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DISCRETE–TIME SLIDING MODE CONTROL OF LINEAR SYSTEMS WITH INPUT SATURATION

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The paper proposes a discrete-time sliding mode controller for single input linear dynamical systems, under requirements of the fast response without overshoot and strong robustness to matched disturbances. The system input saturation is imposed during the design due to inevitable limitations of most actuators. The system disturbances are compensated by employing nonlinear estimation by integrating the signum of the sliding variable. Hence, the proposed control structure may be regarded as a super-twisting-like algorithm. The designed system stability is analyzed as well as the sliding manifold convergence conditions are derived using a discrete-time model of the system in the δ-domain. The results obtained theoretically have been verified by computer simulations.

Keywords: discrete-time sliding mode control, super-twisting controller, input saturation, disturbance compensation.

1. Introduction

Variable structure control systems (VSCSs) (Emelyanov, 1957) operating in sliding mode (SM) (Utkin, 1992), as a robust nonlinear control technique, have attracted a lot of attention in the literature. This is due to the following important features: order reduction of the system dynamics in SM, theoretical invariance to parameter variations and disturbances acting through the control channels (Draženović, 1969) and a simple two-step design procedure (Utkin, 1992). First, the desired SM dynamics is selected by defining the appropriate sliding manifold in the state space, and then a discontinuous control is designed that brings the system state to the manifold in a finite time (Ackermann and Utkin, 1998). Further motion towards the equilibrium takes place along the sliding manifold under the action of the discontinuous control. In addition to these remarkable features, some shortcomings have also been noticed. The first one is related to the discontinuous nature of control. Namely, every real control system has small transport/inertial delays usually present in the control plant and/or in actuators and sensors that are not captured by its mathematical model used in controller design. Such unmodeled dynamics can be excited by discontinuous control inducing high-frequency oscillations (chattering) around the sliding manifold (and the equilibrium state). Chattering cannot be tolerated in most applications, especially in electromechanical systems, where it results in unpleasant sound effects, wear out of mechanical parts, or drive overheating.

In order to mitigate the effects of chattering, the following methods were proposed: the boundary layer method (Slotine, 1984), application of an observer

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(Bondarev *et al.*, 1985) as a bypass for chattering, the implicit method for realization of the signum function (Golo *et al.*, 2000; Huber *et al.*, 2016) and application of higher order sliding modes (HOSMs) (Levant, 1993). In the first method, the discontinuous (signum) function is replaced by a high-gain saturation or a sigmoid function that eliminates or alleviates chattering, but the system robustness is reduced because the system gain becomes limited. In the second approach, the controller governs the observer and not the real plant, thus excluding the plant from the chattering-contaminated loop. The disadvantage is that the plant parameter variations deteriorate robustness and system performance. In the implicit control method, the signum function is treated as a multi-valued function, which reduces chattering. This approach is more complicated for practical implementation and does not result in significantly better results than a method that is the subject of this paper.

HOSMs have become very popular in the past decade. Since the introduction of HOSMs, the former first-order SMs have been called classic SMs. In classic SM, the sliding variable is a continuous function of time, whereas its time derivative is a discontinuous function. In the r-th-order SM, $r-1$ time-derivatives of the sliding variable are continuous, whereas the r-th time derivative is a discontinuous function. The second-order sliding mode certainly attracted most attention because of its practical feasibility (Bartolini *et al.*, 1998). According to Utkin (2016), all HOSMs are ultimately reduced to classical SMs in the sliding variable subspace, and only the super-twisting control (STC) algorithm (Levant, 1993) achieves a true second-order SM. The STC algorithm has become very popular in SM theory and practice. STC was originally developed for single input continuous-time (CT) systems (Levant, 1993). The relative order of the system must be one with respect to the sliding variable, taken as an output.

Modern control systems are implemented using computers or microprocessors, i.e., in the discrete-time (DT) domain. DTSM analysis and development began in the 1980s and 1990s (Milosavljević, 1985; Drakunov and Utkin, 1989; Gao *et al.*, 1995; Bartolini *et al.*, 1995; Bartoszewicz, 1998; Bartoszewicz and Leśniewski, 2014; Bartoszewicz and Adamiak, 2019) in the case of the first-order SM. The DT development of HOSMs began with the paper (Bartolini *et al.*, 2001) and became one of the research fields by well-known authors (Bartoszewicz and Latosinski, 2017; Chakrabarty *et al.*, 2017). However, there are relatively few papers in the field of DT realization of STC (Salgado *et al.*, 2016; 2011; Yan *et al.*, 2015; Koch and Reichhartinger, 2019). An essential feature of CT STC is that the system state reaches exactly the sliding manifold in a finite time. By simple discretization of CT STC by applying the Euler discretization method, this property is lost (Yan *et al.*, 2015). The system state arrives into a vicinity of the sliding manifold in a finite time, and the further motion takes place in a quasi-sliding domain around the manifold even in the absence of disturbances.

This paper discusses DT SMC design for single input linear time-invariant (LTI) systems that provides a fast response without overshoot, but with reduced chattering and strong robustness to matched disturbances. Furthermore, input saturation is imposed on the system considered, which is a realistic situation due to inevitable construction limitations of actuators. Hence, the controller output is saturated, which has required additional stability analysis to determine sliding manifold convergence conditions under saturation. This problem was also discussed by Bartolini *et al.* (1995) and Corradini *et al.* (2014) for classic DT SMC systems, and Golkani *et al.* (2018), Shtessel *et al.* (2012), Castillo *et al.* (2016) for CT STC.

