Int. J. Appl. Math. Comput. Sci., 2025, Vol. 35, No. 1, 129–142
DOI: 10.61822/amcs-2025-0010

EVENT–TRIGGERED FIXED–TIME RESILIENT CONTROL FOR MOBILE SENSOR NETWORKS WITH A SYBIL ATTACK AND INPUT DELAY

DING ZHOU [a,b], LEI HE [a], ZHIGANG CAO [a], AN ZHANG [b], XIAOPENG HAN [a,*]

[a]Endogenous Security Research Center
Purple Mountain Laboratories
9 Mozhou Road, Nanjing, 211111, China
e-mail: zhoudinghawk@163.com,
{helei,caozhigang}@pmlabs.com.cn,
hanxiaopeng@whu.edu.cn

[b]School of Aeronautics
Northwestern Polytechnical University
127 Youyi Road, Xi'an, 710072, China
e-mail: zhangan@nwpu.edu.cn

This study is devoted to the resilient control problem of a mobile sensor network with a Sybil attack and input delay. First, a fixed-time observer is constructed to estimate the state exactly, which makes it possible to calculate the settling time. Then, the delayed system is transformed into a delay-free system by introducing Artstein's transformation, and a confidence metric is used to tackle the Sybil attack problem, which requires no additional data storage beyond signals. Furthermore, a novel distributed event-triggered fixed-time control scheme is proposed, and a triggering function is developed to generate triggering events asynchronously. Using the presented triggering function, each sensor communicates in discrete time, which is fully continuous-communication free. Several sufficient conditions are obtained, and a rigorous proof is given using Lyapunov stability and fixed-time stability theories. Finally, simulation results are presented to demonstrate the efficiency of the theoretical results such as the flocking context.

Keywords: resilient consensus, event-triggered control, Sybil attack, input delay.

1. Introduction

With past decades due to wide sensing, robustness, mobility, mobile sensor networks are more admired than other fields in the networks. A sensor network consists of mobile sensor nodes where each node conducts computation and communicates with neighbors (Liu *et al.*, 2022). In particular, the application areas of mobile sensor networks are found in surveillance, hazardous environment exploration, health monitoring, natural disaster relief operations (Temene *et al.*, 2022; Kandris *et al.*, 2020). Among the challenging problems of mobile sensor networks, the ability to reach consensus has attracted close attention. Effective consensus coordination between sensor nodes requires trust, making them particularly vulnerable to cyber-attacks. Meanwhile, the consensus can be easily disrupted by the presence of a malicious node that gains an influence on the converged value of the network as a whole.

Numerous theoretical studies on mobile sensor networks in various scenarios have been reported in a wide range of academic publications (Temene *et al.*, 2022; Kandris *et al.*, 2020; Wang *et al.*, 2024). It should be pointed out that the following issues may arise from the aforementioned consensus control schemes. One challenge is that mobile sensors are constrained by resources or costs, i.e., each sensor has limited capabilities in communication, measurement, and computing, necessitating a reduction in the frequency of communication and computation to conserve these resources. Despite these limitations, it is essential for each sensor to accurately estimate its state (Zhou *et al.*, 2020a; Yang *et al.*, 2023). For this issue, event-triggered control

*Corresponding author

can be used to reduce the frequency of the communication and computation, and a state observer can be naturally employed to estimate the state exactly (Yang *et al*., 2021; Li *et al*., 2021; Znidi and Nouri, 2024; Gong *et al*., 2023). For additional resources on event-triggered control and state observers, please refer to some recent works (Chen *et al*., 2020; Jenabzadeh *et al*., 2024; Satish Patil and Senthil Kumaran, 2024).

Another issue is that the dynamic characteristics of the detection environment cause delays in the sensor network. The network must quickly and accurately locate the regions of interest with the highest values in the distribution of quality parameters. For this issue, finite/fixed-time control scheme can be used to guarantee that each sensor quickly adjusts its state to adapt the quality parameters. Several works were devoted to the study of finite-time cooperative control methods for sensor networks (Zhang *et al*., 2024) and fixed-time cases (Liang *et al*., 2024). An essential point to note is that the settling time of a finite-time control scheme depends on the initial states of the sensor networks. Fixed-time stability ensures that the settling time has an upper bound independent of the initial state, which facilitates the design of controllers to meet strict time requirements in practical applications (Zhang *et al*., 2024).

Recently, some efforts have been made on fixed-time event-triggered control for first-order systems and fixed-time state observer (Liu *et al*., 2024). In the aforementioned literature, each sensor calculates the triggering condition through continuous communication, which brings a paradox to the purpose of saving communication resources. Fixed-time state observers of Zhang and Duan (2018) were designed based on a bi-limit homogeneous technique, which has proved that the system is fixed-time stable by constructing approximating homogeneous function, but could not give the convergence time. Considering the time delay, Lyapunov–Krasovskii and Lyapunov–Razumikhin functions are usually applied for system stability analysis. Unfortunately, these methods cannot be used to analyze finite/fixed-time stability of time-delayed systems due to the inability to construct corresponding Lyapunov function (Moulay *et al*., 2008).

The third issue is that mobile sensor networks can be easily hacked because optimal quality parameters require shared data to be accurate and trustworthy (Sheng and Li, 2008). A particularly challenging attack on this premise is the so-called "Sybil attack" (Gil *et al*., 2017). To address this issue, several countermeasures, such as software-based attestation, radio resource testing, and key cryptography, have been proposed to mitigate Sybil attacks (Arshad *et al*., 2021; Vasudeva and Sood, 2018). While these approaches reduce the likelihood of attacks, they often incur higher overhead in terms of computation, communication, and data, making them unsuitable for distributed mobile systems due to their lack of scalability (Yaacoub *et al*., 2022). The special needs of mobile sensor networks are often distributed and dynamic, which makes these methods difficult or impossible to implement. Gil *et al.* (2017) attempted to deal with these constraints and the key difference between their and our previous work is that the solution to defend against a Sybil attack is to use the physics of wireless signals without the need for expensive cryptographic key-distribution.

Motivated by the above discussions, the primary research motivation includes the following three aspects: (i) few results about the event-triggered fixed-time resilient control of mobile sensor networks with Sybil attack and input delay are available now, (ii) the existing works on event-triggered fixed-time control of mobile sensor networks are not completely communication-free. This contradicts the primary goal of event-triggered control schemes, which is to conserve communication resources. Finally, (iii) solutions to existing Sybil attacks in mobile sensor networks typically involve high overhead in terms of computation, communication, and data, making a practical implementation challenging.

In view of the above three issues, this paper addresses resilient control of the mobile sensor network with a Sybil attack and time delay by employing a fixed-time event-triggered control scheme and a confidence metric. The main contributions of this paper can be summarized as follows:

(i) A fixed-time observer is presented to estimate each sensor's state, where the settling time can be estimated by subtly constructing a homogeneous Lyapunov function.

(ii) Artstein's transformation is introduced to analyze finite/fixed-time stability of the time delay system. An event-triggered the fixed-time control protocol is constructed, which can effectively guarantee convergence time. A novel threshold is defined, and the triggering function is derived based on the novel threshold, which does not require continuous communication in both controller update and error measurement.

(iii) By introducing a confidence metric, the weight is developed to tackle the Sybil attack problem of mobile sensor networks, which requires no additional data storage beyond the signal.

The remainder of this paper is organized as follows. In Section 2, we address preliminaries and problem formulation. In Section 3, a fixed-time observer is first provided, and then, a novel fixed-time control algorithm is presented. Section 4 gives a simulation example. The conclusions and future work are provided in Section 5.

2. Preliminaries and problem formulation

2.1. Notation and graph theory. We consider the problem of resilient cooperative control for mobile sensor networks. The network can be described by a weighted state-dependent graph $G = \{V, \zeta, A\}$, where G is an undirected graph $\zeta \subseteq \{(i,j), i, j \in V\}$ is the edge set, $V = \{1, 2, \ldots, n\}$ denotes a finite set of node indices for n sensors. $A = [a_{ij}]_{n\times n}$ is the associated adjacency matrix, where $a_{ii}(t) = 0$, and $a_{ij}(t) > 0$ is the weight if $(j,i) \in \zeta$ or $a_{ij}(t) = 0$, otherwise. The neighbor set of i is defined as $N_i = \{j \in V : a_{ij} > 0\}$. Assume that a subset of nodes with indices denoted by the set Ω, $\Omega \subset V$, is malicious. The out-degree of sensor i is defined as $d_{\text{out}}(i) = d_i = \sum_{j=1, j\neq i}^{n} a_{ij}$, the degree matrix is $D = \operatorname{diag}\{d_1, d_2, \ldots, d_n\}$. Then, the Laplacian matrix L can be expressed by $L = D - A$ and L is symmetric. Define $\operatorname{sig}^{\alpha}(r) = [\operatorname{sig}^{\alpha}(r_1), \ldots, \operatorname{sig}^{\alpha}(r_n)]^T$, $\operatorname{sig}^{\alpha}(r_i) = |r_i|^{\alpha}\operatorname{sgn}(r_i)$, where $\operatorname{sgn}(\cdot)$ is the signum function. I_n denotes the identity matrix. $\|\cdot\|$ denotes the 2-norm, $\otimes$ denotes the Kronecker product, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space.

