 16$	$k_2 P_1 = 411$, $k_2 P_2 = 16$	$k_3 P_1 = 246$, $k_3 P_2 = 8$
UATA, $k_i(0) = 10$	$k_1 P_1 = 202$, $k_1 P_2 = 23$	$k_2 P_1 = 202$, $k_2 P_2 = 22$	$k_3 P_1 = 196$, $k_3 P_2 = 22$



(a)

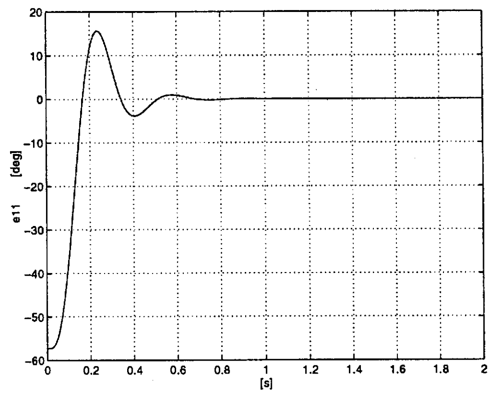


(b)

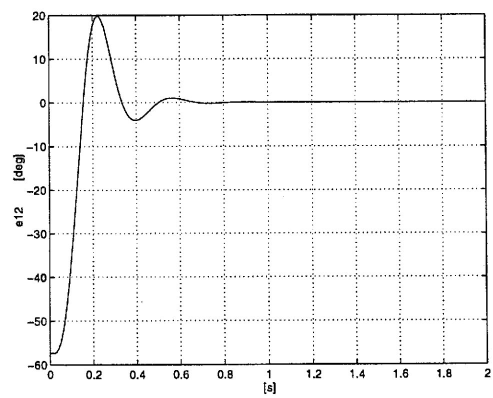


(c)

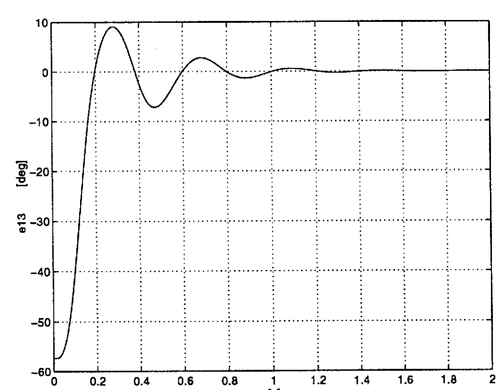
Fig. 2. Tracking errors of the Wen and Bayard algorithm for the desired step trajectory and prescribed value of the performance index $j_i = 0.1$, $i = 1, 2, 3$.



(a)



(b)



(c)

Fig. 3. Tracking errors in the universal adaptive tracking algorithm for the desired step trajectory with $k_i(0) = 0.1$ and with the performance index equal to $j_i = 0.1$.