As in the work of Golo and Milosavljević (2000), the analysis and design of the DT SM controller in this paper have been also carried out using a DT model of the system in the δ-domain, which in this case allows separation of the reaching control component that achieves reaching the sliding manifold and the sliding control component that secures sliding along that manifold. The reaching control of the proposed controller is selected to be linear in order not to excite chattering. However, to provide robustness it is necessary to introduce disturbance compensation. Some methods for it were suggested by Su *et al.* (2000) and Milosavljević *et al.* (2007) for DTSMC systems, both applying linear estimation laws. In this paper, an additional compensation control component is introduced, which is formed by applying nonlinear estimation by integrating the signum of the sliding variable. Thus, the proposed control structure resembles the structure of discretized STC. Finally, in order to avoid integrator wind-up because of the saturation in the system, an anti-wind-up mechanism is embedded into the controller.

The rest of the paper is organized as follows. Section 2 summarizes some basic design aspects of DT SMC and briefly describes the DT realization of an STC. Section 3 carries the main results and proposes a DT SM controller that meets the predefined design tasks. Stability analysis and convergence conditions are also given. Section 4 supports the designed control system by presenting simulation results of an illustrative example. The paper ends with conclusions and the used literature.

2. Preliminaries

Consider a linear time invariant continuous time dynamic system described as

$$\dot{x}(t) = Ax(t) + b(u(t) + d(t)), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control signal, and $A \in \mathbb{R}^{n\times n}$ and $b \in \mathbb{R}^{n\times 1}$ are the state and input matrices, respectively. Also, the system is subjected to a bounded matched disturbance d, $|d(t)| \le d_0 < \infty$. To implement SMC it is necessary (i) to design a sliding manifold in the state space and (ii) to choose a control that establishes a stable SM. The system motion can be divided into two phases: reaching phase and SM. Consequently, there are two control components. The reaching control should bring the system state from any initial state to the sliding manifold, and the SM control should drive the system state along the sliding surface. In CT systems it is possible to have a unique reaching and SM control but it must have a discontinuous nature, which can excite unmodeled dynamics and induce chattering.

As the scope of the paper is DTSMC, some elementary design principles of these systems are summarized below.

2.1. Discrete-time sliding mode control design. Consider a linear time invariant continuous time dynamic system described as

$$x_{k+1} = A_d x_k + b_d (u_k + d_k), \tag{2}$$

$$A_d = e^{AT}, \quad b_d = \int_0^T e^{At} b \, \mathrm{d}t, \tag{3}$$

under the assumption that the sampling period T is sufficiently small and the disturbance is slowly varying, due to which the disturbance can be considered constant over the sampling interval. Note that the time discretization disrupts the matching property (Draženović, 1969) from CT. It was shown by Abidi *et al.* (2007) that the unmatched part of disturbance in DT is of order $O(T^3)$. However, for constant disturbances the matching property is preserved, which is the ground for the main assumption.

The sliding motion should be organized along the sliding manifold

$$s_k = c_d x_k = 0, \tag{4}$$

where the constant vector c_d should provide the desired reduced order SM dynamics, defined by an eigenvalue spectrum

$$\lambda_d = \begin{bmatrix} \lambda_{d,1} & \lambda_{d,2} & \cdots & \lambda_{d,n-1} & 0 \end{bmatrix}. \tag{5}$$

According to the comprehensive approach to the sliding manifold design (Draženović *et al.*, 2013), vector c_d that provides the desired dynamics (5) and common requirement $c_d b_d = 1$ can be easily found using

$$c_d = \begin{bmatrix} k_{de} & 1 \end{bmatrix} \begin{bmatrix} A_d & b_d \end{bmatrix}^{\dagger}. \tag{6}$$

A gain vector k_{de} in (6) is the one that provides the desired spectrum (5) by conventional state feedback in the system (2). Also, the operator $\dagger$ in (6) denotes the matrix pseudo-inversion. The control that establishes the SM along the manifold (4) is obtained by solving equation $s_{k+1} = 0$, using the model (2), as

$$u_{eq,k} = -c_d A_d x_k - d_k. \tag{7}$$

This linear control is the equivalent control, which is in this case both the reaching control and the SM control. Under this control, the system reaches the sliding manifold in one step and slides along it afterwards. It requires the knowledge of disturbance d_k, which is usually unknown. Therefore, only the equivalent control for nominal system ($d_k = 0$) is feasible in practice as

$$u_k = -c_d A_d x_k = -k_{de} x_k. \tag{8}$$

This means that the disturbance should be estimated and compensated as best as possible by an additional control component. Also, there is no possibility of shaping the reaching phase.

2.2. Discrete-time sliding mode control design in the δ-domain. Another design approach is to use a DT model of a system in the δ-domain (Golo and Milosavljević, 2000). A mathematical model of the system (1) in the δ-domain can be obtained using (2) in the following manner:

$$\delta x_k = \frac{x_{k+1} - x_k}{T} = A_\delta x_k + b_\delta (u_k + d_k), \tag{9}$$

$$A_\delta = \frac{1}{T}(A_d - I_n), \quad b_\delta = \frac{b_d}{T}. \tag{10}$$

Now, the sliding variable in the δ-domain is defined as

$$s_{\delta,k} = c_\delta x_k, \quad c_\delta b_\delta = 1. \tag{11}$$

The desired eigenvalues (5) can be mapped into the δ-domain according to (10) as $\lambda_{\delta,i} = (\lambda_{d,i} - 1) T^{-1}$, ($i = 1, \dots, n-1$), which gives the spectrum in the δ-domain

$$\lambda_\delta = \begin{bmatrix} \lambda_{\delta,1} & \lambda_{\delta,2} & \cdots & \lambda_{\delta,n-1} & 0 \end{bmatrix}. \tag{12}$$

