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SLIDING MODE CONTROL WITH DIRECTLY LEARNED FEEDFORWARD COMPENSATION

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A new control method synthesizing direct learning control (DLC) with sliding mode control (SMC) is proposed for the tracking problem of a class of non-linear high order systems. Sliding mode control is used to provide the control system with indispensable robustness in the presence of strong system uncertainties. A new learning control method, the direct learning control method, is developed and used to generate the desired feedforward compensation for SMC. It can be shown that the combined scheme can successfully retain the advantages of both control methods. By virtue of SMC, the global asymptotic stability is ensured. Meanwhile, DLC provides an effective way to anticipate the necessary control signals for a new trajectory in terms of past control profiles which may correspond to different trajectories. In comparison with SMC alone, the synthesized control scheme achieves higher tracking accuracy and smoother control efforts.

1. Introduction

The problems of controlling nonlinear systems with modeling imprecision are often encountered. To solve those problems many control methods have been proposed. Among them, intelligent control and robust control are the two main trends. The main characteristic of intelligent control is the learning or adaptive capability while robustness or insensitivity to disturbances is the main attribute of robust control. To some extent learning or adaptation can be regarded as an *active* way to address system uncertainties in that it tries to identify the system uncertainty so that the control action can be arranged in an optimal way. On the contrary, robust control is a *passive* way in that it considers the worst situation such that the *safest* control arrangement can be made to protect the system from disturbances. Whether choosing intelligent control or robust control depends highly upon the amount of available information concerning the control system.

It is well-known that *sliding mode control* is one of the well-used robust control methods (Slotine and Li, 1991; Utkin, 1978; 1992; Young *et al.*, 1996; Yu and Man, 1998). The main disadvantage of SMC is the "chattering", which is usually inevitable due to the conservative nature of the SMC method itself. In a practical implementation, it is indispensable to smooth the control efforts. There are mainly two ways to do so: the *passive* way and the *active* way. The main characteristic of *passive* smoothing methods is the continuous approximation of the signum function within

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a certain error bound such as a saturation function and a *balance condition* (Slotine and Li, 1991; Xu *et al.*, 1996). The saturation function has a fixed error bound while the *balance condition* tunes the error bound dynamically. However, due to the passive nature of those smoothing schemes, the smoothness can only be achieved at the price of sacrificing the control precision. To better deal with varying control objectives, the *active* way which is trying to identify the system uncertainty and compensate for it in a feedforward manner is highly preferred.

Up to now the feedforward compensation is obtained through estimating the system uncertainty either by a "guess" or using adaptive techniques (Lee *et al.*, 1996). However, the first approach depends highly on the prior knowledge about the system uncertainty. Hence its effectiveness can hardly be guaranteed. The adaptive approach, on the other hand, can only handle unknown constant parameters associated with the *Persistent Excitation* condition and the system must be linear in the parameters. In this paper, a new feedforward method is developed which is based on learning from control signals of previous operation cycles, instead of a "guess" or parameter updating. It is able to avoid the limitations of the existing *active* methods and fully make use of pre-stored control information to effectively reduce the chattering.

The main issue of the learning control methods is to enable the system to do the same work more efficiently in the next cycle of operation. The control information is manipulated quantitatively and a deal of memory components are allocated to store the control signals. There are mainly three types of learning control methods: iterative learning control (ILC), repetitive control (RC) and direct learning control (DLC). ILC and RC require that the trajectories under learning control be exactly identical over repeated operation cycles. Considering the fact that tracking control tasks may change, in this paper DLC is used to generate the desired feedforward compensation. DLC is able to directly generate the desired control profile from existing control inputs without any repeated learning (Xu, 1997; 1998). The ultimate goal of DLC is to fully utilize all the pre-stored control files, even though they may correspond to different motion patterns and be obtained using different control methods such as PID, ILC, adaptive or robust control. In practice, a real control system may have plenty of such control knowledge acquired through the past control actions.

