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THE EFFECT OF VISCOSITY AND HETEROGENEITY ON PROPAGATION OF G–TYPE WAVES

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Earthquakes yield motions of massive rock layers accompanied by vibrations which travel in waves. This paper analyses the possibility of G-type wave propagation along the plane surface at the interface of two different media which is assumed to be heterogeneous and viscoelastic. The upper layer is considered to be viscoelastic and the lower half space is considered to be an initially stressed heterogeneous half space. The dispersion equation, as well as the phase and group velocities, is obtained in closed form. The dispersion equation agrees with the classical Love type wave. The effects of the nonhomogeneity of the parameters and the initial stress on the phase and group velocities are expressed by means of a graph.

Keywords: G-type wave, dispersion equation, heterogeneity.

1. Introduction

Seismology is the study of earthquakes and seismic waves that move through and around the Earth. Seismic waves are the waves of energy that are caused by a sudden breaking of rock within the Earth or an explosion. The study of a G-type wave in a viscoelastic material plays a central role in earth sciences including the construction sector and geophysics. The G-type wave which is a horizontally polarized surface wave with no vertical components is some exceptional type of the Love wave. The duration of the G-type wave is more than the duration of the Love wave and the speed is high compared with the Love wave. The Love wave which propagates for 60 to 300 s is known as a G-type wave. For this particular period, the speed of wave is around 4.4 km/sec. Hool and Kinne (1924) analyzed a reinforced concrete and masonry structure which gave rise to reinforced materials. Belfield *et al.* (1983) studied stress in elastic plates reinforced by fibres lying in concentric circles. Sato (1952) formulated the 6th generation of the Love and other types of SH-waves. In the meantime, Gutenberg (1953) developed the theory of G-type waves. Aki (1964) examined the generation of G-type waves from the Niigata earthquake of June 1964.

Chattopadhyay *et al.* (1986) carefully examined a generation of G-type waves. Chattopadhyay and Singh (2012) studied the propagation of G-type seismic wave in a fibre reinforced layer lying over a fibre reinforced elastic half space. References can be made to Gutenberg (1954), Lehman (1961) or (Mal) 1962, among others. Chattopadhyay *et al.* (2010) studied the propagation of G-type waves in a viscoelastic medium. The dispersion equation and shear wave velocity were obtained using the Laplace transform technique in that paper. Kundu *et al.* (2014) studied the propagation of G-type waves in a heterogeneous layer lying over the same heterogeneous half space. In that paper, it is observed that the initial stress has a dominant effect on the propagation of G-type wave. The term “initial stress” is understood as stresses developed in a medium before it is being used for the study. Kaur *et al.* (2015) discussed the shear wave propagation in a vertically heterogeneous viscoelastic layer over a micro polar elastic half space. Recently, Vishwakarma and Xu (2016) investigated the effect of a rigid boundary on the G-type wave.

In the present paper, we have obtained the dispersion equation for the G-type wave in a viscoelastic layer lying over a heterogeneous elastic half space under an initial stress. The variation in the half space is taken as $\mu_2 = \mu_2(1 - \delta\cos\gamma z)$ and $\rho_2 = \rho_2(1 - \delta\cos\gamma z)$, where δ is a small positive constant and γ is a real depth parameter. With the law of variation, the equation

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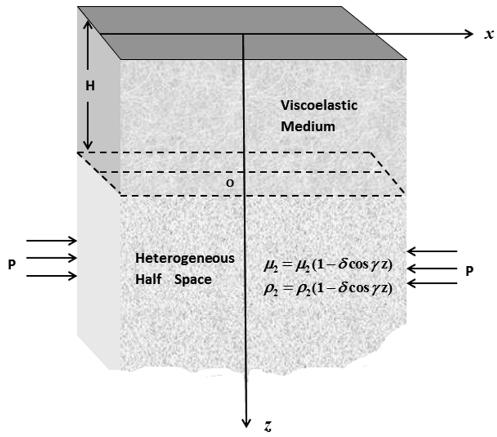

Fig. 1. Geometry of the problem.

of motion reduces to Hill's differential equation with periodic coefficients which has been solved by Valeev's method. Valeev (1961) considered a certain class of systems of linear differential equations with periodic coefficients which have the property that, by means of Laplace transform, they can be converted to systems of linear difference equations, which in turn may be solved by the method of the infinite determinant. The method of solving Hill's differential equation using the technique of the infinite determinant has been successfully applied by Chattopadhyay and Singh (2012) or Kundu *et al.* (2014), among others.

