Appl. Math. and Comp. Sci., 1998, Vol.8, No.1, 135–144

ROBUST IDENTIFICATION BY DYNAMIC NEURAL NETWORKS USING SLIDING MODE LEARNING†

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The problem of identification of continuous, uncertain nonlinear systems in the presence of bounded disturbances is implemented using dynamic neural networks. The proposed neural identifier guarantees a bound for the state estimation error. This bound turns out to be a linear combination of internal and external uncertainty levels. The neural net weights are updated on-line by a learning algorithm based on the sliding mode technique. To the best of the authors' knowledge, such a learning scheme is proposed for dynamic neural networks for the first time. Numerical simulations illustrate its effectiveness, even for highly nonlinear systems in the presence of important disturbances.

1. Introduction

Sliding modes constitute a high speed switching strategy which provides a robust mean for controlling nonlinear plants. Essentially, it utilizes a switching control law to drive the plant state trajectory onto a perspectively sliding surface. This surface is also called the switching surface because if the state trajectory is "above" it, the controller has a gain which switches to a different one if the trajectory drops "below" it. The plant dynamics restricted to this surfaces constitutes the controlled system behavior. By proper design of the sliding surface, it is possible to attain control goals such as stabilization, tracking and/or regulation of nonlinear systems (DeCarlo *et al.*, 1988). Initially, the sliding mode control technique was mainly developed in the former Soviet Union (Utkin, 1978). Due to its robust properties, it is quite attractive for nonlinear system control and optimization (Khalil, 1996; Slotine and Li, 1991; Utkin, 1992).

Recently, it has been proposed to implement sliding mode control for nonlinear systems represented as neural networks. This implementation is, in most cases, carried out as follows: a neural network is adapted on-line in order to minimize the

† The research reported in this paper was supported by CONACYT Project 0652A9506.

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error between its own output and that of a nonlinear system, so the neural network reproduces the dynamic behavior of the system. Then, based on this neural network, a sliding mode controller is synthesized. Initially, such applications were based on radial basis Gaussian networks (Sanner, 1993; Tzirkel-Hancock and Fallside, 1992). Recent works consider other types of neural networks such as single layer perceptrons (Cao *et al.*, 1994), or multi-layer perceptrons for robot control (Safaric *et al.*, 1996). For all these applications, stability is established by means of the Lyapunov approach.

In contrast to neural control applications, the sliding mode technique has almost not been applied to neural network adaptive learning. The first related paper (Sira-Ramirez and Zak, 1991) presents a class of adaptive learning algorithms, based on the theory of quasi-sliding modes in discrete time dynamical systems, for both single and multilayer perceptrons. The convergence is assured through the existence of a quasi-sliding mode on the zero learning error. These algorithms underlie recently proposed identification and control schemes (Colina-Morles and Mort, 1993; Kuschewski *et al.*, 1993). In (Sira-Ramirez and Colina-Morles, 1995), the design of learning strategies in adaptive perceptrons, from the viewpoint of sliding modes in continuous time, is addressed. A unique feature of the sliding mode approach consists in the enhanced insensitivity of the proposed adaptive learning algorithm with respect to bounded external perturbation signals and measurements noises. Again, the convergence is guaranteed by the existence of a zero sliding mode on the zero learning error.

In this paper, we present an application of the sliding mode technique to the adaptive learning of dynamic neural networks, in order to minimize the error between the system to be identified and a neural identifier. The convergence of this error is analysed by means of a Lyapunov function. The structure of the identifier is taken from a previous paper of our research group (Poznyak and Sanchez, 1996). To the best of our knowledge, the proposed learning algorithm constitutes an original contribution, not addressed in the literature yet.

The paper is organized as follows: first, the mathematical models for both the nonlinear system and the neural network are given; then the sliding mode learning algorithm for the neural identifier is developed. The applicability of the proposed scheme is illustrated via simulations. Finally, the relevant conclusions are stated.