The paper is organized in the following manner. Section 2 gives the formulation of the nonlinear high order dynamic system to be discussed and the basic SMC scheme with smoothing. In Section 3, a basic DLC method is first introduced and then used to provide the new feedforward compensation for the SMC controller. Comparative studies are carried out in Section 4 to verify the effectiveness of the synthesized SMC scheme with DLC-based feedforward compensation.

2. Problem Statement

2.1. Nonlinear Dynamics

Consider a class of nonlinear high order systems described by the following differential equations:

$$\dot{x}_i = x_{i+1}, \qquad i = 1, \ldots, n-1 \tag{1}$$

$$\dot{x}_n = f(x_1, \ldots, x_n, t) + b(t)u \tag{2}$$

where $b(t)$ is a time varying function with known bounds

$$b_{\max} \geq b(t) \geq b_{\min} > 0$$

The nonlinear function $f(x_1, \ldots, x_n, t)$ can be expressed as

$$f = \boldsymbol{\theta}(t)^T \boldsymbol{\xi}(\boldsymbol{x}, t)$$

where $\boldsymbol{\xi} = [\xi_1, \ldots, \xi_m]^T$ is a known nonlinear function vector of $\boldsymbol{x} = [x_1, \ldots, x_n]^T$ and t; $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_m]^T$ is a function vector of t associated with known bounds

$$|\theta_i| \leq \theta_{i,\max} \qquad (i = 1, \ldots, m)$$

The control objective is to generate the desired control profile $u_{\mathrm{d}}(t)$ corresponding to a new trajectory

$$\boldsymbol{x}_{\mathrm{d}}(t) = \big[x_{1,\mathrm{d}}(t), \ldots, x_{n,\mathrm{d}}(t)\big] = \big[x_{1,\mathrm{d}}(t), \ldots, x_{1,\mathrm{d}}^{(n-1)}(t)\big], \qquad t \in [0, T]$$

Remark 1. Since both $b(t)$ and $\boldsymbol{\theta}(t)$ are unknown time-varying functions, conventional adaptive control methods are not applicable.

2.2. Basic Sliding Mode Control Scheme

Since all the bounds of the system uncertainties are available, the basic SMC can be easily designed to achieve asymptotic convergence. Define the tracking error as

$$[\tilde{x}_1, \ldots, \tilde{x}_n]^T = [x_1 - x_{1,\mathrm{d}}, \ldots, x_n - x_{n,\mathrm{d}}]^T$$

A typical switching surface σ can be constructed as

$$\sigma = \tilde{x}_n + \sum_{i=1}^{n-1} \lambda_i \tilde{x}_i = \tilde{x}_1^{(n-1)} + \sum_{i=1}^{n-1} \lambda_i \tilde{x}_1^{(i-1)}$$

where λ_i are chosen to be coefficients of a Hurwitz polynomial.

The sliding condition is

$$\sigma\dot{\sigma} \leq -\eta|\sigma| \qquad (\eta > 0)$$

To meet this condition, the switching control law should be constructed as

$$u = \hat{b}^{-1}\Big(\hat{u} - k(\boldsymbol{x})\,\mathrm{sign}(\sigma)\Big) \tag{3}$$

$$\hat{u} = x_{1,\mathrm{d}}^{(n)} - \sum_{i=1}^{n-1} \lambda_i \tilde{x}_1^{(i)} - \hat{f}$$

$$k(\boldsymbol{x}) = \beta(F + \eta) + (\beta - 1)|\hat{u}|$$

associated with

$$
\begin{aligned}
\beta &= \sqrt{\frac{b_{\max}}{b_{\min}}} \\
\hat{b} &= \sqrt{b_{\max} b_{\min}} \\
\hat{f} &= \hat{\boldsymbol{\theta}}^T |\boldsymbol{\xi}|_1 \\
F &= (\boldsymbol{\theta}_{\max} - \hat{\boldsymbol{\theta}})^T |\boldsymbol{\xi}|_1 \\
\hat{\boldsymbol{\theta}} &= \left[\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_m\right]^T \\
\boldsymbol{\theta}_{\max} &= \left[\theta_{1,\max}, \theta_{2,\max}, \ldots, \theta_{m,\max}\right]^T \\
|\boldsymbol{\xi}|_1 &\triangleq \left[|\xi_1|, |\xi_2|, \ldots, |\xi_m|\right]^T
\end{aligned}
$$

where $\hat{\theta}_i$ is an estimate of θ_i.