2. Problem formulation and its solution

Consider a viscoelastic medium of thickness H lying over a heterogeneous half space under an initial stress. The x-axis is taken as the horizontal axis, the z-axis is oriented vertically downwards and its origin is taken at the interface of the layer and the half space. The variation in rigidity and the density for the half space are

$$\begin{aligned} \mu_2 &= \mu_2\left(1-\delta\cos\gamma z\right), \\ \rho_2 &= \rho_2\left(1-\delta\cos\gamma z\right), \end{aligned} \tag{1}$$

respectively, where δ is a small positive constant and γ is a real depth parameter. We assume that the propagation of a horizontally polarized surface wave is of a shear type, propagating along the x-axis, so the displacement components are $u = 0, w = 0$ and $v = v\left(x, z, t\right)$. The displacement components are assumed as (u_1, v_1, w_1) and (u_2, v_2, w_2) for the upper medium and the lower half space, respectively.

The equation of motion for the upper viscoelastic layer is

$$\left(\mu_1 + \mu_1' \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2}\right) = \rho_1 \frac{\partial^2 v_1}{\partial t^2}, \tag{2}$$

where the constant μ_1' is the parameter used for the effect of viscosity. Here v_1, μ_1 and ρ_1 represent the displacement component, rigidity and density for upper media, respectively. The lower half space is considered to be a heterogeneous half space. In the lower heterogeneous half space under the initial stress, the displacement $v_2\left(x, z, t\right)$ satisfies the differential equation

$$\begin{aligned} &\frac{\partial}{\partial x}\left(\mu_2\left(1-\delta\cos\gamma z\right)\frac{\partial v_2}{\partial x}\right) \\ &\quad + \frac{\partial}{\partial z}\left(\mu_2\left(1-\delta\cos\gamma z\right)\frac{\partial v_2}{\partial z}\right) \\ &\quad - \frac{P}{2}\frac{\partial^2 v_2}{\partial x^2} = \rho_2\left(1-\delta\cos\gamma z\right)\frac{\partial^2 v_2}{\partial t^2}. \end{aligned} \tag{3}$$

Assume that

$$v_2\left(x, z, t\right) = V_2\left(z\right) e^{ik(x-ct)} \tag{4}$$

The boundary conditions are the following:
(i) The continuity of displacement requires that, at $z = 0$,

$$v_1 = v_2.$$

(ii) The continuity of stress requires that, at $z = 0$,

$$\bar{\mu}_1 \frac{\partial v_1}{\partial z} = \mu_2\left(1-\delta\cos\gamma z\right)\frac{\partial v_2}{\partial z}.$$

(iii) As the upper surface is stress free, at $z = -H$,

$$\frac{\partial v_1}{\partial z} = 0, \tag{5}$$

where $\bar{\mu}_1 = \mu_1 + i\omega_1\mu_1'$. Using separation of variables twice in (2) for viscoelastic material, we get

$$v_1 = A_1 \cos\left(\zeta\left(z + A_2\right)\right) e^{ik(x-ct)}, \tag{6}$$

where A_1 and A_2 are constants,

$$\zeta = \left(k_1^2 - k^2\right)^{\frac{1}{2}}, \quad k_1^2 = \frac{\rho_1\omega_1^2}{\bar{\mu_1}}$$

and $\omega_1 = kc$, k is the complex wave number and c is the phase velocity. As the upper surface is stress free, using the boundary condition (iii) of (5), we get

$$v_1 = A_1 \cos\left(\zeta\left(z + H\right)\right) e^{i(kx-\omega_1 t)} \tag{7}$$

Using the conditions (i) and (ii) of (5), we have

$$V_2\left(0\right) = A_1 \cos\left(\zeta H\right), \tag{8}$$

$$V_2'\left(0\right) = \frac{-A_1\bar{\mu_1}\zeta\sin\left(\zeta H\right)}{\mu_2^{(0)}}, \tag{9}$$

where $\mu_2^{(0)} = \mu_2(1-\delta)$. Using (3) and (4), the equation of motion for the lower heterogeneous half space may be written as

