Appl. Math. and Comp. Sci., 1998, Vol.8, No.4, 793–813

NONLINEAR BLACK-BOX IDENTIFICATION OF DISTILLATION COLUMN MODELS — DESIGN VARIABLE SELECTION FOR MODEL PERFORMANCE ENHANCEMENT

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Conventional linear system identification techniques fail to capture the strong nonlinearity characteristic of distillation processes. On the other hand, general theories to guide selection of design variables in nonlinear system identification methods, such as the model structure selection, are lacking. In this paper, using the results of a binary distillation column simulation as a basis, problems relating to the proper selection of model structure and input perturbation design for nonlinear system identification are investigated systematically. Three commonly used model structures including the second-order Volterra model, block-structured models and the NARX model are considered. Identification results using a control-relevant technique are also presented where the goodness-of-fit is naturally represented by closed-loop performance requirements.

1. Introduction

Dynamics of distillation columns are often characterized by strong nonlinearity, strong interaction and ill-conditioning. These characteristics make it difficult for conventional linear system identification techniques to deliver satisfactory models (Chien and Ogunnaike, 1992). Most of the recent studies on distillation system identification have focused on the interaction and ill-conditioning issues (see e.g. Gaikwad and Rivera, 1997; Jacobsen, 1994). The nonlinearity problem is usually avoided using some *ad hoc* methods. For example, the steady-state gain of a high-purity distillation column system changes significantly as the operation point changes. As a result of this strong steady-state nonlinearity, a model estimated from linear identification methods may display significant variation even with a small change in the input perturbation design (Chien and Ogunnaike, 1992). To avoid this problem, a common practice is to fit a linear model to the high frequency part of plant response which is less vulnerable to the steady-state process nonlinearity. Otherwise, the input perturbation used in the identification procedure must be restricted to a very limited operation range which may pose some implementation difficulty in practice.

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System identification is the process of estimating system models from input/output measurements. The effectiveness of an estimated model is influenced by several different factors including model structure selection, input perturbation signal design, choice of weighting functions, etc. These factors are often referred as design variables in an identification procedure. The question of how to select these design variables is of fundamental importance in practice. In this paper, we consider to use three most commonly used model structures to describe distillation column dynamics. These are the second order Volterra model, block-structured models and the NARX (Nonlinear AutoRegressive with eXogenous) model. Using the model predicted steady-state responses and step responses of different amplitude as a criterion of model predictive ability, advantages and limitations using these model structures for distillation column system identification are evaluated. Influence of the input perturbation on the final model performance is also studied in the similar fashion.

A mathematical model, no matter how complicated it is, is always a simplified image of a real-life system. Only a portion of the system characteristics can be captured in a model. The issue becomes more evident for nonlinear systems because of the richness of nonlinear world. In this context, it is desired that intended model applications are directly accounted in identification procedures. Hence if the identified model is used for control design, a control-oriented or *control-relevant* method for identification is warranted. The method can be regarded as a weighted prediction error method where the weighting function is determined by the control requirement such as the desired closed-loop speed of response. In this setting, the goodness-of-fit is naturally influenced by closed-loop performance requirements.

The paper is organized as follows. In Section 2, the distillation column model considered in this study (based on an existing pilot-scale column) is described. Section 3 discusses identification results using a second-order Volterra model. Limitations of a second-order Volterra model in describing the column dynamics are discussed. Identification results using block-structured Hammerstein and the Wiener models are presented and compared in Section 4. Some inherent limitations of block-structured models are explained. Influence of the input perturbation design on the final model performance is also discussed. Section 5 presents identification results using a NARX model. Control-relevant identification results are given in Section 6. Finally, Section 7 provides a brief summary and some discussion.

2. Distillation Column Considered in the Study

The distillation column considered in this study is a pilot binary column. It is built of high temperature Pyrex glass, 3 inches in diameter, with 26 sieve trays (without weir), a reboiler and a total condenser. The feed tray is located on the sixteenth tray with the reboiler counted as the first tray. A refractometer mounted in the distillate stream enables on-line top product composition measurement. Thermocouples located on every other trays provide the tray temperature profile measurement. Four control loops were implemented to regulate the process operation. Two liquid level control loops control the reflux flow drum and the column bottom liquid levels. A composition

control loop can be used to control the distillate composition by regulating the reflux flow rate. The reboiler steam flow rate is regulated via a steam pressure control loop. The column is equipped with the Honeywell TDC3000[1] distributed control system on which all the measurement and control loops are implemented. Hence the column provides an ideal testbed for process identification and control studies. Table 1 shows the column normal operation conditions.

Table 1. Normal operation conditions of the column.

Mixture	Methanol/Isopropanol
Feed flow rate (mol/min)	3.0
Feed temperature (K)	303.15
Feed composition (mol %)	Methanol 0.42/Isopropanol 0.58
Reflux flow rate (mol/min)	3.3
Reflux flow temperature (K)	333.15
Reboiler heat duty (J/min)	0.1850e+6
Column top pressure (atm)	1

A rigorous model[2] for the column was developed based on the tray mass and energy balance, phase equilibrium thermodynamics and tray hydraulics (Ling, 1993). In the model, Murphree vapor-phase efficiency estimated from experimental data on the real pilot column was used to correct equilibrium phase compositions. Component phase equilibrium coefficients and the material flow enthalpy were estimated using the RKS (Redlich-Kwong-Soave) equation of state (Soave, 1972). To verify model accuracy, several step and pulse perturbations with amplitude up to $\pm 15\%$ of the normal operation date were applied to the reflux flow rate and corresponding distillate composition and tray temperature responses were measured from both the real pilot column and the model simulation. Comparison of the experimental and the simulation results showed that a prediction error less than 5% was obtained. To aid in understanding the nonlinear characters of this simulated column, column responses to different amplitude step inputs are given in Fig. 1.

We shall use this simulated column to generate the input/output data required in the following identification studies. Only the single input, single output problem will be considered with the reflux flow rate as the input variable and the top distillate composition as the output variable. The noise/disturbance effect is intentionally excluded, i.e., no artificial noise will be added to the input/output data collected.

[1] TDC3000 is a registered trademark of Honeywell Inc.

[2] FORTRAN program for the model is available from the authors upon request.

