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AN H_∞ SLIDING MODE OBSERVER FOR TAKAGI–SUGENO NONLINEAR SYSTEMS WITH SIMULTANEOUS ACTUATOR AND SENSOR FAULTS

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This paper considers the problem of robust reconstruction of simultaneous actuator and sensor faults for a class of uncertain Takagi–Sugeno nonlinear systems with unmeasurable premise variables. The proposed fault reconstruction and estimation design method with H_∞ performance is used to reconstruct both actuator and sensor faults when the latter are transformed into pseudo-actuator faults by introducing a simple filter. The main contribution is to develop a sliding mode observer (SMO) with two discontinuous terms to solve the problem of simultaneous faults. Sufficient stability conditions in terms linear matrix inequalities are achieved to guarantee the stability of the state estimation error. The observer gains are obtained by solving a convex multiobjective optimization problem. Simulation examples are given to illustrate the performance of the proposed observer.

Keywords: fault reconstruction and estimation, simultaneous faults, H_∞ sliding mode observer, uncertain Takagi–Sugeno systems, LMI optimization.

1. Introduction

Fault reconstruction and estimation design can determine the size, location and dynamic behavior of a fault. It is becoming a powerful alternative to the residual fault detection approach. Indeed, it is considered a major problem in modern control theory that has received a considerable amount of attention during the past few years. Especially, thanks to its robustness, some research has exploited the SMO as the best solution to solve the robust fault reconstruction and estimation problem. Up to now, this application has been discussed extensively for both linear and Lipschitz nonlinear systems. In the context of actuator fault estimation, constructing a diagnosis model in order to reconstruct faults is not possible if sensor faults occur simultaneously. The same difficulty is present when trying to estimate sensor faults. Several design methods have been developed in a precise and effective way when actuator and sensor fault reconstruction is divided into two steps:

- If actuator fault reconstruction is considered, fault estimation is possible without sensor faults (Edwards *et al.*, 2000; Ng *et al.*, 2007; Tan and Edwards, 2003b; Dhahri *et al.*, 2012; Raoufi *et al.*, 2010; Xing-Gang and Edwards, 2007a).
- If sensor fault estimation is considered, fault reconstruction is solved without considering actuator faults (Tan and Edwards, 2002; Alwi *et al.*, 2009; Xing-Gang and Edwards, 2007b).

Nevertheless, in practical systems, it is often the case when actuator and sensor fault occur simultaneously. In this framework, reconstruction of simultaneous faults is highly important. So far, only the work of Tan and Edwards (2003a) has addressed the fault reconstruction and estimation problem in a simultaneous actuator and sensor fault scenario. It is worth pointing out that the previous work referred to above considers only certain linear systems. This paper deals with the problem of fault reconstruction and estimation with simultaneous actuator and sensor faults.

The actual physical systems are often more complex and nonlinear. Due to their excellent ability of nonlinear

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system description, very interesting approaches have represented these systems in the Takagi–Sugeno (T–S) form. T–S models have been introduced by Takagi and Sugeno (1985). Roughly speaking, the feature is to understand the overall system behavior by a set of local linear models. Each local model represents the system's operation in a particular area. The local models are then aggregated using an interpolation mechanism by premise variables satisfying the convex sum property.

Taking the T–S representation, several attempts have been oriented to the diagnosis of nonlinear systems (e.g., Ichalal *et al.*, 2010; 2012; Gao *et al.*, 2010; Zhao *et al.*, 2009; Asemani and Majd, 2013; Mechmeche *et al.*, 2012; Bouattour *et al.*, 2011). Special attention has already been paid to the application of SMO design to fault reconstruction and estimation schemes for T–S systems subject to actuator and sensor faults (Akhenak *et al.*, 2007; 2008; Xu *et al.*, 2012). The authors assume that the premise variables are measurable so that they depend on the inputs or the measurement outputs. This requires the development of two different T–S representations of the same system, depending on the reconstruction of sensor or actuator faults. More recently, to overcome this problem, Ichalal *et al.* (2009a; 2009b), Hamdi *et al.* (2012) and Ghorbel *et al.* (2012) have supposed that the premise variables depend on a state variable. This implies that these variables are unmeasurable.

In this paper, we will extend the method of fault diagnosis based on H_∞ optimization, developed for Lipschitz nonlinear systems by Dhahri *et al.* (2012), in order to achieve reconstruction of simultaneous actuator and sensor faults for a T–S system subject to disturbances. It should be noticed that the T–S system is with unmeasurable premise variables which satisfy the Lipschitz constraints. By considering the sensor faults vector as "fictitious" actuator faults, an augmented T–S system is introduced. The main contribution is to construct H_∞ T–S SMO with the generation of two equivalent injection measurement signals to solve the problem of simultaneous faults in actuators and sensors. In this study, we use an LMI optimization approach in which the admissible Lipschitz constant and the disturbance attenuation level are maximized simultaneously through convex multiobjective optimization.

The outline of this paper is as follows. In Section 2, we describe an uncertain T–S system with unmeasurable premise variables in a simultaneous actuator and sensor faults scenario. In Section 3, we propose an H_∞ T–S sliding mode observer design with two discontinuous terms. The stability conditions of the T–S observer are studied via Lyapunov theories and LMI convex multiobjective optimization. Section 4 is devoted to reconstruction of simultaneous actuator and sensor faults. Two simulation examples are described in Section 5, illustrating the effectiveness of the proposed method. Finally, Section 6 presents some concluding remarks.

Notation. $\|A\|$ denotes the Euclidean norm. The symbol I_n illustrates an n-th order identity matrix. $\mathbb{R}^+$ and $\mathbb{C}$ represent the set of nonnegative real numbers and the complex plane, respectively.

2. Problem statement

2.1. Uncertain T–S system description. Consider an uncertain T–S system with unmeasurable premise variables affected both by actuator and sensor faults as follows:

$$\dot{x}(t) = \sum_{i=1}^{k} \mu_i(x(t))\big\{A_i x(t) + B_i u(t) + M_i f_a(t) + D_i \xi(x,u,t)\big\}, \tag{1}$$

$$y(t) = Cx(t) + N f_s(t), \tag{2}$$

where k represents the number of sub-models, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the vector of control inputs and $y(t) \in \mathbb{R}^p$ denotes the output vector. $f_a(t) \to \mathbb{R}^q$ and $f_s(t) \to \mathbb{R}^h$ represent the behaviors of actuator and sensor faults, respectively, which are assumed unknown but bounded by some known constants as $\|f_a(t)\| \le \rho_a$, $\|f_s(t)\| \le \rho_s$. $\xi(x,u,t) : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^+ \mapsto \mathbb{R}^l$ models the uncertainties and external disturbances. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $M_i \in \mathbb{R}^{n\times q}$, $D_i \in \mathbb{R}^{n\times l}$, $C \in \mathbb{R}^{p\times n}$ and $N \in \mathbb{R}^{p\times h}$ are known real matrices with appropriate dimensions. We also assume that the matrices C and N have full row and column ranks, respectively. Here $\mu_i(x(t))$ represent unmeasurable premise variables on the T–S system which satisfy the properties of the sum convex

$$\sum_{i=1}^{k} \mu_i(x(t)) = 1, \qquad 0 \le \mu_i(x(t)) \le 1, \quad \forall i \in \{1,\dots,k\}. \tag{3}$$

In order to transform the sensor faults to fictitious actuator faults affecting the system states, initially we assume that there exists an orthogonal matrix $T_R \in \mathbb{R}^{p\times p}$, obtained by the QR transformation of the sensor fault matrix N, such that

$$T_R y(t) := \begin{cases} y_1(t) = C_1 x(t), \\ y_2(t) = C_2 x(t) + N_1 f_s(t), \end{cases} \tag{4}$$

where $y_2(t) \in \mathbb{R}^h$ and $N_1 \in \mathbb{R}^{h\times h}$ is a nonsingular matrix.

Now define $w(t) \in \mathbb{R}^h$ as a filtered version of the potentially faulty sensor signals $y_2(t)$,

$$\begin{aligned}\dot{w}(t) &= -A_f w(t) + A_f y_2(t) \\ &= -A_f w(t) + A_f C_2 x(t) + A_f N_1 f_s(t),\end{aligned} \tag{5}$$

where $-A_f \in \mathbb{R}^{h\times h}$ is a stable filter matrix.