$$\begin{aligned}
&\frac{\mathrm{d}^2V_2}{\mathrm{d}z^2} + \left(\frac{\rho_2 c^2}{\mu_2} + \left(\frac{P}{2\mu_2} - 1\right)\right) k^2 V_2(z) \\
&\quad + e^{-i\gamma z}\Big(\frac{-\delta\rho_2}{2\mu_2} k^2 c^2 V_2(z) + \frac{\delta}{2} k^2 V_2(z) \\
&\quad - \frac{\delta}{2}\frac{\mathrm{d}^2V_2}{\mathrm{d}z^2} + \frac{\delta\gamma i}{2}\frac{\mathrm{d}V_2}{\mathrm{d}z}\Big) \\
&\quad + e^{i\gamma z}\Big(\frac{-\delta\rho_2}{2\mu_2} k^2 c^2 V_2(z) + \frac{\delta}{2} k^2 V_2(z) \\
&\quad - \frac{\delta}{2}\frac{\mathrm{d}^2V_2}{\mathrm{d}z^2} - \frac{\delta\gamma i}{2}\frac{\mathrm{d}V_2}{\mathrm{d}z}\Big) \\
&\quad = 0.
\end{aligned} \tag{10}$$

This is Hill's differential equation which is solved by Valeev's method. We apply the Laplace transform with respect to z. To this end, multiplying (10) by $e^{-\alpha z}$ and integrating with respect to z from 0 to ∞, we get

$$\begin{aligned}
&\int_0^\infty e^{-(\alpha+i\gamma)z}\left(\frac{-\delta\rho_2}{2\mu_2} k^2 c^2 V_2(z)\right)\mathrm{d}z \\
&+ \int_0^\infty e^{-(\alpha+i\gamma)z}\left(\frac{\delta}{2} k^2 V_2(z) - \frac{\delta}{2}\frac{\mathrm{d}^2V_2}{\mathrm{d}z^2} - \frac{\delta\gamma i}{2}\frac{\mathrm{d}V_2}{\mathrm{d}z}\right)\mathrm{d}z \\
&+ \int_0^\infty e^{-(\alpha-i\gamma)z}\left(\frac{-\delta\rho_2}{2\mu_2} k^2 c^2 V_2(z) + \frac{\delta}{2} k^2 V_2(z)\right)\mathrm{d}z \\
&+ \int_0^\infty e^{-(\alpha-i\gamma)z}\left(-\frac{\delta}{2}\frac{\mathrm{d}^2V_2}{\mathrm{d}z^2} + \frac{\delta\gamma i}{2}\frac{\mathrm{d}V_2}{\mathrm{d}z}\right)\mathrm{d}z \\
&+ \int_0^\infty e^{-\alpha z}\Big[\frac{\mathrm{d}^2V_2}{\mathrm{d}z^2} + \left(\frac{\rho_2 c^2}{\mu_2} + \left(\frac{P}{2\mu_2} - 1\right)\right) k^2\Big] \\
&\times V_2(z)\,\mathrm{d}z = 0.
\end{aligned} \tag{11}$$

Let $w(0) = V_2'(0)$. Define the Laplace transform of $V_2(z)$ as $F(\alpha) = \int_0^\infty e^{-\alpha z} V_2(z)\mathrm{d}z$. Applying the Laplace transform to (11), we get

$$\begin{aligned}
&F(\alpha+i\gamma)\left[\frac{-\delta\rho_2}{2\mu_2}k^2c^2 + \frac{\delta}{2}k^2 - \frac{\delta}{2}(\alpha+i\gamma)^2\right. \\
&\quad \left. + \frac{\delta\gamma i}{2}(\alpha+i\gamma)\right] + F(\alpha-i\gamma)\left[\frac{-\delta\rho_2}{2\mu_2}k^2c^2\right. \\
&\quad \left. + \frac{\delta}{2}k^2 - \frac{\delta}{2}(\alpha-i\gamma)^2 - \frac{\delta\gamma i}{2}(\alpha-i\gamma)\right] \\
&\quad + \left(\alpha^2-\phi^2\right)F(\alpha) = \alpha\tau_1 + \tau_2,
\end{aligned} \tag{12}$$

where $\tau_1 = (1-\delta)V_2(0)$, $\tau_2 = (1-\delta)w(0)$ and

$$\phi^2 = \left(-\frac{\rho_2 c^2}{\mu_2} + \left(1 - \frac{P}{2\mu_2}\right)\right)k^2.$$

