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STATIC AND QUASI–STATIC ROUTING IN INTEGRATED SERVICES DIGITAL NETWORKS[†]

ADAM GRZECH[*], ANDRZEJ KASPRZAK[*]

The aim of this paper is to formulate, discuss and solve some selected problems concerning routings problems in the Integrated Services Digital Network. The problems under consideration are restricted to routes in the ISDN over which both voice calls and data streams are transmitted using circuit– and packet–switched transmission modes. In the paper several optimization problems for static and quasi–static routing are formulated and considered. Solutions of the formulated optimization problems allowed us to design a routing procedures at the ISDN. The procedures are distributed, i.e. at each node of the network, routing variables are assigned independently both for the traffic classes considered and transmission modes applied.

1. Introduction

The concept of the *Integrated Services Digital Network* (ISDN) is a single network that provides universal facilities among many different types of traffic. An efficient ISDN uses a common transmission system for all types of traffic and allows for a standard access to the network for all different types of terminal equipment that it has to serve. The common transmission system is obtained by integrated access, digital transmission and digital switching. Integration within the network can have different meanings, depending on the part of network being considered (Bonatti *et al.*, 1991; Pujolle *et al.*, 1988).

ISDN offers a single network for providing all communication services in order to achieve the economy of sharing. This economy motivates the general idea of an integrated services network. Integration obviates the need for many overlaying networks, which complicate network management and make the introduction and evaluation of services inflexible.

Most of the theoretical and technical problems which occur in the ISDN may be illustrated by considering voice/data integration. This is because the two classes of traffic are representative of interactive and non–interactive traffics, respectively (Sarch, 1976).

In this paper a heterogeneous approach is discussed; both classes of traffic under consideration (it is easy to expand the analysis for more classes) are transmitted simultaneously.

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[*] Institute of Control and Systems Engineering, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland

The advantage of such an approach is that (Sarch, 1976):

- such a network secures potentially the lowest cost at the best performance, since global optimization can be performed for the entire network,
- innovative communication services can be employed wherever they are appropriate,
- consolidating the total pool of communication resources in one network that is available to any subscriber enhances reliability.

The disadvantages of this network are as follows:

- such a network would not be simple to operate, monitor and control,
- routing and congestion control might be difficult, since heterogeneity leads to greater possibilities of congestion wherever traffic flows encounter restrictive circuits or switches.

Integration of voice and data (bulk and/or interactive) is provided by the hybrid–switched multiplexing scheme. This scheme assures both circuit– and packet–switching in the same transmission link by the use of special time–division multiplexing scheme. Transmission consists of fixed–duration frames (composed of a number of equal length time slots) that have two compartments: one part dedicated to voice traffic and the other part dedicated to data traffic. The general idea of compartments in frame is utilized in various versions: with fixed or movable boundaries (Grzech and Kasprzak, 1993; Pujolle *et al.*, 1988).

In this paper it is assumed that the ISDN is used to serve two classes of traffic: voice calls and data. The voice calls are served utilizing circuit–switched technique while the data are transmitted using packet–switched or circuit–switched or both techniques simultaneously. At each digital transmission channel of the network transmission frames are organized. The lengths of frames, equal for all frames, define channels capacities. The frames (i.e. channel capacities) are divided into two compartments: the first is utilized to voice calls or data transmission using circuit–switched while the second is applied to transmit data using packet–switched mode.

Efficient control of voice/data calls transmission in the ISDN requires proper performance measures. For the purpose of analysis provided in this paper, combined performance measures are applied; the quality of voice calls transmission is estimated by blocking probability while the quality of data transmission is measured using average delay per packet (Kleinrock, 1975; 1976). Based on the assumed performance measure, problems connected with flows of voice calls and data streams on route at the ISDN are considered. Moreover, problems of simultaneous flow control and transmission frame division in such networks are also investigated.

Analysis of flows on the route in the ISDN is performed in quasi–static and static cases. The aim of this analysis is to formulate guide lines for the design of routing procedure in the ISDN.

The following flow optimization problems are formulated and considered in this paper:

- for static routing:
  - assignment of distribution of data stream between two transmission modes assuming fixed lengths of frame compartments in each channel of the considered route at the ISDN,
  - simultaneous assignment of distribution of data stream between two transmission modes and division of transmission frame into compartments in each channel of the considered route at the ISDN,
- and for quasi–static routing:
  - assignment of routing variables at any node of the ISDN under assumption that the division of transmission frames is known,
  - simultaneous assignment of routing variables at each node of the ISDN and division of transmission frames in all channels outgoing from the node.

Solutions to the above–mentioned flow optimization problems allowed us to design a routing procedure at the ISDN. The procedure is a distributed one, i.e. at each node of the network, independently of the others nodes, routing variables are assigned both for voice calls and data streams.

2. Voice Calls Blocking Probability Versus Voice Calls Stream Rate

Let consider a route at the ISDN consisting of n digital transmission channels. It is assumed that nodes and transmission channels of the route are numbered sequentially from 1 to $n+1$ and from 1 to n, respectively.

To measure quality of voice calls transmission at the considered route, a voice calls blocking probability is applied. It was shown that it is possible to obtain optimal voice calls blocking probability at the route of ISDN by solving set of state equations. It was also proved that for a route composed of more than two transmission channels, the above–mentioned procedure is impractical because the number of necessary calculations is too large (Grzech *et al.*, 1992). A more convenient solution is to apply a formula which approximates the optimal voice calls blocking probability. Such a solution is also presented in the literature. In (Grzech and Kasprzak, 1993) an analytical formula has been used to obtain voice calls blocking probability depending on the size of transmission frame's compartment reserved for voice calls transmission and on average rate of voice calls stream flowing from the first node to the $(n+1)$–th node of the route. The formula is as follows:

$$P(m_1^c, m_2^c, \dots, m_n^c, \overline{\lambda}^c) = a + \sum_{i=1}^{n} \frac{b_i \overline{\lambda}^c + b_i'}{m_i^c + c_i \overline{\lambda}^c + c_i'} \tag{1}$$

where a, b_i, b_i', c_i, c_i' $(i = 1, 2, \dots, n)$ are constant coefficients; $\overline{\lambda}^c$ – average rate of voice calls stream flowing from the first node to the $(n+1)$–th node of the route; m_i^c – a number of time slots at the transmission frame of the i–th transmission channel, reserved for voice calls transmission using circuit–switched transmission mode.

