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NOISE FILTERING IN MONITORING SYSTEMS[†]

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The solution of the optimal filtering problem for technical state monitoring system of hydrodynamic journal bearing nodes of rotary machines is presented in the paper. The proposed method is based on the dynamic model of the plain journal bearing and corresponding model of variation of technical state symptoms values under supervision of the monitoring system. The Kalman filter design procedure for noise filtering is described. Such a filter enables on–line filtering and prediction of state symptoms values which are modeled by stochastic differential equations.

1. Introduction

Correct and safe operation and operating reliability of rotary machines depends considerably on the technical state of their journal bearing nodes. That is why, nowadays in the industrial practice, the monitoring of transverse vibrations of rotary machines rotors is performed by dedicated automatic monitoring systems. For high–speed rotary machines with rotor supported on plain bearings usually the level of journal vibrations in the bearing bush is estimated. Comparison of the estimated and the criterion values is the base for the technical state classification of the monitored bearing nodes of machines. The vibration measurement enabling estimation of the journal displacement trajectory is carried out with the use of two contactless eddy–current mechanical displacement transducers attached to the bearing bush. The transducers measure vibrations in two mutually perpendicular directions in the plane perpendicular to the rotor axis. Inspection of the journal locus variation relative to stable equilibrium locus areas proved to be a good symptom of failure effects arising in bearing nodes. These effects deal with change of structural parameters of the bearing like: geometrical dimensions, clearance or operational conditions like: viscosity of the lubricating agent, external load and hydromechanical forces. That in turn leads to high amplitude self–excited journal vibrations causing break of the oil-wedge what results in destruction of the bearing.

One of the basic problem in design of bearing node vibration monitoring systems is to provide effective noise filtering methods. Noise in the monitoring system arises due to the measurement noise or due to other disturbing interactions of unknown source and nature. Despite admissible machine technical state, high noise

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level may change values of estimated symptoms, so that in result the system halts the machine operation. There are many examples in the practice of monitoring systems operation of unjustifiable breaks in machines operation due to noise disturbing estimated values of technical state symptoms under supervision.

There are many methods of noise filtering used in machine health monitoring practice. These methods base (Cempel, 1989) on: avareraging of estimated symptoms values, band filtration, adaptive noise reduction and orthogonalisation of state symptoms.

The mentioned methods do not ensure the optimal noise filtering, because there are not any reliable and fully verified models of variation of estimated values of symptoms corresponding to them. That was the reason why many researchers (Batko and Banek, 1989; 1992) undertook the investigation to formulate new methods of noise filtering. These methods should base on a comprehensive description of substantial properties of technical state symptoms, their probabilistic nature and their connection with physical processes reflecting dynamic phenomena arising in bearing journal–bush couple (Barwell, 1979; Muszyńska, 1987). The aim of the research is to find low sensitive to noise methods of estimation of monitored values of symptoms variation.

The solution proposed in this paper is based on theory of optimal filtering for signals which can be modelled by stochastic differential equations in the form introduced by Kalman and Bucy (Anderson and Moore, 1979). This theory, adapted to the considered tasks, forms a base for design and application of new noise filtering systems providing optimal filtering and prediction of estimated technical state of values of symptoms. Such a system could be applied in the journal bearing technical state monitoring systems.

2. Variation Model of Technical State Symptom Values

Technical state monitoring of the hydrodynamic journal bearing of given structural and operational parameters consists in inspection of rotating journal vibrations relative to its stable equilibrium locus. When the amplitude of these vibrations exceeds the admissible value, the stable equilibrium becomes upset what is accompanied by causing self–excited vibrations of the journal. These high–amplitude vibrations often are the main cause of the machine breakdown.