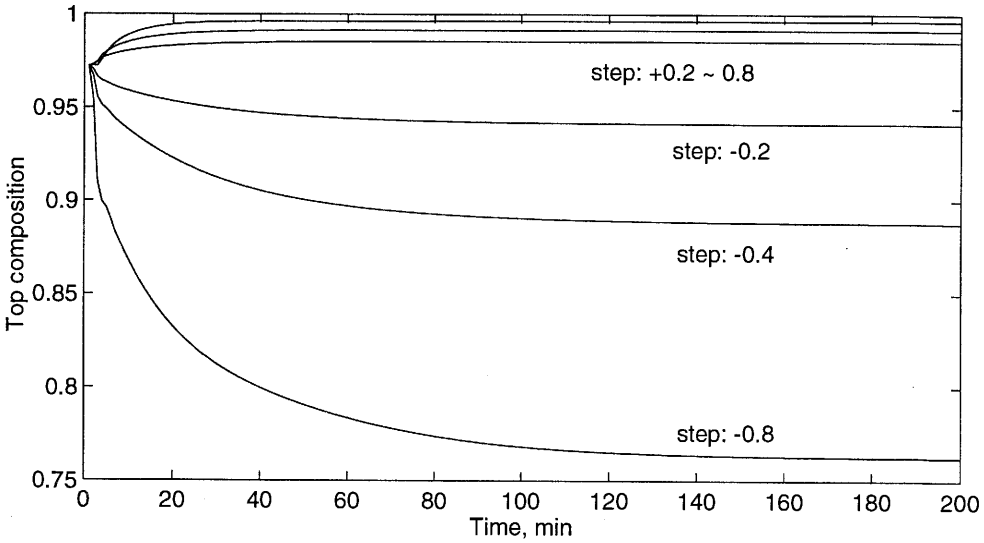

Fig. 1. Distillation column step responses to various input magnitudes.

3. Identification Using a Second-Order Volterra Model

A large class of nonlinear time invariant systems can be represented by Volterra series models (Rugh, 1981; Schetzen, 1980):

$$y(k) = \sum_{n=1}^{\infty} \sum_{i_1=1}^{\infty} \cdots \sum_{i_n=i_{n-1}}^{\infty} h(i_1, i_2, \ldots, i_n) \prod_{j=1}^{n} u(k - i_j) \tag{1}$$

In practice, the sum must be truncated to some finite upper limit and the model is usually restricted to the second or third order due to the number of model parameters involved. In this study, we shall consider the second order Volterra model of the following form:

$$y(n) = \sum_{i=1}^{M} a(i)u(n-i) + \sum_{i=1}^{M} \sum_{j=i}^{M} b(i,j)u(n-i)u(n-j) \tag{2}$$

It is well known that when the third-order moment of the input sequence u is identically zero, i.e.,

$$r_{uuu}(i,j,k) = E\Big[u(n-i)u(n-j)u(n-k)\Big] \equiv 0 \quad \forall i,j,k \tag{3}$$

where E denotes the expectation operation, Volterra kernels in (2) can be estimated as

$$r_{yu}(k) = \sum_{i=1}^{M} a(i) r_{uu}(k-i) \tag{4}$$

$$r_{yuu}(l,m) = \sum_{i=1}^{M}\sum_{j=1}^{M} b(i,j) r_{uuuu}(i,j,l,m) \tag{5}$$

where r represents correlation functions of corresponding sequences. In (4) and (5), estimation of the first- and second-order kernels are decoupled, i.e., only the first-order kernel $a(i)$ is involved in (4) and only the second-order kernel $b(i,j)$ is involved in (5). Equation (4) is exactly the same as the correlation analysis estimator for a linear system (Ljung, 1987).

Many commonly used signals such as Gaussian white noise, a sinusoid wave or any symmetrically distributed sequence satisfy the condition in (3). Usually, u is designed to have white noise-like correlation functions such that the estimation problem for each kernel can be further decoupled. For a zero-mean white noise input, the kernel estimation problem in (4) and (5) gives the following simple closed-form solution (Pearson *et al.*, 1992):

$$a(i) = \frac{r_{yu}(i)}{\sigma^2} \tag{6}$$

$$b(i,j) = \begin{cases} \dfrac{r_{yu}(i,j)}{[(\kappa+2)\sigma^4]} & \text{if } i = j \\ \dfrac{r_{yuu}(i,j)}{(2\sigma^4)} & \text{if } i \neq j \end{cases} \tag{7}$$

where σ^2 is the variance of the input sequence and κ its kurtosis. The accuracy of the kernel estimates can be improved by using white noise input sequences of different probabilic distributions for estimation of different kernel parameters (Pearson *et al.*, 1992). But a trade-off is that more input sequences require a longer experimental time in practice. In this study, we use the following two-input-sequence approach to estimate the Volterra kernels:

1. Excite the plant with a low amplitude pseudo random ternary sequence (PRTS) and estimate the first-order kernel using (7) from the corresponding measurements.

2. Excite the plant with a large power Gaussian white noise sequence and estimate the second order kernel from the corresponding measurements. The power of the Gaussian white noise input is desired to be large but not large enough to excite the plant behavior higher than the third-order.

For a random input, condition (3) is satisfied only if the input sequence is sufficiently long. Using periodic pseudo random input sequences such as the PRTS has

the advantage that their correlation functions can be obtained to their full accuracy by making summation over only one period, i.e.,

$$r_{uuu}(i,j,k) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=-N}^{N} u(n-i)u(n-j)u(n-k)$$

$$= \frac{1}{P} \sum_{n=0}^{P} u(n-i)u(n-j)u(n-k) \tag{8}$$

where P is the periodic length of the pseudo random sequence. The relation in (8) also applies to all the cross- and the auto-correlation functions involved in (4) and (5). But a pseudo-random sequence has the white noise-like even-order auto-correlation functions only up to the second order and its fourth-order auto-correlation function involves some deeper issues (Barker and Pradisthayon, 1970). To simplify the problem, a simple Gaussian white noise sequence is considered here for the second-order kernel estimation.

The PRTS input with an amplitude in the range $[-0.15, 0.15]$, which corresponds to about 5% of the steady state reflux flow rate, is generated using a 5-stage shift register with a switching time 3 minutes (Godfrey, 1993). One cycle of input/output data are collected at a sampling frequency of $1/180$ Hz, but the data collection process is started from the second cycle to avoid the initial state effect. The Gaussian white noise sequence with a variance 0.015 is generated with the same switching time and 1000 data are collected at the same sampling frequency among which the first 700 data are used for kernel estimation and the last 300 data will be used as the cross validation data set.

To estimate the Volterra kernels, model parameter M in (2) must be specified. Tests using step inputs indicate that the plant settling time is less than 90 minutes in the interested operation range. In this context, M should be chosen as 30. Since the number of model parameters involved with the second-order kernel increases significantly as M increases, instead of rigidly adhering to the model structure in (2), we consider to use different M for the two kernels. Actually, we found that by fixing $M = 30$ for the first kernel, the model prediction error based on the cross validation data set remains almost the same when $M > 18$ are selected for the second-order kernel. Hence $M = 30$ for the first-order kernel and 18 for the second-order kernel are determined. Estimated Volterra kernels are shown in Fig. 2.