In (Gerla and Mason, 1978) the following analytical expression for voice calls blocking probability (denoted by P_G), being a function of $m_1^c, m_2^c, \ldots, m_n^c$ and of average rates of voice calls substreams flowing at each transmission channel of the route, has been proposed:

$$P_G = k \sum_{i=1}^{n} \frac{\left((1/\mu)\overline{\lambda}_i^c\right)^2}{\phi m_i^c - (1/\mu)\overline{\lambda}_i^c} \tag{2}$$

where k is constant coefficient, ϕ – length of time slot [bit/slot], $\overline{\lambda}_i^c$ – average rate of voice calls substream flowing at the i–th transmission channel of the route [voice call/sec], $1/\mu$ – average length of the voice calls [bit/voice call].

Analyzing expression (2) it is worth noting that each substream, flowing through transmission channels of the route, contains the stream flowing from the first node to the last node of the route, i.e. $\overline{\lambda}^c \leq \overline{\lambda}_i^c$ for $i = 1, 2, \ldots, n$.

The comparison of results given by expressions (1) and (2) with optimal voice calls blocking probability has been performed in (Grzech and Kasprzak, 1993). The results given by expressions (1) and (2) differ from optimal by 24% and 85% on the average, respectively. This means that the first expression gives better approximation of the voice calls blocking probability than the second one.

3. Performance Measure for Simultaneous Transmission of Voice and Data at the Route of ISDN

In the paper it is assumed that two classes of traffic are served. Voice calls are transmitted using circuit–switched technique, while data may be transferred applying circuit– or packet–switched modes. Blocking probability and average delay per packet are performance measures which are applied to measure the quality of delivered transmission services when the circuit– and packet–switched transmission modes are used, respectively. The goal of flow control in the case under discussion is to minimize both of them. It is always a trade–off between the two performance measures. For further analysis a global performance measure that combines the aforenamed is used: weighted sum of customers blocking probability and average delay per packet times the number of all transmitted packets. The customers blocking probability may be approximated by expressions (1) or (2), while the average delay per packet is given by well–known Kleinrock's formula (Kleinrock, 1964).

Assuming expression (1) as customers blocking probability approximation, the global performance measure of transmission quality at the route of ISDN – denoted by Q – is given as follows:

$$Q = \alpha P(m_1^c, m_2^c, \ldots, m_n^c, \overline{\lambda}^c) + \sum_{i=1}^{n} \frac{\overline{\lambda}_i^p}{\phi m_i^p - \overline{\lambda}_i^p} \tag{3}$$

where α is a weighted coefficient, $\overline{\lambda}_i^p$ – an average rate of total data stream transmitted through the i–th transmission channel using packet–switched mode [bit/sec], m_i^p – a number of time slots at the frame of i–th transmission channel reserved for data transmission using packet–switched mode.

A similar expression for global performance measure may be obtained when, instead of expression (1), the approximation given by (2) is applied.

4. Static Routing

4.1. Distribution of Data Stream Between Two Transmission Modes Assuming Fixed Lengths of Frame Compartments

Let us assume that:

- the data stream arriving at the node of ISDN should be sent to destination node via the assumed node route,
- data may be sent using circuit–switched mode or packet–switched mode or both of them simultaneously.

For further analysis it is also assumed that the transmission frames organized at each channel of the networks are divided into two fixed–length compartments. The compartments are applied to serve the two transmission modes mentioned. It is natural to note that the gain of flow control in such a case is to divide the incoming data stream into two substreams: served using circuit– and packet–switched modes. The global performance measure applied to obtain the incoming stream division is given by expression (3). The following notation is used to formulate corresponding optimization problem:

λ^d – an average rate of incoming data stream which should be transmitted via the route [bit/sec],

λ – an average rate of incoming data substream which is transmitted via the route using circuit–switched transmission mode [bit/sec],

λ^c – an average rate of voice calls substream which is transmitted via the route using packet–switched transmission mode [voice call/sec],

λ_i^p – an average rate of data stream which is transmitted in the i–th transmission channel of the route using packet–switched transmission mode [bit/sec]; this stream does not contain incoming substream described by average rate $(\lambda^d - \lambda)$.

It is easy to note that $\overline{\lambda}_i^p = (\lambda^d - \lambda) + \lambda_i^p$ for $i = 1, 2, \ldots, n$ and that $\overline{\lambda}^c = \mu\lambda + \lambda^c$. The global performance measure for the problem is obtained by substituting these expressions into (3). The optimization problem is formulated as follows:

Given

$$a,\ \phi,\ \alpha,\ n,\ 1/\mu,\ \lambda^c,\ \lambda^d,\ m_i^c,\ m_i^p,\ \lambda_i^p,\ b_i,\ b_i',\ c_i,\ c_i'\ \ for\ \ i = 1, 2, \ldots, n$$

minimize

$$Q = \alpha \left(a + \sum_{i=1}^{n} \frac{b_i(\mu\lambda + \lambda^c) + b_i'}{m_i^c + c_i(\mu\lambda + \lambda^c) + c_i'} \right) + \sum_{i=1}^{n} \frac{\lambda^d - \lambda + \lambda_i^p}{\phi m_i^p - (\lambda^d - \lambda + \lambda_i^p)} \tag{4}$$

with respect to design variable λ, *subject to*

$$\lambda - \lambda^d \le 0 \tag{5}$$

$$(\lambda^d - \lambda + \lambda_i^p) - \phi m_i^p \le 0 \qquad for \quad i = 1, 2, \ldots, n \tag{6}$$

$$\lambda \ge 0 \tag{7}$$

The total performance measure, given by (4), being a sum of strictly concave functions is a strictly concave function of λ. Constraints (5)–(7) are linear, so there is exactly one solution to problem (4)–(7) (Findeisen *et al.*, 1977; Zangwill, 1969). Analysis of the constraints (5)–(7) suggests that the minimum of expression (4) over λ is within an interval $[\widehat{\lambda}, \lambda^d]$, where

$$\widehat{\lambda} = \max\{\widehat{\lambda}_i, \widehat{\lambda}_2, ..., \widehat{\lambda}_n\}, \qquad \widehat{\lambda}_i = \max\{0, \lambda^d + \lambda_i^p - \phi m_i^p\}$$

To solve the optimization problem (4)–(7), the Golden Section method (Zangwill, 1969) may be applied leading to obtaining a minimum value of function (4) for $\lambda \in [\widehat{\lambda}, \lambda^d]$.