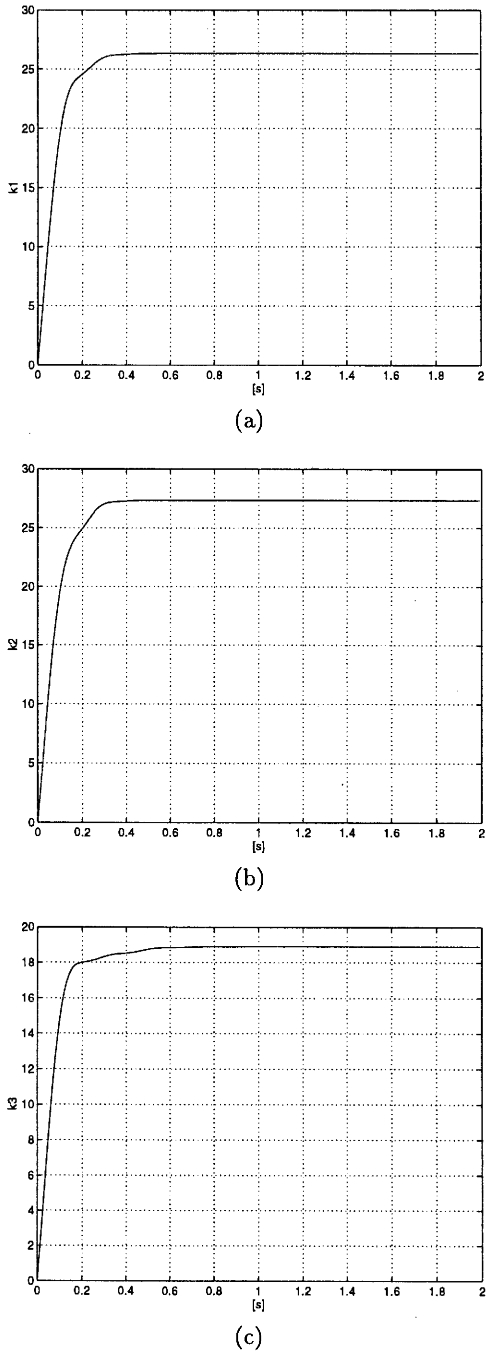
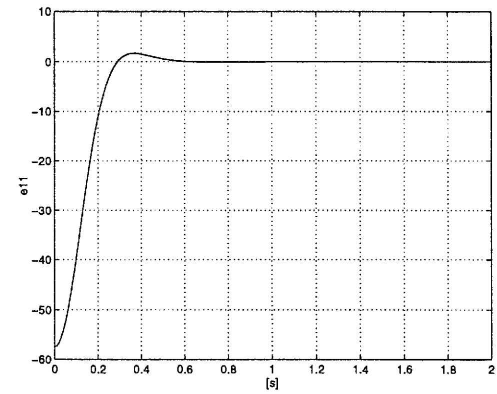
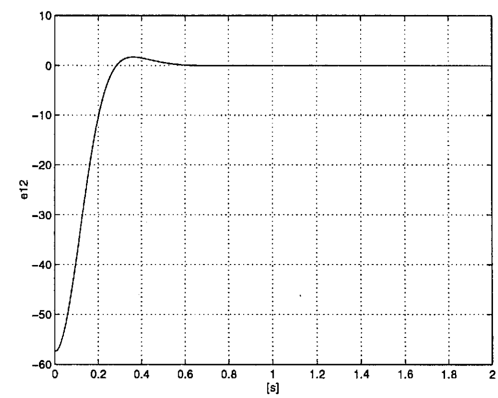


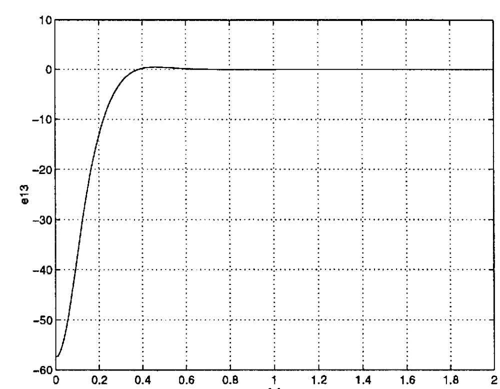
Fig. 4. Gains of the local universal controller obtained while tracking the desired step trajectory with $j_i = 0.1$ and $k_i(0) = 0.1$.



(a)

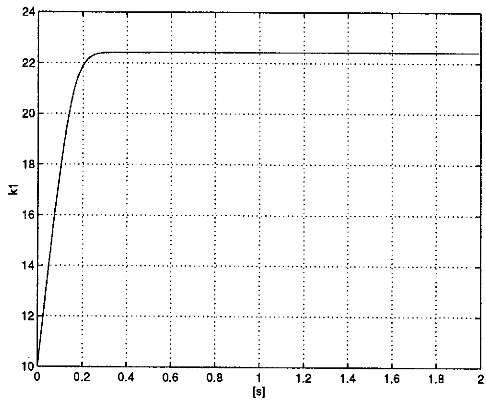


(b)

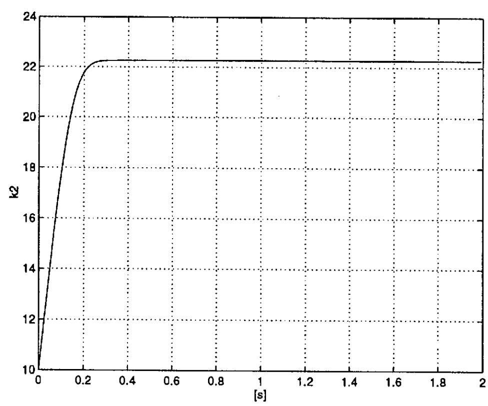


(c)