From the preceding equations, an augmented uncertain T–S system of order $(n+h)$ can be obtained:

$$\underbrace{\begin{bmatrix}\dot{x}(t)\\ \dot{w}(t)\end{bmatrix}}_{\dot{\chi}(t)} = \sum_{i=1}^{k}\mu_i(\chi(t))\Big\{\underbrace{\begin{bmatrix}A_i & 0\\ A_fC_2 & -A_f\end{bmatrix}}_{\mathcal{A}_i}\underbrace{\begin{bmatrix}x(t)\\ w(t)\end{bmatrix}}_{\chi(t)} + \underbrace{\begin{bmatrix}B_i\\ 0\end{bmatrix}}_{\mathcal{B}_i}u(t) + \underbrace{\begin{bmatrix}M_i\\ 0\end{bmatrix}}_{\mathcal{M}_{a,i}}f_a(t) + \underbrace{\begin{bmatrix}0\\ A_fN_1\end{bmatrix}}_{\mathcal{M}_s}f_s(t) + \underbrace{\begin{bmatrix}D_i\\ 0\end{bmatrix}}_{\mathcal{D}_i}\xi(\chi,u,t)\Big\}, \tag{6}$$

$$\underbrace{\begin{bmatrix}y_1(t)\\ w(t)\end{bmatrix}}_{z(t)} = \underbrace{\begin{bmatrix}C_1 & 0\\ 0 & I_h\end{bmatrix}}_{\mathcal{C}}\underbrace{\begin{bmatrix}x(t)\\ w(t)\end{bmatrix}}_{\chi(t)}. \tag{7}$$

The T–S system (6)–(7) with unmeasurable premise variables can be reduced to a T–S system with measurable premise variables as

$$\dot{\chi}(t) = \sum_{i=1}^{k}\mu_i(\hat{\chi}(t))\Big\{\mathcal{A}_i\chi(t) + \mathcal{B}_iu(t) + \mathcal{D}_i\xi(\chi,u,t) + \phi(\chi,\hat{\chi}) + \mathcal{M}_{a,i}f_a(t) + \mathcal{M}_sf_s(t)\Big\}, \tag{8}$$

$$z(t) = \mathcal{C}\chi(t), \tag{9}$$

such that

$$\phi(\chi,\hat{\chi}) := \sum_{i=1}^{k}(\mu_i(\chi(t)) - \mu_i(\hat{\chi}(t)))\Big\{\mathcal{A}_i\chi(t)+\mathcal{B}_iu(t) + \mathcal{M}_{a,i}f_a(t) + \mathcal{M}_sf_s(t) + \mathcal{D}_i\xi(\chi,u,t)\Big\},$$

where $\hat{\chi}(t)$ denote the estimated augmented states, $\mathcal{A}_i \in \mathbb{R}^{(n+h)\times(n+h)}$, $\mathcal{B}_i \in \mathbb{R}^{(n+h)\times m}$, $\mathcal{M}_{a,i} \in \mathbb{R}^{(n+h)\times q}$, $\mathcal{M}_s \in \mathbb{R}^{(n+h)\times h}$, $\mathcal{D}_i \in \mathbb{R}^{(n+h)\times l}$ and $\mathcal{C} \in \mathbb{R}^{p\times(n+h)}$ are the matrices defined for the i-th model, $\forall\, i \in \{1,\dots,k\}$, where $n+h > p \geq q+h$.

2.2. Existence assumptions.

Each local model for the T–S system (8)–(9) must satisfy the following conditions:

Condition C1:

$$\operatorname{rank}(\mathcal{C}\,[\mathcal{M}_{a,i}\ \ \mathcal{M}_s]) = q+h. \tag{10}$$

Condition C2:

$$\operatorname{rank}\begin{bmatrix} sI_{n+h}-\mathcal{A}_i & \mathcal{M}_{a,i} & \mathcal{M}_s\\ \mathcal{C} & 0 & 0\end{bmatrix} = n+h+\operatorname{rank}[\mathcal{M}_{a,i}\mathcal{M}_s], \tag{11}$$

$\forall s \in \mathbb{C}$ such that $\Re(s) \geq 0$.

Condition C3: $\phi(\chi,\hat{\chi})$ satisfies the Lipschitz constraint (Ichalal *et al.*, 2010),

$$\|\phi(\chi,\hat{\chi})\| \leq \gamma\,\|\chi-\hat{\chi}\|, \tag{12}$$

where $\gamma > 0$ is a known scalar called the Lipschitz constant.

Conditions C1 and C2 express the observability properties for each local model of the T–S system (8)–(9). These conditions must be satisfied for each vertex of the original uncertain T–S system (1)–(2).

Furthermore, from (6)–(7), it is easy to see that

$$\mathcal{C}\,[\mathcal{M}_{a,i}\ \ \mathcal{M}_s] = \begin{bmatrix}C_1M_i & 0\\ 0 & A_fN_1\end{bmatrix},\quad \forall\, i \in \{1,\dots,k\}. \tag{13}$$

Premultiplying $\mathcal{C}\,[\mathcal{M}_{a,i}\ \ \mathcal{M}_s]$ in (13) with a nonsingular matrix

$$\begin{bmatrix}I_{p-h} & 0\\ 0 & A_f^{-1}\end{bmatrix},$$

we obtain

$$\begin{bmatrix}C_1M_i & 0\\ 0 & N_1\end{bmatrix},\quad \forall\, i \in \{1,\dots,k\}. \tag{14}$$

It follows that, $\forall\, i \in \{1,\dots,k\}$,

$$\operatorname{rank}(\mathcal{C}\,[\mathcal{M}_{a,i}\ \ \mathcal{M}_s]) = \operatorname{rank}(C_1M_i)+\operatorname{rank}(N_1). \tag{15}$$

Since $N_1 \in \mathbb{R}^{h\times h}$ has full rank, Condition C1 will be satisfied if and only if

$$\operatorname{rank}[C_1M_i] = q,\quad \forall\, i \in \{1,\dots,k\}. \tag{16}$$

The matrix in (16) has $p-h$ rows and q columns. Hence, since by assumption $p \geq q+h$, Condition C1 is fulfilled for each vertex of system (8)–(9) if (16) is satisfied.

In addition, after expressing (11) in terms of the partitioned matrices in (6)–(7), it follows that, $\forall\, i \in \{1,\dots,k\}$,

$$\operatorname{rank}\begin{bmatrix} sI_n-A_i & M_i & 0\\ -A_fC_2 & 0 & A_fN_1\\ C_1 & 0 & 0\end{bmatrix} = n+q+h. \tag{17}$$

Pre-multiplying the matrix in (17) by the following nonsingular matrix:

$$\begin{bmatrix}I_n & 0\\ 0 & T_R^{-1}\end{bmatrix}\begin{bmatrix}I_n & 0 & 0\\ 0 & 0 & I_{p-h}\\ 0 & A_f^{-1} & 0\end{bmatrix}, \tag{18}$$

we have, $\forall\, i \in \{1,\dots,k\}$,

$$\operatorname{rank}\begin{bmatrix} sI_n-A_i & M_i & 0\\ C & 0 & N\end{bmatrix} = n+q+h. \tag{19}$$

Since N has full rank, we obtain, $\forall\, i \in \{1,\ldots,k\}$,

$$\operatorname{rank}\begin{bmatrix} sI_n - A_i & M_i \\ C & 0 \end{bmatrix} = n + \operatorname{rank}(M_i), \tag{20}$$

which must be satisfied for the original T–S system (1)–(2), $\forall s \in \mathbb{C}$ such that $\Re(s) \geq 0$.

The following section presents a T–S sliding mode observer design with two discontinuous terms intended to the reconstruction of simultaneous actuator and sensor faults for the uncertain T–S system (8)–(9).