It is easy to show the convergence property of the above control algorithm (Slotine and Li, 1991). However, this basic SMC will inevitably result in chattering due to the following three reasons. First, the feedback part is conservative in the sense that the control gain $k(\boldsymbol{x})$ has to be designed in terms of the worst case. Second, a discontinuous switching function is used. Third, as β, $\hat{\theta}_i$ and $\hat{b}$ are merely guesses of real ones, the feedforward compensation may not work at all.

2.3. Smoothing of SMC Control Profiles

Since the conventional "active" way fails to work properly, one has to exploit the passive way to eliminate chattering. The simplest way is to adopt the saturation function

$$
\text{sat}\,(\sigma/\Phi) = \begin{cases} \sigma/\Phi & \text{if } |\sigma| \le \Phi \\ \text{sgn}\,(\sigma) & \text{otherwise} \end{cases}
$$

to replace the signum function $\text{sgn}\,(\sigma)$. A new problem is then how to choose an appropriate saturation bound Φ. To improve the tracking accuracy a smaller Φ is preferred. On the contrary, to smooth the control efforts a larger Φ has to be chosen. The trade-off in determining Φ usually depends on the designer's expertise. Moreover, a fixed Φ working well for one control task may not work properly for another, as the control efforts may differ significantly with request to different trajectories.

An improved version of the saturation function is the *balance condition* which tunes the boundary Φ dynamically (Slotine and Li, 1991; Xu *et al.*, 1989). It smooths out the control discontinuity in a thin boundary layer nearby the switching surface

$$
B(t) = \left\{\boldsymbol{x}, |\sigma(\boldsymbol{x}, t)| \le \Phi\right\}, \qquad \Phi > 0
$$

where Φ is the boundary layer thickness. When outside $B(t)$, the robust control law is chosen the same as (3) to guarantee the attractiveness of the boundary layer. Inside the boundary layer $B(t)$ the control input u is interpolated to ensure the smoothness of the control input. To fully exploit the available control "bandwidth", the boundary $\Phi(t)$ is made time-varying. In order to maintain the attractiveness, the distance to the boundary layer should always be guaranteed to decrease

$$\frac{\mathrm{d}}{\mathrm{d}t}[\sigma - \Phi] \leq -\eta, \qquad \sigma \geq \Phi$$
$$\frac{\mathrm{d}}{\mathrm{d}t}[\sigma - (-\Phi)] \geq \eta, \qquad \sigma < -\Phi$$

Thus by combining the above equations, the following equation should be satisfied when outside the boundary layer:

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\sigma^2 \leq (\dot{\Phi} - \eta)|\sigma|, \qquad |\sigma| \geq \Phi \tag{4}$$

Note that the additional term $\dot{\Phi}|\sigma|$ in the above equation reflects the fact that the boundary layer attraction condition is more stringent during boundary layer contraction ($\dot{\Phi} < 0$) and less stringent during boundary layer expansion ($\dot{\Phi} > 0$). The adaptation of $\Phi(t)$ now follows:

$$\begin{aligned} \dot{\Phi} + \lambda\Phi &= \beta k(\boldsymbol{x}_{\mathrm{d}}), & \dot{\Phi} \geq 0 \\ \dot{\Phi} + \frac{\lambda\Phi}{\beta^2} &= \frac{k(\boldsymbol{x}_{\mathrm{d}})}{\beta}, & \dot{\Phi} < 0 \end{aligned} \tag{5}$$

with the initial condition

$$\Phi(0) = \frac{\beta k\big(\boldsymbol{x}_{\mathrm{d}}(0)\big)}{\lambda}$$

It can been seen that the *balance condition* is superior to the fixed boundary Φ method because it can tune the boundary dynamically and make full use of the available bandwidth. However, this smoothing scheme, same as most other passive smoothing methods, gains smoothness but loses the tracking accuracy of the basic SMC scheme. From (5) the boundary Φ increases as the control gain increases, which means that the chattering is avoided by increasing the error bound when the control activity is high. This will inevitably incur a larger tracking error. It is obvious that within the framework of the existing SMC and smoothing schemes, whether passive or active, it is difficult to further improve the control performance. A new paradigm of feedforward compensation is thus needed and DLC meets this need.