Equation (6) for finding the sliding manifold parameters in this case becomes

$$c_\delta = \begin{bmatrix} k_{\delta e} & 1 \end{bmatrix} \begin{bmatrix} A_\delta & b_\delta \end{bmatrix}^{\dagger}, \tag{13}$$

where $k_{\delta e}$ is the state feedback gain vector that provides the desired spectrum (12) in the system (9). According to (11) and (9) it follows that

$$\begin{aligned} \delta s_{\delta,k} &= \frac{s_{\delta,k+1} - s_{\delta,k}}{T} \\ &= c_\delta \delta x_k = c_\delta A_\delta x_k + u_k + d_k. \end{aligned} \tag{14}$$

The equivalent control for the δ-domain representation can be determined from the condition $s_{\delta,k+1} = 0$ by solving the previous equation with respect to u_k. This yields the equivalent control in the following form:

$$u_{eq,k} = -c_\delta A_\delta x_k - \frac{s_{\delta,k}}{T} - d_k. \tag{15}$$

Unlike control (7), in this control form there is the term $s_{\delta,k}T^{-1}$ that plays a role of the reaching control, which becomes zero when the sliding surface is reached. Hence, this approach offers the possibility to influence the reaching phase of the system motion, which will be exploited in this paper for obtaining a suitable control structure that minimizes chattering and increases dynamical properties in real systems. The feasible part of the equivalent control (15) is given by

$$u_k = -c_\delta A_\delta x_k - \frac{s_{\delta,k}}{T} = -\left(k_{\delta e} + \frac{1}{T}c_\delta\right)x_k. \tag{16}$$

By comparing (8) and (16), the equality $\left(k_{\delta e} + \frac{1}{T}c_\delta\right) = k_{de}$ holds. Also, from the design condition $c_\delta b_\delta = c_d b_d = 1$ and $b_d = Tb_\delta$, it follows that $c_\delta = Tc_d$.

2.3. Disturbance compensation via the sliding variable. If the control (8) is applied in the system (2), the future value of the sliding variable s_k, according to (4) and (2), would be $s_{k+1} = d_k$. Hence, $d_{k-1} = s_k$, which means that the past value of the disturbance can be estimated according to the current value of the sliding variable. This knowledge can be employed for disturbance compensation.

A compensation control for slowly varying disturbances was proposed by Milosavljević *et al.* (2007) and Lješnjanin *et al.* (2011) in the form

$$u_{c,k} = u_{c,k-1} - hs_k, \tag{17}$$

with stability condition $0 < hT < 1$. This control is, in fact, an integration of the sliding variable. Due to this integrating property, the compensation control fully rejects stepwise disturbances and significantly decreases the impact of other slowly time-varying disturbances. The overall control can now be rewritten as

$$u_k = -c_d A_d x_k + u_{c,k}, \tag{18a}$$

$$u_{c,k} = u_{c,k-1} - hs_k. \tag{18b}$$

The described method belongs to the class of linear compensators. Further improvements in system compensation can be achieved by employing nonlinear estimation of disturbance.

2.4. Discrete-time realization of the STC. A very popular SMC algorithm, which incorporates a sliding variable based part for disturbance estimation and compensation, is the STC algorithm (Levant, 1993). It belongs to the group of second-order SMC algorithms, developed for CT control systems with relative degree one with respect to the sliding variable. STC is defined by the following equations:

$$u = -k_p|s|^{1/2}\mathrm{sgn}(s) + w, \tag{19a}$$

$$\dot{w} = -k_i \mathrm{sgn}(s). \tag{19b}$$

Here w is the disturbance estimate that is used in (19a) for the compensation to obtain robustness of the ST controller. A DT version of the STC (Koch and Reichhartinger, 2019) is usually obtained by applying the explicit Euler discretization method, yielding

$$u_k = -k_p|s_k|^{1/2}\mathrm{sgn}(s_k) + w_k, \tag{20a}$$

$$w_k = w_{k-1} - k_i T\mathrm{sgn}(s_{k-1}). \tag{20b}$$

Comparison of the controllers (18) and (20) shows that they have identical structure. However, the control components in (18) are linear, whereas in (20) they are realized using nonlinear functions.

It has been noticed by Milosavljević *et al.* (2019) that the factor $|s|^{1/2}$ present in (19a), which provides finite time reaching in CT, loses its importance in the DT realization (20a) since then it is only possible to achieve quasi-SM in real systems. By neglecting the square root, the control (20a) becomes linear. Hence, a combination of (18a) and (20b) was proposed as the controller by Milosavljević *et al.* (2019), whose performance was thoroughly analysed in the control of first order plants with/without unmodeled dynamics.

3. DTSM controller design

The control task is to design a digital controller that provides a fast response without overshoot with significant robustness to disturbances. The controller should also take into account the actuator saturation property that limits the magnitude of the control signal. To meet the given requirements, the designed DTSM controller generally combines in stages two control principles with the proposed modifications in order to eliminate their well-known shortcomings. The first control phase is linear providing a near deadbeat response while a nonlinear control phase, based on a super twisting like algorithm, is activated in the vicinity of the sliding manifold. In this way a quick arrival into the quasi-sliding domain is ensured, avoiding the response overshoot due to integrator windup during the inevitable control saturation arising in the first stage of motion.