Model predicted step responses and the steady-state response are compared with those of the true plant in Figs. 9 and 10. As indicated in these figures, the estimated Volterra model correctly predicted the plant behavior in the operation range approximately covered by the input perturbation signals. But increasing the input signal power will lead to a model with almost the same or worse predictive ability. It suggests that further increasing the input signal power will significantly excite the plant behavior higher than the second or third order. This result can be expected as we examine the steady-state response of a second-order Volterra model. At a steady-state,

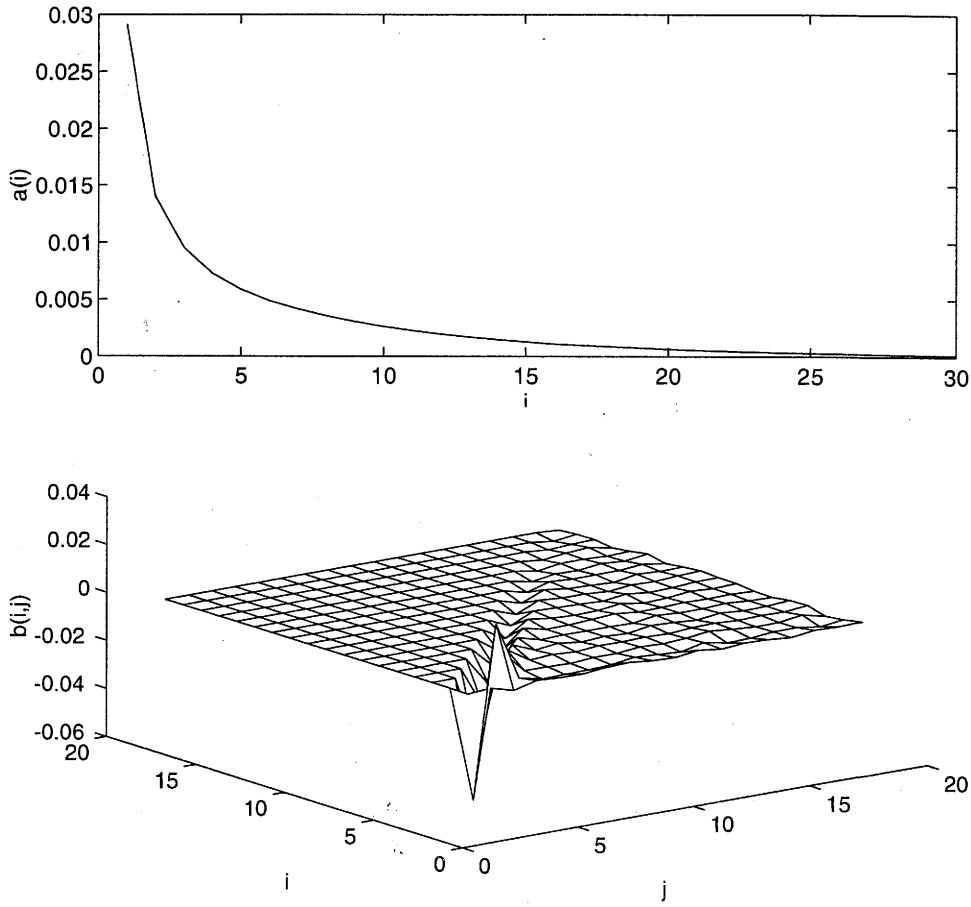

Fig. 2. Kernels of the second-order Volterra model.

model (2) reduces to

$$y_s = u_s \sum_{i=1}^{M} a(i) + u_s^2 \sum_{i=1}^{M} \sum_{j=i}^{M} b(i,j) \tag{9}$$

This corresponds to a symmetric parabola while the steady state response of a distillation column typically displays a shape shown in Fig. 10. If a larger operation range is desired to be covered by a Volterra model, a third- or higher-order model should be used which will drastically increase the number of model parameters. To include the higher-order nonlinearity without introducing too many model parameters, an alternative is to use block-structured models.

4. Identification Using Block-Structured Models

Block-structured models describe nonlinear systems using interconnected linear dynamic elements and memoryless nonlinear elements. Various block-structured model representations have been discussed in (Haber and Unbehauen, 1990). Among them, the simple Hammerstein and the Wiener models give an easy to handle description (Fig. 3).

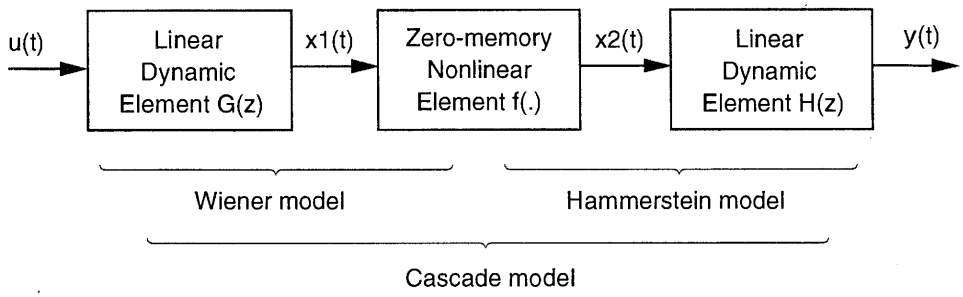

Fig. 3. Diagrammatic representation of typical block-structured models.

Clearly, the block structure is only an idealized image. Few real life systems can be exactly partitioned into those blocks in Fig. 3. Yet block-structured models are preferred in some applications because design and analysis tools for linear systems can be extended for these models. One of the most commonly used block-structured models must be the logarithmic transformation method proposed by many authors in the distillation control studies (see e.g. Skogestad, 1992).

4.1. Effect of the Logarithmic Transformation

Let y represent the distillate composition measurement. A frequently used logarithmic transformation has a form

$$y_t = \log \frac{y}{1-y} \tag{10}$$

Clearly, if the plant can be approximated by a Wiener model with a memoryless nonlinear element given by

$$y = \frac{1}{1+e^{-(a_1 x_1 + a_2)}} \tag{11}$$

where a_1 and a_2 are some constants, the plant nonlinearity can then be effectively reduced by the transformation (10).

For the column in this study, a plot of y_t versus y is given in Fig. 4. The discrepancy between the logarithmic transformation and the true plant state-state nonlinearity is obvious even though the transformation (10) is consistent with the plant response at the sense that $y \to 1$ as $x_1 \to \infty$ (Eskinat *et al.*, 1991).