2.2. Attack model and detecting malicious nodes. The sensor network includes n sensor nodes which are deployed in a region randomly. The sensor is homogeneous (all sensors have unified hardware and software facilities) and each node has a unique identity. Nodes communicate with each other through a WiFi antennas and the radio range of all nodes is the same. Additionally, the sensor network is deployed in an adversary environment; therefore, it might be captured by the adversary. Nodes are not persistently tamper-resistant against interference and if the adversary captures a node, it can access its secret information and reprogram it. One of the first dangerous attacks against mobile sensor networks is the Sybil attack, leading to a further security attack as a black hole and wormhole, as highlighted by Murali and Jamalipour (2020). Therefore, we consider one or more adversarial sensor nodes performing the Sybil attack, where malicious nodes can be mobile.

Definition 1. By the Sybil attack we mean an attack in which malicious nodes can control the value of one or more spoofed nodes in the mobile sensor network by sending false information with unique IDs ($i \in \Omega$) to gain a disproportionate influence in the network. The set V is known but knowledge of which sensors are malicious is not available. If a sensor is a malicious node such that $i \in \Omega$, then its moving position and velocity are denoted by $r_i(t), v_i(t)$, respectively.

In order to measure directional signal profiles, a method is developed by utilizing channel state information of wireless information over each wireless link (Gil *et al.*, 2015; 2017). These profiles measure the signal strength arriving from every direction and signal profiles display two important properties: (i) transmissions originating from the same physical node have very similar profiles and (ii) energy can be measured coming from the direct-line path between physical nodes. Gil *et al.* (2017) quantify two properties, deriving a confidence metric $\sigma_i\{i \in V\} \in (0,1)$ that approaches 1 for legitimate sensors and 0 otherwise.

Lemma 1. (Gil *et al.*, 2017) *Let $V = \Omega \cup \tilde{\Omega}$ be a set of nodes where Ω is the set of malicious node and $\tilde{\Omega}$ is the set of legitimate nodes. The identities of the nodes being malicious is unknown. Let σ_{mali} and σ_{legit} be the confidence metrics of malicious, legitimate node, respectively. The confidence metrics $\sigma_{\text{mali}}, \sigma_{\text{legit}} \in (0,1)$ are bounded by $E[\sigma_{\text{mali}}] \leq \varepsilon_{\text{mali}}$, for each node in Ω and $E[\sigma_{\text{legit}}] \geq 1-\varepsilon_{\text{legit}}$, for each node in Ω, where $\varepsilon_{\text{mali}}$ and $\varepsilon_{\text{legit}}$ are determined by the signal-to-noise ratio (SNR) of the channel, the number of malicious nodes, and channel constants.*

Experimental evaluation of confidence metrics σ_i shows that a threshold of $\sigma_i < 0.5$ performs well to classify nodes as malicious.

2.3. Problem formulation. Consider a continuous time model of n identical sensor nodes in an adversary environment. Meanwhile, there is a time delay for each sensor to the receive the neighbors' information and process after the receipt (Ni *et al.*, 2017; Zhang *et al.*, 2019). The dynamics of the i-th sensor can be described by

$$\begin{cases} \dot{r}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t - \tau_i), \end{cases} \tag{1}$$

where $i \in \{1, 2, \ldots, n\}$ is the ID of sensor node; $r_i(t) \in X_r \subset \mathbb{R}^m, v_i(t) \in X_v \subset \mathbb{R}^m$ denote the position and velocity, respectively; X_r and X_v stands for admissible position set and velocity set, respectively; $u_i \in \mathbb{R}^m$ refers to the control input, and τ_i denotes the input delay. For many practical cases, the whole state information of the sensor is hard to obtain due to the physical or economical restriction. Therefore, the observer design for a networked sensor system is necessary.

The fixed-time resilient consensus is achieved if there exists a fixed-time T such that

$$\lim_{t\to T} \|r_i(t) - r_j(t)\| = 0,$$

$$\lim_{t\to T} \|v_i(t) - v_j(t)\| = 0$$

and

$$\|r_i(t) - r_j(t)\| = 0,$$

$$\|v_i(t) - v_j(t)\| = 0$$

when $t \geq T$, $i, j = 1, 2, \ldots, n$. The settling time T is fixed and bounded, i.e., for any initial states, there is a $T_{\max} > 0$ such that $T \leq T_{\max}$.

We expect to derive weights with the properties that the influence of legitimate sensors on neighbors approaches 1, while the influence of malicious sensors on neighbors approaches 0. Note that this paper focuses on the resilient control of sensor networks suffering from Sybil attacks, rather than the optimization of the network topology. Therefore, we adopt the following reasonable assumption (Gil *et al.*, 2017; 2019).

Assumption 1. The undirected graph is sufficiently connected such that it would remain connected even if malicious sensors were removed. Meanwhile, the wireless channel weights a_{ij} are independent for each link (j, i).

2.4. Some useful definition and lemmas. Consider the following differential equation:

$$\dot{x} = f(t, x), \quad x \in \mathbb{R}^n, \tag{2}$$

where $f(t, x) : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function which may be discontinuous, the solutions of (2) are understood in the sense of Filippov (2013). Suppose that the origin is an equilibrium point of system (2).

Lemma 2. (Polyakov, 2011) *Assume that there exists a continuous radially unbounded function* $V : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ *such that* $V(x) = 0 \Rightarrow x = 0$, $\dot{V}(x(t)) \leq -(\alpha V^p(x(t)) + \beta V^q(x(t)))^k$ *for some* $\alpha, \beta, p, q, k > 0$, $pk < 1$, $qk > 1$ *and for any solution* $x(t)$*; Then the origin of system (2) is globally fixed-time stable with the settling time*

$$T \leq \frac{1}{\alpha^k(1 - pk)} + \frac{1}{\beta^k(qk - 1)}.$$

If $k = 1$*, the origin of system (2) is globally fixed-time stable with settling time* T *bounded by*

$$T \leq T_{\max} := \frac{1}{\alpha(1 - p)} + \frac{1}{\beta(q - 1)},$$

where $\alpha, \beta > 0$, $0 < p < 1$ *and* $q > 1$.

Lemma 3. (Zhou *et al.*, 2020b) *If the undirected graph is connected, then the Laplacian matrix* L *is symmetric. If* $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ *are the eigenvalues of* L*, then* $\lambda_1 = 0$ *and* $\lambda_2 > 0$*. Denote by* a *the algebraic connectivity. If* $\mathbf{1}^T x = 0, x \neq 0$*, then*

$$a = \lambda_2 = \min \frac{x^T L x}{x^T x}$$

and

$$x^T L^2 x \geq a x^T L x.$$

Lemma 4. (Zhou *et al.*, 2020b) *For any nonnegative real numbers* $x_1, x_2, \ldots, x_n$*, the following inequalities hold:*

$$\left(\sum_{i=1}^n x_i\right)^p \leq \sum_{i=1}^n x_i^p \leq n^{1-p}\left(\sum_{i=1}^n x_i\right)^p,$$

where $p \in (0, 1]$*, and*

$$\left(\sum_{i=1}^n x_i\right)^q \geq \sum_{i=1}^n x_i^q \geq n^{1-q}\left(\sum_{i=1}^n x_i\right)^q,$$

where $q \geq 1$.

3. Main results

In this section, we construct (i) a fixed-time observer to estimate information exactly, (ii) the weights of the network, and (iii) a distributed event-triggered control protocol such that a consensus of the sensor network can be achieved.

3.1. Design of a fixed-time observer. For the considered sensor network (1), a fixed-time observer is constructed to accurately estimate information, which is specified as

$$\begin{cases} \dot{\varsigma}_i = \kappa_i - c_1 \mathrm{sig}^{\alpha_1}(\varsigma_i - r_i) \\ \qquad - c_2 \mathrm{sig}^{\alpha_2}(\varsigma_i - r_i), \\ \dot{\kappa}_i = u_i(t - \tau_i) - d_1 \mathrm{sig}^{\tilde{\alpha}_1}(\varsigma_i - r_i) \\ \qquad - d_2 \mathrm{sig}^{\tilde{\alpha}_2}(\varsigma_i - r_i), \end{cases} \tag{3}$$

where ς_i, κ_i are the estimates of r_i, v_i, respectively, c_1, c_2, d_1, d_2 are positive constants such that $\alpha_1 \in (0.5, 1)$, $\alpha_2 \in (1.5, 2)$, $\tilde{\alpha}_1 = 2\alpha_1 - 1$, $\tilde{\alpha}_2 = \alpha_2 + \alpha_1 - 1$. $\alpha_1 \in (0.5, 1)$, $\alpha_2 \in (1.5, 2)$ ensure that $\tilde{\alpha}_1 \in (0, 1)$, $\tilde{\alpha}_2 \in (1, 2)$, which further guarantees that the observer (3) is fixed time stable. Meanwhile, $\alpha_1 \in (0.5, 1)$, $\alpha_2 \in (1.5, 2)$ ensure that $0 < \rho_1 < 1, 1 < \rho_2 < 2$, for additional stable time estimation with homogeneous theory.