The proposed control law is given by the following set of equations:

$$u_k = \begin{cases} U_0\mathrm{sgn}(u_{\Sigma,k}) & \text{if } |u_{\Sigma,k}| > U_0, \\ u_{\Sigma,k} & \text{if } |u_{\Sigma,k}| \le U_0, \end{cases} \tag{21a}$$

$$u_{\Sigma,k} = u_{l,k} - p_{2,k} u_{c,k}, \tag{21b}$$

$$u_{l,k} = -c_\delta A_\delta x_k - [k_{s1} + (1 - p_{2,k}) k_{s2}] \frac{s_{\delta,k}}{T}, \tag{21c}$$

$$u_{c,k} = u_{c,k-1} + k_{\text{int}} T \text{sgn} (s_{\delta,k-1}), \tag{21d}$$

$$p_{1,k} = \begin{cases} 0 & \text{if } |u_{\Sigma,k}| > U_0, \\ 1 & \text{if } |u_{\Sigma,k}| \leq U_0, \end{cases} \tag{21e}$$

$$p_{2,k} = p_{1,k-1}, \tag{21f}$$

$$k_{s1}, k_{s2} > 0, \quad k_{s1} + k_{s2} \leq 1. \tag{21g}$$

The given control strategy can be summarized as follows. The control signal consists of two parts: linear $u_{l,k}$ and nonlinear $u_{c,k}$. For the system state far away from the sliding manifold, the control will be saturated due to the linear control term that tends to bring the system state onto the sliding surface in a nominal system. Hence, a high control magnitude will be generated so the control signal must be limited to the U_0 value, which is acceptable by an actuator. In the sampling instant when the controller output exits saturation, the linear control

$$u_{l,k} = -c_\delta A_\delta x_k - (k_{s1} + k_{s2}) \frac{s_{\delta,k}}{T} \tag{22}$$

is applied during only one sampling period. For the limit case $k_{s1} + k_{s2} = 1$, the control (22) is a deadbeat control that ensures $s_{\delta,k+1} = 0$ in the nominal system ($d_k = 0$). In real cases $k_{s1} + k_{s2} \leq 1$ since $d_k \neq 0$ (including unmodeled dynamics) in order to adjust the width of a quasi-sliding domain. Such control brings the system state to a vicinity of the sliding manifold. In the next sampling period, the gain of the linear part is reduced and the nonlinear component is activated, so the control signal becomes

$$u_k = -c_\delta A_\delta x_k - T^{-1} k_{s1} s_{\delta,k} - u_{c,k}, \tag{23}$$

$$u_{c,k} = u_{c,k-1} + k_{\text{int}} T \text{sgn} (s_{\delta,k-1}). \tag{24}$$

The control (23), (24) represents an ST like structure, which can be obtained from the original ST algorithm by replacing the factor $|s|^{1/2}$ with $|s|$. Neglecting the square root function in the DT realization of the STC is quite acceptable, since only a quasi-sliding mode can be attained by a digital controller. The control component $u_{c,k}$, as an output of the DT integrator, can be understood as a compensational control that tends to cancel disturbance effects on the system behavior, which will be displayed later in the text.

In the light of the given explanation of the controller operation, to prove the stability of the proposed control system means to prove the state trajectory convergence to the sliding manifold for both saturated and unsaturated control phases. The required stability analysis is presented in the subsequent subsections.

3.1. Stability of the system with saturated control.

Since the saturation is invoked by the linear control (22) and the compensational control is activated only in the small vicinity of the sliding manifold, stability analysis during saturation will be analyzed with respect to linear control. From (14), the following equation defines the sliding manifold dynamics:

$$s_{\delta,k+1} = s_{\delta,k} + T c_\delta A_\delta x_k + T (u_k + d_k). \tag{25}$$

The control signal is calculated according to (22). If the calculated control is greater than the allowed actuator limitation, i.e., $u_{l,k} > U_0$, the applied control would be

$$u_k = U_0 \text{sgn} (u_{l,k}) = U_0 \frac{u_{l,k}}{|u_{l,k}|}. \tag{26}$$

The following proposition imposes a condition on the limit value U_0 to get the system out of saturation.

Proposition 1. *The DT system (9) with the controller (21) operating in the saturation will leave this mode in a finite number of sampling periods if*

$$U_0 > |c_\delta A_\delta x_k| + d_0, \quad \forall k \geq 0. \tag{27}$$

Proof. Under the applied saturated control (26), the sliding dynamics becomes

$$\begin{aligned} s_{\delta,k+1} &= s_{\delta,k} + T c_\delta A_\delta x_k + T d_k \\ &\quad - U_0 \frac{T c_\delta A_\delta x_k + (k_{s1} + k_{s2}) s_{\delta,k}}{|u_{l,k}|}. \end{aligned} \tag{28}$$

Let $k_{s1} + k_{s2} + k_{s3} = 1$, where $0 \leq k_{s3} < 1$. Then the sliding dynamics can be rewritten as

$$\begin{aligned} s_{\delta,k+1} &= [(k_{s1} + k_{s2}) s_{\delta,k} + T c_\delta A_\delta x_k] \\ &\quad \times \left(1 - \frac{U_0}{|u_{l,k}|}\right) + k_{s3} s_{\delta,k} + T d_k. \end{aligned} \tag{29}$$