2. Mathematical Models

We consider nonlinear systems in the form

$$\dot{x}_t = f(x_t, u_t, t) + \xi_t \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the system state vector at $t \in R^+ := \{t : t \geq 0\}$, $u_t \in \mathbb{R}^q$ stands for a given control action, $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ denotes an unknown nonlinear function describing the system dynamics, ξ_t is a vector-valued function representing external disturbances, which satisfies the following assumption.

Assumption 1. The function ξ_t is Riemann integrable with bounded norm, i.e.

$$\limsup_{t \to \infty} ||\xi_t|| = \Upsilon < \infty \tag{2}$$

So, in what follows we will consider bounded external disturbances.

Let us select the recurrent neural networks as in (Rovithakis and Christodoulou, 1994):

$$\dot{\widehat{x}}_t = A\widehat{x}_t + W_{1,t}\sigma(\widehat{x}_t) + W_{2,t}\phi(\widehat{x}_t)\gamma(u_t) \tag{3}$$

where $A \in \mathbb{R}^{n\times n}$ is a Hurtwitz matrix, $W_{1,t} \in \mathbb{R}^{n\times n}$ is the weight matrix for nonlinear state feedback, $W_{2,t} \in \mathbb{R}^{n\times n}$ is the input weight matrix, $\widehat{x}_t$ stands for the neural network state.

The matrix function $\phi(\cdot)$ is assumed to be $\mathbb{R}^{n\times n}$ diagonal. The vector-valued functions $\sigma(\cdot)$ and $\gamma(.)$ are assumed to be n-dimensional. The elements of $\sigma(\cdot)$ and $\phi(\cdot)$ are usually selected as sigmoids, i.e.

$$\sigma(x) = \frac{a}{1+e^{-bx}} - c \tag{4}$$

This neural network (Poznyak and Sanchez, 1996) can be classified as a Hopfield-type one.

3. Sliding Mode Learning

We define the identification error as

$$\Delta_t := x_t - \widehat{x}_t \tag{5}$$

According to the sliding mode technique, we would like to obtain the following dynamic behavior:

$$\dot{\Delta}_t = -P\,\mathrm{sign}\,(\Delta_t) + \nu_t \tag{6}$$

where P is a positive diagonal matrix, $P = \mathrm{diag}[P_1, \ldots, P_n]$, $\mathrm{sign}(\Delta_t) := (\mathrm{sign}(\Delta_{1,t}), \ldots, \mathrm{sign}(\Delta_{n,t}))^T$, ν_t is an unmodelled dynamic part which can be evaluated using prior information on the class of uncertainties and on the nonlinear system being considered.

From (1) and (3) it follows that

$$\dot{\Delta}_t = \dot{x}_t - \dot{\widehat{x}}_t = f(x_t, u_t, t) + \xi_t - A\widehat{x}_t - W_{1,t}\sigma(\widehat{x}_t) - W_{2,t}\phi(\widehat{x}_t)\gamma(u_t) \tag{7}$$

Because $f(x_t, u_t, t)$ is unknown, we will use the following approximation:

$$f(x_t, u_t, t) = \frac{x_t - x_{t-\tau}}{\tau} + \delta_t \tag{8}$$

for a sufficiently small $\tau \in R^+$.

The vector δ_t is the approximation error at time t. In view of (1), its norm can be estimated as

$$\begin{aligned} \|\delta_t\| &= \left\|\tau^{-1}(x_t - x_{t-\tau}) - f(x_t,u_t,t)\right\| = \left\|\tau^{-1}\int_{t-\tau}^{t} \dot{x}_s \, \mathrm{d}s - f(x_t,u_t,t)\right\| \\ &= \left\|\tau^{-1}\left[\int_{t-\tau}^{t} f(x_s,u_s,s) - f(x_t,u_t,t)\right]\mathrm{d}s + \tau^{-1}\int_{t-\tau}^{t} \xi_s \, \mathrm{d}s\right\| \\ &\le \tau^{-1}\int_{t-\tau}^{t} \left\|f(x_s,u_s,s) - f(x_t,u_t,t)\right\|\mathrm{d}s + \sup_t \|\xi_t\| \end{aligned} \tag{9}$$

Assumption 2. The condition

$$\left\|f(x_s,u_s,s) - f(x_t,u_t,t)\right\| \le C_\tau + D_\tau |s-t| \tag{10}$$

is valid for any $s,t \in R^+$ and for any $x_s, u_s,\ x_t, u_t$ satisfying (1) (C_τ and D_τ are known nonnegative constants).