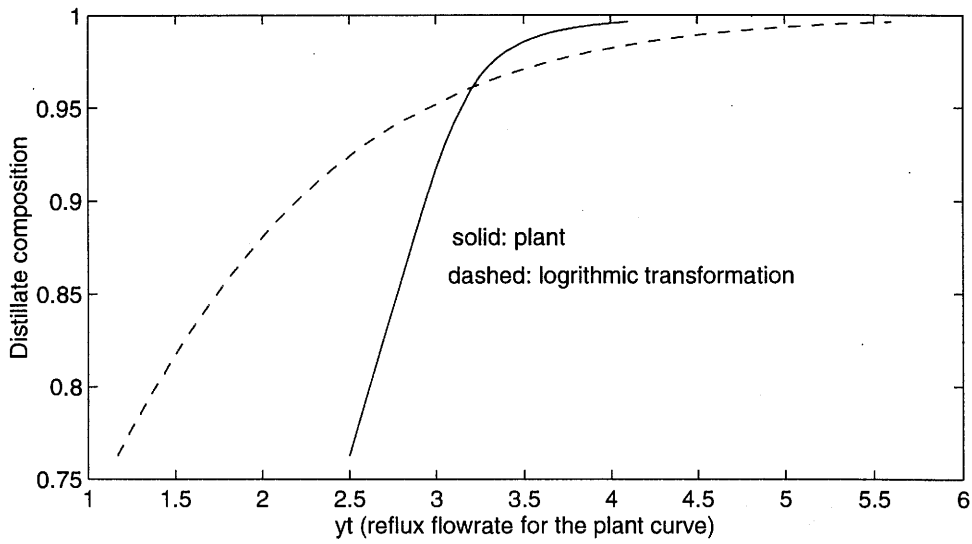

Fig. 4. Effects of the logarithmic transformation block on the steady-state response.

4.2. Identification Using a Wiener Model

An natural extension to the *ad hoc* relation (11) is to use a general Wiener model to approximate the column dynamics. If the memoryless nonlinear element in the Wiener model is analytic, it can be represented as a polynomial. Denote the linear dynamic and the nonlinear memoryless elements respectively by

$$G(z) = \frac{b_1 z^{-1} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}} \tag{12}$$

$$f(x) = x + \gamma_1 x^2 + \gamma_2 x^3 + \gamma_3 x^4 + \cdots + \gamma_l x^{l+1} \tag{13}$$

To make the model unique, the constant term in the polynomial (13) is set to unity. We use the well-known prediction error method to estimate the model parameters. Without noise, we have

$$\theta = \arg \min_{\theta} \sum_{t=1}^{N} \left[y(t) - \hat{y}(t) \right]^2 \tag{14}$$

subject to the constraint that all the poles of $G(z)$ lie in the unit circle. In the estimation problem posed in (14), θ represents the vector of model parameters $\theta = [b_1 \ \cdots \ b_m \ a_1 \ \cdots \ a_n \ \gamma_1 \ \cdots \ \gamma_l]$, y is the output measurement and $\hat{y}$ its model prediction; N is the total number of measurements.

Input/output data are generated using a white noise input perturbation uniformly distributed in the range $[-0.5, 0.5]$ with a sampling frequency of $1/180\,\mathrm{Hz}$

Table 2. Wiener and Hammerstein model parameters.

Model	b_1	a_1	γ_1	γ_2	γ_3
Wiener	0.0235	−0.7792	−17.4903	1.2327	−1.3600
Hammerstein	0.0199	−0.7687	−1.2184	1.2402	−1.3386

and a switching time of 18 minutes (6 sampling intervals) (Fig. 5). 700 data pairs are collected for model parameter estimation and other 300 data are collected as the model cross validation data set. Using a model structure with a first-order transfer function for the linear dynamic element and a polynomial of degree 4 for the memoryless nonlinear element, estimated model parameters are given in Table 2. Increasing the order of linear transfer function or the degree of polynomial will not significantly decrease the mean square prediction error on the basis of the cross validation set.

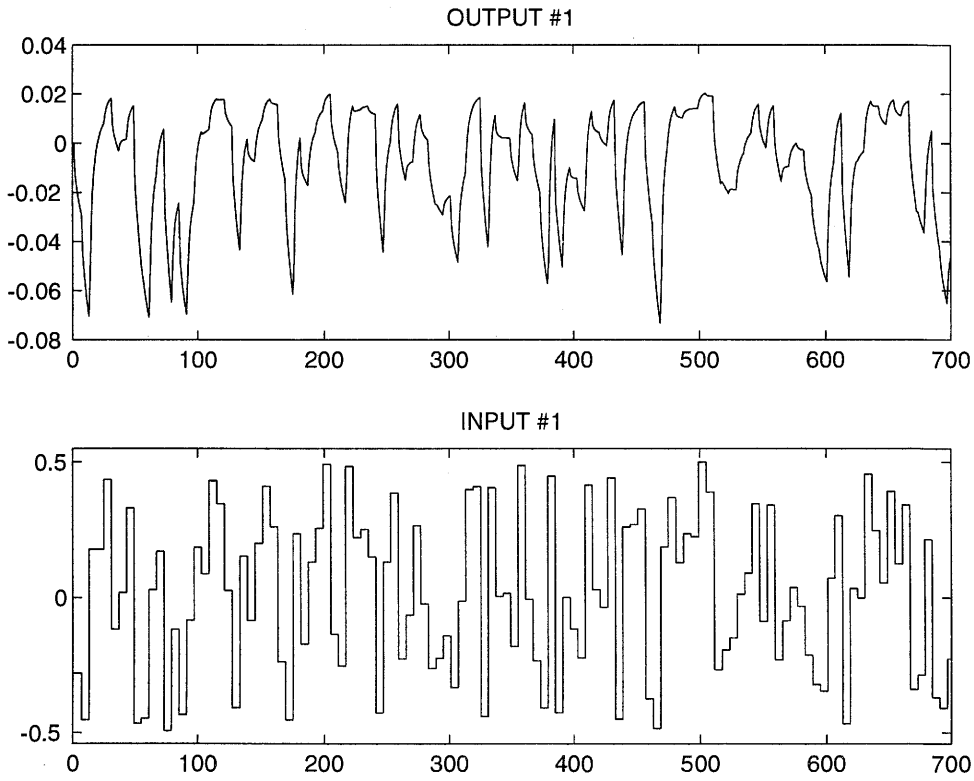

Fig. 5. Input/output simulation data for identification purposes.

We use the steady-state response to evaluate the model predictive ability. In Fig. 6, the model predicted steady-state response (Model 1) is compared with that of the plant. As expected, prediction results of the Wiener model are significantly improved around the normal operation point, compared to the simple logarithmic transformation method. But at a high purity range, it deteriorates rapidly. Changing input signal design, a better result can be obtained. For example, if a Gaussian white noise input perturbation with a variance of 0.1 is applied, the prediction result at the high purity range is significantly improved (Model 2, Fig. 6). Yet for the two input cases the mean square model prediction error based on the cross validation data set remains in the same level. This result reveals the nature of a block-structured model: it can only capture part of the plant characters. What part of the plant characters will be captured by the model depends on what characters are prominently excited by the input perturbation.