Since $|u_{l,k}| > U_0$ we have

$$0 < \left(1 - \frac{U_0}{|u_{l,k}|}\right) < 1.$$

Also, having in mind that $0 \leq k_{s3} < 1$ and $|d_k| \leq d_0$, the following inequality can be obtained:

$$\begin{aligned} |s_{\delta,k+1}| &\leq |(k_{s1} + k_{s2}) s_{\delta,k} + T c_\delta A_\delta x_k| \left(1 - \frac{U_0}{|u_{l,k}|}\right) \\ &\quad + k_{s3} |s_{\delta,k}| + T d_0 \\ &= T |u_{l,k}| \left(1 - \frac{U_0}{|u_{l,k}|}\right) + k_{s3} |s_{\delta,k}| + T d_0 \\ &= T |u_{l,k}| - T U_0 + k_{s3} |s_{\delta,k}| + T d_0 \\ &\leq (k_{s1} + k_{s2}) |s_{\delta,k}| + T |c_\delta A_\delta x_k| - T U_0 \\ &\quad + k_{s3} |s_{\delta,k}| + T d_0 \\ &= |s_{\delta,k}| + T (|c_\delta A_\delta x_k| + d_0 - U_0). \end{aligned} \tag{30}$$

Hence

$$|s_{\delta,k+1}| \le |s_{\delta,k}| + T\left(|c_\delta A_\delta x_k| + d_0 - U_0\right). \quad (31)$$

If the condition (27) holds, then

$$|s_{\delta,k+1}| < |s_{\delta,k}|, \quad (32)$$

which shows that $|s_{\delta,k}|$ decreases monotonically and the system trajectories will be directed towards the sliding manifold. This means that both $s_{\delta,k}$ and x_k will be decreasing, making $u_{l,k}$ decreasing as well. Therefore, it is inevitable that the control signal becomes smaller than the saturation limit and the system exits saturation. ■

Usually, in practice the value U_0 cannot be chosen greater than the actual physical limitation of the actuator. Also, the maximum disturbance magnitude d_0 acting on the system is determined by the system construction and cannot be affected. Therefore, an area in the state space should be determined where prerequisite (27) is valid for the predefined U_0 and d_0. The following remark conservatively finds that area.

Remark 1. The condition (27) is satisfied for the given U_0 and d_0 within a ball around the origin defined by

$$\|x\| < r, \quad r = \frac{U_0 - d_0}{\|c_\delta A_\delta\|}. \quad (33)$$

where $\|x\|$ defines Euclidean vector norm.

Proof. The product $c_\delta A_\delta x_k$ is a scalar, so that $|c_\delta A_\delta x_k| = \|c_\delta A_\delta x_k\|$. Then, inequality (31) can be rewritten as

$$|s_{\delta,k+1}| \le |s_{\delta,k}| + T(\|c_\delta A_\delta x_k\| + d_0 - U_0), \quad (34)$$

Since $\|c_\delta A_\delta x_k\| \le \|c_\delta A_\delta\| \cdot \|x_k\|$, the above inequality becomes

$$|s_{\delta,k+1}| \le |s_{\delta,k}| + T(\|c_\delta A_\delta\| \cdot \|x_k\| + d_0 - U_0). \quad (35)$$

To fulfil the sliding manifold convergence condition (32), it is necessary to have $\|c_\delta A_\delta\| \cdot \|x_k\| + d_0 - U_0 < 0, \forall k \ge 0$, which gives

$$\|x_k\| < \frac{U_0 - d_0}{\|c_\delta A_\delta\|}, \quad \forall k \ge 0. \quad (36)$$

This inequality defines a ball around the origin with the radius r given in (33). ■

It is not always possible in any system to fulfil (27) for a given initial state. Besides the initial state, the fulfilment of (27) depends on the plant dynamics as well as the required sliding mode dynamics. It may happen that for demanding a fast sliding mode dynamics, the condition (27) cannot be fulfilled for a given initial state for the maximum admissible U_0. Then the system slips into instability. It should also be emphasized that the occurrence of an external disturbance further worsens the fulfilment of this condition. Time derivatives of the reference signal must be bounded as well since they are components of the disturbance $d(t)$.

3.2. Stability of the system with unsaturated control. After the control signal exits saturation, by virtue of (21) in the next discretization period the following control acts:

$$u_k = -c_\delta A_\delta x_k - T^{-1}(k_{s1} + k_{s2})\, s_{\delta,k}, \quad (37)$$

so $\delta s_{\delta,k}$ becomes

$$\delta s_{\delta,k} = -T^{-1}(k_{s1} + k_{s2})\, s_{\delta,k} + d_k. \quad (38)$$

Since $\delta s_{\delta,k} = T^{-1}(s_{\delta,k+1} - s_{\delta,k})$, we get

$$s_{\delta,k+1} = (1 - k_{s1} - k_{s2})\, s_{\delta,k} + T d_k. \quad (39)$$

It is interesting to notice that the limit case $k_{s1} + k_{s2} = 1$ gives $s_{\delta,k+1} = T d_k$, which indicates that this control brings the system state into an $O(T)$ vicinity of the sliding manifold whose dimensions depend on the disturbance magnitude. In the nominal case $(d_k = 0)$, $s_{\delta,k+1} = 0$ is achieved, so this is actually the equivalent control providing the deadbeat response.

After the first discretization period of the unsaturated control (37), the controller gain is then reduced $(k_{s2} = 0)$ and the compensational control $u_{c,k}$ is activated, according to (21). Hence, the controller is now described by

$$u_k = -c_\delta A_\delta x_k - T^{-1} k_{s1} s_{\delta,k} - u_{c,k}, \quad (40)$$

$$u_{c,k} = u_{c,k-1} + k_{\text{int}} T \operatorname{sgn}(s_{\delta,k-1}). \quad (41)$$

Then

$$s_{\delta,k+1} = (1 - k_{s1})\, s_{\delta,k} + T(d_k - u_{c,k}), \quad (42)$$

$$u_{c,k+1} = u_{c,k} + k_{\text{int}} T \operatorname{sgn}(s_{\delta,k}). \quad (43)$$

The compensational control $u_{c,k}$ actually represents the disturbance estimated value so let a new variable $z_k = d_k - u_{c,k}$ be introduced, denoting the estimation error. Then the system dynamics can be described as

$$s_{\delta,k+1} = (1 - k_{s1})\, s_{\delta,k} + T z_k, \quad (44)$$

$$z_{k+1} = -k_{\text{int}} T \operatorname{sgn}(s_{\delta,k}) + z_k + \Delta_k, \quad (45)$$

where $\Delta_k = d_{k+1} - d_k$. The stability of the above described system is established by the following proposition.