Theorem 1. *Consider the sensor network (1) suffering from a Sybil attack. The observer (3) can estimate the information in fixed time with the settling time bounded by*

$$T_1 \leq \frac{1}{d_{i1}(1 - \rho_1)} + \frac{1}{d_{i2}(\rho_2 - 1)},$$

where

$$d_{i1} = -\max_{\{x_i : V_{i1}(x_i) = 1\}} L_f V_{i1}(x_i),$$

$$d_{i2} = -\max_{\{x_i : V_{i1}(x_i) = 1\}} L_g V_{i1}(x_i),$$

$$\rho_1 = \alpha_1^2 - \frac{3}{2}\alpha_1 + \frac{3}{2} \in \left[\frac{15}{16}, 1\right),$$

$$\rho_2 = 1 + (2\alpha_1 - 1)(\alpha_2 - 1) \in (1, 2),$$

$V_{i1}(t)$ denotes the Lyapunov function to be designed later, $L_f V_{i1}(x_i)$, $L_g V_{i1}(x_i)$ are Lie derivatives.

Proof. Define the estimate errors $\eta_{ri}(t) = \varsigma_i(t) - r_i(t)$ and $\eta_{vi}(t) = \kappa_i(t) - v_i(t)$, Then we can obtain that

$$\begin{cases} \dot{\eta}_{ri} = \eta_{vi} - c_1 \mathrm{sig}^{\alpha_1}(\eta_{ri}) - c_2 \mathrm{sig}^{\alpha_2}(\eta_{ri}), \\ \dot{\eta}_{vi} = -d_1 \mathrm{sig}^{\tilde{\alpha}_1}(\eta_{ri}) - d_2 \mathrm{sig}^{\tilde{\alpha}_2}(\eta_{ri}). \end{cases} \tag{4}$$

Consider the Lyapunov function candidate as

$$V_{i1}(t) = x_i^T \left(\begin{pmatrix} \chi + 4\varepsilon^2 & -2\varepsilon \\ -2\varepsilon & 1 \end{pmatrix} \otimes I_m \right) x_i, \tag{5}$$

where

$$x_i = \left[\left(\mathrm{sig}^{1/\tilde{\alpha}_1}(\eta_{ri})\right)^T, \left(\mathrm{sig}^{1/\alpha_1\tilde{\alpha}_1}(\eta_{vi})\right)^T \right]^T,$$

$\chi, \varepsilon > 0$. It follows that $V_{i1}(t) \geq 0$ and $V_{i1}(t) = 0$ if and only if $x_i = 0_{2m}$.

Consider the system

$$\begin{cases} \dot{\eta}_{ri} = \eta_{vi} - c_1 \mathrm{sig}^{\alpha_1}(\eta_{ri}), \\ \dot{\eta}_{vi} = -d_1 \mathrm{sig}^{\tilde{\alpha}_1}(\eta_{ri}), \end{cases} \tag{6}$$

with the vector field f. From Definition 3 (Hong *et al.*, 2002), (6) is homogeneous of degree $\alpha_1 - 1$ with respect to $(1, \dots, 1, \alpha_1, \dots, \alpha_1)$. By Bhat and Bernstein (2005), $V_{i1}(t)$ is homogeneous of degree $\frac{2}{2\alpha_1 - 1}$ with respect to $(1, \dots, 1, \alpha_1, \dots, \alpha_1)$. Taking the Lie derivative of $V_{i1}(t)$ along with f yields

$$\begin{aligned} L_f V_{i1} =& \frac{2\left(\chi + 4\varepsilon^2\right)}{\tilde{\alpha}_1} \sum_{j=1}^{m} \eta_{vij} sig^{\frac{2}{\tilde{\alpha}_1} - 1}(\eta_{rij}) \\ &- \frac{4\varepsilon}{\tilde{\alpha}_1} \sum_{j=1}^{m} |\eta_{vij}|^{\frac{1}{\alpha_1\tilde{\alpha}_1} + 1} |\eta_{rij}|^{\frac{1}{\tilde{\alpha}_1} - 1} \\ &- \frac{2c_1\left(\chi + 4\varepsilon^2\right)}{\tilde{\alpha}_1} \sum_{j=1}^{m} |\eta_{rij}|^{\frac{2}{\tilde{\alpha}_1} - 1 + \alpha_1} \\ &+ \frac{4\varepsilon c_1}{\tilde{\alpha}_1} \sum_{j=1}^{m} \mathrm{sig}^{\frac{1}{\tilde{\alpha}_1} - 1 + \alpha_1}(\eta_{rij}) \, \mathrm{sig}^{\frac{1}{\alpha_1\tilde{\alpha}_1}}(\eta_{vij}) \\ &+ \frac{4\varepsilon d_1}{\alpha_1\tilde{\alpha}_1} \sum_{j=1}^{m} |\eta_{rij}|^{\frac{1}{\tilde{\alpha}_1} + \tilde{\alpha}_1} |\eta_{vij}|^{\frac{1}{\alpha_1\tilde{\alpha}_1} - 1} \\ &- \frac{2d_1}{\alpha_1\tilde{\alpha}_1} \sum_{j=1}^{m} \mathrm{sig}^{\tilde{\alpha}_1}(\eta_{rij}) \, \mathrm{sig}^{\frac{2}{\alpha_1\tilde{\alpha}_1} - 1}(\eta_{vij}). \end{aligned} \tag{7}$$

It can be easily obtained that $L_f V_{i1}$ is homogeneous of degree $\frac{2}{2\alpha_1 - 1} + \alpha_1 - 1$ with respect to $(1, \dots, 1, \alpha_1, \dots, \alpha_1)$. Noting that $\frac{2}{2\alpha_1 - 1} > 1 > -(\alpha_1 - 1)$ and the degree of $L_f V_{i1}$ is equal to the degree of $V_{i1}(t)$ plus the degree of system (6), it follows from Theorem 6.2 of Bhat and Bernstein (2005) that the Lie derivative $L_f V_{i1}$ is continuous and negative definite. Then, it follows from Theorem 7.1 of Bhat and Bernstein (2005) that

$$L_f V_{i1} \leq -d_{i1}(V_{i1}(t))^{\rho_1},$$

where

$$\rho_1 = \alpha_1^2 - \frac{3}{2}\alpha_1 + \frac{3}{2} \in \left[\frac{15}{16}, 1\right),$$

$$d_{i1} = -\max_{\{x_i : V_{i1}(x_i) = 1\}} L_f V_{i1}(x_i).$$

Obviously, d_{i1} is positive since $L_f V_{i1}$ is negative definite.

Then, we consider the following system:

$$\begin{cases} \dot{\eta}_{ri} = -c_2 \mathrm{sig}^{\alpha_2}(\eta_{ri}), \\ \dot{\eta}_{vi} = -d_2 \mathrm{sig}^{\tilde{\alpha}_2}(\eta_{ri}), \end{cases} \tag{8}$$

with the vector field g. Similarly, we obtain that (8) is homogeneous of degree $\alpha_2 - 1$ with respect to $(1, \dots, 1, \alpha_1, \dots, \alpha_1)$. Similarly, taking Lie derivative of $V_{i1}(t)$ along with g yields

$$\begin{aligned} L_g V_{i1} =& \frac{4c_2\varepsilon}{\tilde{\alpha}_1} \sum_{j=1}^{m} \mathrm{sig}^{\frac{1}{\tilde{\alpha}_1} - 1 + \alpha_2}(\eta_{rij}) \, \mathrm{sig}^{\frac{1}{\alpha_1\tilde{\alpha}_1}}(\eta_{vij}) \\ &- \frac{2c_2\left(\chi + 4\varepsilon^2\right)}{\tilde{\alpha}_1} \sum_{j=1}^{m} |\eta_{rij}|^{\frac{2}{\tilde{\alpha}_1} - 1 + \alpha_2} \\ &+ \frac{4\varepsilon d_2}{\alpha_1\tilde{\alpha}_1} \sum_{j=1}^{m} |\eta_{rij}|^{\frac{1}{\tilde{\alpha}_1} + \tilde{\alpha}_2} |\eta_{vij}|^{\frac{1}{\alpha_1\tilde{\alpha}_1} - 1} \\ &- \frac{2d_2}{\alpha_1\tilde{\alpha}_1} \sum_{j=1}^{m} \mathrm{sig}^{\tilde{\alpha}_2}(\eta_{rij}) \, \mathrm{sig}^{\frac{2}{\alpha_1\tilde{\alpha}_1} - 1}(\eta_{vij}). \end{aligned} \tag{9}$$

Then, $L_g V_{i1}$ is homogeneous of degree $\frac{2}{2\alpha_1 - 1} + \alpha_2 - 1$ with respect to $(1, \dots, 1,\ \alpha_1, \dots, \alpha)$. Noting that $\frac{2}{2\alpha_1 - 1} > 1 > -(\alpha_2 - 1)$ and the degree of $L_g V_{i1}$ is equal to the degree of $V_{i1}(t)$ plus the degree of system (8), it follows from Theorem 6.2 of Bhat and Bernstein (2005) that the Lie derivative $L_g V_{i1}$ is continuous and negative definite. Then we have

$$L_g V_{i1} \leq -d_{i2}(V_{i1}(t))^{\rho_2},$$

where

$$\rho_2 = 1 + (2\alpha_1 - 1)(\alpha_2 - 1) \in (1, 2),$$

$$d_{i2} = -\max_{\{x_i : V_{i1}(x_i) = 1\}} L_g V_{i1}(x_i).$$

Obviously, d_{i2} are positive due to $L_g V_{i1}$ is negative definite.

