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EHMAC – A NEW SIMPLE TOOL FOR ROBUST LINEAR MULTIVARIABLE CONTROL

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A combination of long range predictive control-originated EHPC and internal model control-structured MAC is shown to produce a new, simple but effective Extended Horizon Model Algorithmic Control (EHMAC). The EHMAC strategy can be used to robustly control open-loop stable non-minimum phase (possibly non-square) MIMO systems under very large model-plant mismatches. Robust EHMAC design is made straightforward by means of a separate selection of a single prediction horizon and an IMC filter parameter, which can be easily auto-tuned.

Keywords: internal model control, model-based predictive control, multivariable control, orthonormal basis functions, robust control

1. Introduction

Long-range or model-based predictive control (Bitmead *et al.*, 1990; Clarke and Mohtadi, 1989; Coelho and Amaral, 1993; Garcia *et al.*, 1989; De Keyser *et al.*, 1989; Soeterboek, 1992) has drawn considerable interest from both the academic and industrial millieu, bringing the two much closer to each other. There are many recognized advantages of long-range predictive control (LRPC), most of them contributing to well-documented robustness. The most general (and complex), unified and generalized predictive controls, UPC and GPC, respectively, can robustly stabilize complex plants, including open-loop unstable non-minimum phase (NMP) ones, in which they can approach, to some extent, LQR/LQG schemes. However, for the majority of process control tasks, involving (possibly NMP) open-loop stable systems, it may be not necessary to employ rather complex UPC, GPC or LQR/LQG algorithms, which in addition may require sophisticated tuning to meet control quality specifications for a specific plant. The control job in many standard cases can be effectively done by much simpler, and yet powerful, predictive control algorithms like EHPC (Latawiec, 1998a; Latawiec and Van Cauwenberghe, 1995; Ydstie *et al.*, 1985), MAC or DMC (Garcia and Morari, 1982; Garcia *et al.*, 1989). The numerical simplicity of a control algorithm can be essential in the adaptive control environment as well as for 'fast' dynamic systems where 'short' sampling periods should be selected.

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Extended Horizon Predictive Control (EHPC) is known to be able to stabilize open-loop stable NMP systems, with a single tuning parameter involved, namely the control prediction horizon. On the other hand, the industrially recognized Model Algorithmic Control (MAC), utilizing the advantages of Internal Model Control (IMC), has been initially designed to control minimum phase systems only (Garcia and Morari, 1982; Rouhani and Mehra, 1982). Some modifications to MAC, intending to cope with NMP systems as well, required a fairly sophisticated GPC-like numerical machinery (Garcia *et al.*, 1989) or quite awkward procedures (Mehra and Rouhani, 1980), thus suppressing the original simplicity of the method.

Here we present a new, simple, linear multivariable control strategy which is an effective combination of EHPC and MAC, called *E*xtended *H*orizon *M*odel *A*lgorithmic *C*ontrol (cf. Latawiec 1996; 1997 for SISO systems). EHMAC is designed to robustly control open-loop stable NMP systems without resorting to multistep, GPC-like control criteria or elegant, but computationally involving, IMC-structured H_∞-control schemes (Murad *et al.*, 1997).

The remainder of this paper is organized as follows. The general, closed-loop EHPC strategy is outlined in Section 2. The basics of IMC, including MAC, are given in Section 3, with new steady-state control accuracy results presented in a general form. In Section 4, the open-loop control design methodology for multivariable EHPC feedback systems is delineated and two feedback structures of the EHPC systems are recalled, namely the 'classical' and 'predictive' ones. In Section 5, the robust EHMAC strategy is readily derived from the classical control structure of open-loop-designed feedback EHPC, which is shown to be closely related to the IMC structure. Specifically, a two-stage EHMAC design procedure is presented, the first stage coming from EHPC and aiming at stabilization of an NMP *model* of a plant, and the other consisting in selecting an IMC filter as in MAC, thus providing the control robustness even for extremely large deviations of plant models from the actual plant. Tuning procedures for the EHMA controller's parameters are also recalled. The results of simulation, examples of Section 6, clearly demonstrate the value of the new control strategy, which is resumed in the conclusions of Section 7.

2. Extended-Horizon Predictive Control

Consider a linear time-invariant discrete-time n_u-input n_y-output system governed by the ARX model

$$A\left(q^{-1}\right) y(t) = q^{-d} B\left(q^{-1}\right) u(t) + e(t), \tag{1}$$

where $y(t) \in \mathbb{R}^{n_y}$, $u(t) \in \mathbb{R}^{n_u}$ and $e(t) \in \mathbb{R}^{n_y}$ are the output, input and zero-mean white noise, respectively, in discrete time t; $A \in \mathbb{R}^{n_y \times n_y}[z]$, $B \in \mathbb{R}^{n_y \times n_u}[z]$ are the left-coprime polynomial matrices of order n and $m \leq n$, respectively; q^{-1} is the backward shift operator and d is the time delay. We will not distinguish between $A(z^{-1})$ and $\underline{A}(z) = z^n A(z^{-1})$, nor between $B(z^{-1})$ and $\underline{B}(z)$, the more so as $A^{-1}(z^{-1})B(z^{-1}) = \underline{A}^{-1}(z)\underline{B}(z)$.