To find $F(\alpha)$, we replace α by $\alpha + i\gamma j$ and then dividing throughout by $(i\gamma j)^n$, $j \neq 0$, we obtain the following infinite system of linear algebraic equation in the quantities $F(\alpha + i\gamma j)$, $j = 1, 2, \ldots$:

$$\begin{aligned}
&(i\gamma j)^{-n} F(\alpha + i\gamma(j+1)) \\
&\quad \times \Big[\frac{-\delta\rho_2}{2\mu_2}k^2c^2 + \frac{\delta}{2}k^2 - \frac{\delta}{2}(\alpha+i\gamma(j+1))^2 \\
&\quad + \frac{\delta\gamma i}{2}(\alpha+i\gamma(j+1))\Big] \\
&\quad + (i\gamma j)^{-n} F(\alpha - i\gamma(j+1)) \\
&\quad \times \Big[\frac{-\delta\rho_2}{2\mu_2}k^2c^2 + \frac{\delta}{2}k^2 - \frac{\delta}{2}(\alpha-i\gamma(j+1))^2 \\
&\quad - \frac{\delta\gamma i}{2}(\alpha-i\gamma(j+1))\Big] \\
&\quad + \left(\alpha^2-\phi^2\right)F(\alpha+i\gamma j) \\
&= (\alpha+i\gamma j)\tau_1 + \tau_2,
\end{aligned} \tag{13}$$

where α may be regarded as a parameter in the coefficients. It should be noted that in order not to consider the special case $j = 0$ separately, we include $(i\gamma j)^{-n} = 1$ when $j = 0$. Solving the system of difference equations, we obtain $F(\alpha)$ as the ratio of two determinants, i.e., $F(\alpha) = \Delta_3/\Delta_4$, where the values of Δ_3 and Δ_4 are given in Appendix.

Neglecting δ^2 and higher powers in Δ_3, we get

$$\begin{aligned}
&s^{2n}\Delta_3 \\
&= (\alpha\tau_1+\tau_2)\left((\alpha+i\gamma)^2-\phi^2\right)\left((\alpha-i\gamma)^2-\phi^2\right) \\
&\quad + (\tau_1(\alpha-i\gamma)+\tau_2)\left((\alpha+i\gamma)^2-\phi^2\right) \\
&\quad \times \left(\frac{\delta\rho_2}{2\mu_2}k^2c^2 - \frac{\delta}{2}k^2 + \frac{\delta\gamma i}{2}(\alpha-i\gamma) + \frac{\delta}{2}(\alpha-i\gamma)^2\right) \\
&\quad + ((\alpha+i\gamma)\tau_1+\tau_2)\left((\alpha-i\gamma)^2-\phi^2\right) \\
&\quad \times \left(\frac{\delta\rho_2}{2\mu_2}k^2c^2 - \frac{\delta}{2}k^2 - \frac{\delta\gamma i}{2}(\alpha+i\gamma) + \frac{\delta}{2}(\alpha+i\gamma)^2\right).
\end{aligned} \tag{14}$$

Neglecting the term δ^2 and higher powers in Δ_4, we get

$$\begin{aligned}
s^{2n}\Delta_4 = &\left((\alpha+i\gamma)^2-\phi^2\right)\left((\alpha-i\gamma)^2-\phi^2\right) \\
&\times\left(\alpha^2-\phi^2\right).
\end{aligned} \tag{15}$$

Therefore, we have

$$\begin{aligned}
&F(\alpha) \\
&= \frac{\alpha\tau_1+\tau_2}{\alpha^2-\phi^2} + \frac{\delta}{2}\frac{V_2(0)(\alpha+i\gamma)+w(0)}{(\alpha^2-\phi^2)\left((\alpha+i\gamma)^2-\phi^2\right)} \\
&\quad \times \left(\frac{\rho_2 k^2 c^2}{\mu_2} - k^2 - \gamma i(\alpha+i\gamma) + (\alpha+i\gamma)^2\right)
\end{aligned}$$

$$+\frac{\delta}{2}\frac{V_2(0)\,(\alpha-i\gamma)+w\,(0)}{(\alpha^2-\phi^2)\left((\alpha-i\gamma)^2-\phi^2\right)} \times\left(\frac{\rho_2k^2c^2}{\mu_2}-k^2+\gamma i\,(\alpha-i\gamma)+(\alpha-i\gamma)^2\right). \tag{16}$$