3. Synthesized DLC and SMC

3.1. Basic Direct Learning Control

To predict and compensate system uncertainties through operations we choose a different feedforward compensation obtained through Direct Learning instead of the conventional estimated one. DLC is a newly developed feedforward-type control scheme which is defined as the generation of desired control input profiles directly from the existing control input profiles without any repeated learning. In contrast to conventional feedforward compensation, DLC uses previous control inputs directly instead of the estimated system parameters to generate the control signal. In practice, a control system may have plenty of prior control knowledge acquired through all the past control actions which correspond to different but highly correlated motion tasks. They may be obtained via any existing control method such as PID, adaptive, robust or intelligent control approaches as long as they are applicable and can achieve accurate results. Here we assume that there are at least $m+1$ pre-stored trajectories $\boldsymbol{x}_i = [x_{1,i}, \ldots, x_{n,i}] \in [0,T]$ for which the corresponding desired control input signals $u_i(t)$ have been achieved *a priori* through learning or other control methods precisely. All the pre-stored trajectories are inherently related with each other through a set of known constants k_i such that $\boldsymbol{x}_i = k_i \boldsymbol{x}_\mathrm{d}$ where $t \in [0,T]$, $k_i \neq 1$ and $k_i \neq k_j$ when $i \neq j$. In other words, trajectories $\boldsymbol{x}_i$ and $\boldsymbol{x}_\mathrm{d}$ are said to be *proportional in magnitude* with scale k_i.

The following theorem characterizes the basic DLC scheme.

Theorem 1. *For the nonlinear plant (1) and (2), the desired control input $u_\mathrm{d}(t)$ with respect to a new trajectory $\boldsymbol{x}_\mathrm{d}(t)$, $t \in [0,T]$ can be directly obtained through past control inputs.*

Proof. Here we show step by step how to generate $u_\mathrm{d}(t)$ from $u_i(t), i \in \{1, \ldots, m+1\}$. The plant model (2) can be written as an n-th order differential equation

$$x_1^{(n)} = \sum_{j=1}^{m} \theta_j(t)\xi_j(\boldsymbol{x},t) + b(t)u$$

Since $b(t)$ is nonzero, the above equation can be rewritten as

$$u = b(t)^{-1}x_1^{(n)} - b(t)^{-1}\sum_{j=1}^{m} \theta_j(t)\xi_j(\boldsymbol{x},t)$$

Since a sufficient number of previous control signals have been obtained *a priori*, we have

$$u_i = b(t)^{-1}x_{1,i}^{(n)} - b(t)^{-1}\sum_{j=1}^{m} \theta_j(t)\xi_j(\boldsymbol{x}_i,t), \qquad i \in \{1, \ldots, m+1\} \tag{6}$$

Because the pre-stored trajectories $\boldsymbol{x}_i$ and the desired trajectory $\boldsymbol{x}_\mathrm{d}$ are *proportional in magnitude*, substituting $\boldsymbol{x}_i$ with $k_i\boldsymbol{x}_\mathrm{d}$ yields

$$u_i = k_i\left(b(t)^{-1}x_{1,\mathrm{d}}^{(n)}\right) + \sum_{j=1}^{m} \xi_j(k_i\boldsymbol{x}_\mathrm{d},t)\left(-b(t)^{-1}\theta_j(t)\right)$$

Note that $b(t)^{-1}x_{1,\mathrm{d}}^{(n)}$ and $b(t)^{-1}\theta_j(t)$, although unknown, are functions of t only, hence remain the same over the period $[0,T]$. In other words, these terms are invariant for all trajectories and can be calculated in a point-wise manner for each $t \in [0,T]$.