Consider

$$x_i = \left[\left(\mathrm{sig}^{1/\tilde{\alpha}_1}(\eta_{ri})\right)^T, \left(\mathrm{sig}^{1/\alpha_1\tilde{\alpha}_1}(\eta_{vi})\right)^T \right]^T$$

with the system (4). Taking the derivative of x_i with respect to t yields

$$\begin{aligned}&\dot{x}_i\\&=\Big(\frac{1}{\tilde{\alpha}_1}\dot{\eta}_{ri1}|\eta_{ri1}|^{\frac{1}{\tilde{\alpha}_1}-1},\cdots,\frac{1}{\tilde{\alpha}_1}\dot{\eta}_{rim}|\eta_{rim}|^{\frac{1}{\tilde{\alpha}_1}-1},\\&\quad\frac{1}{\alpha_1\tilde{\alpha}_1}\dot{\eta}_{vi1}|\eta_{vi1}|^{\frac{1}{\alpha_1\tilde{\alpha}_1}-1},\cdots,\\&\quad\frac{1}{\alpha_1\tilde{\alpha}_1}\dot{\eta}_{vim}|\eta_{vim}|^{\frac{1}{\alpha_1\tilde{\alpha}_1}-1}\Big)^T,\end{aligned}$$

where

$$\begin{aligned}\dot{\eta}_{rij}&=\eta_{vij}-c_1\mathrm{sig}^{\alpha_1}(\eta_{rij})-c_2\mathrm{sig}^{\alpha_2}(\eta_{rij}),\\\dot{\eta}_{vij}&=-d_1\mathrm{sig}^{\tilde{\alpha}_1}(\eta_{rij})-d_2\mathrm{sig}^{\tilde{\alpha}_2}(\eta_{rij}),\end{aligned}$$

$j=1,2,\ldots,m$. Taking the derivative of $V_{i1}(t)$ with respect to time, we have

$$\begin{aligned}\dot{V}_{i1}(t)&=2x_i^T\left(\left(\begin{array}{cc}\chi+4\varepsilon^2&-2\varepsilon\\-2\varepsilon&1\end{array}\right)\otimes I_m\right)\dot{x}_i\\&=L_fV_{i1}(x_i)+L_gV_{i1}(x_i)\\&\le -d_{i1}(V_{i1}(t))^{\rho_1}-d_{i2}(V_{i1}(t))^{\rho_2}.\end{aligned}\tag{10}$$

Thus, based on Lemma 2, the error system (4) is fixed-time stable and the settling time is bounded by $T_1\le\frac{1}{d_{i1}(1-\rho_1)}+\frac{1}{d_{i2}(\rho_2-1)}$. Then, one has $\varsigma_i(t)=r_i(t),\kappa_i(t)=v_i(t)$ when $t>T_1$. We can conclude that the fixed-time observer (3) can estimate the sensors' information within a bounded settling time. This completes the proof. ∎

Remark 1. The design of the fixed-time observer is similar to the finite-time observers provided by Fu and Yu (2018) as well as Hua *et al.* (2017). This observer can estimate the velocity within the settling time T_1 regardless of the initial states, while previous results only used bi-limit homogeneity to prove that the observer is fixed-time stable, but did not give the settling time (Huang and Jia, 2018; Tian *et al.*, 2017; Zhang and Duan, 2018). For many practical cases, the convergence time is required, and the observer (3) can easily satisfy the settling-time requirement while the previous observers (Huang and Jia, 2018; Tian *et al.*, 2017; Zhang and Duan, 2018) cannot give time indicators.

3.2. Event-triggered fixed-time control scheme.

To tackle the time delay, we introduce Artstein's transformation, which takes advantage of the invertible transformation to transform the delayed system (1) into a delay-free system.

$$\begin{cases}e_{ri}=r_i+\int_{-\tau_i}^{0}(-\tau_i-s)\,u_i(t+s)\,\mathrm{d}s\\e_{vi}=v_i+\int_{-\tau_i}^{0}u_i(t+s)\,\mathrm{d}s\end{cases}\tag{11}$$

After Artstein's transformation, we have $\dot{e}_{ri}=v_i-\tau_iu_i(t),\dot{e}_{vi}=u_i(t)$. Defining new variables $e_{\varsigma i}=e_{ri}+\tau_ie_{vi},e_{\kappa i}=e_{vi}$, we have the delay-free system. Meanwhile, introducing the state observer (3), we can obtain that $\varsigma_i=r_i,\kappa_i=v_i$ within T_1. To mitigate the unnecessary communication, a distributed event-triggered control algorithm is proposed as

$$u_i(t)=-\psi_i(t)-\mathrm{sig}(\psi_i(t))^p-\mathrm{sig}(\psi_i(t))^q,\tag{12}$$

where

$$\begin{aligned}\psi_i(t)=\beta\sum_{j=1}^{n}a_{ij}\left(e_{\varsigma i}\left(t_{k_i}^i\right)-e_{\varsigma j}\left(t_{k_j}^j\right)\right)\\+\gamma\sum_{j=1}^{n}a_{ij}\left(e_{\kappa i}\left(t_{k_i}^i\right)-e_{\kappa j}\left(t_{k_j}^j\right)\right),\end{aligned}$$

$\beta,\gamma>0$, $p\in(0,1)$, $q\in(1,2)$, $t\in\left[t_{k_i}^i,t_{k_i+1}^i\right)$, $t_{k_i}^i$ is the latest event-triggered time of sensor i, $k_j\triangleq\arg\min_s\left\{t-t_s^j\,\middle|\,t\ge t_s^j,s\in N\right\}$, i.e., $t_{k_j}^j$ is the latest event-triggered time of sensor j.

Remark 2. The proposed control algorithm (12) is derived from the control algorithms presented by Huang and Jia (2018), Tian *et al.* (2017) or Zhang and Duan (2018). The control algorithms in these references are specifically designed to apply bi-limit homogeneous systems theory. While bi-limit homogeneity can demonstrate that sensor networks are fixed-time stable, it does not provide the settling time, making it unsuitable for scenarios with strict convergence time requirements. To address this limitation, we enhance the control algorithms by Huang and Jia (2018), Tian *et al.* (2017) as well as Zhang and Duan (2018) by incorporating an event-triggered control strategy. Based on the proposed control algorithm, we skillfully construct a Lyapunov function and derive the settling time.

During the time interval $\left[t_{k_i}^i,t_{k_i+1}^i\right)$, define

$$\begin{aligned}\varphi_{\varsigma i}(t)&=e_{\varsigma i}\left(t_{k_i}^i\right)-e_{\varsigma i}(t),\\\varphi_{\kappa i}(t)&=e_{\kappa i}\left(t_{k_i}^i\right)-e_{\kappa i}(t),\\\varphi_i^{\varsigma}&=\sum_{j=1}^{n}a_{ij}\left(\varphi_{\varsigma i}-\varphi_{\varsigma j}\right),\\\varphi_i^{\kappa}&=\sum_{j=1}^{n}a_{ij}\left(\varphi_{\kappa i}-\varphi_{\kappa j}\right),\\y_i(t)&=\sum_{j=1}^{n}a_{ij}\left(e_{\varsigma i}(t)-e_{\varsigma j}(t)\right),\\w_i(t)&=\sum_{j=1}^{n}a_{ij}\left(e_{\kappa i}(t)-e_{\kappa j}(t)\right),\end{aligned}$$

$$y_i\left(t_{k_i}^i\right)=\sum_{j=1}^{n} a_{ij}\left(e_{\varsigma i}\left(t_{k_i}^i\right)-e_{\varsigma j}\left(t_{k_j}^j\right)\right),$$
$$w_i\left(t_{k_i}^i\right)=\sum_{j=1}^{n} a_{ij}\left(e_{\kappa i}\left(t_{k_i}^i\right)-e_{\kappa j}\left(t_{k_j}^j\right)\right).$$

Then we have

$$\begin{aligned}\psi_i(t)&=\beta y_i\left(t_{k_i}^i\right)+\gamma w_i\left(t_{k_i}^i\right)\\&=\beta y_i(t)+\gamma w_i(t)+\beta\varphi_i^{\varsigma}+\gamma\varphi_i^{\kappa}.\end{aligned}$$

Define

$$\begin{aligned}\varphi_{\varsigma}&=\left[\varphi_{\varsigma 1}^T,\dots,\varphi_{\varsigma n}^T\right]^T,\\\varphi_{\kappa}&=\left[\varphi_{\kappa 1}^T,\dots,\varphi_{\kappa n}^T\right]^T,\\\varphi^{\varsigma}&=\left[\varphi_1^{\varsigma T},\dots,\varphi_n^{\varsigma T}\right]^T,\\\varphi^{\kappa}&=\left[\varphi_1^{\kappa T},\dots,\varphi_n^{\kappa T}\right]^T,\\y&=\left[y_1^T,\dots,y_n^T\right]^T,\\w&=\left[w_1^T,\cdots,w_n^T\right]^T.\end{aligned}$$

We have $\varphi^{\varsigma}=(L\otimes I_m)\varphi_{\varsigma}$, $\varphi^{\kappa}=(L\otimes I_m)\varphi_{\kappa}$, $y=(L\otimes I_m)e_{\varsigma}$, $w=(L\otimes I_m)e_{\kappa}$.

A novel distributed triggering function is specified as

$$\begin{aligned}g_i(t)=&-\frac{\delta}{\sum_{j=1}^{n}a_{ij}}\left\|\beta y_i\left(t_{k_i}^i\right)+\gamma w_i\left(t_{k_i}^i\right)\right\|\\&+2\left\|\beta\varphi_{\varsigma i}+\gamma\varphi_{\kappa i}\right\|,\end{aligned}\tag{13}$$

where δ is a positive constant to be determined. The triggering sequence is defined iteratively as $t_{k_i+1}^i=\inf\left\{t>t_{k_i}^i,g_i(t)>0\right\}$.