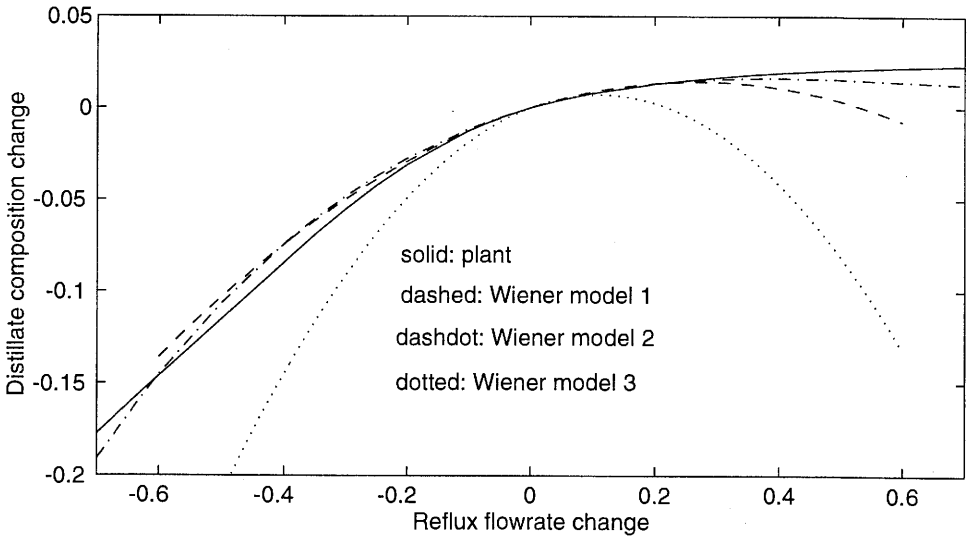

Fig. 6. Comparison of steady-state responses of three Wiener models.

In the input signal design, a quite large switching time is adopted. From a practical standpoint, the larger switching will lead to a longer experiment time if the number of measurement remains the same. But a large switching time is important here for the estimated model to capture the plant steady-state nonlinearity. For example, using a uniformly distributed white noise input of the same power but with a switching time of 3 minutes, and still 700 input/output data pairs are correlated, the resulting model now gives a quite different result as shown in Fig. 6 (Model 3). Intuitively, a larger switching time gives the plant a better chance to display its steady-state behavior.

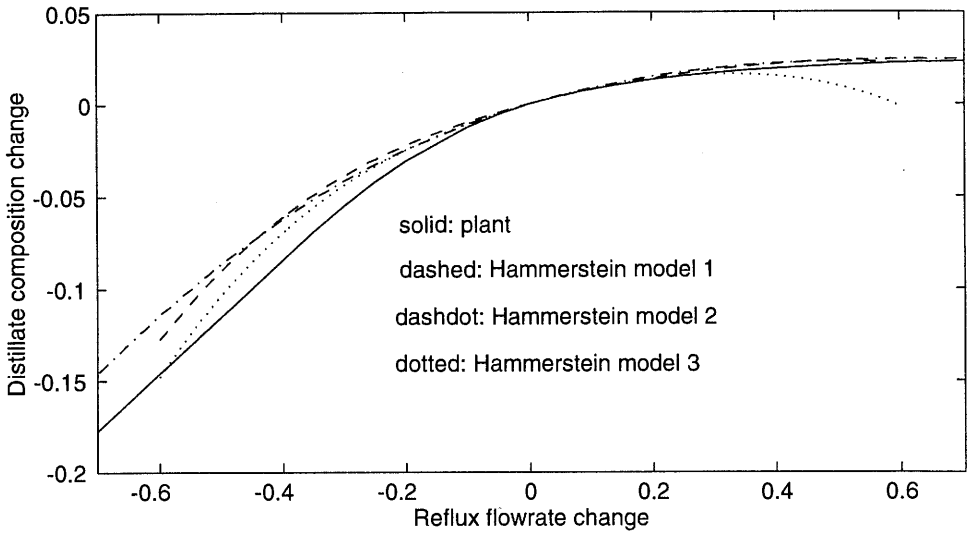

Fig. 7. Comparison of steady-state responses of three Hammerstein models.

4.3. Identification Results Using a Hammerstein Model

The Wiener model can be regarded as a natural extension to the logarithmic transformation method. On the other hand, the second-order Volterra kernel in Fig. 2 shows that its diagonal terms are dominant. This fact suggests that a Hammerstein model can be used to approximate the column dynamics. Using a first-order transfer function for the linear dynamic element, a polynomial of degree 4 for the memoryless nonlinear element, estimated model parameters from the input/output measurements in Fig. 5 are given in Table 2. Motivated by the results of Eskinat *et al.* (1991), Narendra and Gallman's algorithm (1966) was applied for model parameter estimation.

Model-predicted step and the steady-state responses are used to evaluate performance of the estimated model. As indicated in Figs. 9 and 10, the model well approximates the column dynamics in a neighborhood of operation range covered by the input perturbation signal. When compared with the Wiener model, the Hammerstein model better captures the plant behavior at the high purity range (Fig. 7). But the model prediction result becomes a little worse at the low purity range. It is possible to manipulate the design variables, e.g., input signal design such that the model captures the plant low purity behavior better. But consequently, fitting in the high purity range will deteriorate. This result is caused by the inherent limitation of the Hammerstein model structure. The nominal time constant of a Hammerstein model is solely determined by its linear dynamic element and will not change as the operation point changes. On the other hand, the plant nominal time constant is much larger at the low purity range than at the high purity range, as evidenced in Fig. 1. Hence, there is a trade-off between fitting to the high purity and fitting to the low purity range when a Hammerstein model is applied.

Unlike the Wiener model case, a Hammerstein model is more robust to the selection of the input signal switching time. For example, using the same three input perturbations as in the Wiener model case, steady-state responses of the resulting Hammerstein models display much less variation than the Wiener model even though the trade-off between fitting to the high purity and fitting to the low purity ranges mentioned earlier can be observed, as shown in Fig. 7. This robust property of the Hammerstein model may attribute to the fact that a Hammerstein model itself is less flexible than a Wiener model in the sense that step responses of a Hammerstein model will not display quantitatively different behavior for different step amplitudes as the Wiener model does (Pearson, 1994). Consequently, a Hammerstein model is not able to capture very different plant characters even though the input perturbation may have excited them.

It is clear at this point that due to the inherent limitations of a block-structured model, it is unavoidable to make some trade-offs in fitting a model to different plant characters. Even though the difference for the distillation column in this study is not so significant, from the system identification standpoint, a question naturally arisen is that how an optimal trade-off can be obtained. Clearly, the problem depends on the intended application of the estimated model. If the estimated model is for the process control design, we shall address the problem using a control-relevant identification technique in Section 6. But before proceeding to the control-relevant issue, we consider to use another class of more general models to describe the column dynamics, namely the Nonlinear AutoRegressive with eXogenous input (NARX) model.