Using the inverse Laplace transform, we have $V_2\,(z) = \int_{\gamma-\infty}^{\gamma+\infty} F\,(\alpha)\,e^{\alpha z}\,\mathrm{d}\alpha$. The residues at L_1, L_2, L_3 at the poles $r=\phi$, $r=\phi+i\gamma$, $r=\phi-i\gamma$ are given respectively as

$$\begin{aligned} L_1 &= \left(\frac{\phi V_2\,(0)+w\,(0)}{2\phi}\right)\left((1-\delta)+\frac{\delta\phi^2}{\gamma^2+\phi^2}\right)e^{\phi z} \\ &\quad -\frac{\delta M_1}{\gamma^2+4\phi^2}\left(\frac{w\,(0)-\phi V_2\,(0)}{2\phi}\right)e^{\phi z} \\ &\quad +\frac{\delta V_2(0)}{2}\frac{\gamma^2+2\phi^2}{\gamma^2+4\phi^2}e^{\phi z}, \end{aligned} \tag{17}$$

where

$$M_1=\frac{\rho_2k^2c^2}{\mu_2}-k^2.$$

Similarly, we can find the residues at the poles $r=\phi+i\gamma$ and $r=\phi-i\gamma$. The above equation shows that the conditions for a large amount of energy to be confined near the surface are

$$w\,(0)+\phi V_2\,(0)=0,$$

$$w\,(0)-\phi V_2\,(0)=0, \tag{18}$$

$$2\phi^2+\gamma^2=0. \tag{19}$$

From these equations, we have

$$\tan\left(\left(\frac{c^2}{\bar\beta_1^2}-1\right)^{\frac{1}{2}}kH\right)=\frac{\frac{\mu_2^{(0)}}{\bar\mu_1}\left(1-\frac{P}{2\mu_2}-\frac{\rho_2c^2}{\mu_2}\right)^{\frac{1}{2}}}{\left(\frac{c^2}{\bar\beta_1^2}-1\right)^{\frac{1}{2}}}, \tag{20}$$

where $\bar\beta_1=(\bar\mu_1/\rho_1)^{\frac{1}{2}}$. Equating real parts, we get

$$\begin{aligned} &\frac{\tan akH\left(1-(\tanh bkH)^2\right)}{1+(\tan akH+\tanh bkH)^2} \\ &=\frac{\mu_2\,(1-\delta)\left(1-\frac{P}{2\mu_2}-\frac{\rho_2c^2}{\mu_2}\right)^{\frac{1}{2}}(a\mu_1+b\omega\mu_1')}{(a\mu_1+b\omega\mu_1')^2+(a\omega\mu_1'-b\mu_1)^2}, \end{aligned} \tag{21}$$

where

$$a=\sqrt{\left(\frac{F_1\pm\sqrt{F_1^2+F_2^2}}{2}\right)},\quad b=\frac{F_2}{2a},$$

$$F_1=\frac{\rho_1c^2}{\mu_1\left(1+\left(\frac{\omega\mu_1'}{\mu_1}\right)^2\right)}-1$$

$$F_2=\frac{\frac{\rho_1c^2}{\mu_1}\frac{\omega\mu_1'}{\mu_1}}{1+\left(\frac{\omega\mu_1'}{\mu_1}\right)^2}.$$

We consider only the positive sign. Now, from (19), we have

$$kc=\sqrt{\frac{\mu_2}{2\rho_2}\left(2k^2\left(1-\frac{P}{2\mu_2}\right)+\gamma^2\right)}. \tag{22}$$

Then the group velocity is given by

$$U=\frac{\mathrm{d}}{\mathrm{d}k}\,(kc)=\frac{\sqrt{2}k\left(1-\frac{P}{2\mu_2}\right)}{\sqrt{2k^2\left(1-\frac{P}{2\mu_2}\right)+\gamma^2}}. \tag{23}$$

3. Particular cases

Case 1. When $\mu_1'=0$, we have

$$\tan\left(\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}kH\right) =\frac{\mu_2\,(1-\delta)}{\mu_1}\frac{\left(1-\frac{P}{2\mu_2}-\frac{\rho_2c^2}{\mu_2}\right)^{\frac{1}{2}}}{\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}}. \tag{24}$$

Equation (24) represents the dispersion equation for the propagation of the G-type wave in a viscoelastic layer lying over a heterogeneous half space over real k.

