Int. J. Appl. Math. Comput. Sci., 2018, Vol. 28, No. 1, 135–140
DOI: 10.2478/amcs-2018-0010

DECENTRALIZED STABILIZATION OF FRACTIONAL POSITIVE DESCRIPTOR CONTINUOUS–TIME LINEAR SYSTEMS

TADEUSZ KACZOREK [a]

[a] Faculty of Electrical Engineering
Białystok Technical University, Wiejska 45D, 15-351 Białystok, Poland
e-mail: kaczorek@ee.pw.edu.pl

A method for decentralized stabilization of fractional positive descriptor linear systems is proposed. Necessary and sufficient conditions for decentralized stabilization of fractional positive descriptor linear systems are established. The efficiency of the proposed method is demonstrated on a numerical example.

Keywords: positive system, linear system, continuous-time system, descriptor system, decentralized stabilization.

1. Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of the state of the art in positive systems theory was given by Farina and Rinaldi (2000) or Kaczorek (2002; 2014a; 2014b; 2010). Models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc.

A dynamical system is called fractional if it is described by fractional differential equations. Mathematical fundamentals of fractional calculus were given and by Oldham and Spanier (1974), Ostalczyk (2008) and Podlubny (1999). Positive fractional linear systems were investigated by Caputo and Fabrizio (2015), Kaczorek (2013; 2010; 2012), or Losada and Nieto (2015). Stability of positive descriptor systems was addressed by Kaczorek (2011a) and Virnik (2008).

Descriptor (singular) linear systems were considered in many papers and books (Bru *et al.*, 2000; 2003; State, 1976; Dai, 1989; Dodig and Stosic, 2009; Duan, 2010; Fahmy and o'Reill, 1989; Sajewski, 2016a; 2016b). Positive standard and descriptor systems and their stability were analyzed by Dodig and Stosic (2009), Duan (2010), or Sajewski (2016b). Descriptor standard and positive discrete-time and continuous-time nonlinear systems were analyzed by Kaczorek (2014a; 2012; 1997) and Van Dooren (1979). The Drazin inverse was applied to the analysis of descriptor systems by Kaczorek (2013), who also addressed singular standard and positive linear systems (Kaczorek, 2002; 1997), along with minimum energy control of descriptor positive systems (Kaczorek, 2014b) and positive linear systems with different fractional orders (Kaczorek, 2010; 2011b). Positive fractional continuous-time linear systems with singular pencil were also addressed by Kaczorek (2012).

This work is intended as an attempt to characterize decentralized stabilization of fractional positive descriptor continuous-time linear systems. The paper is organized as follows. In Section 2 some definitions and theorems concerning fractional positive linear continuous-time systems are recalled and a method of stabilization of linear systems by state-feedback is proposed. The main result of the paper, a method of decentralized stabilization of fractional descriptor linear systems, is presented in Section 3. In Section 4, concluding remarks are given.

The following notation will be used: $\mathbb{R}$, the set of real numbers; $\mathbb{R}^{n\times m}$, the set of real $n \times m$ matrices; $\mathbb{R}_+^{n\times m}$, the set of $n\times m$ matrices with nonnegative entries and $\mathbb{R}_+^n = \mathbb{R}_+^{n\times 1}$; $\mathbb{Z}_+$, the set of nonnegative integers; $\mathbb{M}_n$, the set of $n \times n$ Metzler matrices (with nonnegative off-diagonal entries); I_n, the $n \times n$ identity matrix; A^T, the transposition of matrix A.

2. Preliminaries

In this paper, the following Caputo definition of the fractional derivative of α order will be used (Kaczorek, 2011a; Oldham and Spanier, 1974; Ostalczyk, 2008;

Podlubny, 1999):

$$ {}_0D_t^\alpha f(t) = \frac{\mathrm{d}^\alpha f(t)}{\mathrm{d}t^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\dot f(\tau)}{(t-\tau)^\alpha}\,\mathrm{d}\tau, \quad 0<\alpha<1, \quad (1) $$

where

$$ \dot f(\tau) = \frac{\mathrm{d}f(\tau)}{\mathrm{d}\tau} $$

and

$$ \Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\,\mathrm{d}t, $$

while $\mathbb{R}(x) > 0$ is the Euler gamma function.