In order to investigate the influence of the hydrodynamic bearing behaviour on the monitored state symptoms the dynamic model of the bearing journal–bush couple should be taken into consideration. The formulated model assumes that the journal is a rigid body moving in the pressure field of the lubricating agent. The admissible operation parameters of the oil filter and the hydrodynamic forces determination algorithm is described in detail in (Kurnik and Starczewski, 1985). Here are the main model assumptions:

- circular contour of the bearing journal and bush,
- constant oil density and temperature,

- pressure distribution in the oil clearance modelled by the simplified Reynold's equation,
- fixed bearing bush.

Distribution of forces acting on the journal according to the considered model is presented in Figure 1.

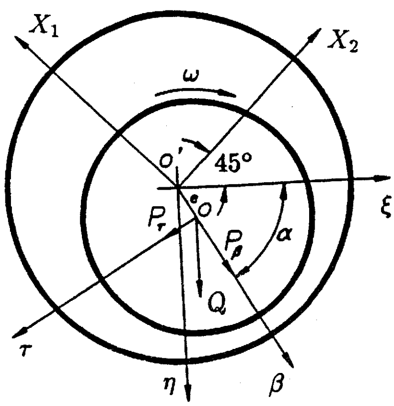

Fig. 1. Forces acting on the journal in analyzed coordinate system.

Dynamic equation of journal motion in coordinate system ξ, η is in the following form:

$$\begin{aligned} m\ddot{\xi} &= P_\beta \cos\alpha - P_\tau \sin\alpha \\ m\ddot{\eta} &= P_\beta \sin\alpha + P_\tau \cos\alpha + Q \end{aligned} \tag{1}$$

where $\ddot{\xi}$, $\ddot{\eta}$ are journal centre accelerations respectively in ξ, η directions, Q is external load, m is journal mass, α is angle between the straight line connecting the centres of journal O and bush O' and ξ axis, and $P_\beta(\omega, \beta, \dot{\beta}, \dot{\alpha})$ and $P_\tau(\omega, \beta, \dot{\beta}, \dot{\alpha})$ are components of hydrodynamic lift force in the oil–wedge, ω is journal rotational speed, β is eccentrity ratio $\beta = e/\epsilon$ ($e = OO'$ – eccentrity, $\epsilon = R_0 - R$ – radial clearance, where: R_0 is the bearing radius and R is the journal radius), $\dot{\beta}$ is journal radial speed, $\dot{\alpha}$ is journal circumferential speed.

According to the simplified integration method of the Reynold's equation, which takes advantage of decomposition of the plain journal motion into component motions, P_β, P_τ are given by the following relationship (Kurnik and Starczewski, 1985):

$$\begin{aligned} P_\beta &= -\frac{12\mu RL}{\delta^2}\left\{\frac{\beta^2(\omega - 2\dot{\alpha})}{S_1 S_2} + \frac{\beta\dot{\beta}}{S_1} + \frac{2\dot{\beta}}{S_1^{\frac{3}{2}}}\operatorname{arctg}\sqrt{\frac{1+\beta}{1-\beta}}\right\} \\ P_\tau &= \frac{6\pi\mu RL}{\delta^2}\frac{\beta(\omega - 2\dot{\alpha})}{S_1^{\frac{1}{2}} S_2} \end{aligned} \tag{2}$$

where $S_1 = 1 - \beta^2$, $S_2 = 2 + \beta^2$, μ is oil viscosity, R is journal radius, L is bearing length, and $\delta = \epsilon / R_0$ is relative clearance.

Expressing β, α, $\dot{\beta}$, $\dot{\alpha}$ in equation (1) by $\xi, \eta, \dot{\xi}, \dot{\eta}$ leads to a very complex relationship, that is why it is better to carry out the coordinate transformation to coordinates β, α according to (3)

$$\ddot{\beta} = f_\beta(\alpha, \beta, \dot{\alpha}, \dot{\beta}) \qquad \ddot{\alpha} = f_\alpha(\beta, \dot{\beta}, \alpha, \dot{\alpha}) \tag{3}$$