This condition can be applied to a wide class of nonlinear functions, including continuous and discontinuous functions with bounded variations, i.e.

$$f(x_t,u_t,t) = f_0(x_t,t) + f_1(x_t,t)\,\mathrm{sign}\,(u_t)$$

where $f_0(x_t,t)$, $f_1(x_t,t)$ are assumed to be continuous.

In general, C_τ is an upper bound estimation for local variations (e.g. in the case of $\mathrm{sign}(u_t)$ we have $C_\tau = 2$). As for D_τ, we can consider it as an upper bound of the cone-condition (as in the Popov criterion for absolute stability of closed-loop systems) valid for the function $f(x_t,u_t,t)$. So, taking into account the bounds (2) and (10) we can obtain directly from (9) that

$$\|\delta_t\| \le C_\tau + \tau D_\tau + \Upsilon \tag{11}$$

After substituting (8) into (7), we conclude that in order to guarantee the sliding mode behavior (6), the following relation has to be satisfied:

$$-P\,\mathrm{sign}\,(\Delta_t) = \frac{x_t - x_{t-\tau}}{\tau} - A\widehat{x}_t - \begin{bmatrix} W_{1,t} & W_{2,t} \end{bmatrix} \begin{bmatrix} \sigma(\widehat{x}_t) \\ \phi(\widehat{x}_t)\gamma(u_t) \end{bmatrix} \tag{12}$$

Accordingly, we obtain

$$\nu_t = \xi_t + \delta_t \tag{13}$$

Selecting the weights $[W_{1,t}\ W_{2,t}]$ such that (12) is fulfilled, we can satisfy the property (6). One possible selection is the least square estimate (Albert, 1972)

$$\begin{bmatrix} \widehat{W}_{1,t} & \widehat{W}_{2,t} \end{bmatrix} = \left[\tau^{-1}(x_t - x_{t-\tau}) - A\widehat{x}_t + P\mathrm{sign}\,(\Delta_t)\right] \begin{bmatrix} \sigma(\widehat{x}_t) \\ \phi(\widehat{x}_t)\gamma(u_t) \end{bmatrix}^+ \tag{14}$$

where $[\,\cdot\,]^+$ stands for the pseudoinverse matrix in the Moore-Penrose sense.

Remark 1. The above learning law is just an algebraic relation depending on Δ_t, which can be directly evaluated.

Taking into account that (Albert, 1972)

$$x^+ = \frac{x^T}{\|x\|^2}, \qquad 0^+ = 0$$

the formula (14) can be rewritten as follows:

$$\left[\begin{array}{cc} \widehat{W}_{1,t} & \widehat{W}_{2,t} \end{array}\right] = \frac{\left[\tau^{-1}\left(x_t - x_{t-\tau}\right) - A\widehat{x}_t + P\mathrm{sign}\left(\Delta_t\right)\right]}{\|\sigma(\widehat{x}_t)\|^2 + \|\phi(\widehat{x}_t)\gamma(u_t)\|^2} \left[\begin{array}{c} \sigma(\widehat{x}_t) \\ \phi(\widehat{x}_t)\gamma(u_t) \end{array}\right]^T \tag{15}$$

Remark 2. Notice that we do not ask for the condition of persistent excitation, which is a requirement for constant parameter identification, because the proposed sliding mode algorithm (15) does not need the convergence of the parameters $\widehat{W}_{1,t}$ and $\widehat{W}_{2,t}$.