5. Identification Using the NARX Model

A polynomial NARX model represents the process output as a polynomial function of the previous inputs and outputs (Leontaritis *et al.*, 1985).

$$\begin{aligned} y(n) &= f\big(y(n-1),\ldots,y(n-n_y),u(n-1),\ldots,u(n-n_u)\big) \\ &= \theta_0 + \sum_{i_1=1}^{L} \theta_{i_1} x_{i_1}(n) + \sum_{i_1=1}^{L}\sum_{i_2=i_1}^{L} \theta_{i_1 i_2} x_{i_1}(n) x_{i_2}(n) + \cdots \\ &\quad + \sum_{i_1=1}^{L} \cdots \sum_{i_L=i_{L-1}}^{L} \theta_{i_1 \cdots i_l} x_{i_1}(n) \cdots x_{i_L}(n) \end{aligned} \tag{15}$$

where

$$L = n_y + n_u$$

and

$$x_1(n) = y(n-1); \quad x_2(n) = y(n-2); \quad \ldots; \quad x_{n_y}(n) = y(n-n_y)$$

$$x_{n_y+1}(n) = u(n-1); \quad \ldots; \quad x_L(n) = u(n-n_u)$$

Equation (15) forms a linear-in-parameters model whose parameters can be estimated using the least squares method in a closed form.

The number of parameters contained in model (15) is given by

$$M = \frac{(L+l)!}{L!l!}$$

where l is the degree of the polynomial. A full order model (15) usually contains too many parameters to be practical. But it can be expected that not all the terms in the model are significant in describing the plant dynamics. It is a common practice in the NARX model estimation to form a combined parameter estimation and model structure determination algorithm based on an orthogonal decomposition least squares method (see e.g. Korenberg *et al.*, 1988; Chen *et al.*, 1989). The model structure determination step sequentially picks up those model terms which are most likely to decrease the model prediction error. The number of model terms included in the final model is usually determined using some information criteria such as the Akaike information criterion (Akaike, 1972).

In this study, the orthogonal least squares method based on the modified Gram-Schmidt decomposition procedure is applied to estimate a distillation column model. In model (15), let $n_u = n_y = 3$ and $l = 4$. Using the input/output measurements in Fig. 5, a model with 7 terms is obtained. Estimated model parameters and the corresponding model terms are given in Table 3.

Table 3. NARX model parameters.

Term	Parameter	Term	Parameter
$x(n-1)$	0.0274	$x(n-1)x(n-2)$	0.0439
$x(n-2)$	−0.0080	$x(n-1)y(n-1)$	−0.3529
$y(n-1)$	0.8054	$x(n-1)x(n-2)y(n-1)$	−0.3381
$x^2(n-1)$	−0.0510		

Again, the model predicted step and the steady-state responses are used to evaluate the model performance. As indicated in Figs. 9 and 10, the plant response is accurately predicted by the NARX model. The accuracy of the estimated model can also be observed from another angle. The linear part of the NARX model is given by

$$F(z) = \frac{0.0274z^{-1} - 0.0080z^{-2}}{1 - 0.8054z^{-1}} \tag{16}$$

In Fig. 8, its impulse response is compared with the first-order Volterra kernel obtained in Section 3. Clearly, the plant linear character has been correctly captured in the NARX model. When compared with the second-order Volterra and the block-structured models, the predictive performance of the NARX model is clearly improved, as expected. But the model is essentially a local model: outside the neighborhood of the operation range covered by the input perturbation signal, the accuracy of the model prediction result deteriorates quickly.

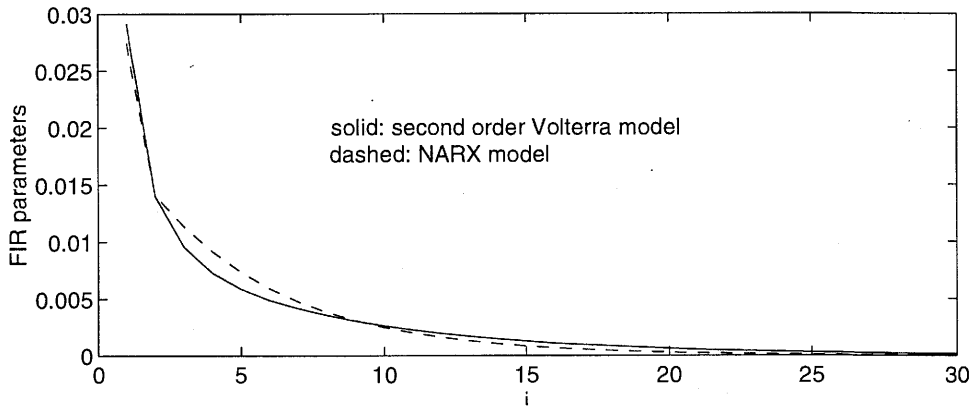

Fig. 8. Comparison of estimated linear models portions, NARX and Volterra models.

Similary to the Wiener model case, the model predicted steady-state response will deteriorate as the switching time of the input signal decreases but not as drastically as in the Wiener model case. If the steady-state nonlinearity is important to the intended model application, for the column in this study, it seems that using a switching time not less than the plant nominal time constant is appropriate.

6. Control Relevant Identification

As noted in the Introduction, a mathematical model, no matter how complicated, is always a simplified image of a real-life system. The situation becomes more apparent when the model's descriptive ability is restricted to a simple model structure such as the block-structured model. Various design variables in an identification procedure can be used to influence the goodness of fit on the estimated model. Clearly, selection of the optimal design variables depends on the intended model application. In this context, it is desired that intended model applications are directly accounted in an identification procedure. In this section, we assume that the estimated model will be used for process control design. A control-relevant identification method proposed by Ling and Rivera (1995; 1996) is considered to estimate a Hammerstein model for the distillation column simulation.

The control-relevant identification method is summarized as follows. The method consists of two steps. In the first step, a Nonlinear AutoRegressive and Moving Average with eXogenous (NARMAX) model is estimated from the input/output measurements in a usual way. For the noise/disturbance-free problem in this study, the NARMAX model reduces to a NARX model. In the second step, a restricted complexity model such as the block-structured model is obtained through a control-relevant model reduction procedure.