We refrain from using the ARMAX model as it is well-known that the polynomial matrix of disturbance parameters is in practice unlikely to be effectively estimated (and it is sometimes used as a control design, 'observer' polynomial matrix instead).

Theorem 1. *Let an LTI discrete-time system be described by the ARX model (1) with $A \in \mathbb{R}^{n_y \times n_y}[z]$ and $B \in \mathbb{R}^{n_y \times n_u}[z]$ left coprime. Then the EHPC1 law $u(t)$, minimizing $\mathrm{E}\{\|y(t+k) - y_{\mathrm{ref}}\|^2\}$ under the assumption $u(t) = u(t+1) = \cdots = u(t+k-d)$ and the model equation constraint, is of the form*

$$u(t) = \left[G'(1)\right]^{\#}\left[y_{\mathrm{ref}} - \underline{H}(q^{-1})y(t) - q^{-1}G''(q^{-1})u(t)\right], \tag{2}$$

where k is the prediction horizon with $k > d$, $C^{\#}$ is (any) generalized inverse of C, and the appropriate polynomial matrices $\underline{F}$ and $\underline{H}$ of orders $k-1$ and $n-1$, respectively, are determined given A and k from the polynomial matrix identity

$$I_{n_y} = \underline{F}A + z^{-k}\underline{H} \tag{3}$$

with

$$\tilde{G}(q^{-1}) = \underline{F}(q^{-1})B(q^{-1}) = G'(q^{-1}) + G''(q^{-1})q^{-k+d-1} \tag{4}$$

and

$$\begin{cases} G'(q^{-1}) = g_0 + g_1 q^{-1} + \cdots + g_{k-d}q^{-k+d}, \\ G''(q^{-1}) = g_1'' + g_2'' q^{-1} + \cdots + g_{m+d-1}'' q^{-m-d+2}. \end{cases} \tag{5}$$

Proof. Pursuing the k-step output predictor, we have

$$y(t+k) = \underline{H}\,y(t) + \tilde{G}q^{k-d}u(t) + \underline{F}e(t+k) = \hat{y}(t+k) + \underline{F}e(t+k),$$

where $\underline{F}$ and $\underline{H}$ satisfy the polynomial matrix identity (3), and $\tilde{G}$, split into two parts (separating the present and future controls from the past ones), is defined by (4) and (5). Now, minimizing the control performance criterion under the aforementioned assumption on future inputs leads to the solution

$$y_{\mathrm{ref}} = \underline{H}(q^{-1})y(t) + \left[G'(1) + q^{-1}G''(q^{-1})\right]u(t)$$

and the result follows. ■

For EHPC1 we will write

$$G'(1) = G_1'(1) = \sum_{i=0}^{k-d} g_i. \tag{6}$$

Theorem 2. *Let an LTI discrete-time system be described by the ARX model (1), with $A \in \mathbb{R}^{n_y \times n_y}[z]$ and $B \in \mathbb{R}^{n_y \times n_u}[z]$ left coprime. Then the EHPC2 law minimizing, with respect to $u(t), u(t+1), \ldots, u(t+k-d)$, the (input 'energy') control effort*

$$\sum_{i=0}^{k-d} \left\|u(t+i)\right\|^2 \tag{7}$$

subject to $E\{y(t+k)-y_{\mathrm{ref}}\}=0$ *and the model equation constraint, is of the form (2), with*

$$G'(1)=G_2'(1)=g_{k-d}^{\#}\sum_{i=0}^{k-d}g_i^2. \tag{8}$$

Proof. Arguments similar to those in the proof of Theorem 1 apply to this case, the only difference being the employment of constrained minimization. ■

Notice that for $k=d$ EHPC specializes to minimum variance control (MVC).

Remark 1. Scaling the control law (2) by the factor $[G'(1)]^{\#}$ affects an EHP controller gain in a quite similar way as the introduction of a control weight in the generalized minimum variance controller. Thus, selection of some $k>d$, with the prediction horizon k being a single design/tuning parameter, enables us to stabilize a control system with an open-loop stable NMP plant (Latawiec, 1998a; Latawiec and Van Cauwenberghe, 1995; Ydstie *et al.*, 1985). Moreover, some open-loop unstable NMP plants can also be closed-loop stabilized under EHPC (Latawiec and Van Cauwenberghe, 1995).