Consider the fractional continuous-time linear system

$$ \frac{\mathrm{d}^\alpha x(t)}{\mathrm{d}t^\alpha} = Ax(t) + Bu(t), \quad 0<\alpha<1, \quad (2a) $$

$$ y(t) = Cx(t), \quad (2b) $$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ are the state, input and output vectors, respectively, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$.

Definition 1. (*Farina and Rinaldi, 2000; Kaczorek, 2002*) The fractional system (2) is called (*internally*) *positive* if $x(t) \in \mathbb{R}_+^n$, $y(t) \in \mathbb{R}_+^p$, $t \ge 0$ for all $x(0) \in \mathbb{R}_+^n$ and every $u(t) \in \mathbb{R}_+^m$, $t \ge 0$.

Theorem 1. (Farina and Rinaldi, 2000; Kaczorek, 2002) *The fractional system (2) is positive if and only if*

$$ A \in \mathbb{M}_n, \quad B \in \mathbb{R}_+^{n\times m}, \quad C \in \mathbb{R}_+^{p\times n}. \quad (3) $$

Definition 2. (*Farina and Rinaldi, 2000; Kaczorek, 2011a*) The positive fractional system (2) for $u(t) = 0$ is called asymptotically stable if

$$ \lim_{t\to\infty} x(t) = 0 \quad \text{for all } x(0) \in \mathbb{R}_+^n. \quad (4) $$

Theorem 2. (Farina and Rinaldi, 2000; Kaczorek, 2011a) *The positive fractional system (2) for $u(t) = 0$ is asymptotically stable if and only if one of the following equivalent conditions is satisfied:*

1. *All coefficients of the characteristic polynomial*

$$ \det[I_n s - A] = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 \quad (5) $$

are positive, i.e., $a_i > 0, i = 0, 1, \dots, n-1$.

2. *All principal minors $\bar M_i > 0, i = 1, \dots, n$, of the matrix $-A$ are positive, i.e.,*

$$ \bar M_1 = |-a_{11}| > 0, \quad \bar M_2 = \begin{vmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{vmatrix} > 0, \quad \dots, \quad \bar M_n = \det[-A] > 0. \quad (6) $$

3. *There exists a strictly positive vector*

$$ \lambda = [\lambda_1, \dots, \lambda_n]^T, \ \lambda_k > 0, \ k = 1, \dots, n, $$

such that

$$ A\lambda < 0. \quad (7) $$

If $\det A \ne 0$ then we may choose $\lambda = A^{-1}c$, where $c \in \mathbb{R}^n$ is strictly positive.

We are looking for a gain matrix

$$ K = ND^{-1}, \quad D = \mathrm{diag}[d_1, \dots, d_n], \quad d_k > 0, \quad k = 1, \dots, n, \quad N \in \mathbb{R}^{m\times n} \quad (8) $$

such that

$$ A + BK \in \mathbb{M}_n \quad (9) $$

is asymptotically stable.

Substitution of (8) into (9) yields

$$ A + BK = (AD + BN)D^{-1}. \quad (10) $$

Using the known (Boyd *et al.*, 1994; Giorgio and Zuccotti, 2015) procedures we choose the matrices D and Nso that

$$ AD + BN \in \mathbb{M}_n, \quad (AD + BN)D^{-1} < 0. \quad (11) $$

If (11) holds, then the matrix (10) is an asymptotically stable Metzler matrix.

To find the matrices D and N, one of the well-known linear programming or LMI procedures can be used (Boyd *et al.*, 1994; Giorgio and Zuccotti, 2015).

Definition 3. The positive system (1) (or, equivalently, the pair (A, B)) is called *stabilizable by the state feedback* if there exists a gain matrix (8) such that the closed-loop system matrix

$$ A_c = A + BK \in \mathbb{M}_n \quad (12) $$

is asymptotically stable.

Remark 1. In a general case, the controllability of the pair (A, B) is not sufficient for the stabilization of a close-loop system with a Metzler matrix. The pair (*A*,*B*) should be stabilizable.