For stationary operation (i.e. for a given rotational speed ω and external load Q) the journal centre locus is described by coordinates values β_0, α_0 and is included in the area $\beta \leq 1$. This locus, being a function of the system parameters p_i:

$$\beta_0 = \beta_0(p_i), \qquad \alpha_0 = \alpha_0(p_i) \tag{4}$$

can be determined as follows (Muszyńska, 1987):

$$\dot{\alpha} = \dot{\beta} = \ddot{\alpha} = 0$$

$$P_{\beta|^{\dot{\alpha}=\dot{\beta}=0}_{\alpha=\alpha_0,\,\beta=\beta_0}} = -Q \sin \alpha_0 = -\frac{12\mu R L \omega \beta_0^2}{\delta^2(1-\beta_0^2)(2+\beta_0^2)}$$

$$P_{\tau|^{\dot{\alpha}=\dot{\beta}=0}_{\alpha=\alpha_0,\,\beta=\beta_0}} = Q \cos \alpha_0 = -\frac{12\mu R L \omega \beta_0}{2\delta^2(2+\beta_0^2)(1-\beta^2)^{\frac{1}{2}}}$$

Above conditions determine journal centre equilibrium loci in terms of ω, Q according to expression (5).

$$\mathrm{tg}\alpha_0 = -\frac{2\beta_0}{\pi(1-\beta_0^2)^{\frac{1}{2}}} \tag{5}$$

These loci depend on structural parameters $[\omega, \mu, R, L, m, Q]$ of system composed of a rotor and a bearing.

Diagnostic analysis of machine dynamic state variation requires consideration of the journal movement relative to the journal equilibrium locus. Let us introduce new relative coordinates:

$$x = \beta - \beta_0; \qquad y = \alpha - \alpha_0 \tag{6}$$

and linearize equation (3) in the neighbourhood of the journal equilibrium locus $(\beta_0, \alpha_0, 0, 0)$. As the result we obtain:

$$\begin{aligned} f_\beta &= \frac{\delta f_\beta}{\delta \beta} x + \frac{\delta f_\beta}{\delta \dot{\beta}} \dot{x} + \frac{\delta f_\beta}{\delta \alpha} y + \frac{\delta f_\beta}{\delta \dot{\alpha}} \dot{y} \\ f_\alpha &= \frac{\delta f_\alpha}{\delta \alpha} x + \frac{\delta f_\alpha}{\delta \dot{\alpha}} \dot{x} + \frac{\delta f_\alpha}{\delta \beta} y + \frac{\delta f_\alpha}{\delta \dot{\beta}} \dot{y} \end{aligned} \tag{7}$$

The linearized dynamic equations in terms of coordinates defined by (6) have the form of:

$$\begin{aligned} &\ddot{x} + k_{11}x + k_{12} + c_{11}\dot{x} + c_{12}\dot{y} = 0 \\ &\ddot{y} + k_{21}x + k_{22} + c_{21}\dot{x} + c_{22}\dot{y} = 0 \end{aligned} \tag{8}$$

with elasticity k_{ij} and damping c_{ij} coefficients given by formulae (9) according to (Kurnik and Starczewski, 1985):

$$\begin{aligned} k_{11} &= \frac{24\mu L\omega}{m\delta^3} \cdot \frac{\beta_0(2+\beta_0^4)}{(1-\beta_0^2)^2(2+\beta_0^2)^2}, & k_{12} &= \frac{6\pi\mu L\omega}{m\delta^3} \cdot \frac{\beta_0}{(1-\beta_0^2)^{\frac{1}{2}}(2+\beta_0^2)^2} \\ k_{21} &= -\frac{6\pi\mu L\omega}{m\delta^3} \cdot \frac{2\beta_0^4-\beta_0^2+2}{\beta_0(1-\beta_0^2)^{\frac{3}{2}}(2+\beta_0^2)^2}, & k_{22} &= \frac{12\mu L\omega}{m\delta^3} \cdot \frac{\beta_0}{(1-\beta_0^2)(2+\beta_0^2)} \\ c_{11} &= \frac{6\pi\mu L}{m\delta^3} \cdot \frac{1}{(1-\beta_0^2)^{\frac{3}{2}}}, & c_{12} &= -\frac{24\mu L}{m\delta^3} \cdot \frac{\beta_0^2}{(1-\beta_0^2)(2+\beta_0^2)} \\ c_{21} &= -\frac{12\mu L}{m\delta^3} \cdot \frac{1}{(1-\beta_0^2)}, & c_{22} &= \frac{12\pi\mu L}{m\delta^3} \cdot \frac{1}{(1-\beta_0^2)^{\frac{1}{2}}(2+\beta_0^2)} \end{aligned} \tag{9}$$