A similar procedure allows us to find a solution of the optimization problem if the voice calls blocking probability in the global performance measure is given by expression (2).

4.2. Simultaneous Distribution of Data Stream Between Transmission Modes and Division of Frame

Let us assume that the division of transmission frame at each transmission channel into two compartments (assigned for circuit– and packet–switched transmission modes, respectively) depends on average rates of voice calls and data streams served via the route under consideration. Moreover, it is assumed that data may be served by circuit–switched or packet–switched modes or by both of them simultaneously.

The optimization problem discussed here is to obtain simultaneously:
- division of transmission frame at each transmission channel into two compartments, and
- partition of data stream into two substreams served by the two transmission modes.

Two different optimization problems are considered. In the first one, the voice blocking probability in the performance measure Q is given by expression (1), while in the other by expression (2).

Problem 1. *Given:*

$$a,\ \alpha,\ \phi,\ n,\ 1/\mu,\ \lambda^c,\ \lambda^d,\ \lambda_i^p,\ m_i,\ b_i,\ b_i',\ c_i,\ c_i' \ \ for\ \ i = 1,2,...,n,$$

minimize the performance measure Q given by expression (4) *with respect to design variables*

$$\lambda,\ m_i^p,\ m_i^c\ (i = 1,2,...,n)$$

subject to (5), (6), (7) *and*

$$m_i^p + m_i^c = m_i \qquad for \quad i = 1,2,...n \tag{8}$$

$$m_i^c \geq 0 \qquad for \quad i = 1,2,...,n \tag{9}$$

$$m_i^p \geq 0 \qquad for \quad i = 1,2,...,n \tag{10}$$

where m_i is a number of time slots in the transmission frame at the i–th transmission channel.

Substituting m_i^p by expression $m_i - m_i^c$ (obtained from (8)) in expressions (4) and (8), and replacing constraint (8) by the following one

$$m_i^c - m_i \leq 0 \qquad \text{for} \quad i = 1, 2, ..., n \tag{11}$$

the obtained optimization problem does not contain variables m_i^p $(i = 1, 2, ..., n)$. Consequently, the minimization may be performed towards to λ and m_i^c $(i = 1, 2, ..., n)$. Then, the values m_i^p $(i = 1, 2, ..., n)$ may be obtained from expression (8).

The problem may be solved in the following way: minimization is carried out first with respect to m_i^c $(i = 1, 2, ..., n)$, keeping λ fixed, and then with respect to λ.

To solve the optimization problem, described by (4), (6) and (9)–(11), with respect to variable m_i^c, the following Kuhn–Tucker optimality conditions are utilized:

$$-\alpha \frac{b_i(\mu\lambda + \lambda^c) + b_i'}{\left(m_i^c + c_i(\mu\lambda + \lambda^c) + c_i'\right)^2} + \frac{\phi(\lambda^d - \lambda + \lambda_i^p)}{\left(\phi(m_i - m_i^c) - (\lambda^d - \lambda + \lambda_i^p)\right)^2} + \psi_i^1 + \psi_i^2 = 0 \qquad \text{for} \quad i = 1, 2, ..., n$$

$$\left((\lambda^d - \lambda + \lambda_i^p) - \phi(m_i - m_i^c)\right)\psi_i^1 = 0 \qquad \text{for} \quad i = 1, 2, ..., n$$

$$(m_i^c - m_i)\psi_i^2 = 0 \qquad \text{for} \quad i = 1, 2, ..., n$$

where ψ_i^1, ψ_i^2 $(i = 1, 2, ..., n)$ are Lagrange multipliers.
Performing a proper transformation, the following expression is obtained:

$$m_i^c = \frac{(\phi m_i - \lambda^d + \lambda - \lambda_i^p)\sqrt{\alpha\left(b_i(\mu\lambda + \lambda^c) + b_i'\right)} - \left(c_i(\mu\lambda + \lambda^c) + c_i'\right)\sqrt{\phi(\lambda^d - \lambda + \lambda_i^p)}}{\phi\sqrt{\alpha\left(b_i(\mu\lambda + \lambda^c) + b_i'\right)} + \sqrt{\phi(\lambda^d - \lambda + \lambda_i^p)}}$$

By substituting the above expression into the assumed performance measure (4), the following problem should be solved with respect to variable λ:

Given

$$a,\ \alpha,\ \phi,\ n,\ 1/\mu,\ \lambda^c,\ \lambda^d,\ \lambda_i^p,\ m_i,\ b_i,\ b_i',\ c_i,\ c_i' \ \textit{for}\ \ i = 1, 2, ..., n$$

minimize

$$Q_1 = a + \sum_{i=1}^{n} \frac{\left(\sqrt{\alpha\phi\left(b_i(\mu\lambda + \lambda^c) + b_i'\right)} + \sqrt{\lambda^d - \lambda + \lambda_i^p}\right)^2}{\phi m_i - \lambda^d + \lambda - \lambda_i^p + \phi c_i(\mu\lambda + \lambda^c) + \phi c_i'} \tag{12}$$

with respect to design variable λ, subject to

$$0 \leq \lambda \leq \lambda^d \tag{13}$$

Function (12) is not concave with respect to variable λ; it is a sum of quasi–concave functions. Therefore, it is possible to solve problems (12) and (13) using, for example, numerical methods to obtain a local minimum. Then, the proper solution, assuring a minimum value of (12) is selected.

Problem 2. *Given*

$$\alpha,\ \phi,\ n,\ 1/\mu,\ \lambda^d,\ \lambda_i^c,\ \lambda_i^p,\ m_i \ \ for \ \ i = 1, 2, ..., n,$$

minimize:

$$Q_2 = \alpha k \sum_{i=1}^{n} \frac{((1/\mu)\lambda_i^c + \lambda)^2}{\phi m_i^c - ((1/\mu)\lambda_i^c + \lambda)} + \sum_{i=1}^{n} \frac{(\lambda^d - \lambda) + \lambda_i^p}{\phi(m_i - m_i^c) - (\lambda^d - \lambda + \lambda_i^p)} \tag{14}$$

with respect to design variables λ, m_i^c $(i = 1, 2, ..., n)$, *subject to* (5)–(7), (9), (11) *and*

$$(1/\mu)\lambda_i^c + \lambda - \phi m_i^c \leq 0 \qquad for \quad i = 1, 2, ..., n \tag{15}$$

In this problem, similarly as in Problem 1, variables m_i^p are not taken into account. The variables m_i^p are easily obtainable from expression (8) if values of m_i^c variables are known. The latter problem is solved likewise the previous one.