Remark 3. In order to connect the Lyapunov function with its derivative, we must establish the connection between $\varphi_{\varsigma i},\varphi_{\kappa i}$ and y_i,w_i. First, the triggering function is designed as $g_i(t)=\|L\|\left\|\beta\varphi_{\varsigma i}+\gamma\varphi_{\kappa i}\right\|-\delta\left\|\beta y_i(t)+\gamma w_i(t)\right\|$, which necessitates ongoing communication to acquire neighbor information. To address this issue, we propose a new threshold $\delta\left\|\beta y_i\left(t_{k_i}^i\right)+\gamma w_i\left(t_{k_i}^i\right)\right\|$ and define the triggering function as $g_i(t)=\|L\|\left\|\beta\varphi_{\varsigma i}+\gamma\varphi_{\kappa i}\right\|-\delta\left\|\beta y_i\left(t_{k_i}^i\right)+\gamma w_i\left(t_{k_i}^i\right)\right\|$. In order to achieve distribution of the event-triggered control scheme, we introduce a novel triggering function (13).

To achieve resilient control of the mobile sensor network suffering from a Sybil attack, the weights are set according to confidence metrics σ_i and Lemma 1.

$$a_{ij}=\begin{cases}\frac{1}{\left(1+e^{-50(\sigma_i-0.5)}\right)\left(1+e^{-50(\sigma_j-0.5)}\right)}, & j\in N_i, i\neq j,\\0, & i=j.\end{cases}\tag{14}$$

By introducing the confidence metric σ_i, the influence of malicious nodes on neighbors approaches zero.

Theorem 2. *Consider the sensor network (1) suffering from a Sybil attack. Assume that the graph for the n sensors is sufficiently connected. With the fixed-time observer (3) and the proposed weight (14), the distributed event-triggered consensus algorithm (12) and triggering function (13) can make the sensor network (1) fixed-time stable if the parameters are selected as*

$$\begin{aligned}&a>\frac{\beta}{\gamma^2},\\&\delta<\min\left\{\frac{a\gamma^2-\beta}{3a\gamma^2-\beta},\sqrt{(mn)^{p-1}},\sqrt{(mn)^{1-q}}\right\}.\end{aligned}$$

Furthermore, the settling time is bounded by

$$\begin{aligned}&\frac{1}{d_{i1}(1-\rho_1)}+\frac{1}{d_{i2}(\rho_2-1)}\\&\quad+\frac{2}{\left(1-\delta(mn)^{\frac{1-p}{2}}\right)\left(a\theta\sqrt{\xi/\lambda}\right)^{p+1}(1-p)}\\&\quad+\frac{2}{\left((mn)^{\frac{1-q}{2}}-\delta\right)\left(a\theta\sqrt{\xi/\lambda}\right)^{q+1}(q-1)}\\&\quad+\max\left\{\tau_i\right\},\end{aligned}$$

where a is the algebraic connectivity.

Proof. The proof is divided into three steps: (i) constructing a relationship between error and state according to the triggering function; (ii) proving that the sensor network can achieve asymptotic stability and (iii) proving that fixed-time stability can be achieved by establishing a relationship between the Lyapunov function and its derivative.

From Theorem 1, the observer (3) can estimate the sensor state within T_1. Then we have

$$\dot{\varsigma}_i(t)=\kappa_i(t),$$
$$\dot{\kappa}_i(t)=u_i(t).$$

Define

$$\tilde{e}_{\varsigma i}=e_{\varsigma i}-\bar{e}_{\varsigma}=e_{\varsigma i}-\frac{1}{n}\sum_{j=1}^{n}e_{\varsigma j}$$

and

$$\tilde{e}_{\kappa i}=e_{\kappa i}-\bar{e}_{\kappa}=e_{\kappa i}-\frac{1}{n}\sum_{j=1}^{n}e_{\kappa j}.$$

Let

$$\tilde{e}_{\varsigma}=\left[\tilde{e}_{\varsigma 1}^T,\dots,\tilde{e}_{\varsigma n}^T\right]^T,$$
$$\tilde{e}_{\kappa}=\left[\tilde{e}_{\kappa 1}^T,\dots,\tilde{e}_{\kappa n}^T\right]^T.$$

Then

$$\tilde{e}_\varsigma = \left[\left(I_n - \frac{1}{n}\mathbf{1}_{n\times n}\right)\otimes I_m\right] e_\varsigma,$$

$$\tilde{e}_\kappa = \left[\left(I_n - \frac{1}{n}\mathbf{1}_{n\times n}\right)\otimes I_m\right] e_\kappa.$$

Further

$$y = (L\otimes I_m)\, e_\varsigma = (L\otimes I_m)\,\tilde{e}_\varsigma,$$

$$w = (L\otimes I_m)\, e_\kappa = (L\otimes I_m)\,\tilde{e}_\kappa.$$

Consider the Lyapunov function $V_2(t)$

$$V_2 = \frac{1}{2}\begin{pmatrix}\tilde{e}_\varsigma\\ \tilde{e}_\kappa\end{pmatrix}^T \left(\begin{pmatrix} 2\beta\gamma L^2 & \beta L\\ \beta L & \gamma L\end{pmatrix}\otimes I_m\right)\begin{pmatrix}\tilde{e}_\varsigma\\ \tilde{e}_\kappa\end{pmatrix}. \tag{15}$$

With $a > \beta/\gamma^2$, one has $V_2(t) \geq 0$ and $V_2(t) = 0$ if and only if $\|\tilde{\varsigma}\| = 0$, $\|\tilde{\kappa}\| = 0$. The derivative of $V_2(t)$ with respect to time is

$$\begin{aligned}&\dot{V}_2(t)\\ &\leq -\beta^2 y^T y - \left(\gamma^2 - \frac{\beta}{a}\right) w^T w\\ &- \left(\beta^2 y^T\varphi^\varsigma + \beta\gamma w^T\varphi^\varsigma + \beta\gamma y^T\varphi^\kappa + \gamma^2 w^T\varphi^\kappa\right)\\ &- (\beta y + \gamma w)^T \mathrm{sig}(\beta y + \gamma w + \beta\varphi^\varsigma + \gamma\varphi^\kappa)^p\\ &- (\beta y + \gamma w)^T \mathrm{sig}(\beta y + \gamma w + \beta\varphi^\varsigma + \gamma\varphi^\kappa)^q.\end{aligned} \tag{16}$$

The Laplacian matrix L is symmetric and

$$\sum_{j=1}^n |l_{ij}| = 2\sum_{j=1}^n a_{ij}.$$

Using Lemma 3 and the triggering function (13), we get

$$\begin{aligned}&\|\beta\varphi^\varsigma(t) + \gamma\varphi^\kappa(t)\|\\ &= \sqrt{\sum_{i=1}^n \left\|\sum_{j=1}^n l_{ij}\left(\beta\varphi_{\varsigma j}(t) + \gamma\varphi_{\kappa j}(t)\right)\right\|^2}\\ &\leq \sqrt{\sum_{i=1}^n\left(\sum_{j=1}^n |l_{ij}|^2\right)\sum_{j=1}^n \|\beta\varphi_{\varsigma j}(t) + \gamma\varphi_{\kappa j}(t)\|^2}\\ &= \sqrt{\sum_{i=1}^n\left(\sum_{j=1}^n |l_{ji}|^2\right)\|\beta\varphi_{\varsigma i}(t) + \gamma\varphi_{\kappa i}(t)\|^2}\\ &\leq \sqrt{\sum_{i=1}^n\left(2\sum_{j=1}^n a_{ij}\right)^2\|\beta\varphi_{\varsigma i}(t) + \gamma\varphi_{\kappa i}(t)\|^2}\\ &\leq \sqrt{\delta^2\sum_{i=1}^n \left\|\beta y_i(t^i_{k_i}) + \gamma w_i(t^i_{k_i})\right\|^2}\\ &= \delta\,\|\psi(t)\|,\end{aligned} \tag{17}$$

where $\psi = \left[\psi_1^T, \psi_2^T, \dots, \psi_n^T\right]^T$. It follows that $\|\psi(t)\| \leq \|\beta y(t) + \gamma w(t)\| + \|\beta\varphi^\varsigma(t) + \gamma\varphi^\kappa(t)\| \leq \|\beta y(t) + \gamma w(t)\| + \delta\,\|\psi(t)\|$. Further, we have $\|\psi\| \leq \frac{1}{1-\delta}\|\beta y + \gamma w\|$. Then, we can obtain

$$\begin{aligned}&-\left(\beta^2 y^T\varphi^\varsigma + \beta\gamma w^T\varphi^\varsigma + \beta\gamma y^T\varphi^\kappa + \gamma^2 w^T\varphi^\kappa\right)\\ &\quad\leq \|\beta y(t) + \gamma w(t)\|\,\|\beta\varphi^\varsigma(t) + \gamma\varphi^\kappa(t)\|\\ &\quad\leq \frac{\delta}{1-\delta}\|\beta y + \gamma w\|^2\\ &\quad\leq \frac{2\delta}{1-\delta}\left(\beta^2 y^T y + \gamma^2 w^T w\right).\end{aligned} \tag{18}$$