3. H_∞ T–S sliding mode observer design

3.1. Observer structure. The proposed sliding mode observer design with two discontinuous terms has the following T–S structure:

$$\begin{aligned}\dot{\hat{\chi}}(t) = \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{&\mathcal{A}_i\hat{\chi}(t) + \mathcal{B}_i u(t) + \mathcal{G}_{l,i} e_z(t) \\ &+ \mathcal{G}_{n,i} v_{a,i}(t) + \mathcal{G}_{n,i} v_{s,i}(t)\Big\},\end{aligned} \tag{21}$$

$$\hat{z}(t) = \mathcal{C}\hat{\chi}(t), \tag{22}$$

where $e_z(t) := z(t) - \hat{z}(t)$ represents the output error estimation.

Assuming that, $\forall\, i \in \{1,\ldots,k\}$, $\mathcal{G}_{n,i}$ have the following structure:

$$\mathcal{G}_{n,i} = \begin{bmatrix} -L_i \\ I_p \end{bmatrix} \mathcal{C}_2^{-1}, \tag{23}$$

where $L_i = \begin{bmatrix} L_{1,i} & 0 \end{bmatrix}$, together with the design matrices $L_{1,i} \in \mathbb{R}^{(n+h-p)\times(p-q-h)}$ will be determined later.

Here $v_{a,i}(t)$ and $v_{s,i}(t)$ are nonlinear discontinuous terms, which compensate $f_a(t)$ and $f_s(t)$, respectively, defined by

$$v_{a,i}(t) := \begin{cases} \eta_{a,i}\dfrac{e_z(t)}{\|e_z(t)\|} & \text{if } e_z(t) \neq 0, \\ 0 & \text{otherwise,} \end{cases} \tag{24}$$

$$v_{s,i}(t) := \begin{cases} \eta_{s,i}\dfrac{e_z(t)}{\|e_z(t)\|} & \text{if } e_z(t) \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{25}$$

Here $\eta_{a,i}$ and $\eta_{s,i}$ must be bounded as $\eta_{a,i} \geq \|\mathcal{C}_2\mathcal{M}_{a2,i}\|\,\rho_a + \eta_{a0,i}$ and $\eta_{s,i} \geq \|\mathcal{C}_2\mathcal{M}_{s2}\|\,\rho_s + \eta_{s0,i}$, respectively, $\forall\, i \in \{1,\ldots,k\}$. $\mathcal{C}_2$, $\mathcal{M}_{a2,i}$ and $\mathcal{M}_{s2}$ will be described formally later, $\forall\, i \in \{1,\ldots,k\}$.

Under Condition C1, there exists a linear change of coordinates such that the matrices $(\mathcal{A}_i, [\mathcal{M}_{a,i}\ \ \mathcal{M}_s], \mathcal{D}_i, \mathcal{C})$ yield, $\forall\, i \in \{1,\ldots,k\}$,

$$\begin{aligned}\mathcal{A}_i &= \begin{bmatrix} \mathcal{A}_{1,i} & \mathcal{A}_{2,i} \\ \mathcal{A}_{3,i} & \mathcal{A}_{4,i} \end{bmatrix}, \\ [\mathcal{M}_{a,i}\mathcal{M}_s] &= \begin{bmatrix} 0 & 0 \\ \mathcal{M}_{a2,i} & \mathcal{M}_{s2} \end{bmatrix}, \\ \mathcal{D}_i &= \begin{bmatrix} \mathcal{D}_{1,i} \\ \mathcal{D}_{2,i} \end{bmatrix}, \\ \mathcal{C} &= \begin{bmatrix} 0 & \mathcal{C}_2 \end{bmatrix},\end{aligned} \tag{26}$$

where $\mathcal{A}_{1,i} \in \mathbb{R}^{(n+h-p)\times(n+h-p)}$, $\mathcal{M}_{a2,i} \in \mathbb{R}^{p\times q}$, $\mathcal{M}_{s2} \in \mathbb{R}^{p\times h}$, $\mathcal{D}_{1,i} \in \mathbb{R}^{(n+h-p)\times l}$ and $\mathcal{C}_2 \in \mathbb{R}^{p\times p}$ is nonsingular.

We also assume that

$$\begin{aligned}\mathcal{A}_{3,i} &= \begin{bmatrix} \mathcal{A}_{31,i} \\ \mathcal{A}_{32,i} \end{bmatrix}, \\ \mathcal{M}_{a2,i} &= \begin{bmatrix} 0 \\ \mathcal{M}_{a0,i} \end{bmatrix}, \\ \mathcal{M}_{s2} &= \begin{bmatrix} 0 \\ \mathcal{M}_{s0} \end{bmatrix},\end{aligned} \tag{27}$$

with $\mathcal{A}_{31,i} \in \mathbb{R}^{(p-q-h)\times(n+h-p)}$, $\mathcal{M}_{a0,i} \in \mathbb{R}^{(q+h)\times q}$ and $\mathcal{M}_{s0} \in \mathbb{R}^{(q+h)\times h}$ having full rank, $\forall\, i \in \{1,\ldots,k\}$.

Remark 1. Condition C2 implies that the invariant zeros for each vertex of the T–S system (8)–(9) are given, $\forall\, i \in \{1,\ldots,k\}$, by the system triple $(\mathcal{A}_i, [\mathcal{M}_{a,i}\ \ \mathcal{M}_s], \mathcal{C})$. By construction, $(\mathcal{A}_{1,i}, \mathcal{A}_{31,i})$ must be detectable and the system zeros are actually the unobservable modes of $(\mathcal{A}_{1,i}, \mathcal{A}_{31,i})$ which must lie in $\mathbb{C}^-\ \forall\, i \in \{1,\ldots,k\}$.

Define $e(t) := \chi(t) - \hat{\chi}(t)$ as the state estimation error. From (8)–(9) and (21)–(22), the dynamics of state estimation error is given by the equation

$$\begin{aligned}\dot{e}(t) = \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{&\mathcal{A}_{l,i}e(t) + \phi(\chi,\hat{\chi}) \\ &+ \mathcal{D}_i\xi(\chi,u,t) + \mathcal{M}_{a,i}f_a(t) - \mathcal{G}_{n,i}v_{a,i}(t) \\ &+ \mathcal{M}_s f_s(t) - \mathcal{G}_{n,i}v_{s,i}(t)\Big\},\end{aligned} \tag{28}$$

where $\mathcal{A}_{l,i} = \mathcal{A}_i - \mathcal{G}_{l,i}\mathcal{C}$.

In order to identify the sliding motion, it is required to apply a further change of coordinates according to

$$T_L := \begin{bmatrix} I_{n+h-p} & L_i \\ 0 & \mathcal{C}_2 \end{bmatrix}. \tag{29}$$

Then, in the new coordinates system, it is straightforward to see that, $\forall\, i \in \{1,\dots,k\}$,

$$\begin{aligned}\mathcal{A}_{L,i} &= \begin{bmatrix} \mathcal{A}_{L1,i} & \mathcal{A}_{L2,i} \\ \mathcal{A}_{L3,i} & \mathcal{A}_{L4,i} \end{bmatrix},\\ [\mathcal{M}_{aL,i}\mathcal{M}_{sL}] &= \begin{bmatrix} 0 & 0 \\ \mathcal{M}_{aL2,i} & \mathcal{M}_{sL2} \end{bmatrix},\\ \mathcal{D}_{L,i} &= \begin{bmatrix} \mathcal{D}_{L1,i} \\ \mathcal{D}_{L2,i} \end{bmatrix},\\ \mathcal{G}_{nL,i} &= \mathcal{C}_L^T = \begin{bmatrix} 0 \\ I_p \end{bmatrix},\\ \mathcal{G}_{lL,i} &= \begin{bmatrix} \mathcal{G}_{lL1,i} \\ \mathcal{G}_{lL2,i} \end{bmatrix},\end{aligned} \tag{30}$$

such that $\mathcal{A}_{L1,i} = \mathcal{A}_{1,i} + L_i\mathcal{A}_{3,i}$ must be stable, $\mathcal{A}_{L3,i} = \mathcal{C}_2\mathcal{A}_{3,i}$, $\mathcal{M}_{aL2,i} = \mathcal{C}_2\mathcal{M}_{a2,i}$, $\mathcal{M}_{sL2} = \mathcal{C}_2\mathcal{M}_{s2}$, $\mathcal{D}_{L2,i} = \mathcal{C}_2\mathcal{D}_{2,i}$ and $\mathcal{G}_{lL,i} = \mathcal{A}_{L4,i} - \mathcal{A}_{s,i}$. $\mathcal{A}_{s,i}$ are stable matrices.