From Kuhn–Tucker optimality conditions the following is obtained:

$$m_i^c = \frac{((1/\mu)\lambda_i^c + \lambda)\left(\phi\sqrt{\alpha k} m_i - \sqrt{\alpha k}(\lambda^d - \lambda + \lambda_i^p) + \sqrt{\lambda^d - \lambda + \lambda_i^p}\right)}{\phi\left(\sqrt{\alpha k}\left((1/\mu)\lambda_i^c + \lambda\right) + \sqrt{\lambda^d - \lambda + \lambda_i^p}\right)}$$

By substituting the above expression into (14), the following problem should be solved on the variable λ:

Given

$$\alpha,\ \phi,\ n,\ 1/\mu,\ \lambda^d,\ \lambda_i^c,\ \lambda_i^p,\ m_i \ \ for \ \ i = 1, 2, ..., n,$$

minimize

$$Q_3 = \sum_{i=1}^{n} \frac{\left(\sqrt{\alpha k}((1/\mu)\lambda_i^c + \lambda) + \sqrt{\lambda^d - \lambda + \lambda_i^p}\right)^2}{\phi m_i - (1/\mu)\lambda_i^c - \lambda^d - \lambda_i^p} \tag{16}$$

with respect to design variable λ, *subject to* (13).

The above task may be solved applying one of the well–known numerical algorithm (Findeisen *et al.*, 1977; Zangwill, 1969).

5. Quasi–Static Routing

In this section, based on quasi–static analysis, a new procedure of flow control in Integrated Services Digital Network is proposed. For the purpose of analysis, the transmission channel between the i–th and the k–th nodes is denoted by $\langle i, k \rangle$. To evaluate the quality of transmission the previously introduced performance measure

Q, described by expression (3), is utilized. The introduced and discussed flow control procedure when applied should minimize value of thc assumed performance measure.

The flow control procedure is associated at each network's node with routing variables. The latter indicate routes which should be selected for voice calls or data transmission between origin and destination nodes. Let $z(\cdot)$ denote routing variables used for flow control of data packets stream, while $w(\cdot)$ denotes routing variables for construction of routes for voice calls stream transmission. It is assumed that data packets should be sent using different routes between origin and destination nodes (datagram service), while the voice calls are sent using one and only one route.

The value of variable $z_{ik}(j)$ describes the part of the stream originated at the i–th node and destined at the j–th node which is sent via transmission channel $\langle i,k\rangle$.

The variable $w_{ik}(j)$ is described as follows:

$$w_{ik}(j) = \begin{cases} 1, & \text{if the transmission channel } \langle i,k\rangle \text{ belongs to the route connecting the } i\text{–th node with the } j\text{–th node when the route is utilized to transmit customers calls between the } i\text{–th and the } j\text{–th nodes,} \\ 0, & \text{otherwise} \end{cases}$$

The variables $z(\cdot)$ and $w(\cdot)$, as variables applied in the flow control procedures, satisfy the following conditions:

i) $z_{ik}(j) > 0,\ \ w_{ik}(j) \in \{0,1\}$ for $i,j,k = 1,2,...,N,$

ii) $z_{ik}(j) = 0,\ \ w_{ik}(j) = 0$ if $i = j$ or $\langle i,k\rangle \notin L,$

iii) $\sum_{k\in A(i)} z_{ik}(j) = 1,\ \ \sum_{k\in A(i)} w_{ik}(j) = 1$ for $i \neq j$ and $i,j = 1,2,...,N,$

iv) between each pair of nodes there is a route which may be used to transfer data or voice calls, i.e. for every pair of i and j nodes there exists a sequence of nodes $i,k,l,...,s,j$ satisfying the conditions below:
- $z_{ik}(j) > 0,\ \ z_{kl}(j) > 0,\ \dots, z_{sj}(j) > 0,$
- $w_{ik}(j) = 1, w_{kl}(j) = 1,\ \dots, w_{sj}(j) = 1,$

where L is a set of all transmission channels in the network, N – a number of all nodes in the network and $A(i) = \{\, j : \langle i,j\rangle \in L\,\}$, i.e. a set of all nodes which are directly connected from the i–th node (adjacent to the i–th node).

Let $\lambda^c(i,j)$ and $\lambda^d(i,j)$ be rates of voice calls and data streams transferred between the i–th and the j–th nodes, respectively.

Let us consider the network states at t_p and $t_p + \Delta t$ times. For the purpose of analysis it is assumed that, during the period of time $[t_p, t_p + \Delta t]$, the rates of voice calls and data are changed from $\lambda^c(i,j)$ by $\delta\lambda^c(i,j)$ and from $\lambda^d(i,j)$ by $\delta\lambda^d(i,j)$, respectively. Changes introduced in the traffic influence changes in flows at the network. To propose some flow control procedures which minimize the introduced performance measure Q in the assumed environment, the routing variables $z(\cdot)$

and $w(\cdot)$ – designed to control the flow of voice calls and data streams – should be evaluated. The routing variables $z(\cdot)$ and $w(\cdot)$ are evaluated step by step to follow the changes in rates of incoming voice calls and data streams. The new evaluated routing variables are valid for the period of time Δt.

Evaluation of routing variables means realization of adaptive routing procedure at the ISDN, similarly as at wide area computer networks. In the latter values of the routing variables are modificated with fixed time–step Δt.

The routing variables are modified only if the incoming streams are increased. If the streams decreased, i.e. if $\delta\lambda^c(i,j) \leq 0$ or $\delta\lambda^d(i,j) \leq 0$ during the time period $[t_p, t_p + \Delta t]$, the routing variable $z_{ik}(j)$ or $w_{ik}(j)$ for $k \in A(i)$ are not modified at the time t_p.