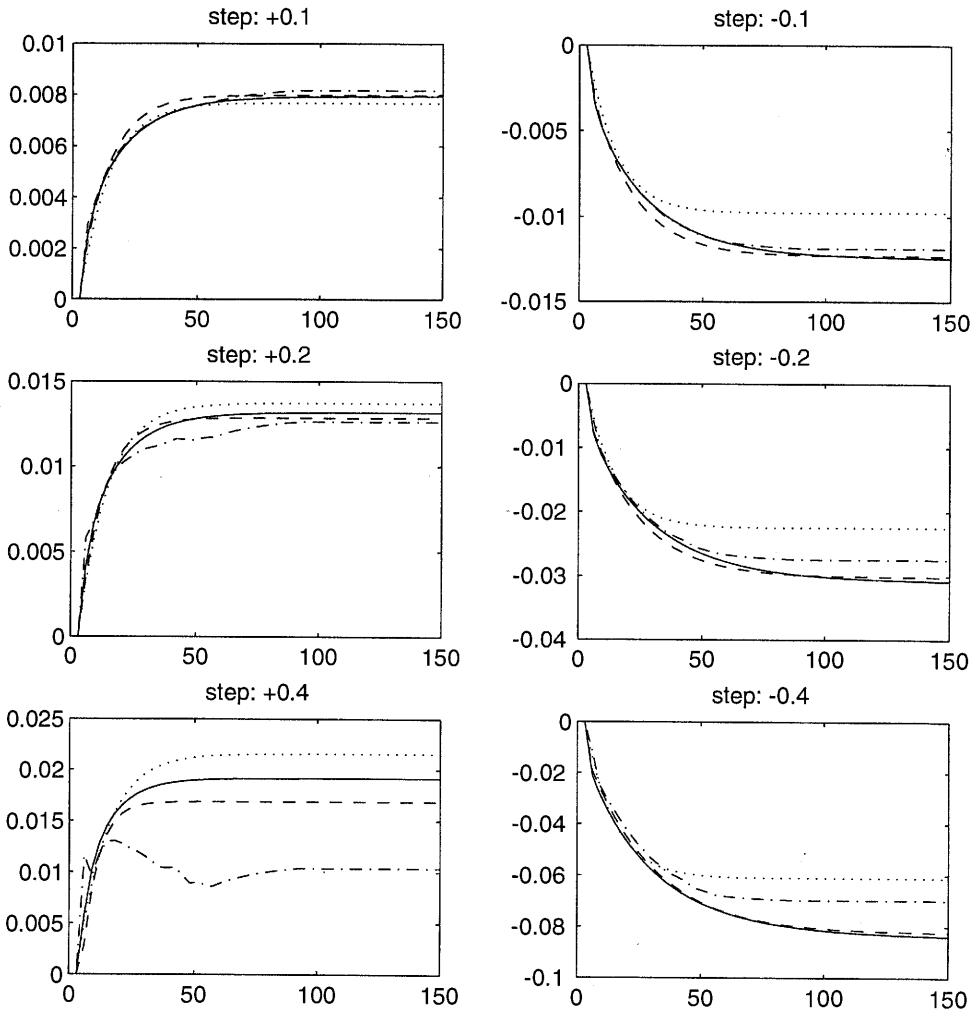

Fig. 9. Step responses of different models (solid: plant, dashdot: Volterra model, dotted: Hammerstein model, dashed: NARX model).

For the case where the restricted complexity model is a Hammerstein model, let both the NARX and the Hammerstein models be represented in their respective Volterra series model forms:

$$p = p_1 + p_2 + \cdots + p_n, \quad \tilde{p} = \tilde{p}_1 + \tilde{p}_2 + \cdots + \tilde{p}_n$$

The closed-loop performance of the estimated model is accessed through the non-linear IMC design in Fig. 11 (Doyle *et al.*, 1995). The control-relevant model reduction

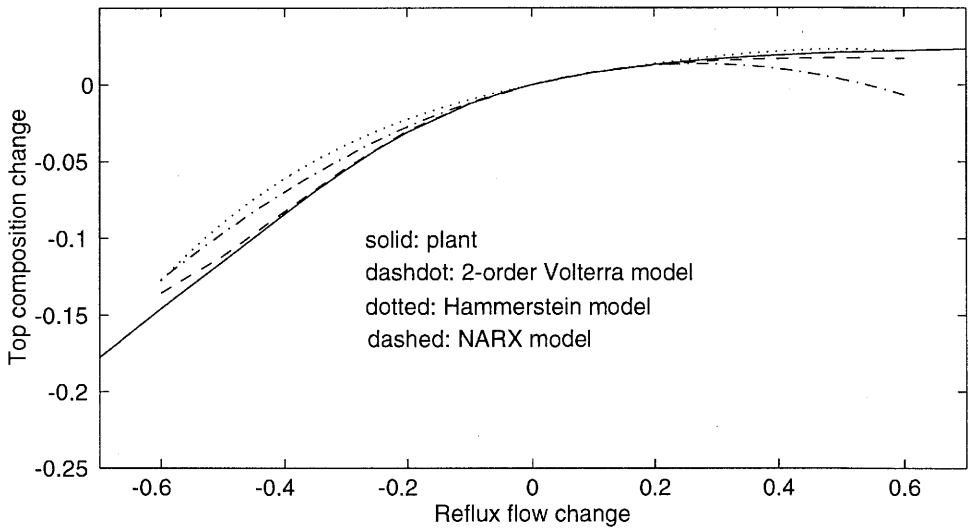

Fig. 10. Steady-state responses of different models (solid: plant, dashdot: Volterra model, dotted: Hammerstein model, dashed: NARX model).

problem is then stated as (Ling and Rivera, 1995):

$$\begin{aligned} \tilde{p}_1 &= \arg \min_{\tilde{p}_1} \left\| \epsilon w_1 [r-d] \right\|_2 \\ \tilde{p}_2 &= \arg \min_{\tilde{p}_2} \left\| \epsilon w_2 [r-d] \right\|_2 \\ &\vdots \\ \tilde{p}_n &= \arg \min_{\tilde{p}_n} \left\| \epsilon w_n [r-d] \right\|_2 \end{aligned} \tag{17}$$

where $\epsilon = I - F$; w_1 is a function of $p_1, \tilde{p}_1$; w_2 is a function of $w_1, p_2, \tilde{p}_2$ and so on (see Ling and Rivera, 1995). Minimization problems in (17) can be numerically effectively solved as discussed in (Ling and Rivera, 1996). An important feature of the estimation problem posed in (17) is that the selection of proper identification design variables is now replaced by parameters related to the closed-loop control performance requirements such as the desired closed-loop speed-of-response (ϵ) and the external signals that the control system is most likely to track or reject $(r - d)$.

The effects of a control-relevant approach were examined on the distillation column simulation. We assume that the control objective is to track a step set point change in Fig. 12 and a first-order linear filter F according to

$$F(z) = \frac{(1-\delta)z^{-1}}{1-\delta z^{-1}}, \quad \delta = \exp(-3/15) \tag{18}$$

is used in the nonlinear IMC design. In the control-relevant approach, a Hammerstein model with a third-degree polynomial memoryless nonlinear element and a first-order

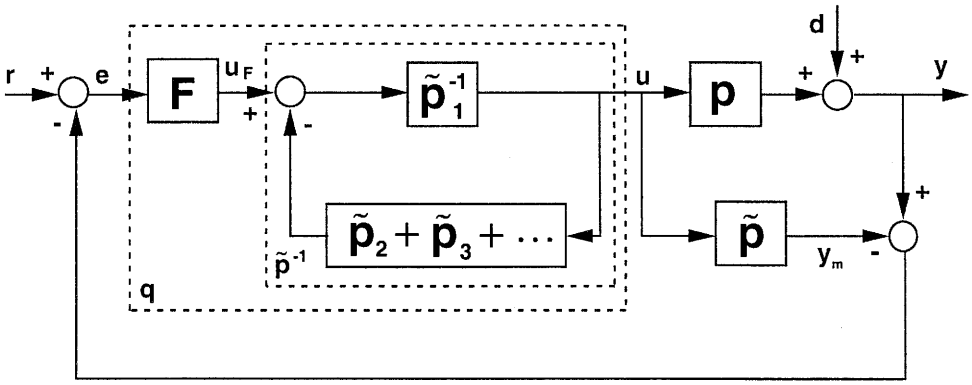

Fig. 11. Nonlinear Internal Model Control structure used to assess model control-relevance.