3. Internal Model Control

We will further assume that a plant is open-loop stable, even though some open-loop unstable systems, in particular those with integrator(s), can also be tractable (Latawiec nad Van Cauwenberghe, 1995). The IMC idea (Garcia and Morari, 1982; Garcia *et al.*, 1989), originally related to the celebrated Smith predictor concept (Brosilow, 1979), has been recognized as a valuable tool for advanced process control (Datta and Ochoa, 1998; De Nicolao *et al.*, 1996; Latawiec, 1997; 1998a; Lopez *et al.*, 1996; Murad *et al.* 1997; Pinchon *et al.*, 1996; Yamada *et al.*, 1996).

The basic IMC structure, for which the control robustness can be easily designed (Garcia and Morari, 1982; Morari and Zafiriou, 1989), is shown in Fig. 1. The structure can empower the control system to act effectively open-loop when there is no difference between a plant and its model, but to tighten the closed-loop in accordance with the model-plant mismatch. It turns out that any conventional feedback controller can be restructured to yield IMC. Furthermore, any IMC can be put into the classical feedback form of Fig. 2 by defining the conventional controller $G_r\in\mathbb{R}^{n_u\times n_y}(z)$ as $G_r=\{[(GG_c)^{-1}-\hat{G}G^{\bar{R}}]G\}^{\bar{R}}$, where $A^{\overline{R}}$ denotes (any) right inverse of A, $G_c\in\mathbb{R}^{n_u\times n_y}(z)$ is the internal model controller, and $G\in\mathbb{R}^{n_y\times n_u}(z)$ and $\hat{G}\in\mathbb{R}^{n_y\times n_u}(z)$ stand for the plant and model representations, respectively. (Notice that for square systems we have $G_r=(G_c^{-1}-\hat{G})^{-1}$). The IMC filter $F\in\mathbb{R}^{n_y\times n_y}(z)$ will be introduced later on. The convenient 'G_c-parametrization' of the controller has been used for a long time, mostly under the heading of IMC. Zames (1981) introduced some fundamental concepts of H_∞-control utilizing the G_c-parametrization. These concepts, originally based on external models, have been widely used in a more general, robust control environment as an alternative to the state space approach, with a

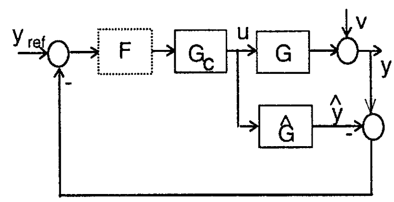

Fig. 1. Basic IMC structure.

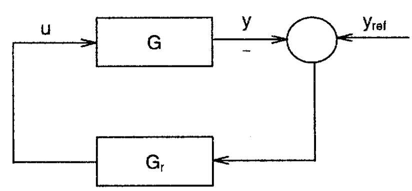

Fig. 2. Classical structure.

sort of compromise solutions being also available from combined schemes (Murad *et al.*, 1997).

Thus, the main advantage of IMC is that the controller $G_c(z)$ is much easier to design than $G_r(z)$ and, in the case of a possible model-plant mismatch, the IMC structure enables us to include the control robustness as a design objective in a very explicit manner.

Another favourable feature of IMC is that neither an integration action nor an incremental model formulation need be introduced, as the IMC structure can inherently provide steady-state error-free servo and regulatory controls in the case of stepwise forcing signals. This well-known SISO and square MIMO feature is now generalized.

3.1. Generalization of Results on Steady-State Control Accuracy

Theorem 3. *Let an open-loop stable linear plant and its (open-loop stable linear) model be governed by the right-invertible transfer-function matrices $G \in \mathbb{R}^{n_y \times n_u}(z)$ and $\hat{G} \in \mathbb{R}^{n_y \times n_u}(z)$, respectively, and let a linear open-loop stable IM controller $G_c \in \mathbb{R}^{n_u \times n_y}(z)$ be $G_c(z) = \underline{\Gamma}_0^{\bar{R}}(z)$, where $\underline{\Gamma}_0^{\bar{R}}(z)$ is the minimum-norm right inverse of some $\underline{\Gamma} \in \mathbb{R}^{n_y \times n_u}(z)$. Then, for stepwise changes in the reference/disturbance, steady-state error-free servo and regulatory controls are provided if*

$$\underline{\Gamma}_0^{\bar{R}}(1)\hat{G}(1) = I. \tag{9}$$

The proofs of the above theorem and the forthcoming corollary were given by Latawiec (1998a) and Latawiec *et al.* (1998).

Remark 2. The *minimum-norm* right inverse, denoted by the subscript '0', is used in eqn. (9) and later on for the uniqueness purpose only. In general, any right inverse can be employed.

Remark 3. The introduction of the $\underline{\Gamma}(z)$ polynomial matrix aims at preparation to the synthesis of IMC-structured, inverse model- related MAC and EHMAC strategies.

Remark 4. It is essential that the condition on the $G_c(z)$ controller to be stable (i.e. stabilizing) can be immediately translated to the minimum phase behaviour requirement for $\underline{\Gamma}(z)$, calling for the need to have a general definition of minimum/nonminimum phase systems, preceded by a sound definition of '*control* zeros' for possibly nonsquare MIMO systems, both the new contributions having been offered by Latawiec (1998a) and Latawiec *et al.* (1999).