Under the circumstances described above, the performance measure Q may be represented as a sum of components $Q(i,k)$ $(i = 1, 2, ..., n,\ k = i+1, ..., n+1)$. Moreover, $Q(i,k)$ is a sum of the appropriate components of voice calls blocking probability $P(i,k)$ and average delay per packet times the number of packets $T(i,k)$; components which correspond to the $\langle i,k\rangle$ transmission channel. If the voice calls blocking probability is approximated by expression (1), the constant a may be treated as a sum of constants $a(i,k)$ which correspond to $\langle i,k\rangle$ transmission channels.

It is easy to observe that each increase in stream rates, $\lambda^c(i,j)$ and $\lambda^d(i,j)$ affect the value of the performance measure Q.

Based on results which are valid for wide area computer network (Gallager, 1977; Kasprzak, 1989), the following should be put down:

$$\frac{\partial Q}{\partial \lambda^d(i,j)} = \sum_{k \in A(i)} z_{ik}(j) \left(\frac{\partial Q(i,k)}{\partial \lambda^d_{ik}} + \frac{\partial Q}{\partial \lambda^d(k,j)} \right) \tag{17}$$

where λ^d_{ik} denotes the rate of total data stream transmitted through the transmission channel $\langle i,k\rangle$.

Therefore, a small increase of rate, i.e. $\delta\lambda^d(i,j)$ influences a small increase of performance measure Q namely δQ. The increase δQ may be represented as below (Gallager, 1977; Kasprzak, 1989):

$$\partial Q \approx \delta\lambda^d(i,j) \frac{\partial Q}{\partial \lambda^d(i,j)} \tag{18}$$

Similarly, based on results presented in (Kasprzak, 1989), the following holds for voice calls streams:

$$\frac{\partial Q}{\partial \lambda^c(i,j)} = \sum_{k \in A(i)} w_{ik}(j) \left(\frac{\partial Q(i,k)}{\partial \lambda^c_{ik}} + \frac{\partial Q}{\partial \lambda^c(k,j)} \right) \tag{19}$$

$$\partial Q \approx \delta\lambda^c(i,j) \frac{\partial Q}{\partial \lambda^c(i,j)} \tag{20}$$

where λ^c_{ik} is the rate of voice calls stream transmitted through the transmission channel $\langle i,k\rangle$.

If the voice calls blocking probability is approximated and given by expression (1), then $\lambda_{ik}^c = \lambda^c(i,j)$ for each $\langle i,k \rangle$ transmission channel belonging to the respective route.

Voice calls and/or data incoming streams, arriving from outside to any node of ISDN should be transmitted over some selected routes to their destinations. It means that at each node a proper decision about the routes is necessary. As it was mentioned earlier, the decision is made based on values of the selected performance measure. It is assumed that the routes for the voice calls and data incoming before t_p time are known. The goal is to evaluate the routes for voice calls and data incoming during the period $[t_p, t_p + \Delta t]$. Such an assumption allows us to minimize the increase in the performance measure Q for increased streams, instead of minimizing the performance measure Q. The routing procedure is obtained based on expressions (17)–(20). The procedure is quasi-static because the applied expressions were obtained based on quasi-static analysis.

The idea of quasi-static routing procedure is as follows. At proper instants of time: $t_1, t_2, ..., t_{p-1}, t_p, t_{p+1}, ...,$ where $t_i - t_{i-1} = \Delta t$ $(i = 2, 3, ..., p-1, p, p+1, ...)$, the routing variables $z(\cdot)$ and $w(\cdot)$ are modified taking into account increments of streams $\lambda^c(\cdot,\cdot)$ and $\lambda^d(\cdot,\cdot)$, respectively. The aim of respective routing variables modification is to obtain a minimum increase of the performance measure value.

Below, a general idea of variables modification at each node of the ISDN is proposed:

Step 1. At instants t_p, the i-th node sends values of derivatives $\partial Q / \partial \lambda^d(i,j)$ and $\partial Q / \partial \lambda^c(i,j)$ $(j = 1, 2, ..., N)$ to all adjacent nodes (i.e. to nodes which belong to the set $A(i)$). The derivatives are computed at instant of time t_{p-1}. It is assumed that

$$\frac{\partial Q}{\partial \lambda^d(i,i)} = \frac{\partial Q}{\partial \lambda^c(i,i)} = 0$$

Step 2. At the i-th node, after receiving (at instant t_p) the derivatives $\partial Q / \partial \lambda^d(k,j)$ and $\partial Q / \lambda^c(k,j)$ for $j = 1, 2, ..., N$ and $k \in A(i)$ (computed at t_{p-1} instant of time) from all adjacent nodes (nodes from the $A(i)$ set), new values of routing variables $w_{ik}(j)$ and $z_{ik}(j)$ $(i \neq j,\ j = 1, 2, ..., N,\ k \in A(i))$ are computed. The new routing variables are computed so as to minimize an increase in the performance measure Q.

Step 3. Derivatives $\partial Q / \partial \lambda^d(i,j)$ and $\partial Q / \partial \lambda^c(i,j)$ for $j = 1, 2, ..., N$ are computed and sent to adjacent nodes at instant of time t_{p+1}. The derivatives are obtained based on the following expressions:

$$\frac{\partial Q^{(t_p)}}{\partial \lambda^d(i,j)} = \sum_{k \in A(i)} z_{ik}^{(t_p)}(j) \left(\left. \frac{\partial Q(i,k)}{\partial \lambda_{ik}^d} \right|_{\lambda_{ik}^d = \lambda_{ik}^{d(t_p)} + \delta \lambda_{ik}^{d(\Delta t_p)}} + \frac{\partial Q^{(t_{p-1})}}{\partial \lambda^d(k,j)} \right) \quad (21)$$

$$\frac{\partial Q^{(t_p)}}{\partial \lambda^c(i,j)} = \sum_{k \in A(i)} w_{ik}^{(t_p)}(j) \left(\left. \frac{\partial Q(i,k)}{\partial \lambda_{ik}^c} \right|_{\lambda_{ik}^c = \lambda_{ik}^{c(t_p)} + \delta \lambda_{ik}^{c(\Delta t_p)}} + \frac{\partial Q^{(t_{p-1})}}{\partial \lambda^c(k,j)} \right) \quad (22)$$

where:

$\frac{\partial Q^{(t_{p-1})}}{\partial \lambda^d(k,j)}$, $\frac{\partial Q^{(t_{p-1})}}{\partial \lambda^c(k,j)}$ – derivatives computed at instant t_{p-1} which are sent to the i–th node from adjacent nodes at instant t_p,