To analyse eqn. (6), we define the Lyapunov function

$$V_t = \frac{1}{2}\|\Delta_t\|^2$$

Its derivative along the trajectories of the differential equation (6) is bounded:

$$\begin{aligned} \dot{V}_t &= \Delta_t^T\dot{\Delta}_t = \Delta_t^T\left(-P\,\mathrm{sign}\left(\Delta_t\right) + \nu_t\right) = -\sum_{i=1}^{n} P_i\left|\Delta_i\right| + \Delta_t^T\nu_t \\ &\le -\min_i P_i\|\Delta_t\| + \|\Delta_t\|\,\|\nu_t\| \end{aligned}$$

Using (13) and applying the bounds given by (11), we deduce that

$$\|\nu_t\| \le \|\xi_t\| + \|\delta_t\| \le \Upsilon + C_\tau + \tau D_\tau$$

and

$$\dot{V}_t \le -\|\Delta_t\|\left[\min_i P_i - \left(\Upsilon + C_\tau + \tau D_\tau\right)\right]$$

If we select

$$\min_i P_i > \Upsilon + C_\tau + \tau D_\tau$$

we will guarantee the property $\Delta_t \to 0$.

Finally, we formulate our main result.

Theorem 1. *Let Assumptions 1 and 2 hold. If the gain diagonal matrix coefficient P in the learning procedure (15) is selected such that*

$$\min_i P_i > \Upsilon + C_\tau + \tau D_\tau \tag{16}$$

then the identification error vector is globally asymptotically stable, i.e.

$$\Delta_t \to 0$$

Remark 3. In order to guarantee the stability condition (16), it is desirable to select τ as small as possible.

4. Simulations

In this section we present simulation results which illustrate the applicability of the theoretical study given above. We consider two illustrative examples. In the first one, we consider a nonlinear system with signum-type elements, and in the second one, we apply the proposed scheme to the Van der Pole oscillator.

Example 1. Let us consider the nonlinear system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -a_1 x_1 \\ -a_2 x_2 \end{bmatrix} + \begin{bmatrix} \beta_1 \operatorname{sign}(x_2) \\ \beta_1 \operatorname{sign}(x_1) \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \xi_{1,t} \\ \xi_{1,t} \end{bmatrix} \tag{17}$$

with $\Upsilon = 0.25$ (see (2)). We will use the following dynamic neural network:

$$\begin{bmatrix} \dot{\widehat{x}}_1 \\ \dot{\widehat{x}}_2 \end{bmatrix} = \begin{bmatrix} -\widehat{a}_1 x_1 \\ -\widehat{a}_2 x_2 \end{bmatrix} + \begin{bmatrix} w_{11}\sigma(\widehat{x}_1) + w_{12}\sigma(\widehat{x}_2) + d_1 u_1 \\ w_{12}\sigma(\widehat{x}_1) + w_{22}\sigma(\widehat{x}_2) + d_2 u_2 \end{bmatrix} \tag{18}$$

As regards the parameters, we select

$$a_1 = \widehat{a}_1 = 5, \quad \beta_1 = 3, \quad d_1 = 1, \quad x_1(0) = 10, \quad \widehat{x}_1(0) = -1$$

$$a_2 = \widehat{a}_2 = 10, \quad \beta_2 = 5, \quad d_2 = 1, \quad x_2(0) = -10, \quad \widehat{x}_2(0) = -2$$

$$\sigma(x) = \frac{2}{1 + e^{-2x}} - 0.5$$

and

$$P = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

In order to adapt on-line the dynamic neural network weights, we use the learning algorithm (15). The input signals are *sine-wave* and *saw-tooth* functions. The corresponding results are shown in Figs. 1 and 2. The solid lines denote the nonlinear system state trajectories, and the dashed line represents neural network outputs. The time evolution for the weight of the neural network is shown in Fig. 3. ♦

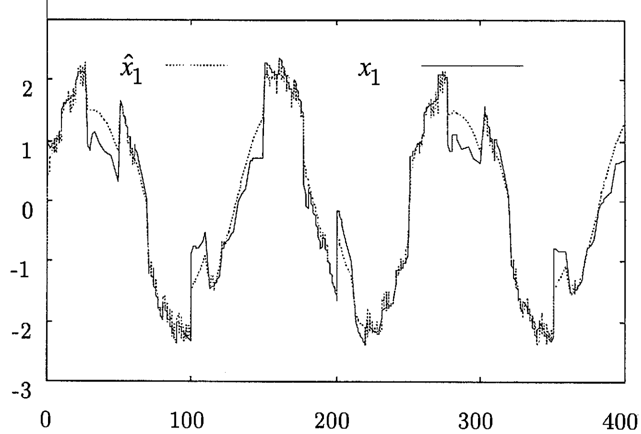

Fig. 1. Output profiles for a sine input (Example 1).

