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ANALYTICAL REDUNDANCY METHODS FOR DIAGNOSING ELECTRIC MOTORS

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This paper deals with two different diagnostic methods for electric motors. The diagnosis can either be used for quality control or on–line monitoring. The most important requirements are a fast and reliable detection and isolation of failures (FDI) of the whole motor. Furthermore, the measurements should be simple to make. These requirements cannot be fulfilled with conventional test systems, but with analytical redundancy methods based on signal and system theory. Two different model based techniques are presented: parameter estimation and a fault sensitive filter bank technique. Finally, evaluations of measurements are given.

1. Introduction

Small power electric motors are employed for several tasks in industry, household and automobiles. These motors can either be d.c., universal or induction. The inspection of these motors is an aspect of technical diagnosis, as a part of quality control. The objective of this diagnosis is a fast and reliable test which includes the detection of failures and their location in all parts of the motor.

Nowadays this diagnosis is done with conventional test systems. For quality control such a conventional test system consists of two major parts:

i) the mechanical construction holding the motor being tested, transducers, a clutch that connects the motor to a brake, and pick ups for measurement purposes.

ii) the electrical equipment consisting of the power supply, displays, the electric units for data acquisition and data handling.

Such a test stand is expensive and the complete diagnosis is very time consuming.

On–line monitoring is often used for large motors, e.g. in power stations or in oil industry. Normally, measurement techniques are based on direct procedures. This means that relevant components of the electric motor are supervised via one or more sensors. The problem is that not all relevant components can be easily measured (like temperature in the rotor) and because of a large number of failure modes and locations, these methods require many sensors and are time consuming.

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In order to avoid these disadvantages signals from acoustic or vibration sensors connected to the motors are evaluated. This is an indirect and integrating technique and a diagnosis of the complete motor is not reliably possible.

The disadvantages of the conventional diagnostic systems for quality control and on-line monitoring of electric motors can be reduced with the use of modern system and signal theory techniques based on analytical redundancy methods. These techniques need easily measurable state variables, which are not directly related to the components being supervised. Nevertheless with the help of *a priori* knowledge failures can be detected and located.

In this paper two analytical redundancy methods are presented: diagnostic techniques based on parameter estimation and fault sensitive filters. Both methods are mathematically described and practical results are presented. In the method based on parameter estimation the diagnosis is accomplished by the determination of physical parameters and their classification. The other method evaluates the residuals of a fault sensitive filter bank by statistical tests.

2. Parameter Estimation Method

Parameter estimation is a well known control theory technique. The task is to estimate the physical parameters of the respective process with the input – output relationship. The structure of parameter estimation is shown in Figure 1.

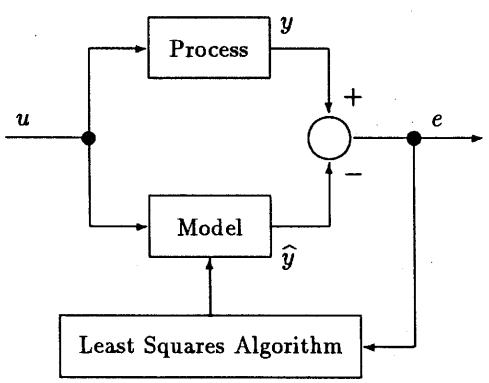

Fig. 1. Structure of parameter estimation.

The algorithm of parameter estimation tries to minimize the residuals e with a special criterion. Three possibilities (degrees of freedom) for optimizing the parameter estimation are provided:

- choice of the estimation algorithm in the time or frequency domain,
- choice of fitted process models,
- choice of the excitation function.

These items will be described as follows.

2.1. Parameter Estimation

2.1.1. Parameter Estimation in the Time Domain

In this paper parameter identification is accomplished by the least square algorithm which is quite common in the literature. The advantage of parameter estimation based on continuous time models, in contrast to the time discrete models, is that the physical significance of the parameters is given. The identification formula is for both methods the same

$$\alpha = [\boldsymbol{x}^T \boldsymbol{x}]^{-1} \boldsymbol{x}^T \boldsymbol{y} \tag{1}$$

The use of continuous time models results in the problem that the measured signals have to be derivated. The emphasis will be on this point. With convolution, which is the same like the application of digital Finite Impulse Response (FIR) filters, it is possible to avoid the direct derivation of state variables. This will be shown in the following deduction. The convolution between two functions $g(t)$ and $f(t)$ will be expressed as

$$g * f = \int_{-\infty}^{\infty} g(t-\tau) f(\tau)\, \mathrm{d}\tau \tag{2}$$

Consider the following equation

$$g * \frac{\mathrm{d}f}{\mathrm{d}t} = \int_{-\infty}^{\infty} g(t-\tau) \frac{\mathrm{d}f(\tau)}{\mathrm{d}\tau}\, \mathrm{d}\tau \tag{3}$$

with a weight function $g(t)$ and a signal function $f(t)$. The rules of partial integration give

$$g * \frac{\mathrm{d}f}{\mathrm{d}t} = g(t-\tau) f(\tau)|_0^t - \int_0^t \frac{\mathrm{d}g}{\mathrm{d}\tau}(t-\tau) f(\tau)\, \mathrm{d}\tau \tag{4}$$

or

$$g * \frac{\mathrm{d}f}{\mathrm{d}t} = g(0) f(t) - g(t) f(0) - \int_0^t \frac{\mathrm{d}g}{\mathrm{d}\tau}(t-\tau) f(\tau)\, \mathrm{d}\tau \tag{5}$$

With the assumptions

$$g(\tau) = 0 \quad \text{for} \quad \tau \notin [0, t] \quad \text{and} \quad g(0) = g(t) = 0$$

equation (5) becomes

$$g * \frac{\mathrm{d}f}{\mathrm{d}t} = -\int_0^t \frac{\mathrm{d}g}{\mathrm{d}t}(t-\tau) f(\tau)\, \mathrm{d}\tau = -\frac{\mathrm{d}g}{\mathrm{d}t} * f \tag{6}$$

This equation contains no derivation of the signal function. The convolution of the n–th derivative of a signal function $f^{(n)}(t)$ with the weighing function $g(t)$ leads to the following general formula

$$g(t) * f^{(n)}(t) = (-1)^n g^n(t) * f(t) \tag{7}$$

The general principle of these filters was described by Metzger (1983) and Elten (1989). The equations (6) and (7) shall be used for a differential equation to demonstrate this procedure. Consider the following differential equation

$$y = a_2 x^{(2)} + a_1 x^{(1)} + a_0 x \tag{8}$$

Convolution of both sides of equation (8) with the weight function $g(t)$ leads to

$$g * y = a_2 g * x^{(2)} + a_1 g * x^{(1)} + a_0 g * x \tag{9}$$

According to equations (6) and (7) it is possible to substitute the convolution with the derivated signal function

$$g * y = a_2 g^{(2)} * x - a_1 g^{(1)} * x + a_0 g * x \tag{10}$$

This equation contains no derivation of the signal function and can be used for the least square algorithm.

2.1.2. Parameter Estimation in the Frequency Domain

When the input and output signals are periodical, the parameters of the system can be determined in the frequency domain. The common procedure to determine the parameters in the frequency domain can be described in the following manner (Bradatsch, 1990; Dreetz, 1989):

i) Transformation of the signals into the frequency domain. This is possible with the Fast Fourier Transformation (FFT).

ii) Application of differential operators or integral operators.

iii) Calculation of system parameters with the least square method.