$w_{ik}^{(t_p)}(j)$, $z_{ik}^{(t_p)}(j)$ – routing variables computed at instant t_p,

$\lambda_{ik}^{c(t_p)}$, $\lambda_{ik}^{d(t_p)}$ – rates of voice calls and data streams flowing through the $\langle i,k\rangle$ transmission channel at instant t_p,

$\delta\lambda_{ik}^{d(\Delta t_p)}$, $\delta\lambda_{ik}^{c(\Delta t_p)}$ – values of voice calls and data streams increases flowing through the transmission channel $\langle i,k\rangle$ during period of time $[t_p, t_p+\Delta t]$,

In the subsequent part of this paper, values of the routing variables $z_{ik}(j)$ and $w_{ik}(j)$ are computed at the instant t_p when the increments of voice calls and data streams over the period $[t_p, t_p+\Delta t]$ are taken into account. For simplicity, in the subsequent parts of this paper the t_p and Δt_p indices are omitted.

5.1. Assignment of Routing Variables for Fixed Transmission Frame Division

This section is devoted to presenting the assignment of routing variables $z(\cdot)$ and $w(\cdot)$ at the instant of time t_p in the i–th node. The assignment is based on information about loading of transmission channels connected to the i–th node and derivatives received from nodes adjacent to the i–th node. The purpose is to discuss the second step of the above procedure in detail.

Let us discuss a case when the transmission frame is divided into compartments which are fixed–length. It is easy to note in this case that the routing variables $z(\cdot)$ and $w(\cdot)$ may be computed independently. This is possible, because in this case the value of $Q(i,k)$ is a sum of two components: $P(i,k)$ depending only on variable $w(\cdot)$ and $T(i,k)$ depending only on variable $z(\cdot)$ (see expression (4)).

5.1.1. Assignment of Routing Variables $w_{ik}(j)$

The performance measure Q introduced previously is valid for one particular route. The increase $\delta\lambda_{ik}^c$ induced an increase in the value of the performance measure Q on some selected route which originated at the i–th node. From the i–th node $|A(i)|$ various routes are originated. Instead of estimating the increments of Q independently at all routes which start from the i–th node, it is convenient to estimate the sum of all increments. The sum is denoted by δQ^w and may be estimated by an expression obtained from descriptions (19) and (20):

$$\delta Q^w \approx \sum_{\substack{j=1 \\ j\neq i}}^{N} \left(\delta\lambda^c(i,j) \sum_{k\in A(i)} w_{ik}(j) \left.\frac{\partial Q(i,k)}{\partial \lambda_{ik}^c}\right|_{\lambda_{ik}^c=\lambda_{ik}^{c(t_p)}} + \frac{\partial Q}{\partial \lambda^c(k,j)} \right) \tag{23}$$

Expression (23) may also be presented as follows:

$$\delta Q^w \approx \sum_{k\in A(i)} \sum_{\substack{j=1\\ j\neq i}}^{N} w_{ik}(j)\delta\lambda^c(i,j) \left(\frac{\partial Q(i,k)}{\partial \lambda_{ik}^c}\bigg|_{\lambda_{ik}^c=\lambda_{ik}^{c(t_p)}} \right)$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{N} \sum_{k\in A(i)} \left(w_{ik}(j)\delta\lambda^c(i,j)\frac{\partial Q}{\partial \lambda^c(k,j)} \right) \tag{24}$$

Let us observe that in expression (24) the first component estimates the global increment of the assumed performance measure at all transmission channels originating at the i-th channel, while the other component approximates an increase in the performance measure over all routes connecting nodes from the $A(i)$ set with destination nodes. The increase induced by an increase in rates of voice calls at the transmission channel $\langle i,k\rangle$ may be described more precisely, than in expression (24), using formula (1) and (3):

$$\delta Q^w(i,j) = \alpha \frac{b_{ik}\left(\lambda_{ik}^c + \sum\limits_{\substack{j=1\\ j\neq i}}^{N} w_{ik}(j)\delta\lambda^c(i,j)\right) + b'_{ik}}{m_{ik}^c + c_{ik}\left(\lambda_{ik}^c + \sum\limits_{\substack{j=1\\ j\neq i}}^{N} w_{ik}(j)\delta\lambda^c(i,j)\right) + c'_{ik}} - \alpha\frac{b_{ik}\lambda_{ik}^c + b'_{ik}}{m_{ik}^c + c_{ik}\lambda_{ik}^c + c'_{ik}} \tag{25}$$

where b_{ik}, b'_{ik}, c_{ik}, c'_{ik} are constant coefficients connected with transmission channel $\langle i,k\rangle$, m_{ik}^c is a number of time slots in the transmission frame at the i-th channel which is reserved for voice calls transmission.

Then, the increase δQ^w may be presented as follows:

$$\delta Q^w = \sum_{k\in A(i)} \delta Q^w(i,k) + \sum_{\substack{j=1\\ j\neq i}}^{N} \sum_{k\in A(i)} w_{ik}(j)\delta\lambda^c(i,j)\frac{\partial Q}{\partial\lambda^c(k,j)} \tag{26}$$

where $\delta Q^w(i,k)$ is given by expression (25).
Variables $w_{ik}(j)$ may be obtained by solving the following optimization problem:

Minimize function (26)

with respect to routing variables $w_{ik}(j)$ $(k\in A(i),\ j=1,2,...,N,\ j\neq i)$

subject to constraints i)–iv) satisfied by variables $w_{ik}(j)$.

The formulated problem is discrete. Its solution is resource consuming. To overcome this inconvenience, the following heuristic procedure, allowing for obtaining variables $w_{ik}(j)$, is proposed:

Algorithm W

Step 1. Let B be a set of all increments $\delta\lambda^c(i,\cdot)$. If $\delta\lambda^c(i,j) \in B$, then $w_{ik}(j) = 0$ for $k \in A(i)$. Go to Step 3.