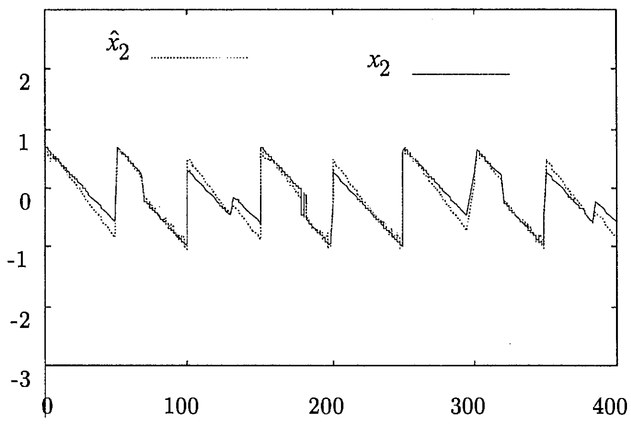

Fig. 2. Output profiles for a saw-tooth input (Example 2).

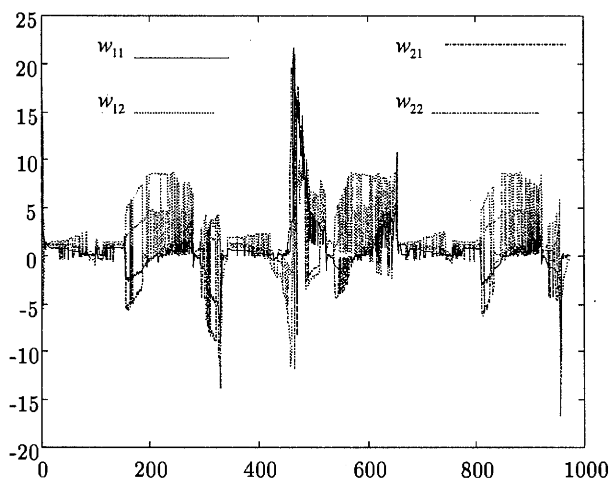

Fig. 3. Time evalution of the NN weights (Example 1).

Example 2. Let us consider the following Van der Pol oscillator with "zero control input":

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1.5 \end{bmatrix} \left[\left(1 - x_1^2\right) x_2 - x_1 \right] \tag{19}$$

The neural net is the same as (18), but with

$$P = \begin{bmatrix} 30 & 0 \\ 0 & 20 \end{bmatrix}$$

The respective results are shown in Figs. 4 and 5. The solid lines correspond to nonlinear system state trajectories, and the dashed line to neural network ones. The time evolution for the weight of the neural network is shown in Fig. 6. The limit circles ((x_1, x_2) and $(\widehat{x}_1, \widehat{x}_2)$) are shown in Fig. 7. ♦

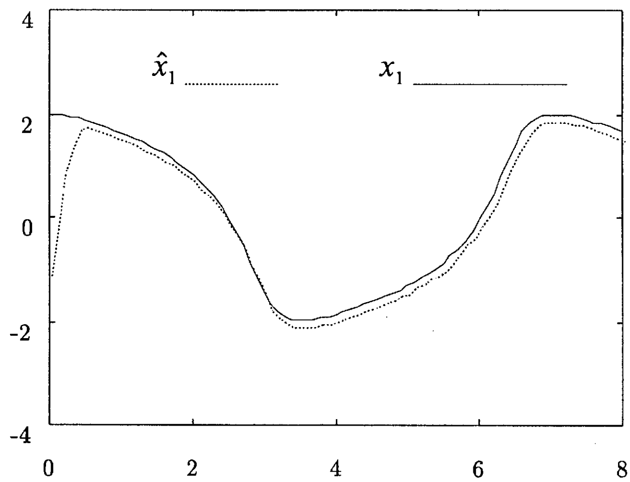

Fig. 4. Output profiles of x_1 (Example 2).