3. Decentralized stabilization of fractional descriptor linear systems

Consider the fractional descriptor linear continuous-time system

$$ E\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = Ax + Bu, \quad (13) $$

where $x = x(t) \in \mathbb{R}^n, u = u(t) \in \mathbb{R}^m$ are respectively the state and input vectors, and $A, E \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$.

Assumption 1. The matrix E possesses $n_1 < n$ linearly independent columns (the remaining columns are zero).

Assumption 2. The pencil of (13) is regular, i.e.,

$$\det[Es - A] \neq 0 \quad \text{for some } s \in \mathbb{C}, \tag{14}$$

where $\mathbb{C}$ is the field of complex numbers.

Defining the new state vector

$$\bar{x} = \bar{x}(t) = P^{-1}x(t) = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix},$$
$$\bar{x}_1 \in \mathbb{R}^{n_1}, \quad \bar{x}_2 \in \mathbb{R}^{n_2}, \quad n_2 = n - n_1 \tag{15}$$

and premultiplying Eqn. (13) by a matrix $Q \in \mathbb{R}^{n\times n}$, we obtain

$$QEPP^{-1}\dot{x} = QAPP^{-1}x + QBu \tag{16}$$

and

$$\frac{\mathrm{d}^\alpha x_1}{\mathrm{d}t^\alpha} = \bar{A}_{11}x_1 + \bar{A}_{12}x_2 + \bar{B}_1 u, \tag{17}$$

$$0 = \bar{A}_{21}x_1 + \bar{A}_{22}x_2 + \bar{B}_2 u, \tag{18}$$

where

$$QEP = \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix}, \tag{19a}$$

$$\bar{A} = QAP = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \tag{19b}$$

$$\bar{A}_{11} \in \mathbb{R}^{n_1\times n_1}, \quad \bar{A}_{22} \in \mathbb{R}^{n_2\times n_2}, \tag{19c}$$

$$\bar{B} = QB = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}, \tag{19d}$$

$$\bar{B}_1 \in \mathbb{R}^{n_1\times m}, \quad \bar{B}_2 \in \mathbb{R}^{n_2\times m}. \tag{19e}$$

The matrices Q and P can be obtained with the use of the following elementary row and column operations (Kaczorek, 2002; 2011a):

1. Multiplication of the i-th row (resp. column) by a real number c. This operation will be denoted by $L[i \times c]$ (resp. $R[i \times c]$).
2. Addition to the i-th row (resp. column) of the j-th row (resp. column) multiplied by a real number c. This operation will be denoted by $L[i + j \times c]$ (resp. $R[i + j \times c]$).
3. Interchange of the i-th and j-th rows (resp. columns). This operation will be denoted by $L[i, j]$ (resp. $R[i, j]$).

From Assumption 1 it follows that the matrix P is a permutation matrix and $P^{-1} = P^T \in \mathbb{R}_+^{n\times n}$. Therefore, if $x(t) \in \mathbb{R}_+^n$, $t \geq 0$, then $P^{-1}x(t) \in \mathbb{R}_+^n$, $t \geq 0$. The matrix $Q \in \mathbb{R}^{n\times n}$ can be obtained by performing an elementary row operation on the identity matrix I_n.

Consider the fractional descriptor unstable positive linear system

$$\begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix} \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u, \tag{20a}$$

where

$$, x_1 \in \mathbb{R}^{n_1},\ x_2 \in \mathbb{R}^{n_2},\ u \in \mathbb{R}^m \text{ and}$$
$$A_{11} \in M_{n_1}, \quad A_{22} \in M_{n_2}, \quad A_{12} \in \mathbb{R}_+^{n_1\times n_2}, \tag{20b}$$
$$A_{21} \in \mathbb{R}_+^{n_2\times n_1}, \quad B_1 \in \mathbb{R}_+^{n_1\times m}, \quad B_2 \in \mathbb{R}_+^{n_2\times m}. \tag{20c}$$

We are looking for a decentralized controller,

$$u = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$
$$K_1 \in \mathbb{R}^{n_1\times n_1}, \quad K_2 \in \mathbb{R}^{n_2\times n_2}, \tag{21}$$

such that the closed-loop system

$$\begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix} \frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_{11} + B_1K_1 & A_{12} \\ A_{21} & A_{22} + B_2K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{22}$$

is positive and asymptotically stable.