Case 2. When $P=0$, that is, when the initial stress is absent, we have

$$\tan\left(\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}kH\right) =\frac{\mu_2\,(1-\delta)}{\mu_1}\frac{\left(1-\frac{c^2}{\beta_2^2}\right)^{\frac{1}{2}}}{\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}}, \tag{25}$$

where $\beta_2=\sqrt{\mu_2/\rho_2}$.

Equation (25) represents the dispersion equation for the propagation of the G-type wave in a viscoelastic layer lying over a heterogeneous half space in the absence of the initial stress.

Case 3. When $\delta=0$, we have

$$\tan\left(\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}kH\right)=\frac{\mu_2}{\mu_1}\frac{\left(1-\frac{c^2}{\beta_2^2}\right)^{\frac{1}{2}}}{\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}}. \tag{26}$$

Equation (26) represents the dispersion equation for the propagation of the G-type wave in a viscoelastic layer lying over a homogeneous half space in the absence of the initial stress.

Case 4. When $\mu_2 = \mu_1$, we have

$$\tan\left(\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}} kH\right) = \frac{\left(1-\frac{c^2}{\beta_2^2}\right)^{\frac{1}{2}}}{\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}}. \tag{27}$$

Equation (27) represents the dispersion equation for the propagation of the G-type waves in a uniform viscoelastic layer lying over a uniform homogeneous half space in the absence of the initial stress.

4. Numerical computation

For graphical representation of the phase velocity of the G-type wave, we have assumed

$$\frac{\rho_2}{\rho_1} = 0.01, \quad \frac{\mu_2}{\mu_1} = 0.1, \quad \frac{\omega\mu_1'}{\mu_1} = \frac{4\pi}{9}.$$

Figures are plotted to show the effect of δ, and $P/2\mu_2$ on the phase velocity. Figure 2 illustrates the consequences of the initial stress on the group velocity with respect to the scaled wave number. It is seen that, as the value of the initial stress increases, the group velocity decreases rapidly for a small variation. Figures 3 and 4 correspond to the occurrence of the phase velocity for the propagation of the G-type wave in a viscoelastic layer lying over a heterogeneous half space. Figure 3 displays the effect of δ on the phase velocity with respect to the dimensionless wave number. Note that when the value of δ increases, the phase velocity decreases. Figure 4 reveals the influence of the initial stress on the phase velocity with respect to the dimensionless wave number. It is observed that, as the value of $P/2\mu_2$ increases for a small variation, the phase velocity decreases. Figures 5 and 6 correspond to the case of the angular frequency and the wave number. Figure 5 renders the effect of δ on the nondimensional angular frequency. It is verified that when the value of δ increases the angular frequency increases. Figure 6 shows the effect of the initial stress on the angular frequency. It is seen that when the initial stress increases, the angular frequency increases.

5. Application

An important application of this work is to use its results in prospecting for oil deposits. This is sometimes used to detect the underlying structure of continental and oceanic crusts. Seismologists study the Earth's interior and its vibrations. These are caused by the explosion and natural ground vibrations which reflect off or are refracted by subsurface features such as bedding planes. In this light, this work will help us to understand the Earth's interior. Most of the ideas of seismic waves are conceptually similar to sensing the world around us using light and sound. The current work complements ongoing efforts to build a numerical model suitable for the study of the crust during an earthquake. The concept of this paper represents the current state of the art in geological or earthquake research and it can be applied successfully in numerous studies of crustal deformations during an earthquake.

6. Conclusion

The dispersion equation for a G-type seismic wave in viscoelastic media lying over a heterogeneous half space has been obtained using a transformation technique and Valeev's method. The study certainly will be helpful in understanding the cause of damages during large earthquakes. It can also be useful to predict the nature of long-period waves. From the numerical results, we may conclude the following:

(i) The phase velocity increases as long as the dimensionless wave number increases.

(ii) The group velocity increases as the scaled wave-number increases.

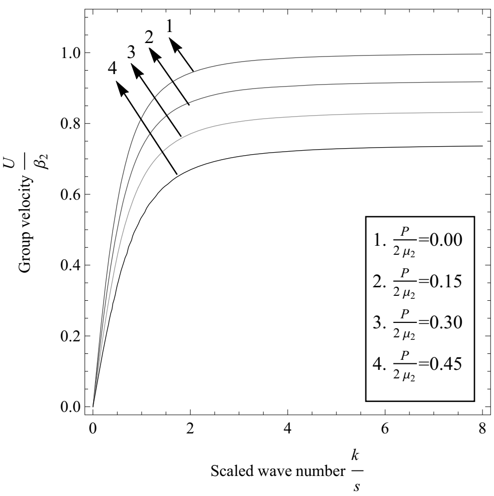

Fig. 2. Variation in $P/2\mu_2$ with respect to the group velocity and the scaled wave number.