It can be easily verified that, in the coordinate system (30), the state estimation error dynamics can be partitioned as

$$\begin{aligned}\dot{e}_1(t) = \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{&(\mathcal{A}_{1,i}+L_i\mathcal{A}_{3,i})e_1(t)\\ &+ (\mathcal{D}_{1,i}+\mathcal{D}_{2,i})\xi(\chi,u,t)\\ &+ [I_{n+h-p}\ \ L_i]\phi_1(\chi,\hat{\chi})\Big\},\end{aligned} \tag{31}$$

$$\begin{aligned}\dot{e}_z(t) = \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{&\mathcal{A}_{s,i}e_z(t)\\ &+ \mathcal{C}_2(\mathcal{A}_{3,i}e_1(t)+\phi_2(\chi,\hat{\chi}))\\ &+ \mathcal{C}_2\mathcal{M}_{a2,i}f_a(t)+\mathcal{C}_2\mathcal{M}_{s2}f_s(t)\\ &- v_{a,i}(t)-v_{s,i}(t)+\mathcal{C}_2\mathcal{D}_{2,i}\xi(\chi,u,t)\Big\}.\end{aligned} \tag{32}$$

The objective of this paper is to present a robust sliding mode observer with two discontinuous terms for estimating both actuator and sensor faults, as well as the T–S system states. It will be shown that sufficient conditions for the stability with H_∞ performances of the observer error (31)–(32) are established by using Lyapunov stability and LMIs.

3.2. Stability of the sliding motion.

Let

$$g(t) = H \begin{bmatrix} e_1(t) \\ e_z(t) \end{bmatrix} \tag{33}$$

stand for the controlled output error estimation system, where H is a full rank design matrix having the following structure:

$$H := \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}. \tag{34}$$

The purpose is to design the observer parameters L_i, $\forall\, i \in \{1,\dots,K\}$, where the observer error dynamics are asymptotically stable with an achieved disturbance attenuation level ς. Hence, the following specified H_∞ norm upper bound is guaranteed:

$$\|g\|_2^2 \le \varsigma^2 \|\xi\|_2^2. \tag{35}$$

The following theorem provides sufficient conditions to ensure the desired properties of stability. It is based on the results of Dhahri *et al.* (2012).

Theorem 1. *The state estimation error is asymptotically stable with simultaneously maximized admissible Lipschitz constant γ^* and minimized gain ς^*, if there exist fixed scalars $0 \le \lambda \le 1$, $\varepsilon > 0$, $\theta > 0$ and $\alpha > 0$, and matrices $P_1 > 0$, $P_2 > 0$, W_i and L_i, such that the following LMI convex multiobjective optimization problem has a solution:*

$$\min\,[\lambda(\alpha+\varepsilon) + (1-\lambda)\theta]$$

subject to

$$\begin{bmatrix} \psi_{11,i} & \mathcal{A}_{3,i}^T\mathcal{C}_2^T P_2 & P_1\mathcal{D}_{1,i} + W_i\mathcal{D}_{2,i} & P_1 & 0 & I_{n+h-p} & 0 \\ (*) & \psi_{22,i} & P_1\mathcal{D}_{2,i} & 0 & P_2 & 0 & I_p \\ (*) & (*) & -\theta I_l & 0 & 0 & 0 & 0 \\ (*) & (*) & (*) & -\varepsilon I_{n+h-p} & 0 & 0 & 0 \\ (*) & (*) & (*) & (*) & -\varepsilon I_p & 0 & 0 \\ (*) & (*) & (*) & (*) & (*) & -\alpha I_{n+h-p} & 0 \\ (*) & (*) & (*) & (*) & (*) & (*) & -\alpha I_p \end{bmatrix} < 0, \tag{36}$$

where $\psi_{11,i} = \mathcal{A}_{1,i}^T P + P\mathcal{A}_{1,i} + W_i\mathcal{A}_{3,i} + \mathcal{A}_{3,i}^T W_i^T + H_1^T H_1$, $\psi_{22,i} = \mathcal{A}_{s,i}^T P_2 + P_2\mathcal{A}_{s,i} + H_2^T H_2$, $\forall\, i \in \{1,\dots,K\}$.

Once the convex multiobjective problem is solved,

$$\begin{aligned}\varsigma^* &= \min(\varsigma) = \sqrt{\theta},\\ \varepsilon^* &= \min(\varepsilon),\\ \alpha^* &= \min(\alpha),\\ \gamma^* &= \max(\gamma) = \frac{1}{\|T_L\|^2\sqrt{\alpha^*\varepsilon^*}},\\ L_i &= P_1^{-1}W_i.\end{aligned}$$

Proof. Write

$$\tilde{e}(t) = \begin{bmatrix} e_1(t) \\ e_z(t) \end{bmatrix}. \tag{37}$$

Then we have

$$\dot{\tilde{e}}(t) = \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{\mathcal{A}_{0,i}\tilde{e}(t)+T_L\phi(\chi,\hat{\chi}) + \mathcal{D}_{L,i}\xi(\chi,u,t) + \mathcal{M}_{aL,i}f_a(t) - \mathcal{G}_{nL,i}v_{a,i}(t) + \mathcal{M}_{sL}f_s(t) - \mathcal{G}_{nL,i}v_{s,i}(t)\Big\}, \tag{38}$$

where

$$\mathcal{A}_{0,i} = \begin{bmatrix} \mathcal{A}_{1,i} + L_i\mathcal{A}_{3,i} & 0 \\ \mathcal{C}_2\mathcal{A}_{3,i} & \mathcal{A}_{s,i} \end{bmatrix}.$$

The proof of this theorem proceeds by using the following quadratic Lyapunov function:

$$V(\tilde{e}(t)) = \tilde{e}^T(t)P\tilde{e}(t), \quad P = P^T > 0, \tag{39}$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \quad P_1 \in \mathbb{R}^{(n+h-p)\times(n+h-p)}, \quad P_2 \in \mathbb{R}^{p\times p}. \tag{40}$$

The time derivative of $V(\tilde{e}(t))$ along the system trajectories is

$$\dot{V}(\tilde{e}(t)) = \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{\tilde{e}(t)(\mathcal{A}_{0,i}^T P+P\mathcal{A}_{0,i})\tilde{e}^T(t) + 2\tilde{e}^T(t)P(T_L\phi(\chi,\hat{\chi})+\mathcal{D}_{L,i}\xi(\chi,u,t) + \mathcal{M}_{aL,i}f_a(t) - \mathcal{G}_{nL,i}v_{a,i}(t) + \mathcal{M}_{sL}f_s(t) - \mathcal{G}_{nL,i}v_{s,i}(t))\Big\}. \tag{41}$$

For actuator faults, from (24) and (30) it follows that

$$\begin{aligned}
&\tilde{e}^T(t)P\mathcal{M}_{aL,i}f_a(t) - \tilde{e}^T(t)P\mathcal{G}_{nL,i}v_{a,i}(t) \\
&= \tilde{e}^T(t)\left(\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}\begin{bmatrix} 0 \\ \mathcal{C}_2\mathcal{M}_{a2,i} \end{bmatrix} f_a(t) - \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}\begin{bmatrix} 0 \\ I_p \end{bmatrix} v_{a,i}(t)\right) \\
&= e_z^T(t)P_2\mathcal{C}_2\mathcal{M}_{a2,i}f_a(t) - e_z^T(t)P_2v_{a,i}(t) \\
&\le \left\|P_2e_z^T(t)\right\|\left(\|\mathcal{C}_2\mathcal{M}_{a2,i}\|\,\|f_a(t)\| - \eta_{a,i}\frac{e_z(t)}{\|e_z(t)\|}\right) \\
&\le \left\|P_2e_z^T(t)\right\|(\|\mathcal{C}_2\mathcal{M}_{a2,i}\|\,\rho_a - \eta_{a,i}) \\
&\le - \left\|P_2e_z^T(t)\right\|\eta_{a0,i} < 0.
\end{aligned} \tag{42}$$