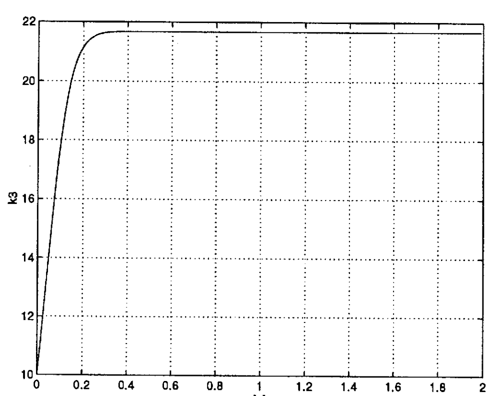
Fig. 5. Tracking errors appearing in the universal adaptive tracking algorithm for the desired step trajectory with $k_i(0) = 10$ and with the performance index equal to $j_i = 0.1$.



(a)



(b)



(c)

Fig. 6. Gains of the local universal controller obtained while tracking the desired step trajectory with $j_i = 0.1$ and $k_i(0) = 10$.

From Table 1 it is clear that the UATA requires considerably smaller values of gains (about ten times smaller) to track the step reference trajectory than the algorithm of Wen and Bayard.

Simulations of a similar nature have been performed for the case of time-varying (periodic) desired trajectory. These trajectories are different for each manipulator joint and chosen as $x_{id}(t) = \sin(it)\,\mathrm{rad}$, $i = 1,2,3$ in order to excite nonlinear dynamic couplings between robot links. We have set the following initial conditions: $x_i(0) = \dot{x}_i(0) = 0$, $i = 1,2,3$. Similarly to the case of the desired step trajectory, we have compared the two algorithms with respect to gains needed to provide the fixed value of the performance index $j_i = 10^{-3}$ and with respect to the behaviour of tracking errors $e_{1i}(t)$.

In Fig. 7, we present the plots of the tracking errors appearing in the Wen and Bayard algorithm during tracking the time-varying desired trajectory.

In Table 2, we show gains K_p and K_v needed to achieve the prescribed value of the performance index $j_i = 10^{-3}$ for each link.

Tab. 2. Controller gains needed to obtain the value $j_i = 10^{-3}$ of the performance index while tracking the time-varying desired trajectory.

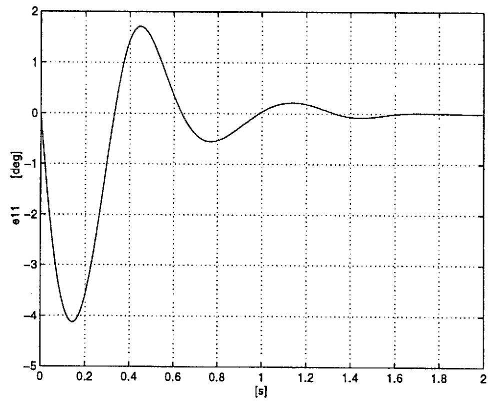
Algorithm	Gains for j_1	Gains for j_2	Gains for j_3
Wen-Bayard	$K_p = 1500$, $K_v = 100$	$K_p = 2500$, $K_v = 100$	$K_p = 11000$, $K_v = 100$
UATA, $k_i(0) = 0.1$	$k_1P_1 = 572$, $k_1P_2 = 11$	$k_2P_1 = 704$, $k_2P_2 = 21$	$k_3P_1 = 1340$, $k_3P_2 = 27$
UATA, $k_i(0) = 10$	$k_1P_1 = 51$, $k_1P_2 = 11$	$k_2P_1 = 105$, $k_2P_2 = 21$	$k_3P_1 = 224$, $k_3P_2 = 21$

Similarly to the case of tracking the step desired trajectory, simulations have been performed for two different initial gains of the local universal adaptive tracking controller, i.e. for $k_i(0) = 0.1$ and $k_i(0) = 10$.