Corollary 1. *Let an open-loop stable linear plant and its (open-loop stable linear) model be governed by full normal rank transfer-function matrices* $G \in \mathbb{R}^{n_y \times n_u}(z)$ *and* $\hat{G} \in \mathbb{R}^{n_y \times n_u}(z)$*, respectively, with* $n_y = n_u$*, and let a linear open-loop stable IM controller* $G_c \in \mathbb{R}^{n_u \times n_y}(z)$ *be* $G_c(z) = \underline{\Gamma}^{-1}(z)$*. Then, for stepwise changes in the reference/disturbance, steady-state error-free servo and regulatory controls are provided by* ***any*** $\underline{\Gamma}(z)$ *if this transfer-function matrix is prescaled by the factor* $\hat{G}(1)\underline{\Gamma}^{-1}(1)$.

Remark 5. The conditions involved in Theorem 3 and Corollary 1 are only sufficient.

Remark 6. It is essential that, for stable square MIMO (including SISO) systems, the very specific structure of IMC enables *any* stable controller to produce $y = y_{\text{ref}}$ in the steady state in response to stepwise changes in the reference/disturbance, provided that the controller's gain is scaled accordingly. Note that the scaling factor involves the (available) gain of a plant *model* and is independent of the actual gain of the plant.

Remark 7. In the case of a non-full normal rank or left-invertible systems the condition (9) of Theorem 3 cannot be obtained, nor Corollary 1 can ever hold true. This means that the steady-state error-free control cannot be reached in such a case.

Latawiec *et al.* (1998) discussed some implications of the above control accuracy results in multivariable predictive control strategies like MAC, EHMAC and GPC. In particular, the basic GPC scheme, involving the common incremental model formulation, was recalled. The GPC scheme was implemented in the IMC structure under a constant reference (Pinchon *et al.*, 1996). It was shown by Latawiec *et al.* (1998) that such a combination of IMC and incremental-formulated GPC is not necessary in a specific application, as suitable scaling of the GP controller would be sufficient. Consequently, a simplified GPC scheme, involving *neither* the incremental model formulation, *nor* a number of prediction horizons, *nor* the control weighting constant, can be effectively used in the IMC structure. But this 'simplified GPC' used in the IMC structure will bring us to our effective EHMAC.

3.2. Model Algorithmic Control

Recall the ARX model of a plant and write

$$\hat{G}(z) = z^{-\hat{d}}\hat{A}^{-1}(z^{-1})\hat{B}(z^{-1}) = z^{-\hat{d}}\sum_{i=0}^{\infty}\hat{g}_i z^{-i} = z^{-\hat{d}}\underline{\hat{G}}(z), \tag{10}$$

where the matrix components $\hat{g}_i$ of the impulse response model are obtained by the 'long division' $\hat{A}^{-1}\hat{B}$. For simplicity, the hats over the parameter estimates will further be omitted.

The basic concept of *M*odel *A*lgorithmic *C*ontrol, pertaining to the IMC structure and related to the Smith predictor, is characterized by the following three crosscoupled features:

1. $\underline{\Gamma}(z) = \hat{\underline{G}}(z)$, which means that MAC is an inverse-model control, with the condition (9) always fulfilled,
2. open-loop stable robustifying filter $F \in \mathbb{R}^{n_y \times n_y}(z)$, with $F(1) = I$, is necessarily used to attenuate the (typical) oversensitivity of the inverse-model control (see Fig. 1), and
3. MAC is asymptotically stable for minimum phase systems only.

It is well-known that the main drawback to MAC, in addition to the minimum phase system limitation, is that it is generally very sensitive to a poorly estimated or a time-varying delay.

4. Open-Loop Design of Feedback EHPC Systems

We assume that a plant is asymptotically stable. To suit the open-loop design formulation, we introduce the alternative (to (1)), convolution description of an n_u-input n_y-output plant

$$y(t) = \sum_{i=0}^{\infty} g_i q^{-i} u(t-d) + v(t), \tag{11}$$

where $v(t) = A^{-1}(q^{-1})e(t)$ is the correlated zero-mean noise vector and the impulse response $\{g_t,\ t = 0, 1, \ldots\}$ corresponds to the right-invertible transfer-function matrix $G \in \mathbb{R}^{n_y \times n_u}(z)$.

Since the plant is asymptotically stable, we can approximate (11) by assuming a finite process memory, $g_i = 0\ \ \forall i > N-1$. We will go on with a Finite Impulse Response (FIR) built up of N matrix components g_i, $i = 0, \ldots, N-1$, with the plant transfer-function matrix $z^{-d}A^{-1}(z^{-1})B(z^{-1})$ approximated by

$$G(z^{-1}) = z^{-d}\left(g_0 + g_1 z^{-1} + \cdots + g_{N-1} z^{-N+1}\right) = z^{-d}\bar{G}(z^{-1}). \tag{12}$$

The general linear open-loop control law is

$$\Gamma(q^{-1})u(t) = y_{\text{ref}}, \tag{13}$$

where $\Gamma \in \mathbb{R}^{n_y \times n_u}(z)$, with $\Gamma(q^{-1}) = \gamma_0 + \gamma_1 q^{-1} + \cdots + \gamma_{n_g} q^{-n_g}$.