In the same way, for sensor faults, from (25) and (30) it follows that

$$\tilde{e}^T(t)P(\mathcal{M}_{sL}f_s(t) - \mathcal{G}_{nL,i}v_{s,i}(t) \le -\left\|P_2e_z^T(t)\right\|\eta_{s0,i} < 0. \tag{43}$$

Applying the inequality

$$2X^TY \le \frac{1}{\varepsilon}X^TX + \varepsilon Y^TY, \tag{44}$$

valid for any scalars $\varepsilon > 0$, and using the Lipschitz constraint in Condition C3, the following inequalities are satisfied:

$$\begin{aligned}
&2\tilde{e}^T(t)PT_L\phi(\chi,\hat{\chi}) \\
&\le \frac{1}{\varepsilon}\tilde{e}^T(t)P^2\tilde{e}(t) + \varepsilon\phi^T(\chi,\hat{\chi})T_L^TT_L\phi(\chi,\hat{\chi}) \\
&= \frac{1}{\varepsilon}\tilde{e}^T(t)P^2\tilde{e}(t) + \varepsilon\|T_L\phi(\chi,\hat{\chi})\|^2 \\
&\le \frac{1}{\varepsilon}\tilde{e}^T(t)P^2\tilde{e}(t) + \varepsilon\|T_L\|^2\gamma^2\|\tilde{e}(t)\|^2 \\
&\le \frac{1}{\varepsilon}\tilde{e}^T(t)P^2\tilde{e}(t) + \varepsilon\tilde{\gamma}^2\|\tilde{e}(t)\|^2,
\end{aligned} \tag{45}$$

where $\tilde{\gamma} = \|T_L\|\,\gamma$.

Substituting (42)–(43) and (45) into (41) yields

$$\dot{V}(\tilde{e}(t)) \le \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{\tilde{e}^T(t)(\mathcal{A}_{0,i}^T P+P\mathcal{A}_{0,i} + \frac{1}{\varepsilon}P^2+\varepsilon\tilde{\gamma}^2)\tilde{e}(t) + 2\tilde{e}^T(t)P\mathcal{D}_{L,i}\xi(\chi,u,t)\Big\}. \tag{46}$$

Now define

$$J(t) := \dot{V}(\tilde{e}(t))+g^T(t)g(t) - \varsigma^2\xi^T(\chi,u,t)\xi(\chi,u,t). \tag{47}$$

We have

$$J(t) \le \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{\tilde{e}^T(t)\Big(\mathcal{A}_{0,i}^T P + P\mathcal{A}_{0,i} + \frac{1}{\varepsilon}P^2+\varepsilon\tilde{\gamma}^2 + H^TH\Big)\tilde{e}(t)+2\tilde{e}^T(t)P\mathcal{D}_{L,i}\xi(\chi,u,t) - \varsigma^2\xi^T(\chi,u,t)\xi(\chi,u,t)\Big\}. \tag{48}$$

Define the new variable

$$\alpha := \frac{1}{\varepsilon\tilde{\gamma}^2} \tag{49}$$

We get

$$\tilde{\gamma} = \frac{1}{\sqrt{\alpha\varepsilon}}, \quad \theta := \varsigma^2.$$

Maximization of $\tilde{\gamma}$ guarantees the stability of the T–S system for any Lipschitz nonlinear function with a Lipschitz constant less than or equal to an unknown constant $\tilde{\gamma}^*$. Maximization of $\tilde{\gamma}$ and minimization of θ

can be accomplished by simultaneous minimization of γ, ε and θ. This leads to a multiobjective optimization.

Therefore, we have

$$\begin{aligned}
J(t) &\le \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{\tilde{e}^T(t)(\mathcal{A}_{0,i}^T P \\
&\quad + P\mathcal{A}_{0,i} + \frac{1}{\varepsilon}P^2 + \alpha^{-1} + H^T H)\tilde{e}(t) \\
&\quad + 2\tilde{e}^T(t)P\mathcal{D}_{L,i}\xi(\chi,u,t) \\
&\quad - \theta\xi^T(\chi,u,t)\xi(\chi,u,t)\Big\} \\
&= \sum_{i=1}^{k} \mu_i(\hat{\chi}(t)) \\
&\quad \times \Big\{\tilde{e}^T(t)\Big(\begin{bmatrix} \mathcal{A}_{1,i}^T + \mathcal{A}_{3,i}^T L_i^T & \mathbf{A}_{3,i}^T \mathcal{C}_2^T \\ 0 & \mathcal{A}_{s,i}^T \end{bmatrix}\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \\
&\quad + \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}\begin{bmatrix} \mathcal{A}_{1,i} + L_i\mathcal{A}_{3,i} & 0 \\ \mathcal{C}_2\mathcal{A}_{3,i} & \mathcal{A}_{s,i} \end{bmatrix} \\
&\quad + \varepsilon^{-1}\begin{bmatrix} P_1^2 & 0 \\ 0 & P_2^2 \end{bmatrix} + \alpha^{-1}\begin{bmatrix} I_{n+h-p} & 0 \\ 0 & I_p \end{bmatrix} \\
&\quad + \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}^T\begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}\Big)\tilde{e}(t) + 2\tilde{e}^T(t) \\
&\quad \times \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}\begin{bmatrix} \mathcal{D}_{1,i} + L_i\mathcal{D}_{2,i} \\ \mathcal{C}_2\mathcal{D}_{2,i} \end{bmatrix}\xi(\chi,u,t) \\
&\quad - \theta\xi^T(\chi,u,t)\xi(\chi,u,t)\Big\} \\
&= \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{\tilde{e}^T(t)\begin{bmatrix} Q_{1,i} & \mathcal{A}_{3,i}^T\mathcal{C}_2^T P_2 \\ P_2\mathcal{C}_2\mathcal{A}_{3,i} & Q_{2,i} \end{bmatrix}\tilde{e}(t) \\
&\quad + 2\tilde{e}^T(t)\begin{bmatrix} P_1\mathcal{D}_{1,i} + P_1 L_i\mathcal{D}_{2,i} \\ P_2\mathcal{C}_2\mathcal{D}_{2,i} \end{bmatrix}\xi(\chi,u,t) \\
&\quad - \theta\xi^T(\chi,u,t)\xi(\chi,u,t)\Big\},
\end{aligned} \tag{50}$$

where

$$\begin{aligned}
Q_{1,i} &= (\mathcal{A}_{1,i} + L_i\mathcal{A}_{3,i})^T P_1 + P_1(\mathcal{A}_{1,i} + L_i\mathcal{A}_{3,i}) \\
&\quad + \varepsilon^{-1}P_1^2 + \alpha^{-1}I_{n+h-p} + H_1^T H_1, \\
Q_{2,i} &= \mathcal{A}_{s,i}^T P_1 + P_1\mathcal{A}_{s,i} + \varepsilon^{-1}P_2^2 + \alpha^{-1}I_p + H_2^T H_2.
\end{aligned}$$

Thus, we obtain

$$J(t) \le \begin{bmatrix} e_1(t) \\ e_z(t) \\ \xi(\chi,u,t) \end{bmatrix}^T \Omega \begin{bmatrix} e_1(t) \\ e_z(t) \\ \xi(\chi,u,t) \end{bmatrix}, \tag{51}$$

with

$$\Omega = \begin{bmatrix} Q_{1,i} & \mathcal{A}_{3,i}^T\mathcal{C}_2^T P_2 & P_1\mathcal{D}_{1,i} + P_1 L_i\mathcal{D}_{2,i} \\ (*) & Q_{2,i} & P_2\mathcal{D}_{2,i} \\ (*) & (*) & -\theta I_l \end{bmatrix}. \tag{52}$$

If $\Omega < 0$, then $J(t) \le 0$ along the system trajectories. The system of the state estimation error (31)–(32) is asymptotically stable with the attenuation level θ and the admissible Lipschitz constant $\tilde{\gamma}^*$.