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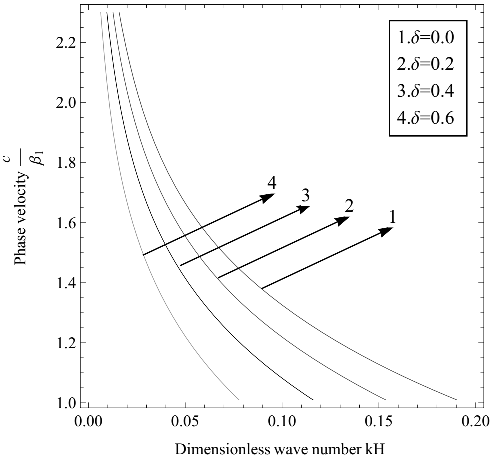

Fig. 3. Variation in δ with respect to the phase velocity and the dimensionless wave number.

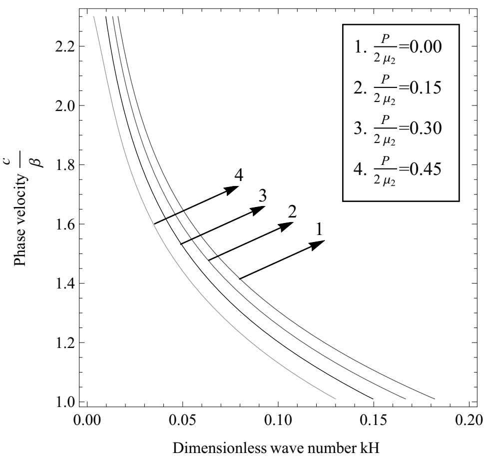

Fig. 4. Variation in $P/2\mu_2$ with respect to the phase velocity and the dimensionless wave number.

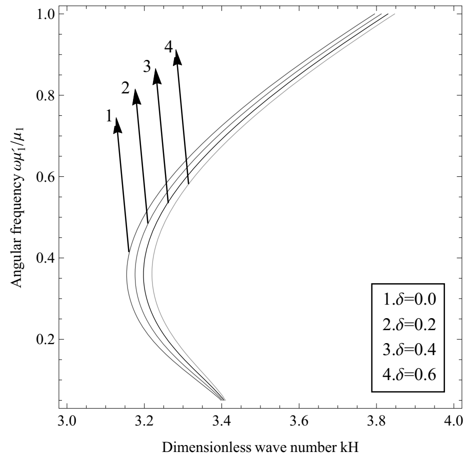

Fig. 5. Variation in δ with respect to $\omega\mu_1'/\mu_1$ and the dimensionless wave number.

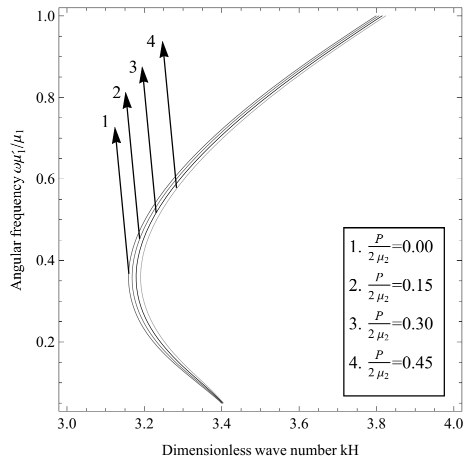

Fig. 6. Variation in $P/2\mu_2$ with respect to $\omega\mu_1'/\mu_1$ and the dimensionless wave number.