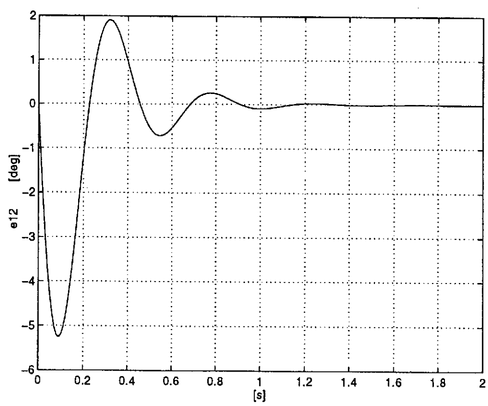
Figure 8 presents the graphs of the tracking errors appearing in the universal adaptive tracking algorithm during tracking the time-varying desired trajectory for $j_i = 10^{-3}$ and $k_i(0) = 0.1$, and Fig. 10 shows the plots of the tracking errors for $k_i(0) = 10$ and $j_i = 10^{-3}$.

The gains $k_i(t)$ of the local universal controllers corresponding to errors from Fig. 8 are presented in Fig. 9, whereas the gains corresponding to errors from Fig. 10 are shown in Fig. 11.

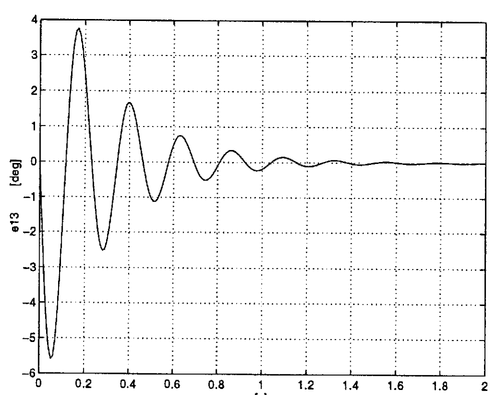
While tracking the time-varying reference trajectory the UATA revealed a faster transient response than the Wen and Bayard algorithm. Additionally, the UATA featured smaller overshoots as well as an exceptionally good damping of the transient response at the third joint in comparison with the algorithm of Wen and Bayard.



(a)



(b)



(c)

Fig. 7. Tracking errors of the Wen and Bayard algorithm for the time-varying desired trajectory and for the value of the performance index equal to $j_i = 10^{-3}$.

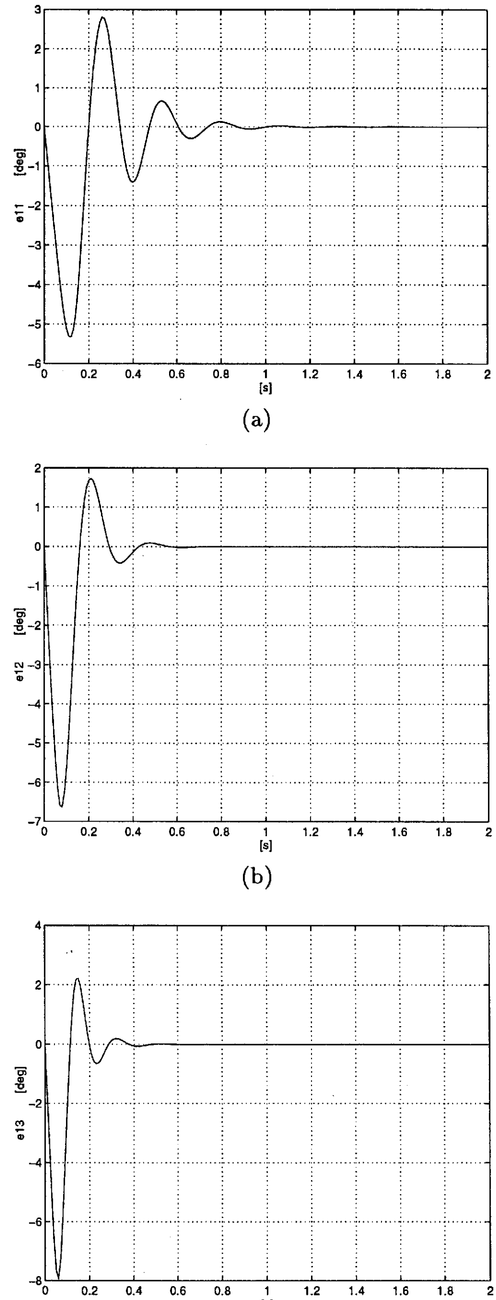
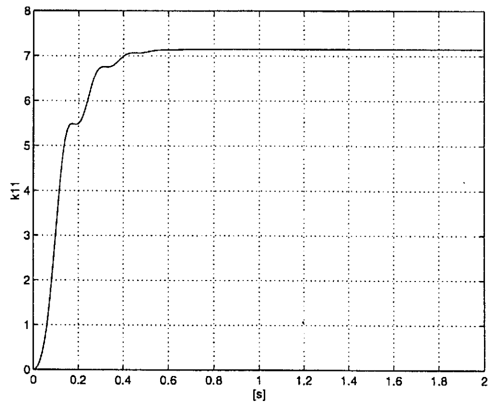
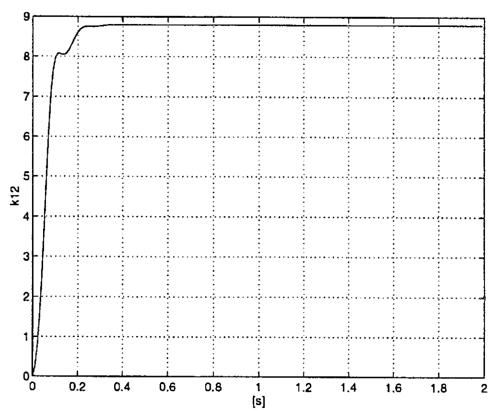