Integrating the expression in (47) from 0 to ∞, we have

$$V(\tilde{e}(\infty)) - V(\tilde{e}(0)) + \|g\|_2^2 - \theta\,\|\xi\|_2^2 \le 0. \tag{53}$$

Together with the zero initial condition $e_1(0) = e_z(0) = 0$, we have

$$\begin{cases} V(\tilde{e}(0)) = 0, \\ V(\tilde{e}(\infty)) = e_1^T(\infty)P_1 e_1(\infty) + e_z^T(\infty)P_2 e_z(\infty) \ge 0. \end{cases} \tag{54}$$

Therefore,

$$\|g\|_2^2 \le \theta\,\|\xi\|_2^2. \tag{55}$$

Notice that $\Omega < 0$ is nonlinear because of the product $P_1 L_i$. This problem can be solved by using the changes of variables $W_i = P_1 L_i$. Thus, applying the Schur complement, the inequality (36) can be obtained.

In addition, it is reported that the designation $(*)$ in (36) satisfies the symmetric property of the LMIs technique. ■

4. Reconstruction of simultaneous actuator and sensor faults

A clear distinction of this paper is the proposed H_∞ sliding mode observer with two discontinuous terms, especially designed for reconstruction of simultaneous faults for a T–S system subject to disturbances. Thus, we are looking forward to generate two equivalent injection measurement signals where each one is designed to compensate a particular fault's class, actuator or sensor.

If all the conditions of the preceding theorem are satisfied and the LMI convex multiobjective optimization is solved, then

$$\|g\|_2^2 \le \theta\,\|\xi\|_2^2. \tag{56}$$

Consequently, the error dynamics of $e_z(t)$ in sliding motion is given by

$$\begin{aligned}
0 &= \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{\mathcal{A}_{3,i}e_1(t) + \phi_2(\chi,\hat{\chi}) \\
&\quad + \mathcal{D}_{2,i}\xi(\chi,u,t) \\
&\quad + \mathcal{M}_{a2,i}f_a(t) - \mathcal{C}_2^{-1}v_{a,i}(t) \\
&\quad + \mathcal{M}_{s2}f_s(t) - \mathcal{C}_2^{-1}v_{s,i}(t)\Big\}.
\end{aligned} \tag{57}$$

This is equivalent to

$$\begin{aligned}
0 &= \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\Big\{\Psi(\chi,u,t) + \mathcal{M}_{a2,i}f_a(t) \\
&\quad - \mathcal{C}_2^{-1}v_{a,i}(t) + \mathcal{M}_{s2}f_s(t) - \mathcal{C}_2^{-1}v_{s,i}(t)\Big\},
\end{aligned} \tag{58}$$

where $\Psi(\chi, u, t) := \mathcal{A}_{3,i} e_1(t) + \phi_2(\chi, \hat{\chi}) + \mathcal{D}_{2,i}\xi(\chi, u, t)$.

Using the Lipschitz constraint (12), $\Psi(\chi, u, t)$ is bounded as follows:

$$\begin{aligned} &\|\Psi(\chi, u, t)\|_2 \\ &\quad \le \|(\mathcal{A}_{3,i} + \gamma)e_1(t)\|_2 + \|\mathcal{D}_{2,i}\xi(\chi, u, t)\|_2 \\ &\quad \le \|(\mathcal{A}_{3,i} + \gamma)\tilde{e}(t)\|_2 + \|\mathcal{D}_{2,i}\xi(\chi, u, t)\|_2 . \end{aligned} \tag{59}$$

Since $\|\tilde{e}(t)\|_2 \le \left\|H^{-1}\right\|_2 \|g(t)\|_2$, it follows that

$$\|\Psi(\chi, u, t)\|_2 \le \epsilon^*, \tag{60}$$

where $\epsilon^* := \left(\left\|(\mathcal{A}_{3,i} + \gamma)H^{-1}\right\|_2 + \|\mathcal{D}_{2,i}\|_2\right) \|\xi(\chi, u, t)\|_2 .$

Therefore, approximately, for some small ϵ^*

$$0 = \sum_{i=1}^{k} \mu_i(\hat{\chi}(t)) \left\{ [\mathcal{M}_{a2,i}\ \mathcal{M}_{s2}] \begin{bmatrix} f_a(t) \\ f_s(t) \end{bmatrix} - \mathcal{C}_2^{-1} \begin{bmatrix} v_{a_{eq},i}(t) \\ v_{s_{eq},i}(t) \end{bmatrix} \right\}, \tag{61}$$

where the equivalent injection measurement signals are

$$v_{a_{eq},i}(t) := \begin{cases} \eta_{a,i} \dfrac{e_z(t)}{\|e_z(t)\| + \delta} & \text{if } e_z(t) \neq 0, \\ 0 & \text{otherwise}, \end{cases} \tag{62}$$

$$v_{s_{eq},i}(t) := \begin{cases} \eta_{s,i} \dfrac{e_z(t)}{\|e_z(t)\| + \delta} & \text{if } e_z(t) \neq 0, \\ 0 & \text{otherwise}, \end{cases} \tag{63}$$

with δ used to obtain a continuous sliding gain capable of estimating both actuator and sensor faults that jointly exist during the T–S system's operation.

Consequently, simultaneous actuator and sensor faults estimation for the T–S system is given by

$$\hat{f}_a(t) = \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\ \mathcal{M}_{a2,i}^{+} \mathcal{C}_2^{-1} v_{a_{eq},i}(t), \tag{64}$$

$$\hat{f}_s(t) = \sum_{i=1}^{k} \mu_i(\hat{\chi}(t))\ \mathcal{M}_{s2}^{+} \mathcal{C}_2^{-1} v_{s_{eq},i}(t), \tag{65}$$

where $\mathcal{M}_{a2,i}^{+}$ and $\mathcal{M}_{s2}^{+}$ represent the pseudo-inverses of $\mathcal{M}_{a2,i}$ and $\mathcal{M}_{s2}$, respectively.

5. Illustrative examples

The proposed design of robust fault reconstruction and estimation is illustrated with two simulation examples.

5.1. Illustrative example 1.

Firstly, let us consider an academic T–S system taken from the work of Ichalal *et al.* (2009a) with the structure of (1)–(2) and the following matrices:

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1 \\ 5 \\ 0.5 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 3 \\ 1 \\ -7 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 7 \\ 5 \\ 2 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \qquad N = \begin{bmatrix} 5 \\ 1 \end{bmatrix}.$$

The parameters are

$$\begin{aligned} \mu_1(x(t)) &= \frac{1 - \tanh(x_1(t))}{2}, \\ \mu_2(x(t)) &= \frac{1 + \tanh(x_1(t))}{2} = 1 - \mu_1(x(t)). \end{aligned}$$

5.1.1. T–S sliding mode observer design.

A suitable choice of the matrix T_R from (4) can be shown to be

$$T_R = \begin{bmatrix} -0.44 & -0.89 \\ 0.89 & -0.44 \end{bmatrix},$$

where we have $N_1 = -6.70$, $C_1 = \begin{bmatrix} 0.44 & -0.44 & 0.44 \end{bmatrix}$ and $C_2 = \begin{bmatrix} 0.78 & -0.19 & 0.78 \end{bmatrix}$. The filter matrix A_f from (5) is chosen as $A_f = 1$. Hence, the design T–S system (6)–(7) can be obtained.

It was found that $\operatorname{rank}(C_1 M_i) = q = 1$, so that Condition C1 is fulfilled. In addition, we also assume that Condition C2 is satisfied. Therefore, the proposed observer design (21)–(22) exists for the uncertain T–S system (8)–(9).

The aim of the following study is to simulate states by the proposed H_∞ T–S sliding mode observer, and then estimate the actuator and sensor faults in the simultaneous scenario. We assume that

$$\begin{aligned} H_1 &= 5I_2, \\ H_2 &= 2I_2, \\ \mathcal{A}_{s,i} &= \mathcal{A}_s = \operatorname{diag}\{-2, -3\}, \\ \lambda &= 0.99. \end{aligned}$$

The T–S sliding mode observer is designed by using the Matlab LMI toolbox. Once the convex multiobjective problem is solved, we get

$$\mu^* = 5.96,$$
$$\varepsilon^* = 5.55,$$
$$\alpha^* = 5.52,$$
$$\gamma^* = 0.99,$$
$$P_1 = \begin{bmatrix} 1.54 & -0.18 \\ -0.18 & 2.60 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.87 & 0.01 \\ 0.01 & 1.27 \end{bmatrix}.$$

For a given Lipschitz constant in the uncertain T–S system $\gamma = 0.65$ and the maximum admissible Lipschitz $\gamma^* = 0.99 \geq \gamma$, the maximization of γ guarantees the stability of the error for any Lipschitz nonlinear function.