By defining

$$u = [u_1, \ldots, u_{m+1}]^T$$

$$d = \left[\begin{array}{cccc} b(t)^{-1}x_{1,\mathrm{d}}^{(n)}, & -b(t)^{-1}\theta_1(t), & \ldots, & -b(t)^{-1}\theta_m(t) \end{array}\right]^T$$

$$K = \left[\begin{array}{cccc} k_1 & \xi_1(k_1 x_\mathrm{d}, t) & \ldots & \xi_m(k_1 x_\mathrm{d}, t) \\ \vdots & & & \vdots \\ k_{m+1} & \xi_1(k_{m+1} x_\mathrm{d}, t) & \ldots & \xi_m(k_{m+1} x_\mathrm{d}, t) \end{array}\right]$$

eqn. (6) can be rewritten in matrix form

$$u = Kd$$

It can be seen that if the known matrix K has full rank for all $t \in [0, T]$, the vector d can be calculated directly through $d = K^{-1}u$.

Now, by defining a known vector $v = [1, \xi_1(x_\mathrm{d}, t), \ldots, \xi_m(x_\mathrm{d}, t)]^T$, the desired control input can be expressed as

$$u_\mathrm{d}(t) = b(t)^{-1}x_{1,\mathrm{d}}^{(n)} + \sum_{j=1}^{m} \xi_j(x_\mathrm{d}, t)\left(-b(t)^{-1}\theta_j(t)\right) = v^T d = v^T K^{-1} u \triangleq u_\mathrm{dlc} \qquad (7)$$

Equation (7) clearly shows that the desired $u_\mathrm{d}(t)$ can be directly learned using $m+1$ pre-stored control profiles. ■

Remark 2. To avoid a potential singularity associated with the matrix K, one can either use more pre-stored trajectories or rearrange the matrix in terms of the function ξ_i (Xu and Song, 1997).

3.2. Incorporation of DLC Based Feedforward Compensation into SMC

By incorporating the DLC-based feedforward compensation into the basic SMC scheme, the new SMC becomes

$$u = u_\mathrm{dlc} - k(x)\,\mathrm{sign}(\sigma) \qquad (8)$$

where

$$k(x) = \frac{b_\mathrm{max}|u_\mathrm{dlc}| + \theta_\mathrm{max}^T|\xi|_1 + \left|\sum_{i=1}^{n-1}\lambda_i \tilde{x}_1^{(i)} - x_{1,\mathrm{d}}^{(n)}\right| + \eta}{b_\mathrm{min}} \qquad (9)$$

The convergence property of the synthesized control scheme is shown by the theorem below.

Theorem 2. *For the nonlinear plant (1) and (2), the new synthesized SMC method, (8) and (9), ensures asymptotic tracking performance.*

Proof. By differentiating the sliding surface $\sigma = \tilde{x}_1^{(n-1)} + \sum_{i=1}^{n-1} \lambda_i \tilde{x}_1^{(i-1)}$, we get

$$\begin{aligned}\dot{\sigma} &= \tilde{x}_1^{(n)} + \sum_{i=1}^{n-1} \lambda_i \tilde{x}_1^{(i)} = x_1^{(n)} - x_{1,\mathrm{d}}^{(n)} + \sum_{i=1}^{n-1} \lambda_i \tilde{x}_1^{(i)} \\ &= \theta^T \xi + \sum_{i=1}^{n-1} \lambda_i \tilde{x}_1^{(i)} - x_{1,\mathrm{d}}^{(n)} + bu\end{aligned}$$

Therefore,

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\sigma^2 = \dot{\sigma}\sigma = \Big(\theta^T \xi + \sum_{i=1}^{n-1} \lambda_i \tilde{x}_1^{(i)} - x_{1,\mathrm{d}}^{(n)} + bu_{\mathrm{dlc}}\Big)\sigma - bk|\sigma|$$