As it was mentioned before, the determination of the parameters is carried out in the frequency domain and is based on the application of the Fast Fourier Transformation algorithm. It is noted here that the FFT is only a fast discrete Fourier transformation.

If a signal $x(t)$ is sampled with N points per period the Fourier coefficients are calculated with the following formulae

$$x_c(k) = \frac{2}{N} \sum_{n=0}^{N-1} x(n) \cos(k\omega_0 n\ \Delta t/2\pi) \tag{11}$$

$$x_s(k) = \frac{2}{N} \sum_{n=0}^{N-1} x(n) \sin(k\omega_0 n\ \Delta t/2\pi) \tag{12}$$

From the sampled N points per period one obtains a spectrum with $K = N/2$ significant lines. The frequency resolution can be calculated as follows

$$\Delta f = \frac{1}{N\,\Delta t} \tag{13}$$

In this expression Δt is the sampling interval, Δf is the frequency resolution and N is the number of sampled data. This must be an integer power of two.

The problem of leakage arises because the observation of the signal must be limited to a finite interval. This is an equivalent to multiplying the sampled signal with a rectangular window in the time domain. If the observation interval is not a multiple of the signal period, the phenomenon of spectral leakage will appear and may cause errors in the harmonic determination of the components. In order to avoid large range leakage, first a suitable window has to be applied instead of a rectangular window in the time domain, and second the observation interval must be more than one multiple of the signal period. For the parameter estimation in the frequency domain a Blackman window has been used because of the very strong attenuated side lopes. The starting point for the derivation of the least square solution in the frequency domain is the model description added with an error signal $e(t)$

$$y(t) = \sum_{i=1}^{n} \alpha_i x_i(t) + e(t) \tag{14}$$

The sought after coefficients can be determined by minimizing the loss function

$$L = \int_0^\infty e^2(t)\ \mathrm{d}t \equiv \text{ Minimum} \tag{15}$$

Setting the derivation of L with respect to the coefficients α_i to zero

$$\frac{\partial}{\partial \alpha_i}\Big(\int_0^\infty e^2\ \mathrm{d}t\Big) = 0 \tag{16}$$

results in

$$\int_0^\infty -2y(t)x_i(t)\ \mathrm{d}t + \int_0^\infty 2\sum_{j=1}^{M} \alpha_j x_i(t) x_j(t)\ \mathrm{d}t = 0 \quad \text{with } i = 1, ..., n \tag{17}$$

The obtained system has M normal equations, allowing the determination of the parameters, which are being sought after. If the signals are periodical as mentioned before they can be developed into the Fourier series:

$$y = \sum_{k=0}^{\infty} [y_{ks} \sin(k\omega_0 t) + y_{kc} \cos(k\omega_0 t)] \tag{18}$$

$$x_j = \sum_{k=0}^{\infty} [x_{jks} \sin(k\omega_0 t) + x_{jkc} \cos(k\omega_0 t)] \tag{19}$$

where y_{ks}, y_{kc}, x_{jks}, x_{jkc} are Fourier coefficients. Substitution of the Fourier series (18) and (19) into the normal equations with respect to the orthogonality principle gives

$$\frac{1}{2}\sum_{k=1}^{\infty} y_{ks}x_{iks} + \frac{1}{2}\sum_{k=1}^{\infty} y_{kc}x_{ikc} + y_0 x_{i0c} - \ldots$$

$$\ldots - \sum_{j=1}^{M} \alpha_i \Big(\frac{1}{2}\sum_{k=1}^{\infty} x_{jks}x_{iks} + \frac{1}{2}\sum_{k=1}^{\infty} x_{jkc}x_{ikc} + x_{j0c}x_{i0c} \Big) = 0 \qquad (20)$$

The next step is to simplify the normal equation for the determination of the coefficients by a complex notation

$$y_k = \frac{1}{\sqrt{2}}\Big(y_{kc} - j y_{ks}\Big) \qquad \text{for } k \geq 1; \quad y_0 = y_{0c} \qquad (21a)$$

$$x_{ik}^{*} = \frac{1}{\sqrt{2}}\Big(x_{ikc} + j x_{iks}\Big) \qquad \text{for } k \geq 1; \quad x_{i0} = x_{i0c} \qquad (21b)$$

$$x_{ik} = \frac{1}{\sqrt{2}}\Big(x_{ikc} - j x_{iks}\Big) \qquad \text{for } k \geq 1; \quad x_{i0} = x_{i0c} \qquad (21c)$$

According to the complex notation the normal equation becomes

$$\sum_{k=0}^{\infty}\Big[\mathrm{Re}[y_k x_{ik}^{*}] - \sum_{j=1}^{M} \alpha_j \mathrm{Re}[x_{jk}x_{ik}^{*}]\Big] = 0 \qquad (22)$$

In matrix notation it is

$$\mathrm{Re}[\boldsymbol{x}^T \boldsymbol{y}^*] - \mathrm{Re}[\boldsymbol{x}^T \boldsymbol{x}^*]\boldsymbol{\alpha} = 0 \qquad (23)$$

Note that the matrices $\boldsymbol{x}$, $\boldsymbol{x}^*$ and $\boldsymbol{y}$ contain complex notation of spectral lines. Equation (23) can be re–expressed as

$$\boldsymbol{\alpha} = [\mathrm{Re}(\boldsymbol{x}^T \boldsymbol{x}^*)]^{-1}[\mathrm{Re}(\boldsymbol{x}^T \boldsymbol{y}^*)] \qquad (24)$$

which is very similar to the least square solution in the time domain. $\boldsymbol{x}^{-1}$ means the inverse of $\boldsymbol{x}$, $\boldsymbol{x}^T$ is the transponate, $\boldsymbol{x}^*$ is the conjugate complex and Re( ) the real part of the elements. The necessary differentiation is a simple operation. According to the Fourier series equation (18) and (19) we obtain

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \sum_{k=0}^{\infty} -k\omega_0 x_{kc}\sin(k\omega_0 t) + k\omega_0 x_{ks}\cos(k\omega_0 t) \qquad (25)$$

This operation in which cosine and sinus terms were derived is very easy. Thus, differentiation is achieved with the following simple operations

$$\frac{\mathrm{d}x_{ks}}{\mathrm{d}t} = -k\omega_0 x_{kc} \qquad (26a)$$

$$\frac{\mathrm{d}x_{kc}}{\mathrm{d}t} = k\omega_0 x_{ks} \qquad (26b)$$

It is noted here that this operation has been carried out for all complex spectral lines.

2.2. Modelling

The models which are used for the parameter estimation are parametric models. Due to simple mathematics there is the assumption that the models are linear and time invariant. But this assumption can not be fulfilled in all cases. For the parameter estimation the models must describe the technical process very exactly. If the description is not perfect, there can be a serial correlation in the residuals (see below) and a correlation of the residuals with the excitation. This model error causes a parameter bias and parameter deviations.

Because of these connections it is important that there is an advanced modelling process to reduce the model error. This process is divided into two steps:

- development of models,
- verification of the models.