The T–S siding mode observer gains are

$$\mathcal{G}_{l,1} = \begin{bmatrix} 2.23 & -2.37 \\ 1.67 & 0.25 \\ 5.03 & 6.54 \\ -6.75 & 4.65 \end{bmatrix},$$
$$\mathcal{G}_{n,1} = \begin{bmatrix} 3.35 & 1.67 & 0.55 & 0 \\ 0.25 & -1.18 & -1.44 & 1 \end{bmatrix}^T,$$
$$\mathcal{G}_{l,2} = \begin{bmatrix} -0.39 & 0.81 \\ 4.77 & -1.39 \\ 2.07 & 0.88 \\ -2.54 & 0.73 \end{bmatrix},$$
$$\mathcal{G}_{n,2} = \begin{bmatrix} 1.15 & 0.82 & 0.32 & 0 \\ -0.24 & 0.83 & 0.45 & 1 \end{bmatrix}^T.$$

5.1.2. Reconstruction of simultaneous faults. In the corresponding simulations, we assume that $x_0 = [0.1, 0.2, 0.1]$, $\eta_{a,i} = \eta_a = 25$, $\eta_{s,i} = \eta_s = 35$, $\delta = 0.001$, and the scenario with simultaneous actuator and sensor faults starting at $t = 7$ s. The simulation was carried out with the input signal $u(t) = 0.5\sin(t)$ and uncertainty $\xi(x, u, t) = 0.1\sin(0.2t)$.

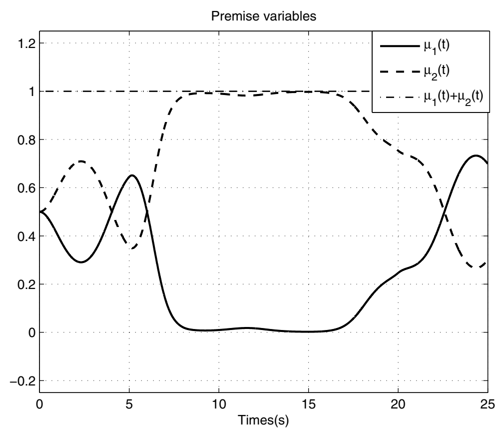

Fig. 1. Premise variables.

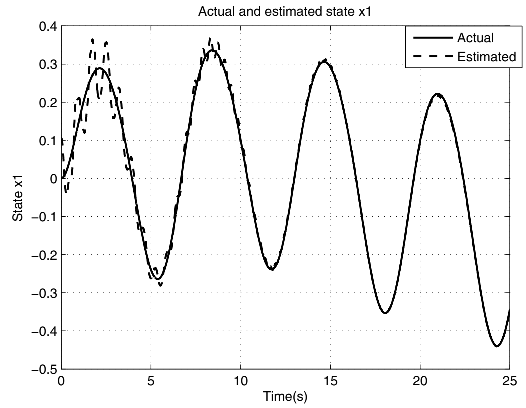

Fig. 2. Trajectories of state $x_1(t)$ and its estimate $\hat{x}_1(t)$.

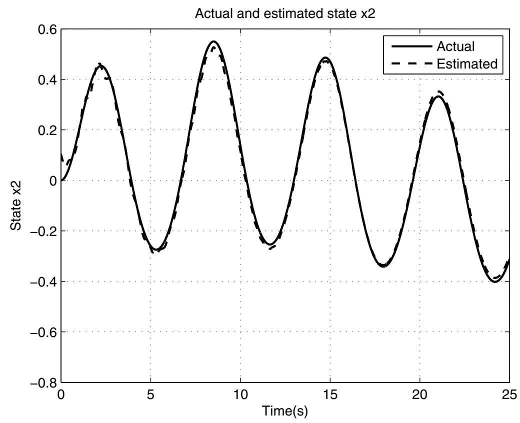

Fig. 3. Trajectories of state $x_2(t)$ and its estimate $\hat{x}_2(t)$.

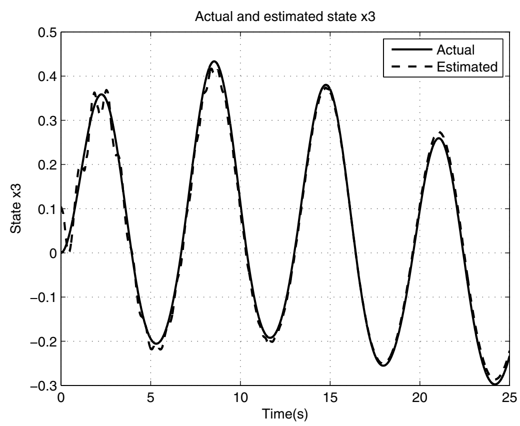

Fig. 4. Trajectories of state $x_3(t)$ and its estimate $\hat{x}_3(t)$.

Figure 1 describes the behaviors of the premise variables such that $0 \leq \mu_1(x(t)) \leq 1$, $0 \leq \mu_2(x(t)) \leq 1$ and $\mu_1(x(t)) + \mu_2(x(t)) = 1$ along the system trajectories.

Figures 2–4 show the states $x_1(t)$, $x_2(t)$ and $x_3(t)$ plotted for comparison against the estimated values $\hat{x}_1(t)$,

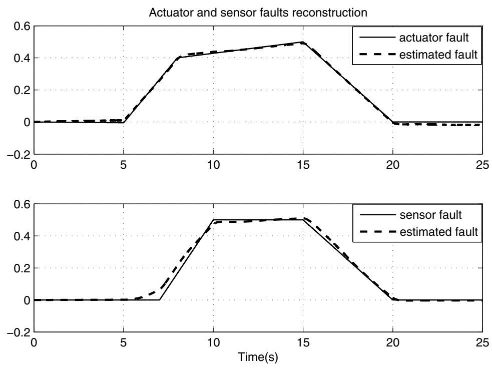

Fig. 5. Reconstruction of simultaneous actuator and sensor faults.

$\hat{x}_2(t)$ and $\hat{x}_3(t)$, respectively. It can be seen that the estimated states can converge towards the original states.

In order to see the effectiveness of the proposed fault reconstruction and estimation design for T–S system with unmeasurable premise variables, Fig. 5 shows that the T–S sliding mode observer faithfully reconstructs faults simultaneously occurring in the actuator and sensor in spite of the presence of uncertainties.

5.2. Illustrative example 2. In this example, an application of the proposed reconstruction design for simultaneous actuator and sensor faults is illustrated by the nonlinear model of a single link flexible joint robot arm, taken from the work of Ichalal *et al.* (2010), whose model is defined by

$$\begin{cases}\dot{\theta}_m = \omega_m,\\ \dot{\omega}_m = \frac{k}{J_m}(\theta_l - \theta_m) - \frac{B_\nu}{J_m}\omega_m + \frac{K_\iota}{J_m}(u(t) - f_a(t)),\\ \dot{\theta}_l = \omega_l,\\ \dot{\omega}_l = \frac{k}{J_l}(\theta_l - \theta_m) - \frac{mgh}{J_l}\sin(\theta_l),\end{cases}$$

where θ_m and ω_m are the position and angular velocity of the DC motor, respectively, θ_l and ω_l represent the position and angular velocity of the link. The DC motor is excited with $u(t) = \sin(t)$. We choose $x_1 = \theta_m$, $x_2 = \omega_m$, $x_3 = \theta_l$, and $x_4 = \omega_l$.