Using Lemma 3, it follows from $\psi = \beta y + \gamma w + \beta\varphi^\varsigma + \gamma\varphi^\kappa$ that

$$\begin{aligned}&-(\beta y + \gamma w)^T \mathrm{sig}^p\left(\beta y + \gamma w + \beta\varphi^\varsigma + \gamma\varphi^\kappa\right)\\ &\quad= (\beta\varphi^\varsigma + \gamma\varphi^\kappa)^T \mathrm{sig}^p(\psi(t))\\ &\qquad- (\psi(t))^T \mathrm{sig}^p(\psi(t))\\ &\quad\leq \|\beta\varphi^\varsigma + \gamma\varphi^\kappa\|\,\|sig^p(\psi(t))\|\\ &\qquad- \sum_{i=1}^n\sum_{j=1}^m |\psi_{ij}(t)|^{p+1}\\ &\quad\leq \delta\,\|\psi(t)\|\sqrt{\sum_{i=1}^n\sum_{j=1}^m \left(|\psi_{ij}(t)|^p\right)^2}\\ &\qquad- \sum_{i=1}^n\sum_{j=1}^m |\psi_{ij}(t)|^{p+1}\\ &\quad\leq \delta(mn)^{\frac{1-p}{2}}\|\psi(t)\|^{p+1} - \sum_{i=1}^n \|\psi_i(t)\|^{p+1}\\ &\quad\leq -\left(1 - \delta(mn)^{\frac{1-p}{2}}\right)\|\psi(t)\|^{p+1}.\end{aligned} \tag{19}$$

Similarly, from Lemma 3 it follows that

$$\begin{aligned}&-(\beta y + \gamma w)^T \mathrm{sig}(\beta y + \gamma w + \beta\varphi^\varsigma + \gamma\varphi^\kappa)^q\\ &\qquad\qquad\leq -\left((mn)^{\frac{1-q}{2}} - \delta\right)\|\psi(t)\|^{q+1}.\end{aligned} \tag{20}$$

Combining (14), (16) (17) and (18) yields

$$\begin{aligned}\dot{V}_2(t) \leq &-\beta^2\left(1 - \frac{2\delta}{1-\delta}\right) y^T y\\ &- \left(\gamma^2 - \frac{\beta}{a} - \frac{2\delta\gamma^2}{1-\delta}\right) w^T w\\ &- \left(1 - \delta(mn)^{\frac{1-p}{2}}\right)\|\psi(t)\|^{p+1}\\ &- \left((mn)^{\frac{1-q}{2}} - \delta\right)\|\psi(t)\|^{q+1}.\end{aligned} \tag{21}$$

From

$$\delta < \min\left\{\frac{a\gamma^2 - \beta}{3a\gamma^2 - \beta}, \sqrt{(mn)^{p-1}}, \sqrt{(mn)^{1-q}}\right\},$$

$a > \beta/\gamma^2$, we deduce that $\dot{V}_2(t) < 0$, which implies that $(\tilde{e}_\varsigma, \tilde{e}_\kappa)^T$ asymptotically converges to $\left(0_{mn}^T, 0_{mn}^T\right)^T$. Further, the following analysis shows that the equilibrium point $\left(0_{mn}^T, 0_{mn}^T\right)^T$ is fixed-time stable.

Before a consensus is achieved, we have $\|\tilde{e}_\varsigma\| \neq 0, \|\tilde{e}_\kappa\| \neq 0$ and $\|\psi\| > 0$. During the time interval $\left[t_{k_i}^i, t_{k_i+1}^i\right)$, $\psi_i(t) = \beta y_i\left(t_{k_i}^i\right) + \gamma w_i\left(t_{k_i}^i\right) = \beta y_i(t) + \gamma w_i(t) + \beta\varphi_i^\varsigma + \gamma\varphi_i^\kappa$ is constant; if $\psi_i(t) = 0_m$, then $\psi_i(t) = \beta y_i\left(t_{k_i}^i\right) + \gamma w_i\left(t_{k_i}^i\right) = 0_m$, $\dot{e}_{\kappa i}(t) = 0_m$, $\dot{w}_i(t) = 0_m$, $\dot{\varphi}_i^\kappa(t) = 0_m$. Assume that at least one of $y_i\left(t_{k_i}^i\right)$ and $w_i\left(t_{k_i}^i\right)$ is not 0_m. Further, $\dot{\psi}_i(t) = \beta\dot{y}_i\left(t_{k_i}^i\right) + \gamma\dot{w}_i\left(t_{k_i}^i\right) = \beta w_i(t) + \beta\varphi_i^\kappa(t) + \gamma\dot{w}_i(t) + \gamma\dot{\varphi}_i^\kappa(t) = 0_m$.

It follows that $w_i\left(t_{k_i}^i\right) = w_i(t) + \varphi_i^\kappa(t) = 0_m$ and $\beta y_i\left(t_{k_i}^i\right) = -\gamma w_i\left(t_{k_i}^i\right) = 0_m$, which contradicts with that at least one of $y_i\left(t_{k_i}^i\right)$ and $z_i\left(t_{k_i}^i\right)$ is not 0_m.

Therefore, if $\psi_i(t) = 0_m$, then we have that $y_i\left(t_{k_i}^i\right) = w_i\left(t_{k_i}^i\right) = 0_m$, $i = 1, 2, \ldots, n$. Let

$$\varepsilon_i(t) = \left(\beta y_i^T\left(t_{k_i}^i\right), \gamma w_i^T\left(t_{k_i}^i\right)\right)^T.$$

Then $\psi_i(t) = \begin{pmatrix} 1 & 1 \end{pmatrix} \otimes I_m \varepsilon_i(t)$. Before a consensus is achieved, it follows that

$$\psi^T(t)\psi(t) \geq \xi \sum_{i=1}^{n} \left(\beta^2 \left\|y_i\left(t_{k_i}^i\right)\right\|^2 + \gamma^2 \left\|w_i\left(t_{k_i}^i\right)\right\|^2\right),$$

where

$$\xi = \min_{\frac{\varepsilon(t)}{\|\varepsilon(t)\|}} \left(\frac{\varepsilon(t)}{\|\varepsilon(t)\|}\right)^T \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \otimes I_{mn} \left(\frac{\varepsilon(t)}{\|\varepsilon(t)\|}\right) > 0,$$

$$\varepsilon(t) = \left[\varepsilon_1^T(t), \varepsilon_2^T(t), \ldots, \varepsilon_n^T(t)\right]^T.$$

We have proved that $(\tilde{e}_\varsigma, \tilde{e}_\kappa)^T$ will asymptotically converge to $\left(0_{mn}^T, 0_{mn}^T\right)^T$. It is easy to see that $\|y\|$, $\|w\|$ asymptotically converge to 0 with $\|y\| \leq \|L\| \|\tilde{e}_\varsigma\|$, $\|w\| \leq \|L\| \|\tilde{e}_\kappa\|$. Further, we have $\left\|y_i\left(t_{k_i}^i\right)\right\| \geq \left\|y_i(t)\right\|$ and $\left\|w_i\left(t_{k_i}^i\right)\right\| \geq \left\|w_i(t)\right\|$ during each interval $\left[t_{k_i}^i, t_{k_i+1}^i\right)$. Then, we have

$$\begin{aligned} &\frac{\left(1 - \delta(mn)^{\frac{1-p}{2}}\right) \|\psi(t)\|^{p+1}}{V_2(t)^{\frac{p+1}{2}}} \\ &\geq \frac{\left(1 - \delta(mn)^{\frac{1-p}{2}}\right) \|\psi(t)\|^{p+1}}{\lambda^{\frac{p+1}{2}} \left\|\tilde{e}_\varsigma^T \tilde{e}_\varsigma + \tilde{e}_\kappa^T \tilde{e}_\kappa\right\|^{\frac{p+1}{2}}} \\ &\geq \frac{a^{p+1}\left(1 - \delta(mn)^{\frac{1-p}{2}}\right) \left\|\xi\theta^2 \sum_{i=1}^{n} \left(y_i^T y_i + w_i^T w_i\right)\right\|^{\frac{p+1}{2}}}{\lambda^{\frac{p+1}{2}} \left\|y^T y + w^T w\right\|^{\frac{p+1}{2}}} \\ &= \left(1 - \delta(mn)^{\frac{1-p}{2}}\right)\left(a\theta\sqrt{\xi/\lambda}\right)^{p+1}, \end{aligned} \tag{22}$$

where $\theta = \min\{\beta, \gamma\}$ and λ is the maximum eigenvalue of the matrix

$$\begin{pmatrix} \alpha\beta L^2 & \alpha L/2 \\ \alpha L/2 & \beta L/2 \end{pmatrix}.$$

Similarly, we get

$$\begin{aligned} &\frac{\left((mn)^{\frac{1-q}{2}} - \delta\right) \|\psi(t)\|^{q+1}}{V_2(t)^{\frac{q+1}{2}}} \\ &\qquad \geq \left((mn)^{\frac{1-q}{2}} - \delta\right)\left(a\theta\sqrt{\xi/\lambda}\right)^{q+1}. \end{aligned}$$