Proposition 2. *The SM convergence condition for the DT system described by (44) and (45), with the controller parameters $0 < k_{s1} \le 1$ and $k_{int} > 0$, is satisfied within the area defined by*

$$|s_{\delta,k}| > \frac{k_{int} T^2}{k_{s1}}. \quad (46)$$

Proof. By using $\operatorname{sgn}(s_{\delta,k}) = s_{\delta,k}/|s_{\delta,k}|$, the above nonlinear system (44), (45) can be rewritten in a pseudo-linear form (Koch and Reichhartinger, 2019; Ghane and Menhaj, 2015)

$$\sigma_{k+1} = \Lambda(s_{\delta,k})\,\sigma_k + p_k, \tag{47}$$

$$\sigma_k = \begin{bmatrix} s_{\delta,k} \\ z_k \end{bmatrix},$$
$$\Lambda(s_{\delta,k}) = \begin{bmatrix} 1-k_{s1} & T \\ \frac{-k_{int}T}{|s_{\delta,k}|} & 1 \end{bmatrix},$$
$$p_k = \begin{bmatrix} 0 \\ \Delta_k \end{bmatrix}.$$

The characteristic equation of the pseudo-linear system (45) can be found as

$$F(z) = \det(zI - \Lambda) = 0, \tag{48}$$

which yields

$$F(z) = a_2 z^2 + a_1 z + a_0(s_{\delta,k}) = 0, \quad a_2 > 0, \tag{49}$$

$$a_2 = 1, \quad a_1 = k_{s1} - 2,$$
$$a_0 = 1 - k_{s1} + \frac{k_{\text{int}}T^2}{|s_{\delta,k}|}.$$

The convergence area can be determined using the Jury stability test that examines the system characteristic equation. Stability conditions for the second order system (47) are given through the following inequalities $F(1) > 0$, $F(-1) > 0$ and $|a_0| < a_2$. In the case of (49) these conditions are expressed by the following requirements

$$\begin{aligned} &\frac{k_{\text{int}}T^2}{|s_{\delta,k}|} > 0, \\ 4 - 2k_{s1} + &\frac{k_{\text{int}}T^2}{|s_{\delta,k}|} > 0, \\ &\frac{k_{\text{int}}T^2}{|s_{\delta,k}|} < k_{s1}. \end{aligned} \tag{50}$$

Since $k_{\text{int}} > 0$ and $k_{s1} \le 1$, the first two conditions are always fulfilled. The convergence area can be determined from the third condition as (46). ∎

Therefore, the system (47) is stable within the space defined by the condition (46), i.e., the system trajectories are directed towards the sliding manifold $s_\delta = 0$. It is important to notice that in the case of constant and slowly varying disturbances, for which can be assumed that $d_k = d_{k-1}$, the system (47) is autonomous since the external input $\Delta_k = 0$. Then the system trajectories will reach the convergence boundary $|s_{\delta,k}| = k_{\text{int}}T^2/k_{s1}$.

To determine a quasi-sliding domain in the case of arbitrary disturbances, a worst-case analysis is applied. Suppose that the system state reached the convergence boundary $|s_{\delta,k}| = k_{\text{int}}T^2/k_{s1}$. Let the compensational control be $u_{c,k} = 0$ (the integrator is empty or reset) and let the system be affected by a disturbance having a maximal magnitude of $|d_k| = d_0$. Then the sliding variable dynamics (42) is described by

$$s_{\delta,k+1} = (1 - k_{s1})\,s_{\delta,k} + T d_k. \tag{51}$$

Hence

$$\begin{aligned} |s_{\delta,k+1}| &\le (1-k_{s1})\,|s_{\delta,k}| + T\,|d_k| \\ &= (1-k_{s1})\,\frac{k_{\text{int}}T^2}{k_{s1}} + T d_0. \end{aligned} \tag{52}$$

Thus, according to (46) the quasi-sliding domain is obtained as

$$\frac{k_{\text{int}}T^2}{k_{s1}} < |s_{\delta,k+1}| \le T\left[\frac{(1-k_{s1})\,k_{\text{int}}T}{k_{s1}} + d_0\right], \tag{53}$$

and the system accuracy is of order

$$O\left(T^2\right) < |s_{\delta,k+1}| \le O(T). \tag{54}$$

The obtained upper bound (53) of $|s_\delta|$ is valid only when disturbance affects the system $(d_0 \neq 0)$ and represents the maximal possible deviation from the sliding manifold. However, the effective width of the quasi-sliding domain is much smaller, since the integrator output u_c has been accumulated after some time to compensate the disturbance.