From (8) and (9) it follows that the above equation can be rewritten as

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\sigma^2 &= \Big((\theta^T\xi)\sigma - \frac{b}{b_{\min}}\theta_{\max}^T|\xi|_1|\sigma|\Big) \\ &+ \left(\Big(\sum_{i=1}^{n-1} \lambda_i \tilde{x}_1^{(i)} - x_{1,\mathrm{d}}^{(n)}\Big)\sigma - \frac{b}{b_{\min}}\Big|\sum_{i=1}^{n-1} \lambda_i \tilde{x}_1^{(i)} - x_{1,\mathrm{d}}^{(n)}\Big||\sigma|\right) \\ &+ \Big(bu_{\mathrm{dlc}}\sigma - b\frac{b_{\max}}{b_{\min}}|u_{\mathrm{dlc}}|\,|\sigma|\Big) - \eta|\sigma|\end{aligned}$$

Since $0 < b_{\min} \le b \le b_{\max}$, the first three terms on the right-hand side of the above equation are obviously less than zero. This leads to

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\sigma^2 \le -\eta|\sigma| \tag{10}$$

which ensures the asymptotic convergence of the new control scheme. ■

With the DLC-based feedforward compensation, we are in a position to address the chattering issue again by using the smoothing scheme without loss of tracking accuracy. The previously introduced *balance condition* method is used here to ensure the smoothness of control input profiles. The resulting synthesized DLC and SMC method now takes the following form:

$$u = u_{\mathrm{dlc}} - \bar{k}(\boldsymbol{x})\,\mathrm{sat}\Big(\frac{\sigma}{\Phi(t)}\Big)$$

where

$$\bar{k}(\boldsymbol{x}) = k(\boldsymbol{x}) - k(\boldsymbol{x}_{\mathrm{d}}) + \frac{\lambda\Phi}{\beta} \tag{11}$$

and $\Phi(t)$ adapts according to (5). It can be clearly seen that, as u_{dlc} approaches the desired control input $u_{\mathrm{d}}(t)$, the $\mathrm{sat}(\sigma/\Phi(t))$ part will take a very small value, and hence reduce the feedback control efforts.

4. Simulation Results

Comparative simulations have been carried out using the following dynamics as a prototyping model:

$$\begin{aligned}\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} &= a\sin(x) + bu \\ a &= 21 \\ b &= b_0\big(1 + 0.5\sin(2\pi t)\big) \\ b_0 &= 0.8571\end{aligned}$$

The state vector is defined as $\boldsymbol{x} = [x_1, x_2]$. The actual value of a and b are assumed to be unknown to the controller except for their bounds,

$$a_{\min} = 0.5a, \qquad a_{\max} = 1.5a, \qquad b_{\min} = 0.5b_0, \qquad b_{\max} = 1.5b_0$$

The pre-stored trajectories are available as

$$\begin{aligned}& x_{1,i}(t) = k_i\Big[x_0 + (x_0 - x_f)(15t^4 - 6t^5 - 10t^3)\Big] \\ & k_1 = 2, \quad k_2 = 3, \quad i = 1,2\end{aligned}$$

where $x_0 = 10°$, $x_f = 30°$; $t \in [0,1]$.

The desired trajectory is

$$x_{1,\mathrm{d}}(t) = x_0 + (x_0 - x_f)(15t^4 - 6t^5 - 10t^3)$$

Three cases are simulated and compared:

- Basic sliding mode control

$$\begin{aligned}u &= \hat{b}^{-1}\big(\hat{u} - k(\boldsymbol{x})\,\mathrm{sign}(\sigma)\big) \\ \hat{u} &= \ddot{x}_{1,\mathrm{d}} - \lambda\dot{\tilde{x}}_1 - \hat{a}\sin x_1 \\ k(\boldsymbol{x}) &= \beta\big(a_{\max}|\sin x_1| + \eta\big) + (\beta - 1)|\hat{u}|\end{aligned}$$

  where all controller parameters are designed according to (3).

- SMC with *balance condition*

  In addition to the basic SMC, the boundary $\Phi(t)$ is now dynamically tuned according to (5).