2.2.1. Development of Models

The modelling will be demonstrated on the example of a low power d.c. motor (Filbert, 1991). The first approach to development of appropriate models for the respective process (motor) is to describe the physical relations of the process. For a low power d.c. motor with permanent magnets two equations are given in the literature (eqs. 27 and 28)

$$v = Ri + c_1\omega \tag{27}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{c_2}{J}i - \frac{c_3}{J}\omega \tag{28}$$

The first equation describes the voltage balance where v represents voltage, R resistance, i current, c_1 the parameter proportional to the e.m.f and ω rotational speed. Normally, the voltage drop of the brushes and the inductivity of the coil are taken into account. However, in a small power motor these terms are very small and they can be omitted.

The torque balance with internal losses proportional to the speed is described by equation (28). The parameter J represents the moment of inertia, c_2 the flux parameter and c_3 the mechanical load parameter.

These equations describe an ideal motor, but the behaviour of real motors is not ideal. Hence, there must be a second step where the models can be improved for the specific type of d.c. motor.

2.2.2. Model Criteria

First of all, in order to improve the models, criteria for a good model must be found. The most important requirement for the diagnostic system and also for the process model is the ability to find failures. They must be detected as sensitively as possible, while disturbances must be suppressed. For the development of accurate models, the analysis of the residuals (model error) is one of the best methods. The structure of the residual analysis is shown in Figure 2.

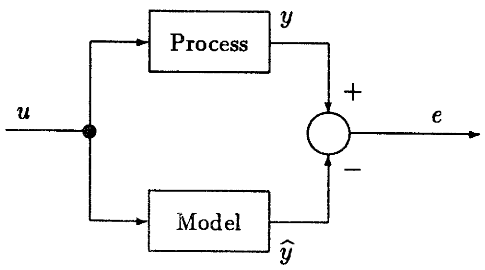

Fig. 2. Structure of the residual analysis.

The residuals are defined as the difference between the process output and estimated outputs. If the model comes up to the process exactly and if there are no disturbances, the estimated output $\hat{y}$ is the same like the process output y and the residuals e are zero. Normally, the measurements are noisy and the condition for the residuals is white noise. Test criteria for the residuals are:

- small rms value of the residuals,
- no serial correlation in the residuals,
- no correlation between residuals and excitation.

From deterministic structures in the residuals it is possible to one can deduce the missing or wrong terms in the model equations.

2.2.3. An Example of Modelling

Evaluations of measurements on a low power d.c. motor will give an example of the residual analysis. Figure 3 shows the residuals of the electrical model (27). They are drawn versus the output v.

A strong dependence of the residuals on the voltage is obvious. The additional disturbances in the residuals are due to the commutator ripple and the ripple of the tachometer for the speed measurement. With two further models it was attempted to reduce the deterministic structure

$$v = Ri + c_1\omega + c_{10}v^2 \tag{29}$$

$$v = Ri + c_1\omega + v_0 \tag{30}$$

The remaining residuals of these models are shown in Figures 4 and 5.

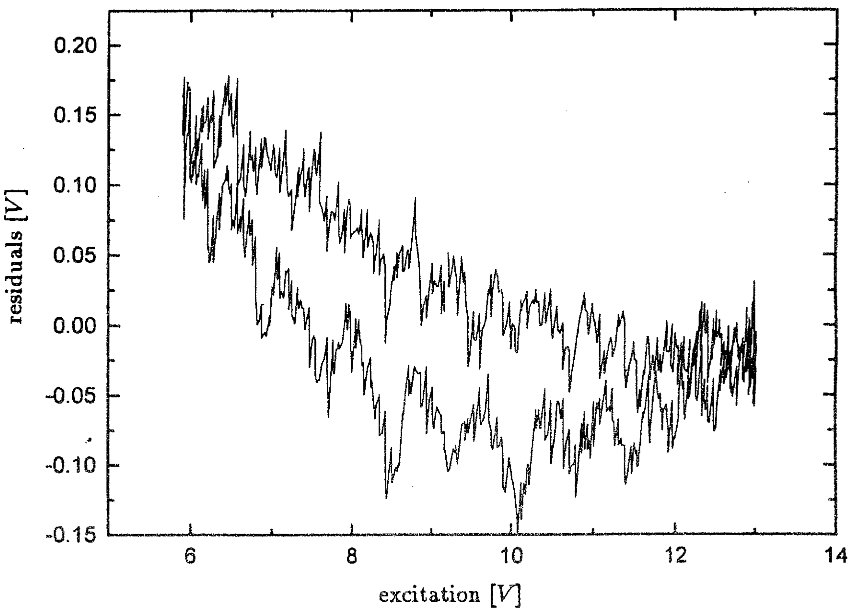

Fig. 3. Residuals of model equation (27).

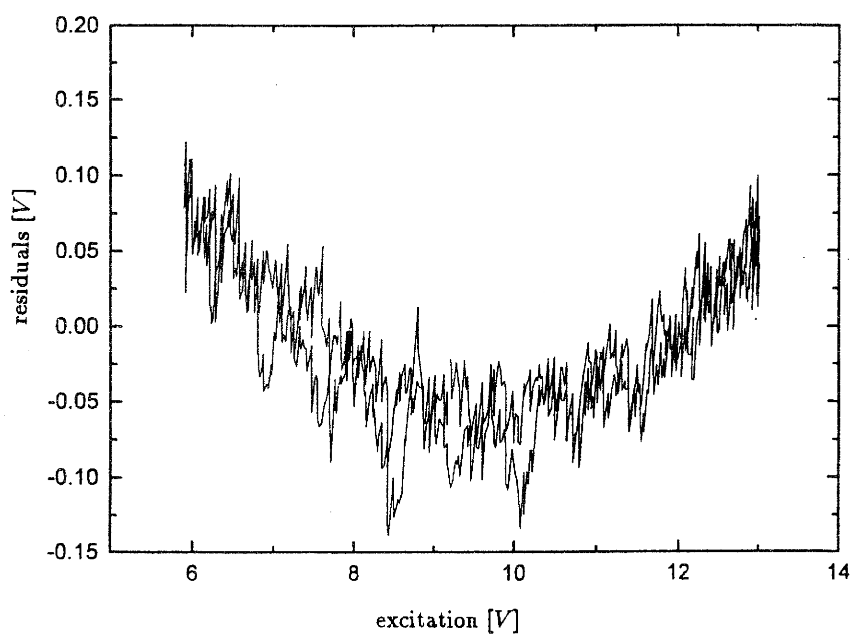

Fig. 4. Residuals of model equation (29).

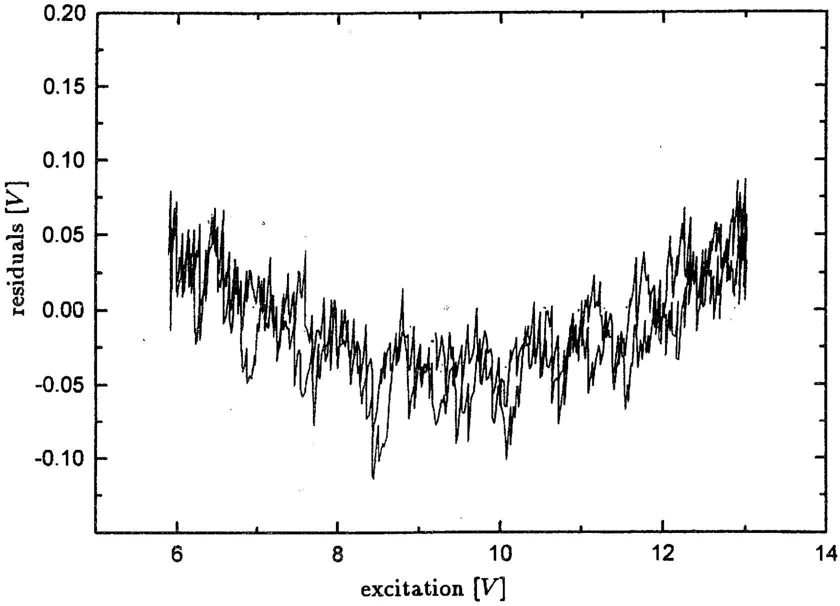

Fig. 5. Residuals of model equation (30).