4. Design example

The proposed control structure has been examined in an illustrative example and verified through simulation tests. For that purpose, let an arbitrary (academic) fifth order unstable controllable linear plant model (1) be defined by

$$A = \begin{bmatrix} 1 & 2 & 3 & -5 & 6 \\ -2 & 6 & -3 & -4 & -7 \\ 2 & -4 & 6 & -10 & 12 \\ -8 & -6 & -4 & 3 & 1 \\ 4 & 12 & -6 & -8 & -14 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \\ 2 \end{bmatrix},$$

with the initial state $x(0) = \begin{bmatrix} 10 & -5 & 0 & -10 & 5 \end{bmatrix}^T$. The corresponding discrete-time δ-model (9) for the sampling time $T = 0.001$ s has the following matrices:

$$A_\delta = \left[\begin{matrix} 1.0097 & 2.0521 & 2.9996 \\ -1.9801 & 5.9919 & -2.9920 \\ 2.0272 & -3.9198 & 6.0112 \\ -8.0161 & -6.0213 & -4.024 \\ -3.9601 & 11.9839 & -5.984 \end{matrix}\right.$$
$$\left.\begin{matrix} -5.0531 & 5.9695 \\ -3.97 & -6.9979 \\ -10.0902 & 11.967 \\ 3.0528 & 0.9676 \\ -7.94 & -13.9958 \end{matrix}\right],$$

$$b_\delta = \begin{bmatrix} 1.0115 \\ -2.0165 \\ 3.031 \\ -1.0046 \\ 1.967 \end{bmatrix}.$$

The control task is to bring the system state from the initial state into the equilibrium (the origin) with the prescribed dynamics by organizing SM along the appropriate manifold. Let the desired SM dynamics be defined by the eigenvalue spectrum $\lambda = \begin{bmatrix} -1 & -2 & -3 & -4 & 0 \end{bmatrix}$ in the CT domain. Using $\lambda_{\delta,i} = \left(e^{\lambda_i T} - 1\right)/T, i = 1, \dots, n-1$, the δ-domain spectrum is obtained as

$$\lambda_\delta = \begin{bmatrix} -0.9995 & -1.998 & -2.9955 & -3.992 & 0 \end{bmatrix}$$

To provide the SM mode dynamics, the required sliding manifold vector c_δ is calculated using (13), which yields

$$c_\delta = \begin{bmatrix} 0.4437 & 0.5794 & 0.3072 & -0.6614 & 0.063 \end{bmatrix}$$

In order to accomplish the control task, the proposed controller (21) is employed, whose parameters are set as follows: $k_{s1} = 0.9$, $k_{s2} = 0.1$ and $k_{\text{int}} = 100$. Also, let the actuator saturation be defined by $U_0 = 150$.

To check the analytically predicted system behaviour, let the system first be subjected to an action of step external disturbance $d(t) = 100h(t-3)$, where $h(t)$ is the Heaviside function. The next four figures show the simulation results of this case. Evolution of the state variables is presented in Fig. 1(a), which shows that the state variables asymptotically reach the origin despite the action of the disturbance. The controller output is given in Fig. 2(a). It can be noted that the control signal is initially saturated, but exits saturation after a short period of time. Figure 1(b) indicates that the condition (27), which ensures convergence during saturation and consequently provides saturation termination, is satisfied during the whole system motion. The dashed line in Fig. 1(a) denotes U_0. Also, Fig. 2(b) shows that the compensation control component u_c accurately estimates the disturbance, and thus eliminates its impact.

According to the sliding variable, which is presented in Fig. 3(a) and enlarged in Fig. 3(b), it can be concluded that the DTSM occurs in a very short time, providing desired dynamics and robustness to matched disturbances. It can be noticed in Fig. 3(b) that the disturbance initial action forces the system state to leave the sliding manifold, but the controller compensation part restores sliding motion.

It is important to emphasize that the sliding motion takes place in some vicinity of the sliding manifold due to the discrete nature of the controller. Proposition 2 finds convergence area defined by (46). This means that there is no convergence in the immediate vicinity of the sliding manifold up to the bounds given by (46). For

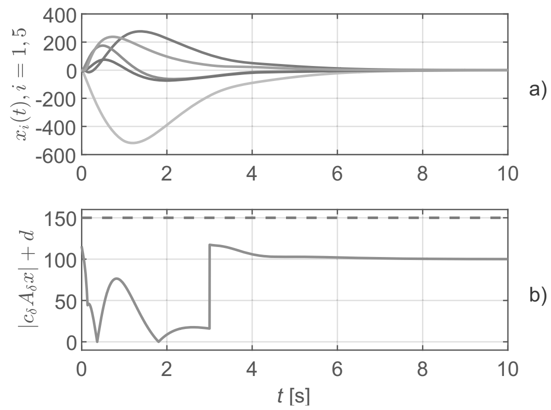

Fig. 1. System response: state coordinates (a), the fulfilment of the condition (27) (b).

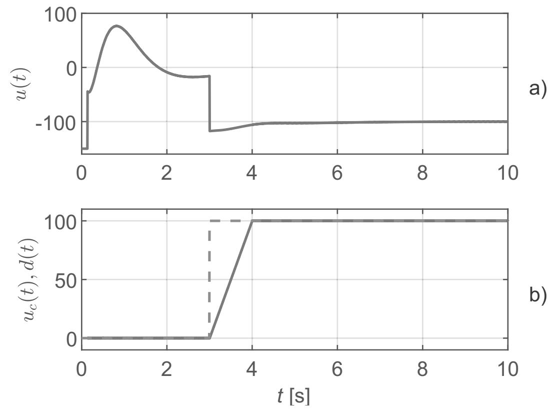

Fig. 2. Output of controller (21): the overall control signal (a), compensation control component u_c and disturbance d (b).

constant disturbances, such as in this case, it is shown that the system trajectory should reach the convergence boundary $|s_{\delta,k}| = k_{int}T^2/k_{s1}$. This claim is confirmed by the zoomed details of the sliding variable, given in Fig. 4, where the dashed lines denote the convergence boundaries and the circles represent the sliding variable in discrete-time instants. It is evident that for all the circles in the convergence area, the consequent system trajectory is directed toward the sliding manifold. This property does not exist in the space between the two dashed lines, which confirms the correctness of Proposition 2.