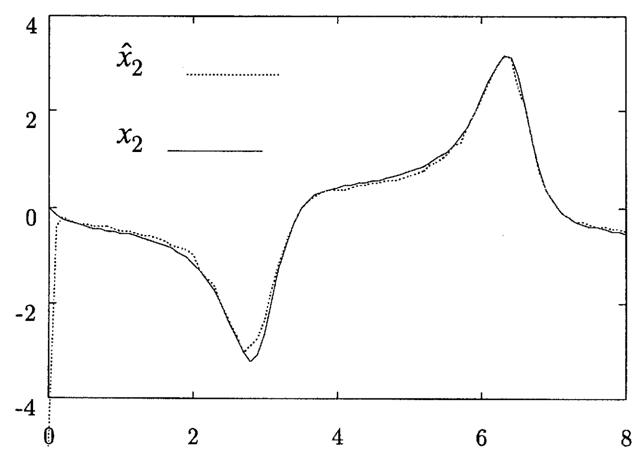

Fig. 5. Output profiles of x_2 (Example 2).

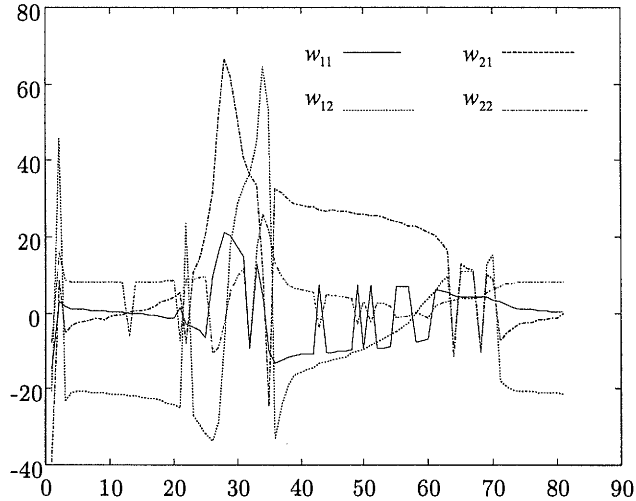

Fig. 6. Time evolution of the NN weights (Example 2).

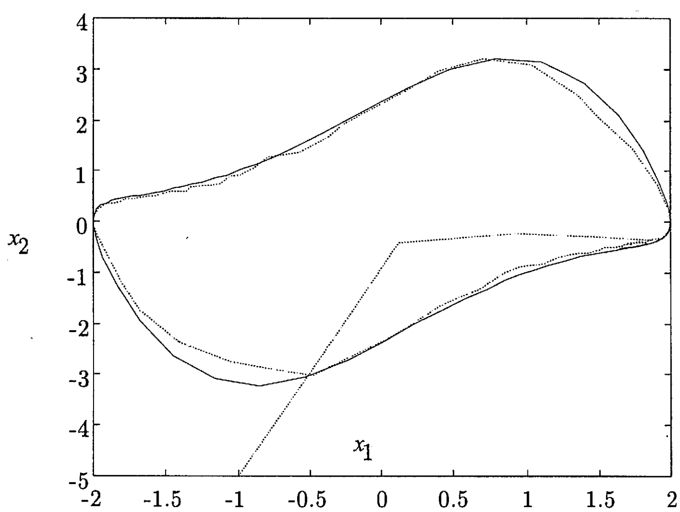

Fig. 7. Limit cycles.

5. Conclusion

We have discussed an application of the sliding mode techniques to learning algorithms of dynamic neural networks which are utilized to implement a neural identifier. The global convergence of the identification error to zero is established via the Lyapunov approach. In order to guarantee the existence of a sliding mode, we propose a new learning law to adapt on-line the weights of the neural network identifier. This law has a sliding mode structure.

The applicability of the proposed scheme is illustrated by two examples which were executed via simulations. The results show the excellent performance of the proposed neural network identifier with sliding mode on-line learning.

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