Table 4. Control-relevant (C-R) vs. open-loop optimal (O-L) Hammerstein model parameters. The open-loop error (O-LE) is the 2-norm of the prediction error. The closed-loop error (C-LE) is the integral square error for a −0.03 setpoint change.

Model	b_1	a_1	γ_1	γ_2	O-LE	C-LE
Model 4 (C-R)	0.0217	−0.7768	−2.0833	1.7878	0.2613	5.5256e-5
Model 5 (O-L)	0.0204	−0.7660	−1.3775	0.8977	0.2593	9.5340e-5

linear dynamic element is estimated using as a starting point the polynomial NARX model estimated in Section 5. Estimated model parameters (Model 4) are given in Table 4. Correspondingly, the Hammerstein model parameters which minimize the open-loop prediction error to the identification data are obtained (Model 5 in Table 4).

In Fig. 13, the closed-loop performance of both models is simulated for the case of a −0.03 setpoint change. From the standpoint of open-loop criteria, the control-relevant model appears less adequate; however, comparison of the closed-loop error (Table 4) indicates that this model provides an improvement in closed-loop performance. Figure 13 shows that the control-relevant model reduces overshoot and decreases settling time while displaying a less aggressive manipulated variable response than that obtained from the open-loop model-based controller.

7. Summary and Discussion

Nonlinear black box identification of a simulated binary distillation column has been investigated. Three commonly used model structures including the second-order

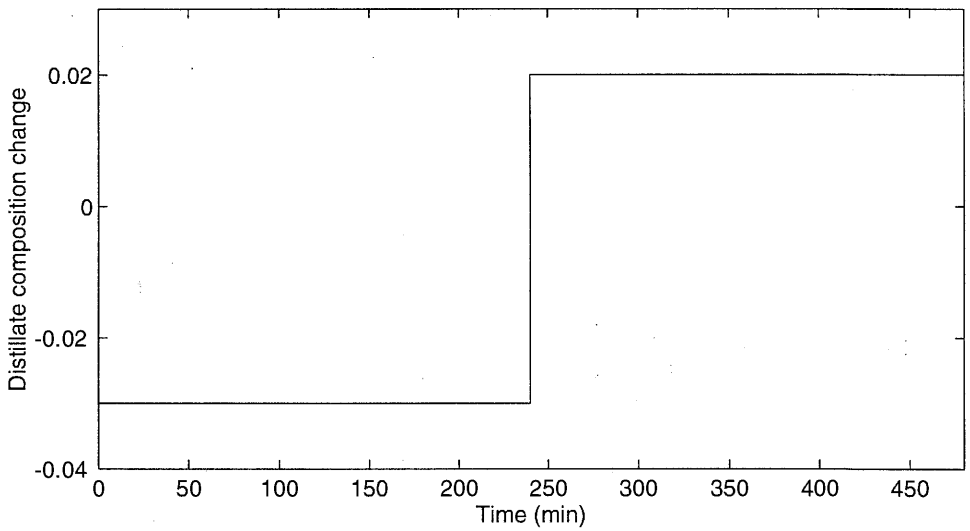

Fig. 12. Set point changes assumed in control-relevant estimation, Hammerstein Model 4.

Volterra model, block-structured models and the NARX model were used to describe the column dynamics. The performance of these models is evaluated based on their steady-state response and step responses of different amplitudes. Among them, the NARX model provides the most accurate approximation to the column dynamics. A second-order Volterra model, despite the fact that it can display the asymmetry response phenomenon (Pearson *et al.*, 1992) which is clearly observed in the distillation column response, can correctly capture the column dynamics only in a quite limited operation range. For the block-structured Wiener and the Hammerstein models, their limitations and some trade-offs among fittings to different plant characters were explained. It is pointed out that if the identification design variables such as the input design are selected properly a block-structured model can well approximate the column dynamics. But the problem how to select those design variables must be accessed in an *ad hoc* manner. When the estimated model was used for control design, a control-relevant identification method was applied to overcome this difficulty. In the control-relevant method, those identification design variables are represented by the closed-loop performance requirements such as the desired closed-loop speed-of-response. Closed-loop performance requirements naturally influence the goodness-of-fit in the model.

For all the model structures, it was emphasized that if the system steady-state nonlinearity is important for the intended model application, as it does for most process control applications, the input signal switching time should be made sufficiently long. For the column considered in this study, it seems that a switching time not less than the plant nominal time constant is appropriate. Because most distillation

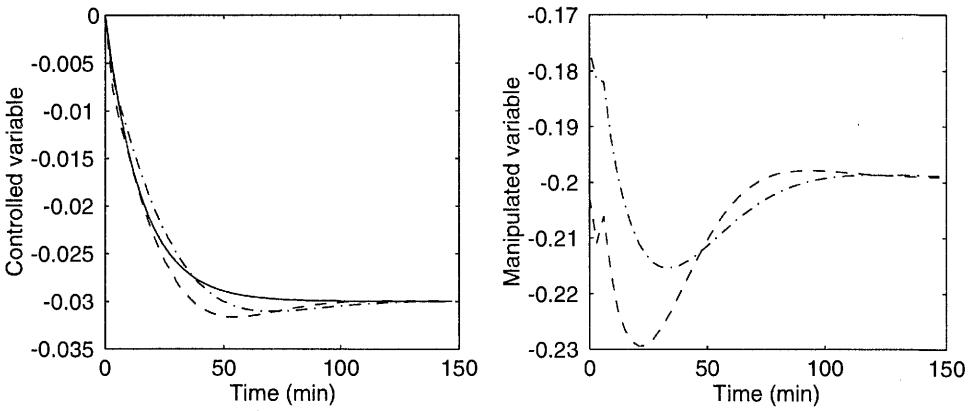

Fig. 13. Closed-loop responses (solid: reference, dashdot: control-relevant Hammerstein model (Model 4), dashed: open-loop optimal Hammerstein model (Model 5)).

processes display a slow open loop response, this corresponds to a quite long switching time.

Acknowledgments

The financial support from the Dupont Educational Aid Program and the ASU-Honeywell Control System Engineering Program is gratefully acknowledged.

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