The problem can be solved with the use of the following procedure.

Procedure 1.

Step 1. Given $A_{22} \in M_{n_2}$ and $B_2 \in \mathbb{R}_+^{n_2\times m}$, compute a gain matrix K_2 such that

$$\hat{A}_{22} = A_{22} + B_2K_2 \in \mathbb{M}_{n_2} \tag{23}$$

and is asymptotically stable.

Then, from (22), we have

$$x_2 = -\hat{A}_{22}^{-1}A_{21}x_1 \in \mathbb{R}_+^{n_2} \iff x_1 \in \mathbb{R}_+^{n_1}, \quad t \geq 0, \tag{24}$$

where $-\hat{A}_{22}^{-1}A_{21} \in \mathbb{R}_+^{n_2\times n_1}$ since, by Theorem 2, $-\hat{A}_{22}^{-1} \in \mathbb{R}_+^{n_2\times n_2}$ and $A_{21} \in \mathbb{R}_+^{n_2\times n_1}$.

Substituting (24) into

$$\hat{x}_1 = (A_{11} + B_1K_1)x_1 + A_{12}x_2, \tag{25}$$

we obtain

$$\dot{x}_1 = (\hat{A}_{11} + B_1K_1)x_1, \tag{26}$$

where

$$\hat{A}_{11} = A_{11} - A_{12}\hat{A}_{22}^{-1}A_{21} \in \mathbb{R}_+^{n_1\times n_1}. \tag{27}$$

Step 2. Knowing $\hat{A}_{11} \in \mathbb{R}_+^{n_1 \times n_1}$ and $B \in \mathbb{R}_+^{n_1 \times m}$, compute a gain matrix K_1 such that

$$\hat{A}_{11} + B_1 K_1 \in \mathbb{M}_{n_1} \tag{28}$$

is asymptotically stable.

If the condition (28) is satisfied, then from (26) we have

$$x_1 \in \mathbb{R}_+^{n_1}, \quad \lim_{t\to\infty} x_1(t) = 0, \tag{29}$$

which implies

$$x_2 \in \mathbb{R}_+^{n_2}, \quad \lim_{t\to\infty} x_2(t) = 0. \tag{30}$$

It is well known that the eigenvalues of the matrix $A_c = A + BK$ can be arbitrarily assigned to the state-feedback matrix K if and only if the pair (A, B) is stabilizable. Therefore, the following theorem has been proved.

Theorem 3. *The positive descriptor system (20) can be stabilized by the decentralized controller* (21) *if and only if the pairs* (A_{22}, B_2), $(\hat{A}_{11}, B_1)$ *are stabilizable.*

Remark 2. Note that the closed-loop system is positive and asymptotically stable if the gain matrices K_1 and K_2 are chosen so that the matrices $A_{22} + B_2K_2$ and $\hat{A}_{11} + B_1K_1$ are asymptotically stable Metzler matrices, but the matrix

$$\begin{bmatrix} A_{11} + B_1K_1 & A_{12} \\ A_{21} & A_{22} + B_2K_2 \end{bmatrix} \tag{31}$$

is not necessarily asymptotically stable.

Example 1. Consider the positive descriptor system (20) with the matrices

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & -3 & 0 & 1 \\ 1 & 1 & -2 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (n_1 = n_2 = 2). \tag{32}$$

The system is unstable since the matrices $A_{11}$and A_{22} have positive diagonal entries. ♦

Find a decentralized controller (21) which is stabilized for the system (20) with (32).

Using Procedure 1, we obtain what follows.

Step 1. Note that the pair

$$(A_{22}, B_2) = \left(\begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

is controllable, and using (8) we obtain that

$$K_2 = N_2 D_2^{-1} = \begin{bmatrix} 2 & -3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -3 \end{bmatrix} \tag{33}$$

and the matrix

$$\hat{A}_{22} = A_{22} + B_2K_2 = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \in \mathbb{M}_2 \tag{34}$$

is asymptotically stable.