Combining the process model equation (12) with (13), we obtain the familiar, open-loop control system equation

$$y(t) = q^{-d}\bar{G}(q^{-1})\Gamma_0^{\bar{R}}(q^{-1})y_{\text{ref}}, \tag{14}$$

where $\Gamma_0^{\bar{R}}$ is the minimum-norm right inverse of Γ.

Now, assuming an open-loop stable plant, the control system described by (14) is stable iff the controller $\Gamma_0^{\bar{R}}(q^{-1})$ is stable. This implies the well-known fact that the control system with the inverse-model controller $\Gamma(q^{-1}) = \bar{G}(q^{-1})$ is stable for stably (generalized-)invertible or minimum-phase systems only. We can also see from (14) that, when the control system is stable, the error-free servo behaviour (for a constant reference) can be obtained if $\Gamma(1) = \bar{G}(1)$.

Consider the EHP control objective under the constraint

$$y(t) = \sum_{i=0}^{N-1} g_i q^{-i} u(t-d) + v(t). \tag{15}$$

It is well-known that the long-range output predictor for all the EHPC versions is of the same form, i.e.

$$\hat{y}(t+k) = \Gamma(0)u(t) + \sum_{i=k-d+1}^{N-1} g_i q^{-i} u(t+k-d), \tag{16}$$

with $k > d$ as before and $\Gamma(0) = \gamma_0$.

The EHPC open-loop control law is now of the form (13) with

$$\Gamma\left(q^{-1}\right) = \Gamma(0) + \sum_{i=k-d+1}^{N-1} g_i q^{-i+k-d}, \tag{17}$$

where $\Gamma(0) = G'(1)$ as in (6) or (8). Note that for $k = d$ we obtain an MVC.

The key step in our design is the demand on the closed-loop control system, represented by the equation $y(t) = G_{cl}(q^{-1})y_{\text{ref}}$, to have the transfer-function matrix $G_{cl}(z^{-1})$ identical to that of the open-loop control system, i.e.

$$G_{cl}\left(z^{-1}\right) = G\left(z^{-1}\right)\Gamma_0^{\bar{R}}\left(z^{-1}\right). \tag{18}$$

In this way, the design specifications for the feedback control system could be expressed in terms of the Γ polynomial matrix of the open-loop controller, which contains the components of the FIR of the plant.

The so-called 'classical' and 'predictive' control structures of the closed-loop SISO system for which the open-loop EHPC design technique can be employed, were examined by Latawiec (1995). The 'predictive' control structure gives rise to the solution to the control problem as presented in Section 2. Here we focus our interest on the alternative, classical (MIMO) structure depicted in Fig. 2.

Pursuing the equivalence of the closed-loop control system of Fig. 2 and the EHPC open-loop one, we equate $(I + GG_r)^{-1}GG_r = G\Gamma_0^{\bar{R}}$ according to (18). Hence we obtain

$$G_r = \Gamma_0^{\bar{R}}\left(I - G\Gamma_0^{\bar{R}}\right)^{-1}. \tag{19}$$

An interesting specific result is obtained for square MIMO systems $G_r = (\Gamma - G)^{-1}$, so that the feedback control law is

$$(\Gamma - G)u(t) = y_{\text{ref}} - y(t). \tag{20}$$

5. Robust EHMAC

Since we arrive at effectively open-loop control in the feedback control system (from the viewpoint of tracking the reference), this implies the aforementioned equivalence relation to IMC. In fact, sensitivity and complementary sensitivity functions can be easily shown to be precisely the same as for IMC. This means that the control structure of Fig. 2, with the controller as in (19), is equivalent to IMC.

Now, we can immediately recognize (20) as the common IMC control law if we realize that, referring to the IMC structure, G is the transfer-function matrix of the plant *model* denoted by $\hat{G}$, so that $\hat{y}(t) = \hat{G}(q^{-1})u(t)$ is the model output or the plant output predictor. (Note that we can now identify $\bar{G}(\cdot)$ with $\hat{\underline{G}}(\cdot)$). Then we have $\Gamma(q^{-1})u(t) = y_{\text{ref}} - y(t) + \hat{y}(t)$ which, when supplemented with the (open-loop stable) IMC filter $F \in \mathbb{R}^{n_y \times n_y}(z)$, finally gives the *E*xtended *H*orizon *M*odel *A*lgorithmic *C*ontrol law

$$\Gamma(q^{-1})u(t) = F(q^{-1})\,[y_{\text{ref}} - y(t) + \hat{y}(t)] \tag{21}$$

with the output predictor $\hat{y}(t)$ computed either from the ARX or FIR plant models.