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Appendix

$$\Delta_1 = \begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & (i\gamma)^{-n}\left((\alpha+i\gamma)^2-\phi^2\right) & (i\gamma)^{-n}\left((\alpha+i\gamma)\tau_1+\tau_2\right) & 0 & \cdots \\ \cdots & \frac{-\delta\rho_2}{2\mu_2}k^2c^2+\frac{\delta k^2}{2}+\frac{\delta\gamma i}{2}(\alpha+i\gamma)-\frac{\delta}{2}(\alpha+i\gamma)^2 & \alpha\tau_1+\tau_2 & \frac{-\delta\rho_2}{2\mu_2}k^2c^2+\frac{\delta k^2}{2}-\frac{\delta\gamma i}{2}(\alpha+i\gamma)-\frac{\delta}{2}(\alpha+i\gamma)^2 & \cdots \\ \cdots & 0 & (i\gamma)^{-n}\left((\alpha-i\gamma)\tau_1+\tau_2\right) & (i\gamma)^{-n}\left((\alpha-i\gamma)^2-\phi^2\right) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix}, \tag{A1}$$

$$\Delta_2 = \begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & (i\gamma)^{-n}\left((\alpha+i\gamma)^2-\phi^2\right) & (i\gamma)^{-n}\left(-\frac{\delta\rho_2k^2c^2}{2\mu_2}+\frac{\delta k^2}{2}-\frac{\delta\gamma i\alpha}{2}-\frac{\delta\alpha^2}{2}\right) & 0 & \cdots \\ \cdots & \frac{-\delta\rho_2}{2\mu_2}k^2c^2+\frac{\delta k^2}{2}+\frac{\delta\gamma i}{2}(\alpha+i\gamma)-\frac{\delta}{2}(\alpha+i\gamma)^2 & \alpha^2-\phi^2 & \frac{-\delta\rho_2}{2\mu_2}k^2c^2+\frac{\delta k^2}{2}-\frac{\delta\gamma i}{2}(\alpha+i\gamma)-\frac{\delta}{2}(\alpha+i\gamma)^2 & \cdots \\ \cdots & 0 & (i\gamma)^{-n}\left(-\frac{\delta\rho_2k^2c^2}{2\mu_2}+\frac{\delta k^2}{2}+\frac{\delta\gamma ir}{2}-\frac{\delta\alpha^2}{2}\right) & (i\gamma)^{-n}\left((\alpha-i\gamma)^2-\phi^2\right) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix}, \tag{A2}$$

$$\Delta_3 = \begin{vmatrix} (i\gamma)^{-n}\left((\alpha+i\gamma)^2-\phi^2\right) & (i\gamma)^{-n}\left((\alpha+i\gamma)\tau_1+\tau_2\right) & 0 \\ \frac{-\delta\rho_2}{2\mu_2}k^2c^2+\frac{\delta k^2}{2}+\frac{\delta\gamma i}{2}(\alpha+i\gamma)-\frac{\delta}{2}(\alpha+i\gamma)^2 & \alpha\tau_1+\tau_2 & \frac{-\delta\rho_2}{2\mu_2}k^2c^2+\frac{\delta k^2}{2}-\frac{\delta\gamma i}{2}(\alpha+i\gamma)-\frac{\delta}{2}(\alpha+i\gamma)^2 \\ 0 & (i\gamma)^{-n}\left((\alpha-i\gamma)\tau_1+\tau_2\right) & (i\gamma)^{-n}\left((\alpha-i\gamma)^2-\phi^2\right) \end{vmatrix}, \tag{A3}$$

$$\Delta_4 = \begin{vmatrix} (i\gamma)^{-n}\left((\alpha+i\gamma)^2-\phi^2\right) & (i\gamma)^{-n}\left(-\frac{\delta\rho_2k^2c^2}{2\mu_2}+\frac{\delta k^2}{2}-\frac{\delta\gamma i\alpha}{2}-\frac{\delta\alpha^2}{2}\right) & 0 \\ \frac{-\delta\rho_2}{2\mu_2}k^2c^2+\frac{\delta k^2}{2}+\frac{\delta\gamma i}{2}(\alpha+i\gamma)-\frac{\delta}{2}(\alpha+i\gamma)^2 & \alpha^2-\phi^2 & \frac{-\delta\rho_2}{2\mu_2}k^2c^2+\frac{\delta k^2}{2}-\frac{\delta\gamma i}{2}(\alpha+i\gamma)-\frac{\delta}{2}(\alpha+i\gamma)^2 \\ 0 & (i\gamma)^{-n}\left(-\frac{\delta\rho_2k^2c^2}{2\mu_2}+\frac{\delta k^2}{2}+\frac{\delta\gamma i\alpha}{2}-\frac{\delta\alpha^2}{2}\right) & (i\gamma)^{-n}\left((\alpha-i\gamma)^2-\phi^2\right) \end{vmatrix}. \tag{A4}$$

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