Taking into account that

$$-\hat{A}_{22}^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \tag{35}$$

we obtain

$$\hat{A}_{11} = A_{11} - A_{12}\hat{A}_{22}^{-1}A_{21} = \begin{bmatrix} 1 & 2 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 5 & 10 \\ 4 & -4 \end{bmatrix}. \tag{36}$$

Note that the matrix (36) is unstable and the pair $(\hat{A}_{11}, B_1)$ is controllable.

Step 2. Using (8), we obtain that

$$K_1 = N_1 D_1^{-1} = \begin{bmatrix} -4 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -3 \end{bmatrix} \tag{37}$$

and the matrix

$$\hat{A}_{11} + B_1K_1 = \frac{1}{3} \begin{bmatrix} 5 & 10 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} -4 & -3 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -7 & 1 \\ 4 & -4 \end{bmatrix} \in \mathbb{M}_2 \tag{38}$$

is asymptotically stable.

Note that the closed-loop system (22) with the matrix

$$\begin{bmatrix} A_{11} + B_1K_1 & A_{12} \\ A_{21} & A_{22} + B_2K_2 \end{bmatrix} = \begin{bmatrix} -\frac{7}{3} & \frac{1}{3} & 1 & 0 \\ \frac{4}{3} & -\frac{4}{3} & 0 & 1 \\ 1 & 1 & -2 & 1 \\ 0 & 2 & 1 & -2 \end{bmatrix} \tag{39}$$

is positive and asymptotically stable although the Metzler matrix (39) is not asymptotically stable.

4. Concluding remarks

A method for decentralized stabilization of fractional positive descriptor continuous-time linear systems has been proposed. Necessary and sufficient conditions for decentralized stabilization of fractional positive descriptor systems have been established (Theorem 3). A procedure for computation of decentralized feedbacks has been proposed. The effectiveness of the procedure has been demonstrated on a numerical example. The considerations discussion can be easily extended to the fractional positive descriptor discrete-time linear systems and to fractional positive linear systems described by the new definitions of fractional derivatives (Caputo and Fabrizio, 2015; Losada and Nieto, 2015). An open problem is the extension of the method to positive nonlinear systems.

Acknowledgment

This work was supported by the National Science Center in Poland under the grant no. 2014/13/B/ST7/03467.

References

Boyd, S. El Ghaoui, L. Feron, E. and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, PA.

Bru, R., Coll, C. and Sanchez E. (2000). About positively discrete-time singular systems, *in* M.E. Mastorakis (Ed.), *System and Control: Theory and Applications*, World Scientific and Engineering Society, Athens, pp. 44–48.

Bru, R. Coll, C. Romero-Vivo, S. and Sanchez, E. (2003). Some problems about structural properties of positive descriptor systems, *in* L. Benvennnuti *et al.* (Eds.), *Positive Systems,* Lecture Notes in Control and Information Science, Vol. 294, Springer, Berlin, pp. 233–240.

State, N.J. (1976). Applications of the Drazin inverse to linear systems of differential equations with singular constructions, *SIAM Journal on Applied Mathematics* **31**(3): 411–425.

Caputo, M. and Fabrizio, M. (2015). A new definition of fractional derivative without singular kernel, *Progress in Fractional Differentiation and Applications* **1**(2): 1–13.

Dai, L. (1989). *Singular Control Systems*, Springer-Verlag, Berlin.

Dodig, M. and Stosic, M. (2009). Singular systems state feedbacks problems, *Linear Algebra and Its Applications* **431**(8): 1267–1292.

Duan, G.R. (2010). *Analysis and Design of Descriptor Linear Systems*, Springer, New York, NY.

Fahmy, M.M. and O'Reill, J. (1989). Matrix pencil of closed-loop descriptor systems: Infinite-eigenvalues assignment, *International Journal of Control* **49**(4): 1421–1431.

Farina, L. and Rinaldi, S. (2000). *Positive Linear Systems*, Wiley, New York, NY.

Giorgio, G. and Zuccotti, C. (2015). Metzlerian and generalized Metzlerian matrices: Some properties and economic applications, *Journal of Mathematics Research* **7**(2): 42–55.

Kaczorek, T. (1997). Positive singular discrete time linear systems, *Bulletin of the Polish Academy of Sciences: Technical Sciences* **45**(4): 619–631.