Thus, substituting (22) into (21) yields

$$\begin{aligned} &\dot{V}_2(t) + \left(1 - c(mn)^{\frac{1-p}{2}}\right)\left(a\theta\sqrt{\xi/\lambda}\right)^{p+1} V_2(t)^{\frac{p+1}{2}} \\ &\quad + \left((mn)^{\frac{1-q}{2}} - c\right)\left(a\theta\sqrt{\xi/\lambda}\right)^{q+1} V_2(t)^{\frac{q+1}{2}} \leq 0. \end{aligned} \tag{23}$$

Based on Lemma 2,

$$\left(\tilde{e}_\varsigma^T, \tilde{e}_\kappa^T\right)^T$$

will converge to

$$\left(0_{mn}^T, 0_{mn}^T\right)^T$$

with the settling time $T \leq T_1 + T_2$, where T_2 is bounded by

$$\begin{aligned} T_2 \leq &\frac{2}{\left(1 - \delta(mn)^{\frac{1-p}{2}}\right)\left(a\theta\sqrt{\xi/\lambda}\right)^{p+1}(1-p)} \\ &+ \frac{2}{\left((mn)^{\frac{1-q}{2}} - \delta\right)\left(a\theta\sqrt{\xi/\lambda}\right)^{q+1}(q-1)}. \end{aligned} \tag{24}$$

As a result,

$$\lim_{t \to T_1 + T_2} \tilde{e}_\varsigma = 0_{nm},$$

$$\lim_{t \to T_1 + T_2} \tilde{e}_\kappa = 0_{nm}$$

and

$$\tilde{\varsigma} = 0_{nm}, \quad \tilde{\kappa} = 0_{nm}$$

when $t \geq T_1 + T_2$, which implies $e_{\varsigma 1} = \cdots = e_{\varsigma n} = \bar{e}_\varsigma$, $e_{\kappa 1} = \cdots = e_{\kappa n} = \bar{e}_\kappa$. It follows from (11) and Lemma 4 that $\int_{-\tau_i}^{0} u_i(t+s)\,\mathrm{d}s$ will asymptotically converge to 0_m when $t \geq T_1 + T_2 + \max\{\tau_i\}$, which results in $r_1 = r_2 = \cdots = r_n$ and $v_1 = v_2 = \cdots = v_n$. Then, we can conclude that the proposed control scheme can solve the fixed-time consensus problem of the mobile sensor network.

Before a consensus is achieved, by Theorem 2 there exists a positive constant ϖ such that $\left\|\beta y_i\left(t_{k_i}^i\right) + \gamma w_i\left(t_{k_i}^i\right)\right\| \geq \varpi > 0$. When the triggering condition is satisfied, an event is generated and the measurement errors $\left\|\varphi_{\varsigma i}\left(t_{k_i}^i\right)\right\|, \left\|\varphi_{\kappa i}\left(t_{k_i}^i\right)\right\|$ are reset to zero. Let $s_v = \sup\{\|e_{\kappa 1}\|, \|e_{\kappa 2}\|, \ldots, \|e_{\kappa n}\|\}$ and

$s_{\dot{v}} = \sup\{\|\dot{e}_{\kappa 1}\|, \|\dot{e}_{\kappa 2}\|, \dots, \|\dot{e}_{\kappa n}\|\}$. During the time interval $\left[t_{k_i}^i, t_{k_i+1}^i\right)$, we have

$$\begin{aligned}\|\varphi_{\varsigma i}(t)\| &\le \Big\|\int_{t_{k_i}^i}^{t} \dot{\varphi}_{\varsigma i}(s)\,\mathrm{d}s\Big\| \\ &\le \int_{t_{k_i}^i}^{t} \|\dot{\varphi}_{\varsigma i}(s)\|\,\mathrm{d}s \\ &= \int_{t_{k_i}^i}^{t} \|e_{\kappa i}(s)\|\,\mathrm{d}s \\ &\le s_v\left(t - t_{k_i}^i\right)\end{aligned}$$

and

$$\begin{aligned}\|\varphi_{\kappa i}(t)\| &\le \Big\|\int_{t_{k_i}^i}^{t} \dot{\varphi}_{\kappa i}(s)\,\mathrm{d}s\Big\| \le \int_{t_{k_i}^i}^{t} \|\dot{\varphi}_{\kappa i}(s)\|\,\mathrm{d}s \\ &= \int_{t_{k_i}^i}^{t} \|\dot{e}_{\kappa i}(s)\|\,\mathrm{d}s \le s_{\dot{v}}\left(t - t_{k_i}^i\right).\end{aligned}$$

Further,

$$\|\beta\varphi_{\varsigma i}(t) + \gamma\varphi_{\kappa i}(t)\| \le (\beta s_v + \gamma s_{\dot{v}})\left(t - t_{k_i}^i\right).$$

It follows from the triggering function (13) that an event is triggered if $g_i(t) > 0$, which implies

$$\begin{aligned}\frac{\delta}{\sum_{j=1}^{n} a_{ij}} \left\|\beta y_i\left(t_{k_i}^i\right) + \gamma w_i\left(t_{k_i}^i\right)\right\| &< 2\left\|\beta\varphi_{\varsigma i} + \gamma\varphi_{\kappa i}\right\| \\ &\le 2(\beta s_v + \gamma s_{\dot{v}})\left(t_{k_i+1}^i - t_{k_i}^i\right).\end{aligned}$$

Then, at each event-triggered instant $t_{k_i+1}^i$, we can obtain that

$$t_{k_i+1}^i - t_{k_i}^i > \frac{(\delta\varpi)}{2(\beta s_v + \gamma s_{\dot{v}})\sum_{j=1}^{n} a_{ij}} > 0.$$

Further, we can conclude that the proposed event-triggered control scheme can exclude the Zeno behavior for mobile sensor networks. This completes the proof. ■

Remark 4. From the conditions of Theorem 2, the control parameter γ can be set on the basis of $\gamma > \sqrt{\beta/a}$. Increasing the parameter β will increase the convergence rate, while increasing the parameter γ will reduce the convergence rate for the fixed β. Thus, γ is better selected over

$$\left(\sqrt{\frac{\beta}{a}}, 1 + \sqrt{\frac{\beta}{a}}\right).$$

Meanwhile, decreasing p and increasing q will improve the convergence rate. But, according to

$$\delta < \min\left\{\sqrt{(mn)^{p-1}}, \sqrt{(mn)^{1-q}}\right\},$$

the selection of p, q should avoid δ being too small. p is usually selected in $(0.5, 1)$ and q is usually selected in $(1, 1.5)$.

Remark 5. The major difficulties of this paper can be summarized as follows:

(i) The control schemes by Huang and Jia (2018), Tian *et al.* (2017) or Zhang and Duan (2018) all have a specific structure designed to apply bi-limit homogeneous systems theory, but they lack the capability to estimate the settling time. Estimating the settling time is crucial for ensuring fixed-time stability.

(ii) As suggested by Guo and Chen (2020), the ongoing topic is continuous communication. To prevent constant communication and updates entirely, this paper introduces a novel event-triggered function threshold, a rather challenging task.

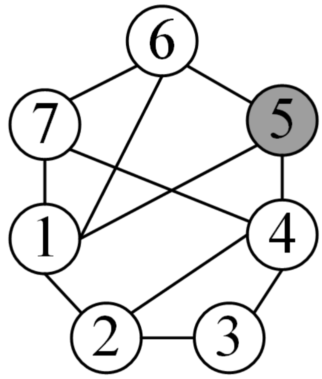

Fig. 1. Topology of the sensor network, where Sensor 5 is a malicious node.

4. Simulation results

To confirm the validity of the main results, we utilize the proposed control scheme to deal with the flocking problem of environmental monitoring in $\mathbb{R}^2$. We present simulation for a sensor network of six legitimate sensor nodes and one malicious node using WiFi signals to communicate. The sensor topology is an undirected sufficiently connected graph (see Fig. 1). Based on Theorem 1, we design the observer with $\alpha_1 = 0.8$, $\tilde{\alpha}_1 = 0.6$, $\alpha_2 = 1.55$, $\tilde{\alpha}_2 = 1.35$, $c_1 = d_1 = 5$, $c_2 = d_2 = 3$. We choose controller parameters as $p = 0.8$, $q = 1.2$, $\beta = 0.15$, $\gamma = 0.1$, $\delta = 0.22$. Time delays are chosen as $\tau_i = 0.1s$. This simulation applies the Sybil attack detection algorithm by Gil *et al.* (2017) to calculate the confidence metric σ_i. Further, the weights obtained according to (14). The initial positions are randomly generated in the region $[-10, 10] \times [-10, 10]$

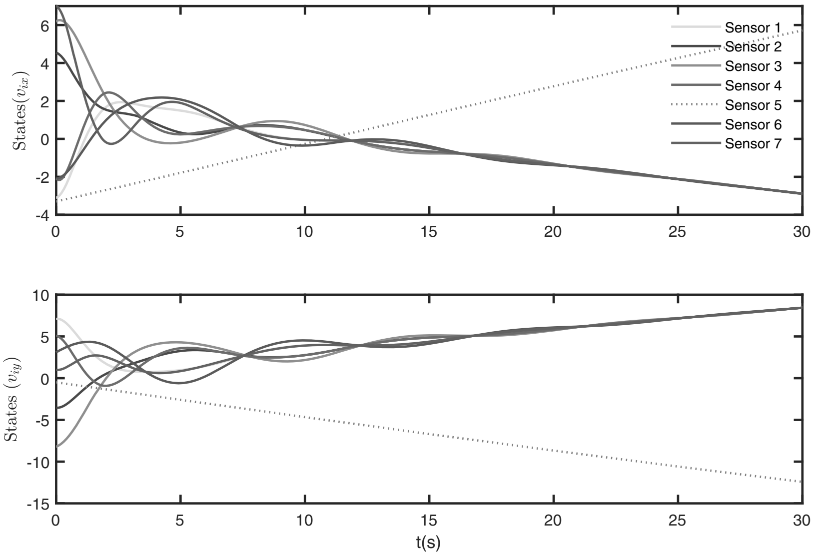

Fig. 2. Position trajectories under the controller (12) and the observer (3).