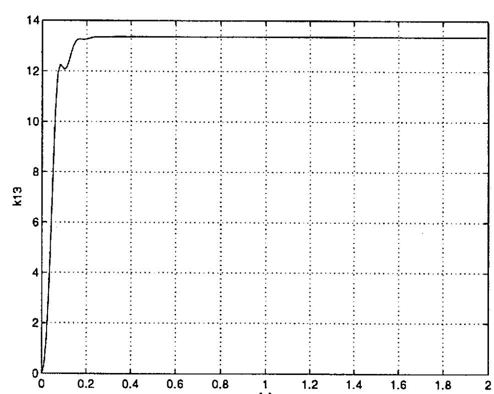
Fig. 8. Tracking errors of the universal adaptive tracking algorithm for the time-varying desired trajectory with $k_i(0) = 0.1$ and with the performance index equal to $j_i = 10^{-3}$.



(a)

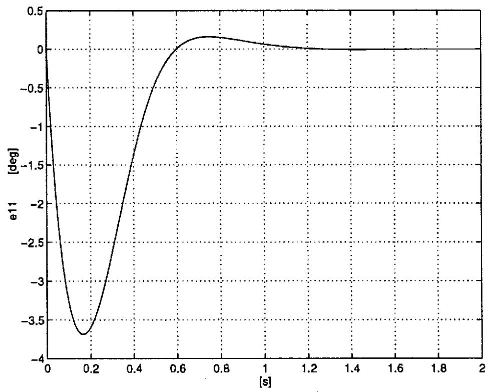


(b)

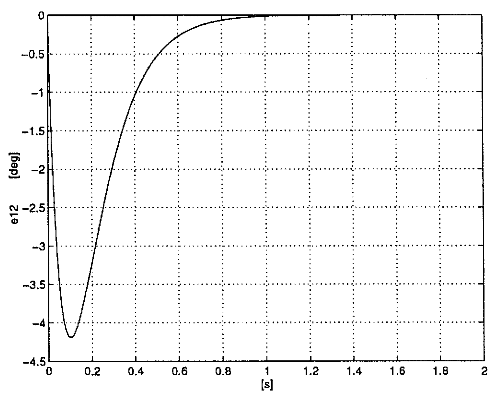


(c)

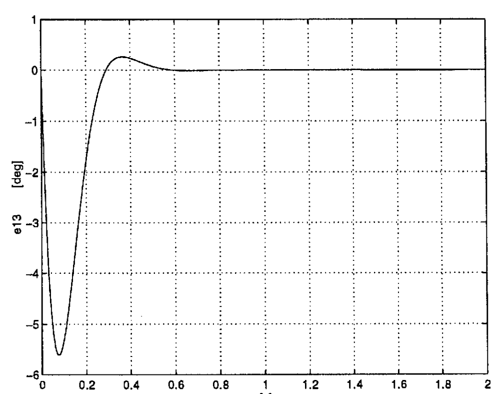
Fig. 9. Gains of the local universal controllers obtained while tracking the time-varying desired trajectory with $j_i = 10^{-3}$ and $k_i(0) = 0.1$.