For the two models (29) and (30) the residuals and the dependence of the residuals on the excitation become much smaller. A better statement about the residuals can be made with the correlation coefficient of the respective residuals with the excitation voltage. These correlation coefficients c_{eu} are shown in Table 1.

Tabl. 1. Correlation coefficients.

model	(27)	(29)	(30)
c_{eu}	−0.75	−0.2	0.01

2.2.4. Verification of Models

While the process models can be developed with the residual analysis, the residuals do not provide the information, which of the developed models supplies the best description of the process. Therefore, in the second step, the models must be verified. For the verification of the parameters several conditions of the parameters can be used:

- equality of estimated parameters with the physical parameters of the motor,
- correlation between parameters,
- standard deviation of parameters.

The first requirement can be fulfilled by comparing the estimated parameters with conventional measured parameters. The second item can be tested by conducting several measurements, estimation of the parameters and computing the correlation coefficients between the parameters. This coefficient has to be very small due to the separation of several failure classes. The third requirement means that the standard deviations of the parameters have to be small, which is a criterion for the reliability of diagnostic systems.

As an example the standard deviations of the parameters of twenty measurements are shown in Table 2.

Tabl. 2. Standard deviations of parameters [%].

model	R	c_1	u_0	c_{10}
(27)	0.30	0.06	–	–
(29)	0.34	0.13	–	4.87
(30)	0.23	0.07	4.9	–

The standard deviation of model (30) compared with model (27) for the parameter R is some 23% smaller. The standard deviation of the flux constant c_1 is a little bit bigger. The standard deviation of parameter u_0 as compared with parameters R and c_1 is very large, which is due to the small value of u_0 compared with the terms $R \cdot i$ and $c_1 \cdot \omega$ in the model equation. In model (29) the standard deviations are larger than in the models (27) and (30).

The modelling procedure shows, that the electrical model (30) describes the special type of d.c. motor best. The same procedure was conducted for the mechanical part of the motor. The result of the modelling was the mechanical model (31)

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{c_2}{J} i - \frac{c_3}{J} \sqrt{\omega} \tag{31}$$

2.3. Excitation

The choice of the excitation function is after the estimation algorithm (Section 2.1) and the process model (Section 2.2) the third degree of freedom of the parameter estimation. In the case of on–line monitoring the normal excitation of the process should be used, while in the case of quality control the excitation function can be freely chosen. In both cases the excitation must meet special demands. The main demand is, that the system must be sufficiently excited. Due to this criterion the excitation must persistently affect the system at all eigenvalues. A further demand is that in the case of a d.c. motor the motor must be excited only in one rotational direction, because other parameters will result if the motor is excited in another direction.

For the evaluation of measurements of a low power d.c. motor in Section 2.4 a sinusoidal riding on a d.c. is used as excitation.

2.4. Fault Detection

In the case of parameter estimation for on–line monitoring or quality control the classification is very elegant, because the results of the parameter estimation are the physical parameters of the process. Hence, failures can be detected and located only with a comparison of the parameters of the device under test (d.u.t.) and nominal parameters. Good results give a weighted distance classificator

$$d = \frac{\theta_F - \theta_N}{s_N} \tag{32}$$

The absolute distances $(\theta_F - \theta_N)$ between the d.u.t. parameters and the nominal parameters are related to the measured standard deviation s_N of several motors. Hence, the vector d represents multiples of the standard deviation of all parameters for the d.u.t.. If the level of significance is given, the threshold, which has to be exceeded in a failure case, can be determined; e.g. for a level of significance of 95% the threshold "2" has to be exceeded.

Measurements with four different fault classes were made to test the diagnostic system. The fault classes are:

i) bearing without grease,
ii) hook not connected,
iii) half magnetised,
iv) wrong direction.

The resulting distances (32) from the nominal class to the respective failure class are shown in Table 3.

Tabl. 3. Resulting distances (32) for four failure classes.

failure class	R	C_1	U_0	C_2/J	C_3/J
(1)	−1.1	1.7	1.1	0.5	−4.4
(2)	2.2	3.0	−4.7	−0.8	1.5
(3)	−1.7	−13.8	2.8	−15.7	−3.3
(4)	−1.8	3.9	−5.1	0.2	1.5

In the case of a motor without grease a large deviation of the parameter c_3/J is obvious. This is due to the smaller load of the bearing. The deviations of the other parameters are small. For a motor with a disconnected hook there are deviations in the electrical part of the motor. The large deviation of the parameter u_0 has no direct physical meaning, but it shows a failure in the electrical part of the motor. In the case of a half magnetized motor the flux parameters c_1 and c_2/J show strong negative deviations, which are easily understood. Furthermore, the load parameter c_3/J is smaller because of the higher speed. The failure class "wrong direction" cannot be directly detected, because the speed is measured with a phase-locked-loop, which is not direction sensitive. Nevertheless, it can be detected because the motor is not symmetrical and therefore the parameters will vary depending on the motor direction. For this special type of motor the parameters c_1 and u_0 show a large deviation.

These examinations show that the parameter estimation with the diagnosis vector (32) provides not only the detection of failures but also the determination of the failure type and location.

3. Fault Sensitive Filter Bank

The diagnosis system based on fault sensitive filters (Gühmann, 1991; Röpke, 1992) can be used for on-line monitoring of large power motors or for fault detection in small power motors which are produced in series. In comparison to parameter identification diagnosis or observer based systems (Gühmann, 1989) only the output signal is necessary for the diagnosis.

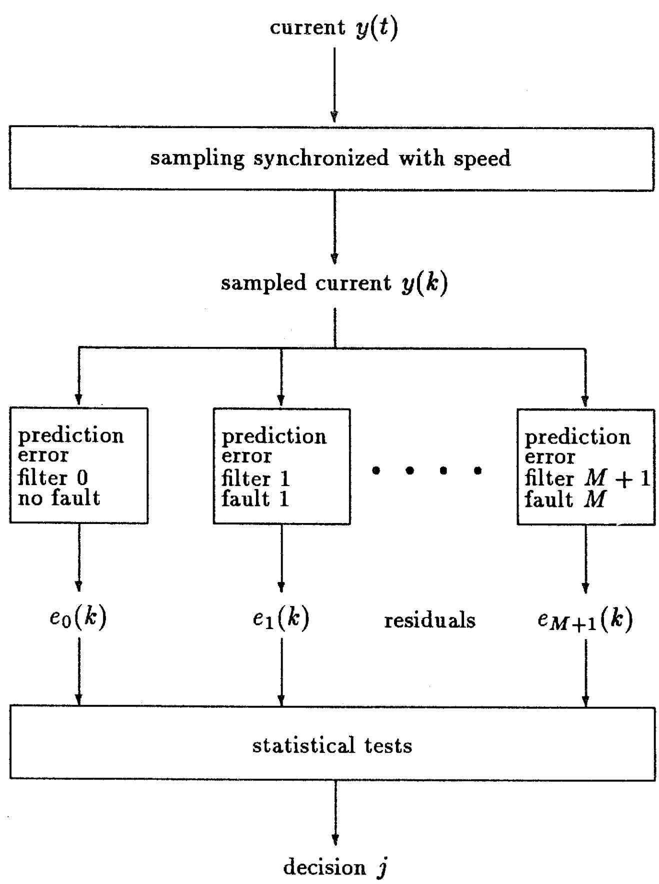

Fig. 6. Fault diagnosis system.