Kaczorek, T. (2002). *Positive 1D and 2D Systems*, Springer-Verlag, London.

Kaczorek, T. (2010). Positive linear systems with different fractional orders, *Bulletin of the Polish Academy of Sciences: Technical Sciences* **58**(3): 453–458.

Kaczorek, T. (2011a). *Selected Problems of Fractional Systems Theory*, Springer-Verlag, Berlin.

Kaczorek, T. (2011b). Singular fractional discrete-time linear systems, *Control and Cybernetics* **40**(3): 753–761.

Kaczorek, T. (2012). Positive fractional continuous-time linear systems with singular pencil, *Bulletin of the Polish Academy of Sciences: Technical Sciences* **60**(1): 9–12.

Kaczorek, T. (2013). Application of the Drazin inverse to analysis of descriptor fractional discrete-time linear systems with regular pencils, *International Journal of Applied Mathematics and Computer Science* **23**(1): 29–33, DOI: 10.2478/amcs-2013-0003.

Kaczorek, T. (2014a). Descriptor positive discrete-time and continuous-time nonlinear systems, *Proceedings of SPIE* **92902Q**.

Kaczorek, T. (2014b). Minimum energy control of fractional descriptor positive discrete-time linear systems, *International Journal of Applied Mathematics and Computer Science* **24**(4): 735–743, DOI: 10.2478/amcs-2014-0054.

Losada, J. and Nieto, J. (2015). Properties of a new fractional derivative without singular kernel, *Progress in Fractional Differentiation and Applications* **1**(2) 87–92.

Oldham, K.B. and Spanier, J. (1974). *The Fractional Calculus*, Academic Press, New York, NY.

Ostalczyk, P. (2008). *Epitome of the Fractional Calculus: Theory and Its Applications in Automatics*, Łódź University of Technology Press, Łódź, (in Polish).

Podlubny, I. (1999). *Fractional Differential Equations*, Academic Press, San Diego, CA.

Sajewski, Ł. (2016a). Descriptor fractional discrete-time linear system and its solution—Comparison of three different methods, *in* R. Szewczyk *et al.* (Eds.), *Challenges in Automation, Robotics and Measurement Techniques*, Advances in Intelligent Systems & Computing, Vol. 440, Springer, Cham, pp. 37–50.

Sajewski, Ł. (2016b). Descriptor fractional discrete-time linear system with two different fractional orders and its solution, *Bulletin of the Polish Academy of Sciences: Technical Science* **64**(1): 15–20.

Van Dooren, P. (1979). The computation of Kronecker's canonical form of a singular pencil, *Linear Algebra and Its Applications* **27**: 103–140.

Virnik, E. (2008). Stability analysis of positive descriptor systems, *Linear Algebra and Its Applications* **429**(10): 2640–2659.

Tadeusz Kaczorek received his MSc, PhD and DSc degrees in electrical engineering from the Warsaw University of Technology in 1956, 1962 and 1964, respectively. In the years 1968–1969 he was the dean of the Electrical Engineering Faculty, and in the period of 1970–1973 he was a deputy rector of the Warsaw University of Technology. In 1971 he became a professor and in 1974 a full professor at the same university. Since 2003 he has been a professor at the Białystok University of Technology. In 1986 he was elected a corresponding member and in 1996 a full member of the Polish Academy of Sciences. In the years 1988–1991 he was the director of the Research Center of the Polish Academy of Sciences in Rome. In 2004 he was elected an honorary member of the Hungarian Academy of Sciences. He has been granted honorary doctorates by 13 universities. His research interests cover systems theory, especially singular multidimensional systems, positive multidimensional systems, singular positive 1D and 2D systems, as well as positive fractional 1D and 2D systems. He initiated research in the field of singular 2D, positive 2D and positive fractional linear systems. He has published 28 books (8 in English) and over 1100 scientific papers. He has also supervised 69 PhD theses. He is the editor-in-chief of the *Bulletin of the Polish Academy of Sciences: Technical Sciences* and a member of editorial boards of ten international journals.

Received: 2 March 2017
Revised: 25 May 2017
Accepted: 28 August 2017