Step 2. Select increment $\delta\lambda^c(i,j)$ satisfying the following condition

$$\delta\lambda^c(i,j) = \max_{\delta\lambda^c(i,l)\in B} \delta\lambda^c(i,l)$$

Next perform $B = B - \{\delta\lambda^c(i,j)\}$ and set: $w_{ik}(j) = 1$ for $k \in A(i)$. Compute $\delta Q^w(i,k)$ using expression (25) for $k \in A(i)$. Select such k for which the following is satisfied:

$$\delta Q^w(i,k) + \delta\lambda^c(i,j)\frac{\partial Q}{\partial\lambda^c(k,j)} = \min\left\{\delta Q^w(i,l) + \delta\lambda^c(i,j)\frac{\partial Q}{\partial\lambda^c(l,j)};\ l \in A(i)\right\}$$

Next perform $w_{ik}(j) = 0$ for $l \in A(i)$, $l \neq k$.

Step 3. If $B = \emptyset$, then the algorithm terminates. Otherwise, go to Step 2.

5.1.2. Assignment of Routing Variables $z_{ik}(j)$

Assignment of routing variables $z_{ik}(j)$ for $k \in A(i)$, $j = 1,2,...,n$ and $j \neq i$ is performed similarly to the assignment of variables $w_{ik}(j)$. The aim of assignment of variables $z_{ik}(j)$ is to minimize the total increase in the performance measure Q which is denoted by δQ^z. The total increase is given as below:

$$\delta Q^z = \sum_{k\in A(i)} \frac{\lambda^d_{ik} + \sum\limits_{\substack{j=1\\ j\neq i}}^{N} z_{ik}(j)\delta\lambda^d(i,j)}{\phi m^p_{ik} - \left(\lambda^d_{ik} + \sum\limits_{\substack{j=1\\ j\neq i}}^{N} z_{ik}(j)\delta\lambda^d(i,j)\right)} - \sum_{k\in A(i)} \frac{\lambda^d_{ik}}{\phi m^p_{ik} - \lambda^d_{ik}}$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{N} \sum_{k\in A(i)} \left(z_{ik}(j)\delta\lambda^d(i,j)\frac{\partial Q}{\partial\lambda^d(i,j)}\right) \tag{27}$$

To obtain optimal values of the routing variables $z_{ik}(j)$, the following optimization problem should be solved:

Minimize function (27)

with respect to routing variables $z_{ik}(j)$ for $k \in A(i)$, $j = 1,2,...,n$, $j \neq i$

subject to constraints

$$\sum_{k\in A(i)} z_{ik}(j) = 1 \qquad \text{for} \quad j = 1,2,...,n \text{ and } j \neq i$$

$$\lambda^d_{ik} + \sum_{\substack{j=1\\ j\neq i}}^{N} z_{ik}(j)\delta\lambda^c(i,j) - \phi m^p_{ik} \leq 0 \qquad \text{for} \quad k \in A(i)$$

$$z_{ik}(j) \geq 0 \qquad \text{for} \quad k \in A(i),\ j = 1,2,...,n \text{ and } j \neq i$$

Function (27), being a sum of concave and linear functions, is a concave function with respect to variables $z_{ik}(j)$.

The optimization problem formulated above may be solved using the FD method (Fratta *et al.*, 1973). In this case Phase 1 of this method is very simplified. To solve the problem a heuristic algorithm may also be applied. The idea of this algorithm is based on conclusions from the analysis of Kuhn–Tucker optimality conditions. They are as follows:

$$l_{ik}(j) - \Theta(i,j) - \Theta_{ik}(j) = 0 \qquad \text{for} \quad k \in A(i),\ j = 1,2,\dots,n,\ j \neq i$$

$$\Theta_{ik}(j)\big(-z_{ik}(j)\big) = 0 \qquad \text{for} \quad k \in A(i),\ j = 1,2,\dots,n,\ j \neq i$$

$$\Theta(i,j)\Big(1 - \sum_{k \in A(i)} z_{ik}(j)\Big) = 0 \qquad \text{for} \quad j = 1,2,\dots,n,\ j \neq i$$

where $\Theta_{ik}(j)$ and $\Theta(i,j)$ are Lagrange's multipliers and

$$l_{ik}(j) = \frac{\phi m^p_{ik}\delta\lambda^d(i,j)}{\left(\phi m^p_{ik} - \left(\lambda^d_{ik} + \sum\limits_{\substack{r=1 \\ r\neq i}}^{N} z_{ik}(r)\delta\lambda^d(i,r)\right)\right)^2} + \delta\lambda^d(i,j)\frac{\partial Q}{\partial \lambda^d(k,j)} \tag{28}$$

Expression (28) is the "length" of route from the i–th one to the j–th node which contains the transmission channel $\langle i,k\rangle$. Let $D \subset A(i)$ be the set of such nodes l from the set $A(i)$ for which $z_{il}(j) > 0$. It follows from the Kuhn–Tucker optimality conditions that for optimal values of variables $z_{ik}(j)$ the following conditions are satisfied:

$$l_{ik}(j) = l_{ir}(j) < l_{ip}(j) \quad \text{for every} \quad k,r \in D,\ p \in \big(A(i) - D\big)$$

This means that the increase $\delta\lambda^d(i,j)$ should be sent over routes for which values of $l_{ik}(j)$ are minimum. Moreover, $l_{ik}(j)$ computed for routes which are applied to transfer data stream with rate $\delta\lambda^d(i,j)$ must be equal. This forms the basis of the heuristic algorithm proposed for assignment of routing variables $z_{ik}(j)$.

Algorithm Z

Step 1. If $\delta\lambda^d(i,j) > 0$ then perform $z_{ik}(j) = 0$ for $k \in A(i)$, $j = 1,2,\dots,n$ and $j \neq i$. Perform $j = 0$ and $a = |A(i)|$.

Step 2. a) Perform $j = j + 1$.

b) If $\delta\lambda^d(i,j) \leq 0$ or $j = i$, then go to Step 2a.

c) If $j > n$, then the algorithm terminates.

d) Compute $l_{ir}(j)$ for every $r \in A(i)$ using expression (28).

Introduce the set $B = \{l^{(1)}_{ir}(j), l^{(2)}_{ip}(j), \dots, l^{(a)}_{ik}(j)\}$. The set contains values $l_{ik}(j)$ in increasing order, i.e. $l^{(k)}_{il}(j) \leq l^{(k+1)}_{ir}(j)$ for $k = 1,2,\dots,a-1$. Routing variables $z_{ik}(j)$ connected with value $l^{(s)}_{ik}(j)$ are temporarily denoted by $z^{(s)}_{ik}(j)$.

Let $g = 2$.