The most important information about the condition of a motor is included in the current signal and/or the vibration signal. However, in practice it is easier to measure the current signal than the vibrations, so that in the following application the current signal is used for the diagnosis. After sampling the current the features are extracted by filtering through a filter bank determined in advance. The filter bank contains $M+1$ linear prediction error filters. The generated error also processes termed residuals $\{e(k)\}$, that are defined as the difference between the system output $\{y(k)\}$ (sampled current signal) and the expected system output $\{\widehat{y}(k)\}$, which is based on a model and the previous output data

$$e(k) = y(k) - \widehat{y}(k) \tag{33}$$

The idea is that one predictor represents the motor under normal conditions and the remaining predictors represant the motor under faulty conditions. Under the assumption that the tested motor is faultless, the residual of the filter EP0 is "small" and corresponds to the fluctuation in the output since all the systematic trends are eliminated by the model. However, under the assumption that the motor is faulty, the residual process $\{e_0(k)\}$ is "large" and contains systematic trends because the model is no longer representing the system adequately. But, on the other hand, another residual process is "small" and the appropriate filter represents the motor with its fault. A "small" process is, as will be shown later, a white noise process with minimal variance. In comparison, a "large" error process is a correlated process.

In most cases of on-line monitoring only the behaviour of the motor in good condition is known. Then the filter bank is reduced to one filter and there are only two decision possibilities: the motor has a fault or does not have a fault.

3.1. Prediction Error Filter Design

The filter bank is built by supervised learning. A human expert has the task to classify the produced motors. The basis of the prediction error filter design is either in the time domain the estimated autocorrelation sequence or in the frequency domain the estimated periodogram. In this paper only the time domain method is discussed. To increase the statistical certainty as many autocorrelation sequences as possible of each class are averaged. Subsequently the filter coefficients are determined.

The starting point for the prediction error filter design is the hypothesis, that the signal, which is to be classified, could be described with an autoregressive (AR) model (34) shown in Figure 7.

$$\begin{aligned} y(k) &= -\sum_{i=1}^{P} a_i y(k-1) + q(k) \quad \text{with} \quad E\{q(1)\} = 0 \\ \text{and} \quad R_{qq}(l) &= \begin{cases} \sigma_q^2 & \text{if } l = 0 \\ 0 & \text{if } l \neq 0 \end{cases} \end{aligned} \tag{34}$$

In this equation P is the order of the process $\{y(k)\}$, a_i are the AR coefficients of the process $\{y(k)\}$, $E\{\}$ is the expected value operator, R_{qq} is the autocorrelation sequence of $\{q(k)\}$ and σ_q^2 is the variance of the process $\{q(k)\}$.

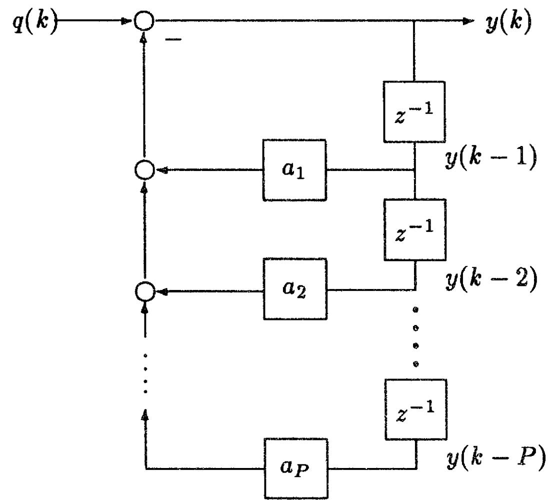

Fig. 7. Autoregressive process.

The prediction error filter matching the current signal eliminates the correlated parts of the signal. In this case, the filter output (prediction error) is a white noise sequence.

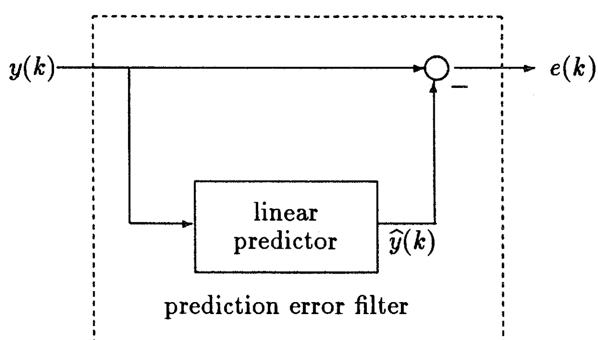

Fig. 8. Prediction error filter.

The prediction error filter design is an equivalent to the determination of a linear predictor (see Fig. 8). In this method (Makhoul, 1975) the actual sampled signal $y(k)$ is predicted from a linear weighted summation of past values

$$\hat{y}(k) = \sum_{i=1}^{P} a_i y(k-i) \tag{35}$$

With the least squares method the parameters a_i are obtained as a result of the minimization of the mean squared error with respect to each of the parameters

$$Q = \sum_{m=-\infty}^{\infty} e(m)^2 = \sum_{m=-\infty}^{\infty} [y(m) - \hat{y}(m)]^2$$

$$= \sum_{m=-\infty}^{\infty} \left[y(m) - \sum_{i=1}^{P} a_i y(m-i)\right]^2 \rightarrow \text{ Minimum} \tag{36}$$

If the derivation of Q is set to zero

$$\frac{\partial Q}{\partial a_i} = 0 \tag{37}$$

the relation between the model parameters and the autocorrelation sequence is found by

$$\sum_{k=1}^{P} a_i R_{yy}(i-k) = -R_{yy}(i) \tag{38a}$$

The parameter vector $\boldsymbol{a}$ is the solution of the following set of equations

$$\begin{bmatrix} R_{yy}(0) & R_{yy}(1) & \dots & R_{yy}(P-1) \\ R_{yy}(1) & R_{yy}(0) & \dots & R_{yy}(P-2) \\ R_{yy}(2) & R_{yy}(1) & \dots & R_{yy}(P-3) \\ \vdots & \vdots & \dots & \vdots \\ R_{yy}(P-1) & R_{yy}(P-2) & \dots & R_{yy}(0) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_P \end{bmatrix} = - \begin{bmatrix} R_{yy}(1) \\ R_{yy}(2) \\ R_{yy}(3) \\ \vdots \\ R_{yy}(P) \end{bmatrix} \tag{38b}$$

Because there is not an infinite number of sampled data, the signal is windowed with e.g. a Hanning window

$$w(n) = \begin{cases} w(n) \neq 0 & \text{for } n = 1, ..., N \\ w(n) = 0 & \text{otherwise} \end{cases} \tag{39}$$