Theorem 4. *Let an open-loop stable linear plant and its (open-loop stable linear) model be governed by full-normal rank transfer-function matrices $G \in \mathbb{R}^{n_y \times n_u}(z)$ and $\hat{G} \in \mathbb{R}^{n_y \times n_u}(z)$, respectively, with $n_y = n_u$, and let the (linear open-loop stable) EHMAC law be given by (21), where the filter F is open-loop stable. Then the feedback EHMAC system is asymptotically stable iff all the roots of the characteristic equation*

$$\det\left\{\left[\Gamma + F(G - \hat{G})\right]\right\} = 0 \tag{22}$$

lie inside the unit circle.

Proof. The closed-loop control system equation can be easily shown to have the form (in terms of the $\mathcal{Z}$-transfer function) $Y = G[\Gamma + F(G - \hat{G})]^{-1}FY_{\text{ref}}$. By standard stability arguments, the result follows. ∎

Now, for a given model-plant mismatch, a filter F can be selected so that (22) has all its roots inside the unit circle. Typically, the filter is of the form $F(z^{-1}) = [(I - \alpha z^{-1})^{-1}(I - \alpha)]^r$, where the filter order r is usually taken as 1 or 2, and the filter parameter matrix α is most often selected as $\alpha = \text{diag}\,[\alpha_1, \dots, \alpha_{n_y}]$, with $\alpha_i \in (0, 1)$ $\forall i = 1, \dots, n_y$, or, which is exploited in EHMAC, with $\alpha_1 = \alpha_2 = \cdots = \alpha_{n_y} = \underline{\alpha}$. In practice, the transfer-function matrix G of an actual plant is usually unknown, so that the selection of a filter F for the closed-loop stability is made on the basis of some measure of the model-plant mismatch (Latawiec, 1998a).

The EHMAC strategy, combining the advantages of EHPC and MAC, is thus composed of the following *separate* stages:

1. determination of the polynomial matrix Γ as in EHPC, i.e. selection of an appropriate horizon k so that possibly an NMP *model* of the plant could be stabilized,
2. selection of the IMC filter parameter(s) to make the control system *robust* with respect to a possible model-plant mismatch.

Remark 8. It is interesting that the polynomial matrix Γ can be interpreted as the 'best' (in the sense of the EHPC criterion) minimum-phase approximation to the NMP model factor $\underline{\hat{G}}$, the approximation having repeatedly been sought after by (not only) IMC designers (Garcia and Morari, 1982; Garcia *et al.*, 1989).

Remark 9. Note that, inherently, EHMAC based on EHPC1 *always* satisfies the conditions of Theorem 3 due to the specific form of Γ as in (17) and (6). Also observe that in case the error-free control conditions are satisfied, the IMC filter should, as usual, be such that $F(1) = I$.

Remark 10. It is interesting that the EHMAC law (21) can be used as a sort of 'suboptimal' one for nonsquare systems as well. In such cases, the last remark is still valid whenever Γ is right-invertible.

Remark 11. The above one-degree-of freedom scheme can be readily extended to the two-degree-of-freedom one.

Remark 12. EHMAC can produce the whole spectrum of quality controls, from a high accuracy-oriented MVC $(G = \hat{G},\ k = d,\ F(z) = I)$ to a robustness-oriented control $(G \neq \hat{G},\ k > d,\ F(z) \neq I)$. The latter case is of special practical interest, in particular for large model-plant mismatches.

5.1. Robustness to Parametric Uncertainties

The expected IMC-originated robustness of EHMAC against disturbances and *extreme* parametric uncertainties, including over/underestimation of a plant gain as well as of orders n and m, has been verified by Latawiec (1995; 1996; 1997; 1998a) and Latawiec and Van Cauwenberghe (1995). Another distinguishing feature of EHMAC, which is not inherited from MAC, is indicated in the forthcoming subsection.

5.2. Robustness to Uncertainties in Estimation of the Time Delay

It is important that the EHPC-originated, *long-range predictive feature* of EHMAC makes it robust to either a poorly estimated or a time-varying delay d, the outstanding advantage removing the main inconveniency of MAC. The issue has been carefully examined by Latawiec and Domek (1998) and Latawiec *et al.* (2000a; 2000b). Here we wish to emphasize that the value of EHMAC in this respect can even be more appreciated for MIMO systems, in particular for multiple-delay cases, i.e. when the distribution of time delays throughout various entries of $\hat{G}(z)$ is diversified. In fact, it is sufficient for such systems to choose the EHMAC prediction horizon $k > d_{\max}$, where $d_{\max}$ is the maximum expected time delay out of all (possibly time-varying) delays of the entries, in order to skip over the initial, trouble-making, dead-time intervals and arrive at a robust EHMAC law (Latawiec, 1998a).