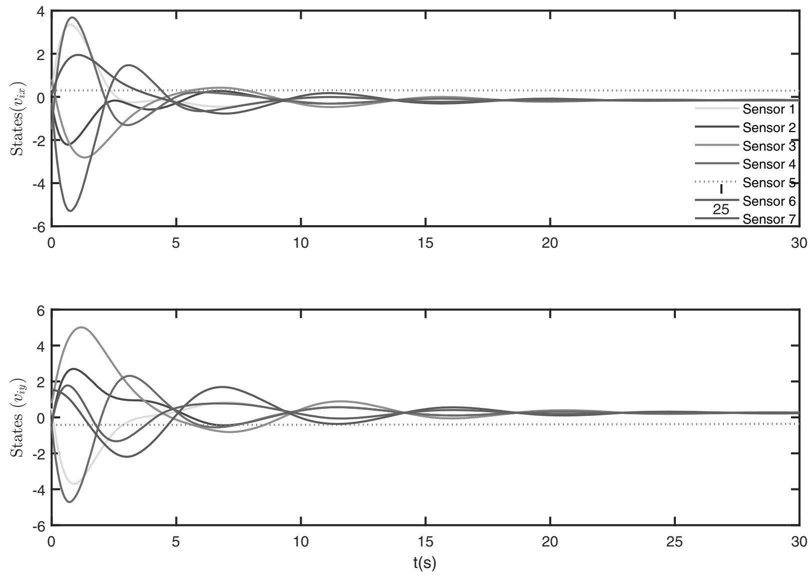

Fig. 3. Velocity trajectories under the controller (12) and the observer (3).

and the velocities are randomly generated in the interval $[-2.5, 2.5]$.

By using the fixed-time observer (3), the confidence metrics weights (14) and the event-triggered control algorithm (12), we implement the simulations and plot the time responses of the position and velocity trajectories in Figs. 2 and 3, respectively. It can be seen that the position and velocity of legitimate nodes on the X and Y axes converge rapidly from their respective initial states to the same states, which satisfies $t < T_1 + T_2 + \max\{\tau_i\}$ of Theorem 2. The malicious node moves in a deceptive state and has little impact on other nodes in the network. Figures 2 and 3 show the result of implementing our resilient control protocol in a flocking with time delay context where legitimate sensors must achieve an average heading value.

The observer's output state errors $e_{ri} = \|\varsigma_i - r_i\|, e_{vi} = \|\kappa_i - v_i\|$ are shown in Fig. 4. It follows that each observer's output state errors rapidly converge to zero on the x and y axes. The simulation results demonstrate that the observer (3) can accurately estimate each node's states. The triggering events are

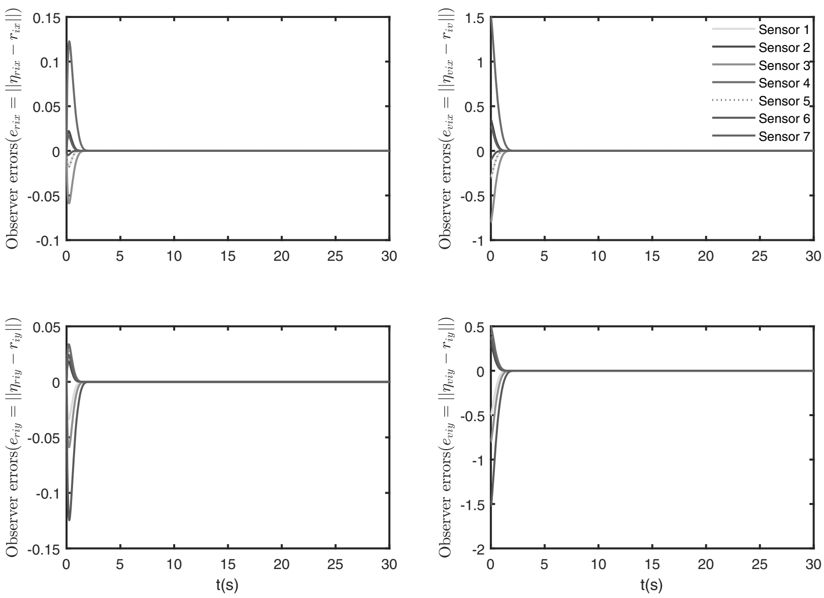

Fig. 4. Time evolution of the observer output state error on the x and y axes: $e_{ri} = \|\varsigma_i - r_i\|$, $e_{vi} = \|\kappa_i - v_i\|$.

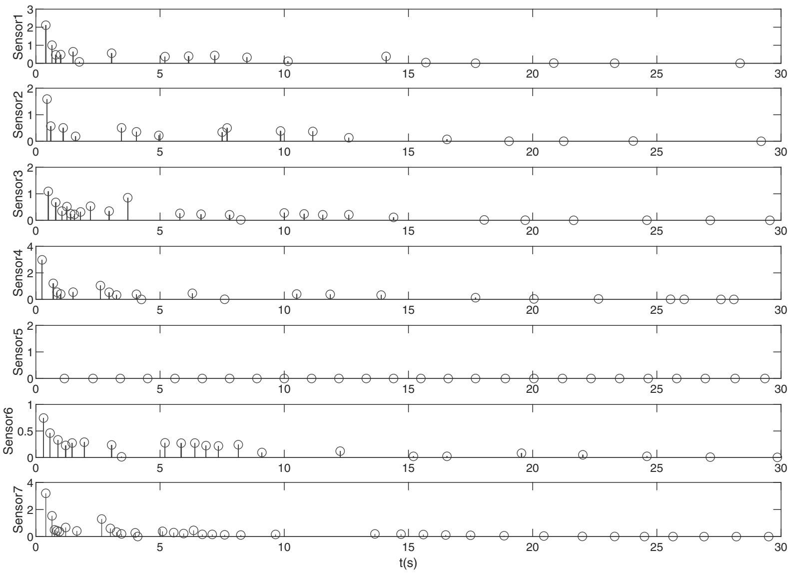

Fig. 5. Triggering events by using the triggering function (13).

shown in Fig. 5. It can be seen that the triggering events of legitimate nodes are asynchronous and the Zeno behavior does not take place.

5. Conclusion

In this paper, we presented a novel control scheme for resilient consensus in sensor networks against the Sybil attack. We proposed a fixed-time observer to estimate the state and constructed a novel homogeneous Lyapunov function to estimate the settling time. By introducing Artstein's transformation, we transformed the delayed system into a delay-free system. A novel threshold is defined, and the triggering function is derived based on the novel threshold. An event-triggered resilient control scheme was employed not only to mitigate the effects of malicious nodes but also avoid continuous communication. With the constructed triggering function, the Zeno behavior was excluded. The validity of the proposed method is proved by the ingenious construction of inequalities. Simulation validated the analysis, wherein the sensor network converges to the average of the legitimate node values and avoids continuous communication.

It is worth noting that the settling time given by Theorem 2 is relatively large compared with simulations. Recently, as noted by Liu *et al*. (2024) and Zhao *et al*. (2016), settling time estimation has attracted increasing interest. Accurate settling time estimation methods have significant influence on the application of the fixed-time consensus. Future topics will focus on resilient optimized control for sensor networks with settling time estimation.

Acknowledgment

This work was funded in part by the National Key R&D Program of China under the grant 2022YFB3104300.

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Ding Zhou received his BS and PhD degrees in control engineering from the Northwestern Polytechnical University of China, Xi’an, in 2014 and 2020, respectively. He is currently an assistant research fellow of control engineering with Purple Mountain Laboratories. His present research interests include multi-agent systems, formation control and mobile sensor networks.

Lei He received his PhD degree in systems engineering from the Information Engineering University of China, Zhengzhou, in 2000. He is currently a research fellow of control engineering with Purple Mountain Laboratories. His research interests include multi-agent systems, nonlinear control systems and intelligent control.

Zhigang Cao received his PhD degree in systems engineering from the University of Science and Technology of China, Hefei, in 2021. He is currently an assistant research fellow of control engineering with Purple Mountain Laboratories. His present research interests include nonlinear control systems and intelligent control.

An Zhang received his MS degree in systems engineering in 1986, and his PhD degree in control theory and control engineering from the Northwestern Polytechnical University of China, Xi’an, in 1999. He is currently a full-time professor of control engineering with Northwestern Polytechnical University. His present research interests include multi-agent systems, nonlinear control systems, intelligent control, and UAV control.

Xiaopeng Han received his PhD degree from Wuhan University, Wuhan, China, in 2019. Now, he is contributing to research at Purple Mountain Laboratories in Nanjing, China, as a system architecture researcher, focusing on machine learning, network situation awareness, and deep learning. He is involved in cutting-edge work at the intersection of artificial intelligence and network technologies.

Received: 3 April 2024
Revised: 11 July 2024
Accepted: 11 October 2024