In the general case for any structure of the bearing the elasticity and damping factors might be determined with the use of the perturbation method or the generalized perturbation method algorithm developed by Kicinski, (1989, in press). The latter method consists in determination of the bearing reaction forces due to small variations of the perturbation vector components and differentiation of the determined expressions with respect to the perturbation vector. In practice it resolves itself to Taylor's series expansion in the static equilibrium locus of all the terms reflecting mutual relationship between forces and displacements (relationship between the pressure and the oil clearance shape) with respect to perturbations: $\Delta\alpha$, $\Delta\dot{\alpha}$, $\Delta\beta$, $\Delta\dot{\beta}$. Above formalization might be useful to model the technical state symptom values variations for a given monitoring system. It requires only the transformation of the considered state vector to a coordinate system determined by the directions of the monitoring system transducers axes.

3. Optimal Filtering and Prediction of Diagnostic Signals

Investigating the problem of noise minimization in monitoring systems we can consider the system as the measurement system performing the inspection of bearing journal–bush couple dynamic behaviour. The stochastic model of the monitoring system is determined by disturbances arising due to noise in the measuring circuit and some other effects not included in the model description. So, we deal with a signal composed of its useful part and disturbance part of some statistic properties. For optimal noise filtering algorithm to be used in monitoring systems the theory of Kalman–Bucy optimal filtering might be applicable. The base for formulation of the optimal filtering algorithm is the dynamic model of the bearing journal vibrations relative to equilibrium locus derived in the previous section.

The journal vibration measurement is carried out in two mutually perpendicular directions determined by space orientation of the vibration transducers (Fig. 1.) attached to the bearing bush. Let us define the measurement system with the axes (X_1, X_2) located at the axes of the vibration transducers and transform the journal vibration model into that coordinate system according to the following relationship:

$$\begin{aligned} X_1 &= -e\cos(45-\alpha) = -\frac{\sqrt{2}}{2}\varepsilon\beta(\cos\alpha+\sin\alpha) = g(\alpha,\beta) \\ X_2 &= e\sin(45-\alpha) = \frac{\sqrt{2}}{2}\varepsilon\beta(\cos\alpha-\sin\alpha) = h(\alpha,\beta) \end{aligned} \tag{10}$$

The journal centre stabile equilibrium locus (α_0, β_0) in the measurement coordinates X_1, X_2 is given by X_{10}, X_{20} values and the movement of the journal centre relative to this locus is described in the relative coordinates: $x_1 = X_1 - X_{10}$, $x_2 = X_2 - X_{20}$.
As:

$$\begin{aligned} dX_1 &= \frac{\delta g}{\delta\alpha}\,d\alpha + \frac{\delta g}{\delta\beta}\,d\beta \\ dX_2 &= \frac{\delta h}{\delta\alpha}\,d\alpha + \frac{\delta h}{\delta\beta}\,d\beta \end{aligned} \tag{11}$$

so for the small deviation of the journal centre locus from the stable equilibrium locus we obtain:

$$X_1 = \frac{1}{\beta_0}X_{10}x - X_{20}y, \qquad X_2 = \frac{1}{\beta_0}X_{20}x - X_{10}y \tag{12}$$

$$x = ax_1 + bx_2, \qquad y = cx_1 + dx_2 \tag{13}$$

where:

$$\begin{aligned} a &= \frac{\beta_0 X_{10}}{X_{10}^2 + X_{20}^2}, \qquad & b &= \frac{\beta_0 X_{10}}{X_{10}^2 + X_{20}^2} \\ c &= -\frac{X_{20}}{X_{10}^2 + X_{20}^2}, \qquad & d &= \frac{X_{10}}{X_{10}^2 + X_{20}^2} \end{aligned} \tag{14}$$