5.3. Tuning in EHMAC

In (the basic version of) EHMAC, there are two controller parameters to be tuned, namely the prediction horizon and the IMC filter parameter. It is essential that, owing to the separation of the EHPC and MAC contributors to EHMAC, the tuning of the two parameters is handled separately. Since the prediction horizon belongs to the set of integers and the filter parameter is selected from the interval $(0, 1)$, it is relatively easy to arrange for auto-tuning of both the parameters, which is of great practical importance. Bearing in mind various implementation-oriented issues, reliable auto-tuning procedures, employing rule based reasoning and fuzzy sets, have been developed (Latawiec, 1998a).

5.4. Extensions

Efficient adaptive versions of EHMAC are available, both for SISO and MIMO systems (Latawiec, 1997; 1998a). Also, a computationally effective modification of EHMAC, based on orthonormal basis functions (OBF) modelling, has recently been developed (Latawiec *et al.*, 2000a; 2000b). Moreover, OBF-based EHMAC has been designed to operate under input constraints (Latawiec *et al.*, 2000b). Surprisingly, the constraint EHMAC strategy is based on an *analytical*, rather than typical, quadratic-programming solution, so it is computationally very effective. Current research is focused on an industrial application of EHMAC.

6. Simulation Studies

The value of EHMAC is illustrated with simulation experiments below. See also Latawiec (1995; 1996; 1997), Latawiec and Van Cauwenberghe (1995), Latawiec *et al.* (2000a; 2000b) for SISO simulation examples, as well as Latawiec (1998a; 1998b), Latawiec and Domek (1998) for MIMO simulations. For transparency of simulation runs, we present the performance of EHMAC in a disturbance-free case only. The aforementioned references, especially related to adaptive EHMAC, cover a stochastic case as well. Of course, both MVC and MAC are unstable for the two examples to follow.

Example 1. In what follows, we consider the following settings:
Plant (2-input 2-output, nonminimum phase):

$$y(t) - a_1 y(t-1) + a_2 y(t-2) = b_0 u(t-2) + b_1 u(t-3) + b_2 u(t-4)$$

with

$$b_0 = \begin{bmatrix} -1.0 & 0.2 \\ -0.1 & -0.8 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1.5 & 0.3 \\ 0.1 & 0.9 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0.2 & 0.05 \\ 0.1 & 0.15 \end{bmatrix}$$

$$a_1 = \begin{bmatrix} 1.3 & 0.2 \\ 0.25 & 0.9 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 0.43 & 0.09 \\ 0.14 & 0.24 \end{bmatrix},$$

$$K = A^{-1}(1)B(1) = \begin{bmatrix} 7.7570 & 6.6822 \\ 2.8037 & 2.8972 \end{bmatrix}.$$

Model (minimum phase, lower order):

$$y(t) - a_1 y(t-1) = b_0 u(t-3) + b_1 u(t-4)$$

with

$$b_0 = \begin{bmatrix} 1.0 & 0.2 \\ 0.3 & 0.9 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.6 \end{bmatrix}, \quad a_1 = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}, \quad \hat{A}(1) - \text{singular!}$$

In spite of a very large model-plant mismatch, the EHMAC law (21) provides robust stabilization of the feedback control system under the prediction horizon $k = 13$ and the IMC filter with $r = 1$ and $\alpha = 0.99I$. The effective EHMA control of the above *square* plant is still not very surprising, even with the poor-quality model. More impressive is the control performance for a nonsquare plant as in the forthcoming example, where EHMAC is a suboptimal strategy.

Example 2. Consider now the following data:
Plant (3-input 2-output, nonminimum phase):

$$y(t) - a_1 y(t-1) + a_2 y(t-2) = b_0 u(t-2) + b_1 u(t-3) + b_2 u(t-4)$$

with

$$b_0 = \begin{bmatrix} -1.0 & -0.3 & 0.2 \\ -0.1 & -0.5 & -0.8 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1.5 & 0.4 & 0.2 \\ 0.3 & 0.7 & 0.9 \end{bmatrix},$$

$$b_2 = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.15 \end{bmatrix}, \quad a_1 = \begin{bmatrix} 1.3 & 0.2 \\ 0.25 & 0.9 \end{bmatrix},$$

$$a_2 = \begin{bmatrix} 0.43 & 0.09 \\ 0.14 & 0.24 \end{bmatrix}, \quad K = \begin{bmatrix} 8.4424 & -25.109 & 6.1526 \\ 3.6137 & -6.947 & 2.7259 \end{bmatrix}.$$

Model (nonminimum phase, lower order):

$$y(t) - a_1 y(t-1) = b_0 u(t-3) + b_1 u(t-4)$$

with

$$b_0 = \begin{bmatrix} 1.0 & 0.2 & 0.1 \\ 0.1 & 0.5 & 0.7 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0.8 & 0.5 & 0.2 \\ 0.2 & 0.6 & 0.7 \end{bmatrix},$$

$$a_1 = \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.5 \end{bmatrix}, \quad \hat{K} = \hat{A}^{-1}(1)\hat{B}(1) = \begin{bmatrix} 5.3333 & 3.1667 & 2.3889 \\ 1.6667 & 2.8333 & 3.2778 \end{bmatrix}.$$