- DLC based SMC with balance condition

$$u = u_{\mathrm{dlc}} - \overline{k}(\boldsymbol{x})\,\mathrm{sat}\left(\frac{\sigma}{\Phi}\right)$$

  where $\overline{k}(\boldsymbol{x})$ takes the form of (11) and

$$k(\boldsymbol{x}) = \frac{a_{\max}|\sin(x_1)| + |\lambda\dot{\tilde{x}}_1 - \ddot{x}_{1,\mathrm{d}}| + b_{\max}|u_{\mathrm{dlc}}| + \eta}{b_{\min}}$$

In all cases $\lambda = 20$, $\Phi_0 = 0.05$ and $\eta = 0.01$ are chosen.

To show the effectiveness of the proposed synthesized SMC control scheme, four sets of simulations are carried out:

(a) The system is controlled using the basic SMC method alone.

(b) The system is controlled using the SMC method with *balance condition*.

(c) The system is controlled using the synthesized SMC method with DLC feedforward compensation. Past control inputs stored for DLC are obtained using the SMC scheme with *balance condition*.

(d) The system is controlled using the synthesized SMC method with DLC feedforward compensation. Past control inputs stored for DLC are obtained using the basic SMC. Pre-filtering is applied to smooth the stored control signals. The filter takes the form $F(s) = 1/(T_{\text{filter}}s + 1)$, where T_{filter} is chosen to be twenty times the sampling period 1 ms.

The simulated control inputs are given in Fig. 1 and the corresponding tracking errors are shown in Fig. 2, respectively. It can be observed from Figs. 1(a) and 2(a) that, although the basic sliding mode control can work satisfactorily, it is achieved at the price of extremely high control activity. It can also be observed in Fig. 2(b) that the SMC with *balance condition* alone cannot achieve perfect tracking due to the trade-offs made to smooth control inputs. In Fig. 1(b) we can observe a discrepancy between the desired and actual control input profiles. In Fig. 2(c), it can be seen that even with imperfect control input profiles, the DLC-feedforward part can still work properly and the tracking performance has been improved compared with the result of SMC without DLC-based feedforward compensation. Finally, although the filtered VSC control signals have chattering and distortion as shown in Fig. 1(d), the tracking performance shown in Fig. 2(d) is nevertheless improved, which clearly illustrates the excellent learning capability of the DLC scheme.

5. Conclusion

In this paper, a synthesized Sliding Mode Control method with DLC-based feedforward compensation is proposed and applied to nonlinear high order systems. In this synthesized method, sliding mode control is used to equip a control system with necessary robustness in the presence of strong system uncertainties while direct learning control is developed and used to generate the desired feedforward compensation for SMC. It has been shown that the synthesized scheme can successfully inherit the advantages of both the methods, i.e. the robustness from SMC and the direct generation of control input from DLC. Simulation work shows the superiority of the synthesized control scheme over SMC or DLC alone.