In the measurement coordinate system (X_1, X_2) the dynamic model of the journal vibrations in the bearing bush given by equation (8) takes the form:

$$\begin{aligned} \ddot{x}_1 + A_1\dot{x}_1 + B_1\dot{x}_2 + C_1x_1 + D_1x_2 &= 0 \\ \ddot{x}_2 + A_2\dot{x}_2 + B_2\dot{x}_1 + C_2x_1 + D_2x_2 &= 0 \end{aligned} \tag{15}$$

where

$$
\begin{aligned}
A_1 &= X_{20}(ac_{21} + cc_{22}) - \frac{X_{10}}{\beta_0}(ac_{11} + cc_{12}) \\
B_1 &= X_{20}(bc_{21} + dc_{22}) - \frac{X_{10}}{\beta_0}(bc_{11} + dc_{12}) \\
C_1 &= X_{20}(ak_{21} + ck_{22}) - \frac{X_{10}}{\beta_0}(ak_{11} + ck_{12}) \\
D_1 &= X_{20}(bk_{21} + dk_{22}) - \frac{X_{10}}{\beta_0}(bk_{11} + dk_{12}) \\
A_2 &= -\frac{X_{10}}{\beta_0}(bc_{11} + dc_{12}) - X_{10}(bc_{21} + dc_{22}) \\
B_2 &= -\frac{X_{10}}{\beta_0}(ac_{11} + dk_{12}) - X_{10}(ac_{21} + cc_{22}) \\
C_2 &= -\frac{X_{10}}{\beta_0}(ak_{11} + ck_{12}) - X_{10}(ak_{21} + ck_{22}) \\
D_2 &= -\frac{X_{10}}{\beta_0}(bk_{11} + dk_{12}) - X_{10}(bk_{21} + dk_{22})
\end{aligned}
\tag{16}
$$

The state space formulation of (15) in the following form:

$$
\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x_1 \\ u_1 \\ x_2 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -C_1 & -A_1 & -D_1 & -B_1 \\ 0 & 0 & 0 & 1 \\ -C_2 & -B_2 & -D_2 & -A_2 \end{pmatrix}\begin{pmatrix} x_1 \\ u_1 \\ x_2 \\ u_2 \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} \tag{17}
$$

describes variations of the monitored technical state symptom values $[x_1, u_1 = \dot{x}_1,\ x_2, u_2 = \dot{x}_2]$ accurate to some Gauss vector process $W = [w_1, w_2, w_3, w_4]^T$ of white noise properties.

$$
\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ u_1 \\ x_2 \\ u_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} \tag{18}
$$

Formula (18) describes the measurement process with addition of the vector of random measurement noise $V = [v_1, v_2, v_3, v_4]^T$ of assumed white noise properties. This formula possesses the form suitable for design of the Kalman–Bucy optimal filter (Anderson and Moore, 1979). For the partially observed state vector $\boldsymbol{X} = [x_1(t), u_1(t), x_2(t), u_2(t)]^T$ the optimal filter enables to determine the optimal estimator:

$$
\widehat{\boldsymbol{X}}_t = E(\boldsymbol{X}_t|Z_t) \tag{19}
$$

which constitutes the base for the journal bearing technical state estimation with the use of the measurement signals $[z_1(t), z_3(t)]$ reflecting the technical state. The optimal estimator is given by the following equation:

$$\mathrm{d}\widehat{\boldsymbol{X}}_t = (\boldsymbol{A} - \boldsymbol{H})\widehat{\boldsymbol{X}}_t \,\mathrm{d}t + \boldsymbol{R}\boldsymbol{H}^T \,\mathrm{d}Z \tag{20}$$

where $\boldsymbol{R}(t)$ is the Riccati's equation solution:

$$\dot{\boldsymbol{R}} = \boldsymbol{A}\boldsymbol{R} + \boldsymbol{R}\boldsymbol{A}^T - \boldsymbol{R}\boldsymbol{H}\boldsymbol{R} \tag{21}$$

with $\boldsymbol{R}(0) = \boldsymbol{R}_0 = E[\boldsymbol{X}_0 - E(\boldsymbol{X}_0)][\boldsymbol{X}_0 - E(\boldsymbol{X}_0)]^T$ being a covariance matrix of the state vector X_1 estimation error for the time moment $t = 0$. The components of the estimator equation:

$$\begin{aligned}
\mathrm{d}\widehat{x}_1 &= (-\widehat{x}_1 + \widehat{u}_1)\,\mathrm{d}t + r_{11}\,\mathrm{d}z_1 + r_{13}\,\mathrm{d}z_3 \\
\mathrm{d}\widehat{u}_1 &= (-C_1 - \widehat{x}_1 - A_1\widehat{u}_1 - D_1\widehat{x}_2 - B_1\widehat{u}_2)\,\mathrm{d}t + r_{21}\,\mathrm{d}z_1 + r_{23}\,\mathrm{d}z_3 \\
\mathrm{d}\widehat{x}_2 &= (-\widehat{x}_2 - \widehat{u}_2)\,\mathrm{d}t + r_{31}\,\mathrm{d}z_1 + r_{33}\,\mathrm{d}z_3 \\
\mathrm{d}\widehat{u}_2 &= (-C_2 - \widehat{x}_1 - B_2\widehat{u}_1 - D_2\widehat{x}_2 - A_2\widehat{u}_2)\,\mathrm{d}t + r_{41}\,\mathrm{d}z_1 + r_{43}\,\mathrm{d}z_3
\end{aligned} \tag{22}$$

describe a design procedure of the optimal filter for monitored signals $[x_1, x_2]$. The system of equations (22), in which r_{ij} are the solutions of equation (21) and (z_1, z_3) are journal centre locus variations determined during measurement enables on–line estimation of $[x_1, x_2]$ value variation. Recurrent form of the filter formulation suitable for applications can be derived from equation (22). Moreover, the solution of (20), (21), (22) gives the possibility of prediction of the technical state basing on filtered monitored signals. We can find the prediction state vector:

$$\boldsymbol{X}(t,s) = [x_1(t,s),\ u_1(t,s),\ x_2(t,s),\ u_2(t,s)]^T$$

where $0 \leq s \leq t$, $[0, s]$ is the fixed observation interval, and $[0, t-s]$ is the prediction horizon. For the case where s is fixed and t varies the predicted states are given by the equation:

$$\frac{\mathrm{d}\boldsymbol{X}(t,s)}{\mathrm{d}t} = \boldsymbol{A}\boldsymbol{X}(t,s) \tag{23}$$

with initial conditions:

$$\boldsymbol{X}(s,s) = \widehat{\boldsymbol{X}}_s \tag{24}$$

where $\boldsymbol{X}_s$ is the output from the Kalman filter.

4. Concluding Remarks

The proposed method of noise filtering in hydrostatic journal bearing vibration monitoring system differs considerably from commonly used in machine health monitoring practice. The method refers to the current comprehensive research and model formulation of dynamics of supported on plane journal bearing rotors. It takes advantage of mathematical formulation of the optimal filtering and prediction of signals modelled by stochastic differential equations. This method opens new research areas for investigation of monitored diagnostic signals filtering and prediction methods for the particular system under consideration. The proposed optimal filtering design procedure might be easily applied in practice.

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