The data windowing yields an unbiased estimation of the autocorrelation sequence

$$\hat{R}_{yy}(i) = \sum_{n=0}^{N-1-i} y(n)\, w(n)\, y(n+i)\, w(n+i) \tag{40}$$

The parameters are also an estimation and given by

$$\sum_{i=1}^{P} \hat{a}_i \hat{R}_{yy}(k-i) = -\hat{R}_{yy}(k) \tag{41}$$

The parameters found in (41) are the sought after prediction error filter coefficients. The filter has a finite impulse response and possesses the following structure

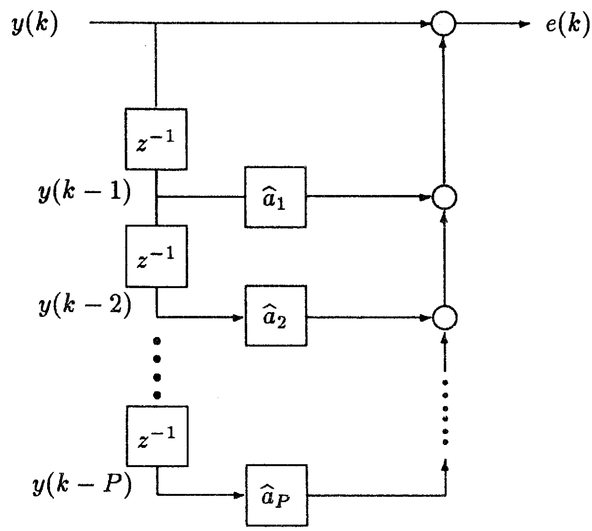

Fig. 9. Prediction error filter as finite impulse response filter.

with the difference equation

$$e(k) = \sum_{i=0}^{P} \hat{a}_i \, y(k-i) \qquad \text{with} \quad \hat{a}_0 = 1 \tag{42}$$

The methods for the prediction error filter design have been improved constantly in the past years. A detailed survey is given by Marple (1987) or by Kay (1988).

3.1.1. Model Order Selection

The order P of an AR process is not known *a priori*. Hence, there is a need for a criterion which has a minimum if the order P increases from 1 to N (N–number of samples). The simplest way is to consider the estimated prediction error variance in dependence on P

$$\hat{\sigma}_P^2 = \frac{1}{N-1} \sum_{n=1}^{N} (e_P - \bar{e}_P)^2 \tag{43}$$

The problem is that this variance decreases monotonically with increasing order. The variance cannot be a criterion for the best order. A simple check concerning proper order selection is to perform a periodogram analysis or an autocorrelation analysis of the prediction error. The error should be approximately white, so that an approximately flat diagram or a Kronecker delta should be obtained.

Many objective order selection criteria (Marple; 1987) have been developed in the past years. The application of the Final Prediction Error, Akaike's Information Criterion and the Baysian Information Criterion to the current signal has had similar results.

3.2. Classification

If the $M+1$ filters are designed by supervised learning, the diagnostic system can be used for the classification. The filtering process is done by equation (42). The main problem is now the evaluation of the residuals and finding the matched filter. The solution of this problem can be reduced to a statistical test of whiteness.

3.2.1. Tests of Whiteness

It has been mentioned before that this system can be used for quality assurance and on–line monitoring. In case of quality control the correct filter has to be chosen. If the appropriate filter has been chosen there will be zero autocorrelation in the errors. In checking the adequacy of a fit it is therefore logical to study the sample autocorrelation of the filter residuals. If the system is used for on–line monitoring a significant change of the filter residual has to be detected.

3.2.1.1. Measure of Fit in the Time Domain

An ideal white noise process has zero autocorrelation. That means all k–th autocorrelation coefficients are zero, with $k \neq 0$:

$$R_{ee}(k) = \begin{cases} \sigma_{ee}^2 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases} \tag{44}$$

Box and Pierce (1970) pointed out that if the filter was appropriate and if p parameters were estimated the function (Box–Pierce Portmanteau statistic)

$$Q := N \sum_{\tau=1}^{m} r_\tau^2, \qquad \text{with} \qquad r_\tau = \frac{R_{ee}(\tau)}{R_{ee}(0)} \tag{45}$$

would be distributed as χ^2_{m-p} for large N. Therefore an appropriate test for lack of fit is yielding, since the residuals are normally distributed. N is the length of the time series, m the number of the observed autocorrelation coefficients which are taken as $\sqrt{N}$.

Unfortunately, not all residuals comply with the condition of being normally distributed in practical operations. However, the results of Anderson and Walker (1964) and Ljung and Box (1970) show that the Gaussian distribution of the residuals is not required. Q is the variance times N of the autocorrelation sequence, but only if the expected value of the autocorrelation sequence is zero.

In the case of quality control the residuals of an appropriate filter do not have to be totally independent. This is a result of the sample deviations of the motor currents in one class. For every class the Q is computed and the classification is made by comparing the different values of the test function Q. The lower the value, the greater is the similarity of the residual to a white noise. A border of Q can be defined, so that a new class is opened if Q crosses this border at every class.

In the case of on–line monitoring there are no sample deviations, since the fault sensitive filter was generated only for one motor. Therefore, the distribution of Q can be used for the decision whether there is a fault or not, respective whether the residual is uncorrelated or not. The problem reduces the null hypothesis to testing

H_0: the residuals are uncorrelated, i.e. it is a white noise

against the alternative hypothesis

H_1: the residuals are correlated, i.e. it is not a white noise.

The null hypothesis is rejected with the level of significance α. For that purpose Q is compared with the upper α–tail of a χ^2 distribution with $m-p$ degrèes of freedom.

3.2.1.2. Measure of Fit in the Frequency Domain

There is also a possibility of testing the power spectral density (psd) of the residuals. It is well known that the psd of a white noise constantly equals one. A simple estimation of the psd is the periodogram, which can be computed by transforming the autocorrelation sequence into the frequency domain, using the Fast Fourier Transformation algorithm (FFT). The periodogram of a finite sequence of white noise always has accidental variations around one. This leads to the test of the null hypothesis

H_0: the residuals from which the periodogram is calculated are serially independent and therefore the variations of the periodogram are accidental.

Then the alternative hypothesis is

H_1: the residuals are not independent and therefore the variations of the periodogram are deterministic.

The distribution of a white noise periodogram (Schlittgen, 1991) is known as an exponential distribution with the density function $f(x) = \theta e^{-\theta x}$, but θ is unknown. Therefore, a goodness– of–fit test cannot be applied, but a transformation may be used to produce uniform observations. The result of this transformation is the accumulated periodogram S_r

$$S_r = \frac{\sum_{k=1}^{r} E(\omega_k)}{\sum_{k=1}^{M} E(\omega_k)}, \qquad \text{with} \quad M = \frac{N}{2} \tag{46}$$

where S_r is the normalized integration of the periodogram at index r and indicates the part of the spectral quantity within the first r spectral lines.