Remark 13. This example additionally justifies the need for (not only) EHMAC to rely on a sound definition of the NMP behaviour for a *nonsquare* plant model, inevitably related to our 'control zeros' (and not to any other type of multivariable zeros, in particular transmission ones, see Latawiec, 1998a; Latawiec *et al.*, 1999). In fact, the stabilization of the NMP model of a plant necessitates a proper choice of the prediction horizon at the first stage of EHMAC so that the control polynomial matrix Γ could be stable. Now, with Γ being the 'best' minimum phase approximant to the NMP factor $\underline{\hat{G}}$, we need to refer EHMAC to the NMP property, just as it is the case for MVC or MAC.

The plant in Example 2 has no transmission zeros but is nonminimum phase (control zeros outside the unit circle). Again, the model-plant mismatch is extremely large. A robust EHMAC can still be obtained for $k = 12$, $r = 1$ and $\alpha = 0.98I$. Of course, with a large value of the filter parameter, which is necessary due to the large model-plant mismatch, the feedback control system is 'slow'. The performance of the EHMA control system can be evaluated from Fig. 3 where the plots of the input/output variables and the setpoints are shown. (Note: The minimum-norm right inverse is employed to compute the control vector according to the control law (22).) Of course, smoother control plots can be obtained through further increasing the IMC filter parameter (but this would result in making the control system more sluggish).

7. Conclusions

Extended Horizon Model Algorithmic Control may be an attractive alternative to the more computationally involved GPC/UPC, LQR/LQG or H_∞ methods for robust control of open-loop stable NMP systems, especially for 'fast' systems where 'short' sampling intervals are desirable, as well as in adaptive control applications. The combination of EHPC and MAC preserves the conceptual simplicity, whilst utilizing the essential advantages of LRPC on one hand and the IMC structure on the other. We emphasize two well-known advantages of IMC. First, we introduce the general control accuracy theorem for IMC, with significant implications for the quality control of square MIMO systems. Specifically, steady-state error-free control under stepwise forcing inputs can be obtained by *any* stabilizing IM controller, provided that its gain is scaled accordingly. Second, EHMAC is designed at two *separate* stages, the first of which is aimed at the stabilization of a closed-loop system incorporating a (possibly NMP) *model* of the plant, and the other providing the control *robustness* to a possible model-plant mismatch. A very nice feature is that two EHMA controller parameters, namely the prediction horizon and the IMC filter parameter, can be separately tuned, thus facilitating the construction of auto-tuning procedures. A simple EHMAC has been demonstrated in simulations to be robust, even subject to unreasonably large model-plant mismatches. Moreover, OBF- based (constraint/unconstrained) EHMAC has been indicated to offer considerable computational gains.

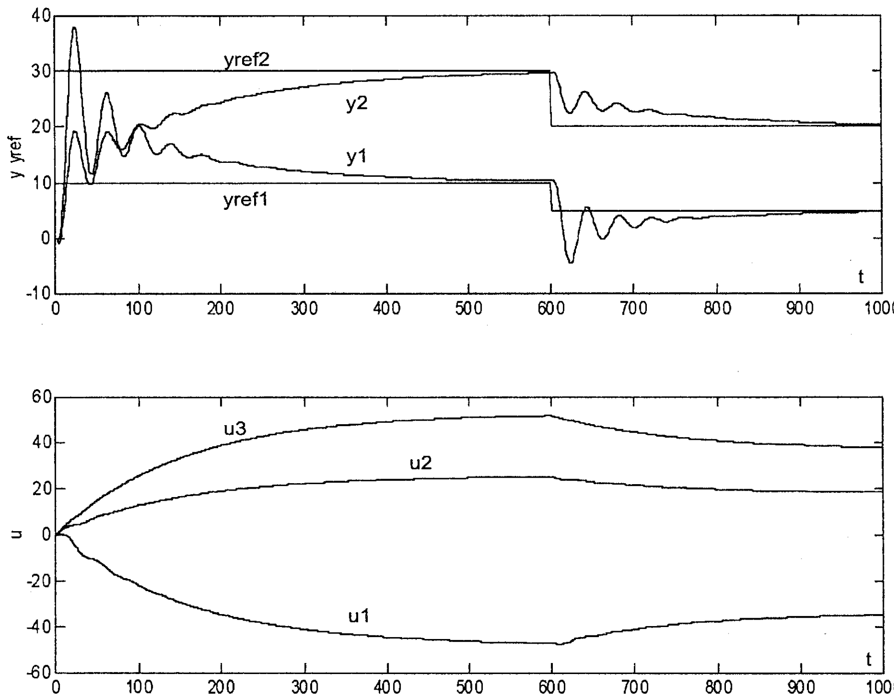

Fig. 3. EHMAC performance in Example 2.

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