Step 3. a) Let $s = 1$.
b) If $s = g$, then go to Step 3d.
c) Compute

$$z_{ik}^{(s)}(j) = \min\left\{1, \frac{1}{\delta\lambda^d(i,j)}\left(\phi m_{ik}^p - \lambda_{ik}^d - \sum_{\substack{r=1\\ r\neq i,k}}^{n} z_{ik}(r)\delta\lambda^d(i,r) - \sqrt{\frac{\phi m_{ik}^p \delta\lambda^d(i,j)}{l_{ir}^{(g)}(j) - \delta\lambda^d(i,j)\frac{\partial Q}{\partial\lambda^d(k,j)}}}\right)\right\}$$

If $\sum_{r\in A(i)} z_{ir}(j) = 1$, then go to Step 2.
Otherwise, perform $s = s + 1$ and go to Step 3b.
d) Perform $g = g + 1$. If $g = a$, then perform for every $k \in A(i)$

$$z_{ik}(j) = z_{ik}(j) + \frac{u_{ik}}{\sum_{r\in A(i)} u_{ir}}\left(1 - \sum_{r\in A(i)} z_{ir}(j)\right)$$

where

$$u_{ik} = \phi m_{ik}^p - \left(\lambda_{ik}^d + \sum_{\substack{r=1\\ r\neq i}}^{N} z_{ik}(r)\delta\lambda^d(i,r)\right)$$

and go to Step 2.
Otherwise, perform $z_{ik}(j) = 0$ for every $k \in A(i)$ and go to Step 3a.

The expression applied at the third step is received during calculation of variable $z_{ik}(j)$ from the first Kuhn–Tucker optimality condition assuming that $\Theta_{ik}(j) = 0$ and $\Theta(i,j) = l_{ir}^{(g)}(j)$.

5.2. Assignment of Routing Variables for Movable Transmission Frame Division

In this section the division of the transmission frame between voice calls and data is calculated simultaneously with the assignment of flows at the transmission channels. The second step of previously presented quasi–static algorithm is presented in details. As in Section 5.1, in order to find the best values of the routing variables the sum of performance measure increments is minimized. Applying a procedure similar to the procedure which leads to expression (26), the sum of increments of performance measure Q may be written down in the following form:

$$\delta Q^s = \sum_{k\in A(i)} \delta Q^s(i,k) + \sum_{\substack{j=1\\ j\neq i}}^{N} \sum_{k\in A(i)} \left(w_{ik}(j)\delta\lambda^c(i,j)\frac{\partial Q}{\partial\lambda^c(k,j)} + z_{ik}(j)\delta\lambda^d(i,j)\frac{\partial Q}{\partial\lambda^d(k,j)}\right) \tag{29}$$

The case under consideration has been discussed previously in the Section 4.2 where simultaneously routing (flow) and transmission frame's division are assigned; the increase $\delta Q^s(i,k)$ may be described using expressions (12) or (16). For example, from expression (12) the following is received:

$$\delta Q^s(i,k) =$$

$$= \frac{\left(\alpha\phi\left(b_{ik}\left(\lambda^c_{ik} + \sum_{\substack{j=1\\ j\neq i}}^{N} w_{ik}(j)\delta\lambda^c(i,j)\right) + b'_{ik}\right)^{\frac{1}{2}} + \left(\lambda^d_{ik} + \sum_{\substack{j=1\\ j\neq i}}^{N} z_{ik}(j)\delta\lambda^d(i,j)\right)^{\frac{1}{2}}\right)^2}{\phi m_{ik} - \lambda^d_{ik} - \sum_{\substack{j=1\\ j\neq i}}^{N} z_{ik}(j)\delta\lambda^d(i,j) + \phi c_{ik}\left(\lambda^c_{ik} + \sum_{\substack{j=1\\ j\neq i}}^{N} w_{ik}(j)\delta\lambda^c(i,j)\right) + \phi c'_{ik}}$$

$$- \frac{\left((\alpha\phi(b_{ik}\lambda^c_{ik} + b'_{ik}))^{\frac{1}{2}} + (\lambda^d_{ik})^{\frac{1}{2}}\right)^2}{\phi m_{ik} - \lambda^d_{ik} + \phi c_{ik}\lambda_{ik} + \phi c'_{ik}} \qquad (30)$$

From the above expression it follows that the routing variables $z_{ik}(j)$ and $w_{ik}(j)$ for $k \in A(i)$, $j = 1,2,...,N$ have to be computed simultaneously. To obtain the optimal values of the routing variables, the following optimization problem is formulated:

Minimize function (29)

with respect to routing variables $w_{ik}(j)$ and $z_{ik}(j)$ $k \in A(i)$, $j = 1,2,...,N$, $j \neq i$

subject to the following constraints:

$$\sum_{k\in A(i)} z_{ik}(j) = 1 \qquad \text{for } j = 1,2,...,N \text{ and } j \neq i$$

$$\sum_{k\in A(i)} w_{ik}(j) = 1 \qquad \text{for } j = 1,2,...,N \text{ and } j \neq i$$

$$z_{ik}(j) \geq 0, \quad w_{ik}(j) \in \{0,1\} \quad \text{for } j = 1,2,...,N,\ j \neq i \text{ and } k \in A(i)$$

The above optimization problem is a mixed integer optimization one. To solve it the branch-and-bound method and FD algorithm may be applied. To decrease the necessary computing time the following heuristic algorithm is proposed:

Step 1. Compute routing variables $w(\cdot)$ using the algorithm W under assumption that all $z(\cdot)$ variables are equal to zero.

Step 2. Compute routing variables $z(\cdot)$ using an algorithm similar to that applied at the Algorithm Z under assumption that routing variables $w(\cdot)$ are equal to the values obtained at the Step 1.

6. Conclusions

In the paper some selected problems of data/voice flows optimization in the ISDN are solved. The aim of the investigations, which are reported in this paper, was to obtain a distributed routing procedure for the ISDN. Based on the analysis in quasi–static case, a new method leading to obtain routing variables at each ISDN node was proposed. Moreover, new procedures to reach simultaneously both routing variables and to divide transmission frames into two compartments were introduced and examined. The latter procedure is based on the knowledge of average rates of voice calls and data streams (occuring at periods between sequential actualizations of routing variables) and estimates of performance measures received from adjacent nodes. The algorithm, proposed and presented in this paper, is used for further investigations of routing problems at the ISDN.

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