The plot S_r versus r should be close to the bisecting line of an angle in case of white noise. Beyond that, the distribution of S_r is known as uniform. There are some test statistics which can be used to detect uniform distribution. Stephens (1970) provides an overview. Here the Kolmogorow–Smirnow statistic is taken

$$C := \max_r \left| S_r - \frac{r}{M} \right| \tag{47}$$

With a given level of significance α the upper $(1-\alpha)$–tail is computed with the empirically developed function

$$c = \frac{\sqrt{-\frac{1}{2}\ln\frac{\alpha}{2}}}{\sqrt{M-1}+0,2+\frac{0,68}{\sqrt{M-1}}} - \frac{0,4}{M-1} \tag{48}$$

This test can be made graphically, so that a visual decision is possible. Two parallel lines are plotted to the bisecting line of an angle $s = x$ with the equations $s = x + c$ and $s = x - c$. The accumulated periodogram of a white noise is inside these two lines with the probability $1-\alpha$. Therefore, the null hypothesis H_0 is rejected, if the curve of S_r crosses one of the two parallel lines. Figure 10 shows the accumulated periodogram of a white noise sequence with the 5% level of significance.

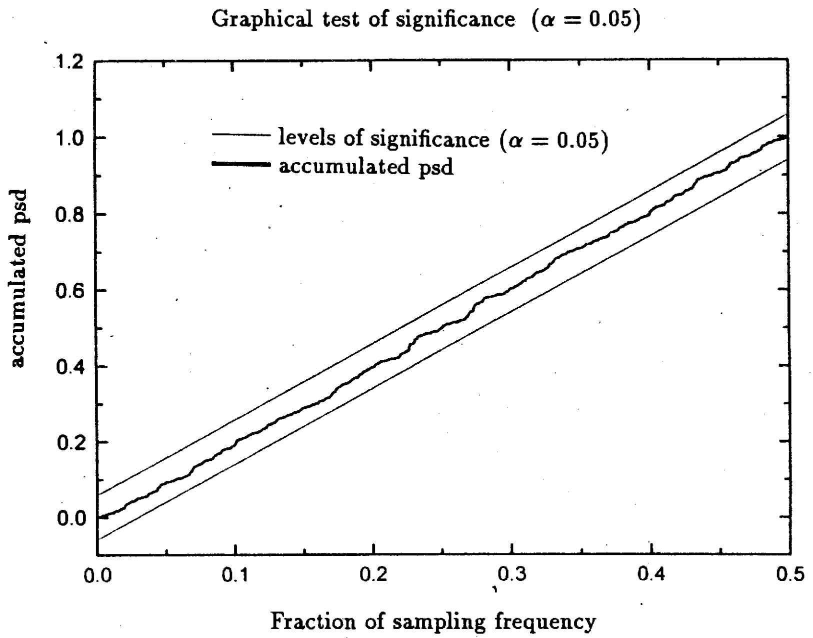

Fig. 10. Accumulated periodogram of white noise.

The statistical test is only used for on–line monitoring, as for quality control the sample deviations are too large. Instead of the statistical test, the values of C are used for the classification.

3.3. An Example of a Practical Application

An example of application for quality assurance is given in the following section. The currents of 61 mass produced universal motors were examined with the fault sensitive filter system. To design the correct filters and to compare the results, the diagnoses of human experts were used. They made their decision by hearing the noise of the motors and using simple test instruments for the following partition:

- motor has no fault (20 samples),
- motor has a strange noise (15 samples),
- motor has a disturbance voltage (11 samples),
- motor has too much vibration (15 samples).

There are two clusters with mechanical faults and one cluster with electric ones. The subjectivity of human experts is an instability factor, because it is not certain that all motors are classified by the same standards. Furthermore, the same fault may have different causes and therefore it might be wrong to put all motors with this fault into one class. A wrong noise may be caused by a vibration or by a bearing fault or something else leading to different changes of the current.

These faults were then detected by using only the current signal of a motor. The current signal analysis has the advantage, that it is a simple measurement and the results are stable. The cost of this data acquisition is low and a complicated measuring of vibration signals is not necessary.

3.3.1. Acquisition of the Current Signal

The information about the motor's condition is contained by the periodical components of the motor current. To compare the different current signals, it is necessary to sample the same number of times for each rotation. Otherwise, spectral lines containing the same information would be located at different subscripts of the spectrum. Moreover, the leakage effect is decreased when sampling during the whole number of rotations. Since all motors do not have the same speed, it is necessary to select an individual sample frequency. Therefore, the speed of every motor has to be known. With the knowledge of the number of slots, the speed can be computed using the ripple frequency peak.

If the motor is driven by alternating current, there are modulations at the speed peak and its multiples at $50Hz$ intervals. The result of these modulations are two speed peaks (and multiples) instead of one. For data acquisition a two step technique is used: With a fixed sample frequency the motor current is measured and afterwards the speed is detected. Knowing both these individual sample frequency can be computed and the individual data acquisition begins. However, it is not possible to correct variations of the speed during the data acquisition. Therefore, a data fitting is necessary.

3.3.2. Fitting Techniques in Time and Frequency Domain

There are different techniques for fitting data. It is possible to interpolate the constantly sampled current signal only to sample it afterwards with the individual sample frequency again. Another possibility are dynamic fitting methods, e.g. the dynamic time warping (DTW). In the case of a phaseshift between the different current signals the time warping is rather complicated.

Another technique fits all spectra in the frequency domain (Röpke, 1992). It is known that all speed peaks even of different spectra do not alter their position if they are sampled correctly. Therefore, it is possible to fit the spectra in order to make them comparable. Linear or dynamic functions (e.g. DTW) may be used.

3.3.3. Results

The fault sensitive filter bank generates four filter residuals for every motor–current. These residuals have to be examined to decide the correct filter as was described in Section 3.1. Figures 11–14 show the autocorrelation sequences (acs) of the four residuals.

It is obvious that the residuals of filter number two and three are correlated. However, it is not easy to decide whether the residual from filter number one or four is more uncorrelated. In Table 4 the values of the test functions (47) and (48) are given

Tabl. 4. Values of the test functions for a faultless motor.

	1. cluster: faultless	2. cluster: noise	3. cluster: voltage fault	4. cluster: vibration
Box-Pierce Portmanteau	36.29	115.12	989.83	70.40
Kolmogoroff–Smirnow	0.033	0.09688	0.264	0.063

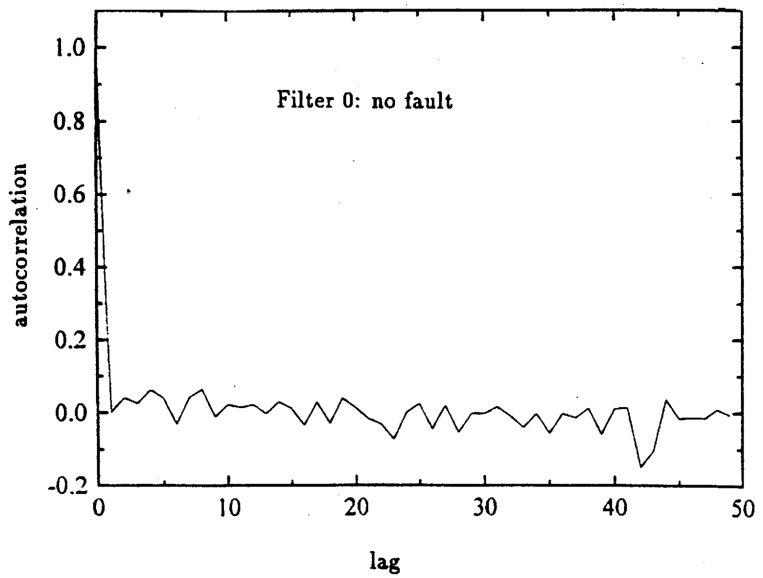

Fig. 11. Acs of the residuals – no fault.