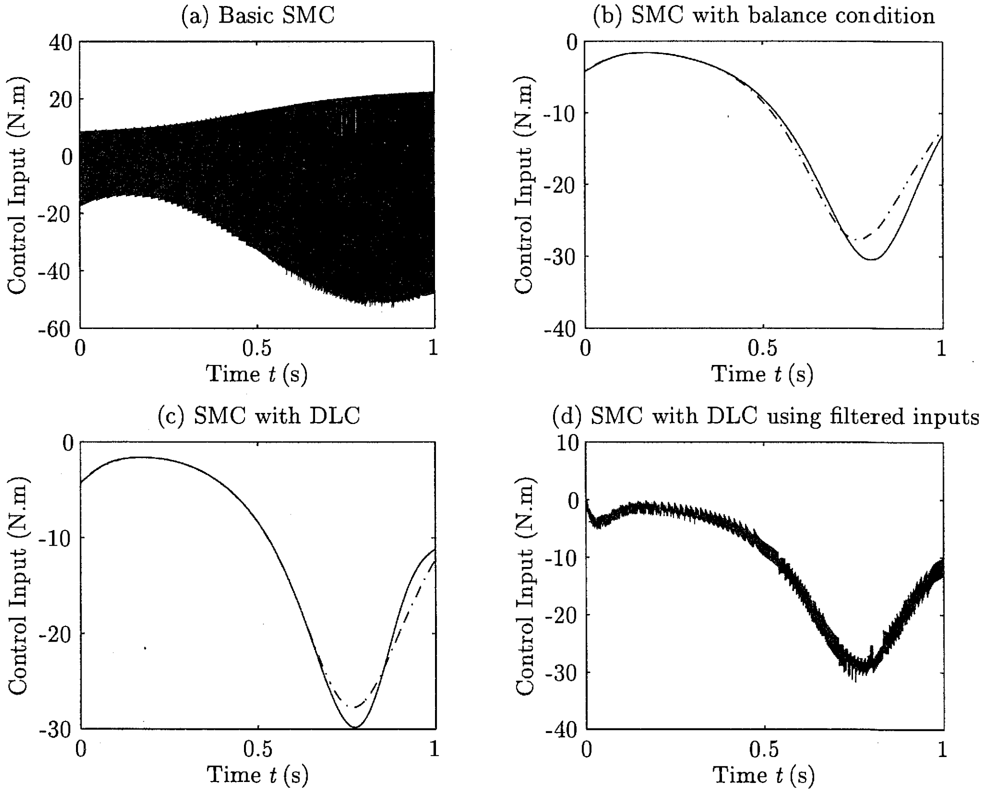

Fig. 1. Control inputs (solid line: actual input, dash-dotted line: desired input).

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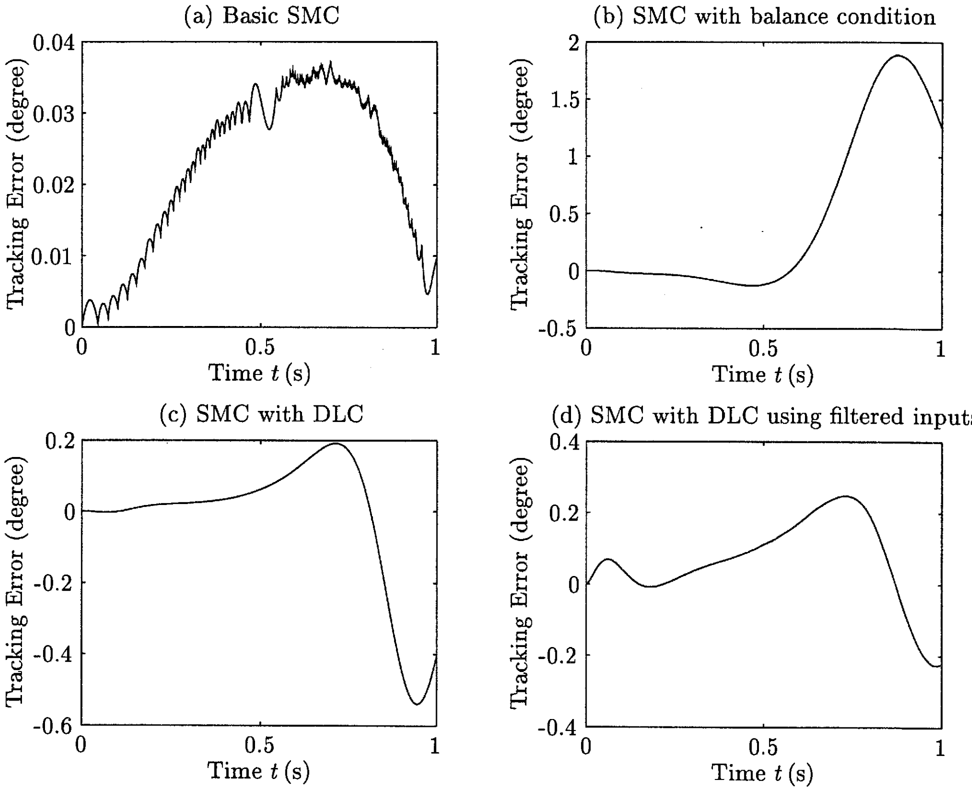

Fig. 2. Tracking error.

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