(a)

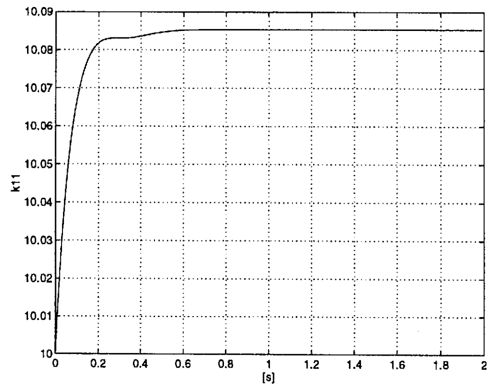


(b)

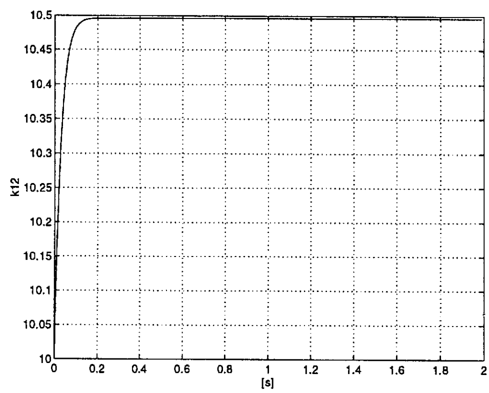


(c)

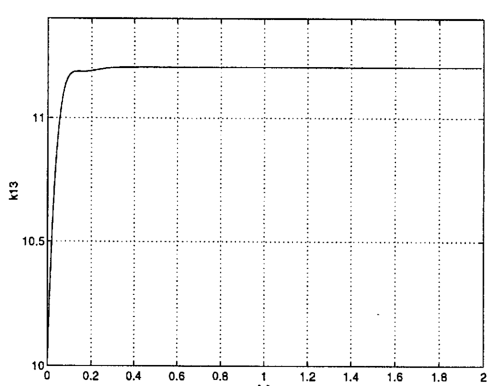
Fig. 10. Tracking errors of the universal adaptive tracking algorithm for the time-varying desired trajectory with $k_i(0) = 10$ and with the performance index equal to $j_i = 10^{-3}$.



(a)



(b)



(c)

Fig. 11. Gains of the local universal controllers obtained while tracking the time-varying desired trajectory with $j_i = 10^{-3}$ and $k_i(0) = 10$.

Minimal gains needed to obtain the prescribed value $j_i = 10^{-3}$ for each joint of the performance index are collected in Table 2. We can observe again that the UATA requires considerably smaller gains (about 30–40 times smaller) to track the time-varying desired trajectory with a better transient response than the algorithm of Wen and Bayard.

5. Concluding Remarks

In this paper, we have shown that in the case of the universal adaptive tracking algorithm the knowledge of the manipulator's model along the desired trajectory is sufficient to ensure the exponential convergence to zero of both the position tracking error $e_1(t)$ and the velocity tracking error $e_2(t)$.

This type of stability is called the semiglobal stability, i.e. it is always possible to adjust controller parameters $k_i(0), P_1, P_2 > 0$, depending on the initial conditions $x(0)$, $\dot{x}(0)$ and the desired trajectory, which enlarge arbitrarily the domain of attraction.

Moreover, we have proved that the universal adaptive controller gains $k_i(t)$ tend to finite values, so they remain bounded during control.

The basic goal of simulations was to compare the UATA with the algorithm given by Wen and Bayard. Both the algorithms share a similar structure, although the latter applies constant gains.

Simulation results have shown that the UATA overperforms the Wen and Bayard algorithm. First of all, the UATA reaches the same values of the performance index as the Wen and Bayard algorithm starting from much smaller gains for both the step and the periodic reference trajectory. The UATA algorithm has also other advantages: it produces a fast transient response and small overshoots in the case of tracking the step trajectory, and negligible oscillations when tracking the time-varying trajectory.

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