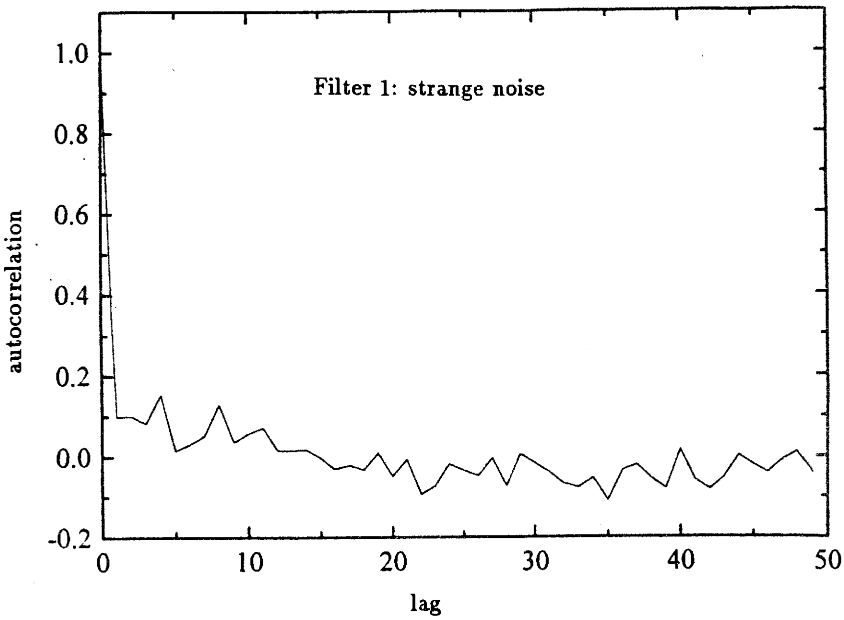

Fig. 12. Acs of the residuals – strange noise.

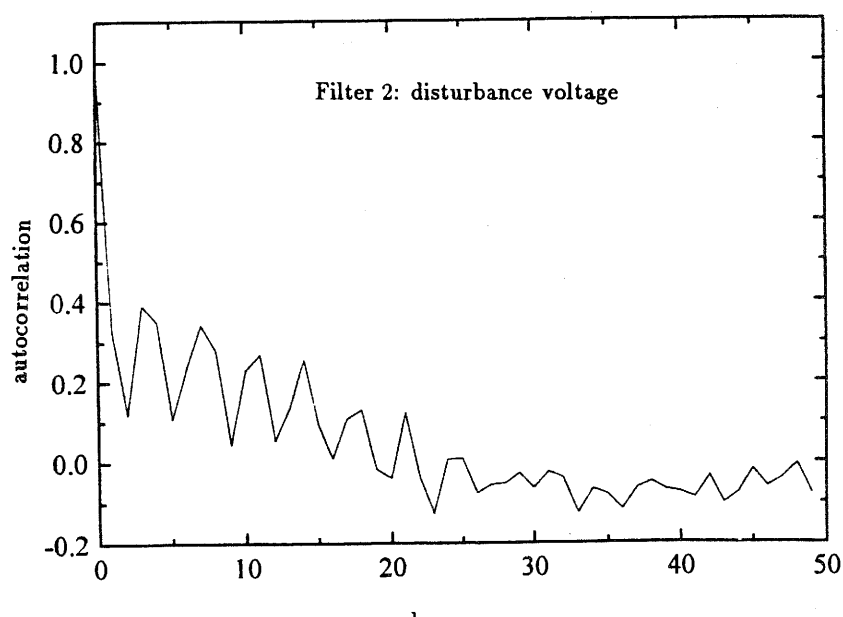

Fig. 13. Acs of the residuals – voltage fault.

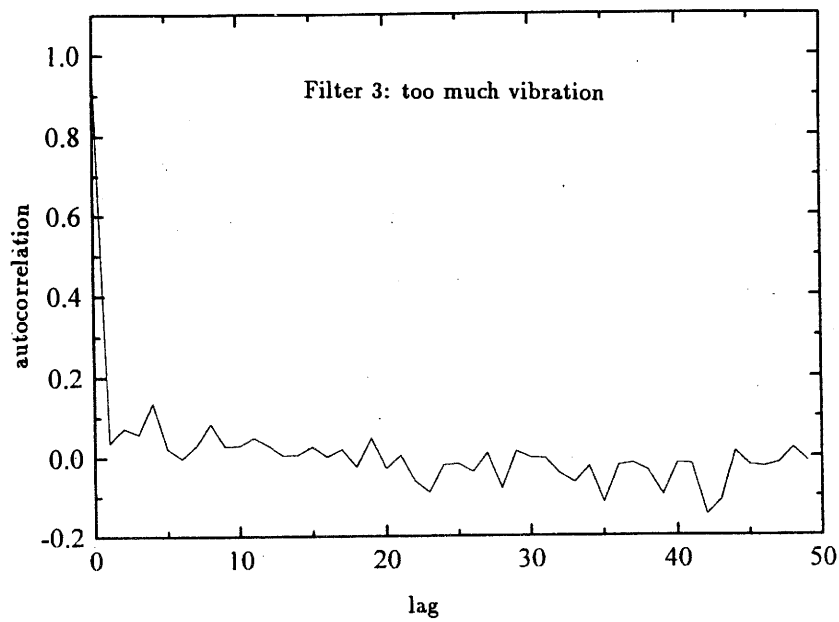

Fig. 14. Acs of the residuals – vibration.

As expected, both functions detect the "faultless" filter, because the motor is indeed faultless. Due to the lack of many objects in each class, a reclassification was performed. Table 5 shows the results. If only the "go or no-go"–decision is considered, there is a close agreement between the expert's and the system's diagnosis. 84% of the motors, which are labelled as faultless by the experts, are also regarded as faultless by the system. The agreement at the faulty motors is 92%. The complete result of the classification is shown in Table 6.

Tabl. 5. "go or no-go"–decisions from the expert and the system.

expert's diagnosis		diagnosis with fault sensitive filters	
		good	bad
good	20	16	4
bad	41	3	38

Electric faults can be easily classified. In comparison, the mechanical faults cannot be so easily classified, because the noise class and the vibration class overlap. The reason for this can be the expert diagnosis, on which the generation of the filters was based. A vibration can cause a noise and therefore the subdivision may be difficult for experts. Overlapping between the two classes results.

Tabl. 6. Comparison of the single decisions.

expert's diagnosis		diagnosis with fault sensitive filters			
		faultless	noise	voltage fault	vibration
faultless	20	16	–	–	4
noise	15	–	11	–	4
voltage fault	11	–	1	9	1
vibration	15	3	1	–	11

It can be said that the fault sensitive filter bank system gives satisfying results if the filters are generated correctly. Therefore, the problem in using fault sensitive filters is to choose correct motors to establish correct classes.

4. Final Remarks

In this paper two methods for technical diagnosis of low power electric motors were presented. The parameter estimation technique and the diagnostic technique with a fault sensitive filter bank were mathematically described and experimental results of measurements on small power motors were shown.

The algorithms for evaluation were implemented on an IBM PC (386) where the computing time was smaller than 10 seconds for both techniques. The investigations showed that these techniques described, based on analytical redundancy can be used for quality control and on–line monitoring of electric motors, because of a fast and reliable detection and isolation of faults and failures.

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