The flexible joint robot arm system is described in the nonlinear form as

$$\begin{cases}\dot{x}(t) = Ax(t) + Bu(t) + \Gamma(x,u,t) + Mf_a(t) + D\xi(x,u,t),\\ y(t) = Cx(t) + Nf_s(t),\end{cases}$$

with the matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0\\ -48.64 & -1.25 & 48.64 & 0\\ 0 & 0 & 0 & 1\\ 1.95 & 0 & -1.95 & 0\end{bmatrix},$$

$$B = M = \begin{bmatrix} 0\\ 21.62\\ 0\\ 0\end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\end{bmatrix},$$

$$\Gamma(x,u,t) = \begin{bmatrix} 0\\ 21.62u(t)\\ 0\\ -3.33\sin(x_3(t))\end{bmatrix}.$$

The variable $f_a(t)$ denotes the signal of the actuator faults. The potentially faulty sensor signal, which affects the first output system, is $f_s(t)$, with $N = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^T$. $\Gamma(x,u,t)$ encapsulates the nonlinearities present in the D-C motor.

As described by Ichalal *et al.* (2010), the flexible joint robot arm system can be formulated in the T–S representation (1)–(2), where $k = 2$, with the system matrices

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0\\ -48.64 & -1.24 & 48.64 & 0\\ 0 & 0 & 0 & 1\\ 1.95 & 0 & -22.83 & 0\end{bmatrix}, \quad B_1 = \begin{bmatrix} 0\\ 21.62\\ 0\\ 0\end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0\\ -48.64 & -1.24 & 48.64 & 0\\ 0 & 0 & 0 & 1\\ 1.95 & 0 & -18.77 & 0\end{bmatrix}, \quad B_2 = \begin{bmatrix} 0\\ 21.62\\ 0\\ 0\end{bmatrix}.$$

The parameters $\mu_i(x(t))$ are given by

$$\mu_1(x(t)) = \frac{\vartheta(t) + 0.21}{1.21},$$

$$\mu_2(x(t)) = \frac{1 - \vartheta(t)}{1.21}$$

where

$$\vartheta(t) = \frac{\sin(x_3(t))}{x_3(t)}.$$

The matrices T_R and A_f are chosen respectively as

$$T_R = \begin{bmatrix} 0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$$

and $A_f = 1$. Hence, the designed T–S system (6)–(7) can be obtained.

It was found that Conditions C1 and C2 are satisfied. Therefore, the T–S observer design (21)–(22) exists due to the T–S system (8)–(9).

After solving the optimization problem with LMI technique, for a given Lipschitz constant in the T–S system $\gamma = 0.33$ and the maximum admissible Lipschitz

$\gamma^* = 0.96 \geq \gamma$, the obtained observer matrices are given by

$$\mathcal{G}_{l,1} = \begin{bmatrix} 0 & 1 & 0 \\ 0.07 & 1.75 & 0 \\ 1 & 0 & 0 \\ -1.99 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad \mathcal{G}_{l,2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1.75 & 0 \\ 1 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

$$\mathcal{G}_{n,1} = \mathcal{G}_{n,2} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T.$$

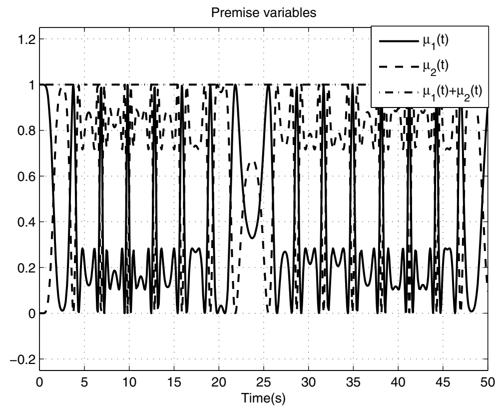

Fig. 6. Premise variables.

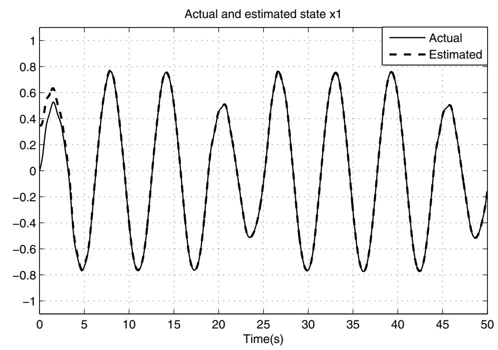

Fig. 7. Trajectories of state $x_1(t)$ and its estimate $\hat{x}_1(t)$.

Figure 6 describes the behaviors of the premise variables, such that they satisfy the properties of the sum convex.

From the following simulation results, cf. Figs. 7–11, it clearly appears that the proposed robust fault reconstruction and estimation method is effective in estimating simultaneously the actuator/sensor faults and states for the T–S system with unmeasurable premise variables subject to uncertainties.

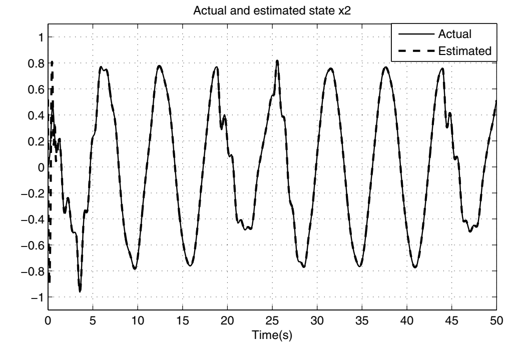

Fig. 8. Trajectories of state $x_2(t)$ and its estimate $\hat{x}_2(t)$.

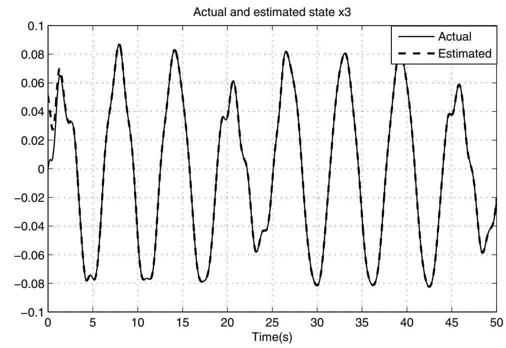

Fig. 9. Trajectories of state $x_3(t)$ and its estimate $\hat{x}_3(t)$.

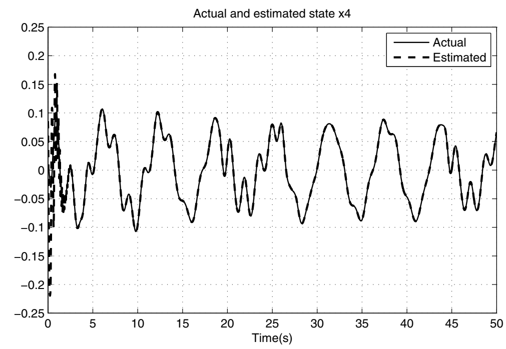

Fig. 10. Trajectories of state $x_4(t)$ and its estimate $\hat{x}_4(t)$.

6. Conclusion

This paper presented a robust H_∞ fault reconstruction and estimation scheme for a T–S system with unmeasurable premise variables subject both to actuator/sensor faults and disturbances. An augmented system was constructed by assuming the sensor faults to be auxiliary actuator faults. Hence, the proposed T–S sliding mode observer with two discontinuous terms was constructed through the search for suitable Lyapunov matrices in order to decouple simultaneous faults. The LMIs conditions were formulated by using convex multiobjective optimization techniques for maximizing simultaneously the admissible

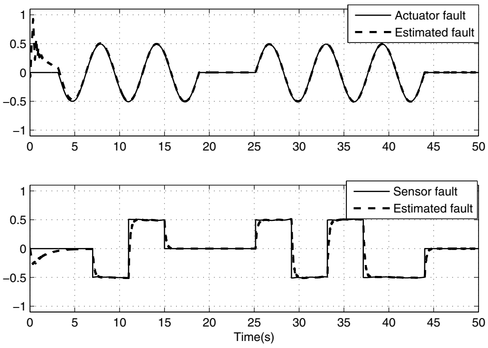

Fig. 11. Simultaneous reconstruction of actuator and sensor faults.

Lipschitz constant and the disturbance attenuation level. Finally, simulation results were presented to verify the effectiveness of the proposed method in this design framework. The extension of our work to fault tolerant control of T–S systems is under study.

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