In constructing the proposed control structure, one of the goals was to eliminate the main shortcoming of the STC algorithm (the appearance of an overshoot), while reducing the chattering. Therefore, in the next simulation test, a comparison of the performances of the proposed and the discretized ST algorithm is performed. Parameters of the discretized ST controller (20) are set as $k_p = 100$

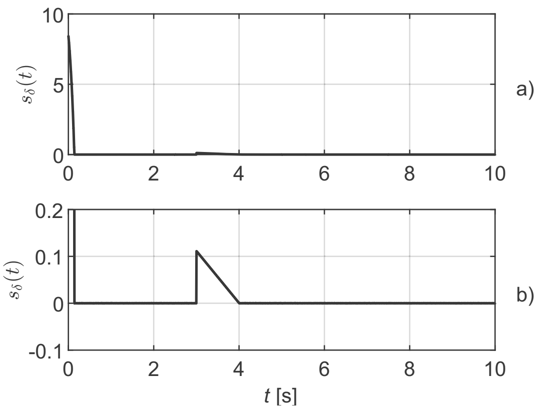

Fig. 3. Sliding variable (enlarged scale in (b)).

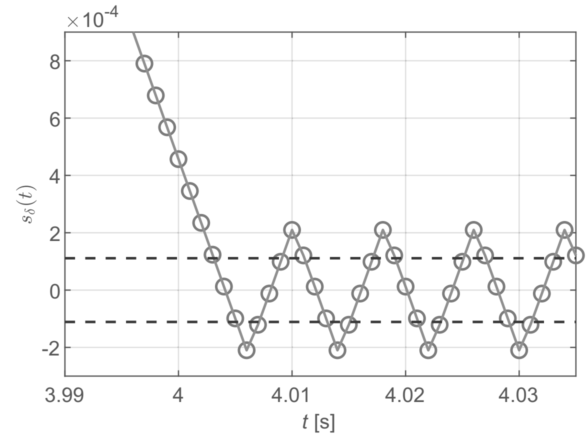

Fig. 4. Sliding variable (zoomed details).

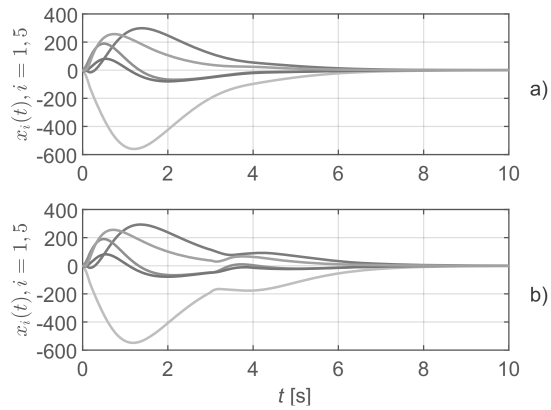

Fig. 5. State variables: the proposed controller (21) (a), the ST controller (20) (b).

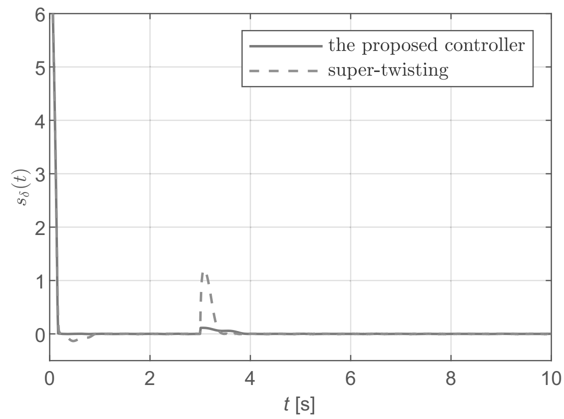

Fig. 6. Sliding variables: solid line—the proposed controller (21), dashed line—the ST controller (20).

and $k_i = 200$. Since there are no specific methods of parameter tuning of the ST controller, these values were adopted by the criterion to obtain dynamics as close as possible to the sliding manifold reaching ones compared with the proposed controller. Now, the systems are subjected to a more complex disturbance, consisting of sinusoidal and constant parts, i.e., $d(t) = 10\sin(4\pi t) + 100h(t-3)$. The other conditions and settings in the simulation are unchanged from the previous case.

Figure 5 shows evolution of the state variables of the both controllers. The proposed controller provides the desired dynamics, while a deviation can be observed in the response of the STC. The reason for such behaviour can be identified according to the sliding variables, which are presented in Fig. 6. It is obvious that the STC produces an overshoot in reaching the sliding manifold and is more sensitive to abrupt changes in disturbance, which occurred in $t = 3$ s. The performance of the proposed controller is consistent with the previous case.

Both controllers give similar control signals, presented in Fig. 7, although a more pronounced chattering can be observed in the ST controller output. This advantage of the proposed controller is more visible in the enlarged plot of the sliding variables in Fig. 8. The proposed controller creates a narrower quasi-sliding domain around the sliding manifold, and the chattering is hardly visible. The width of the resulting quasi-sliding domain is greater than the convergence boundary, since the external disturbance contains the varying component.

5. Conclusion

Based on the conducted simulation tests, it can be concluded that the designed DT SMC system has completely satisfied the predefined design requirements. A fast response and good elimination of disturbances were achieved. The overshoot, observed in the response of the DT ST controller, was successfully avoided by the proposed controller. The integrator wind-up did not occur, either. Moreover, the proposed system induced a much

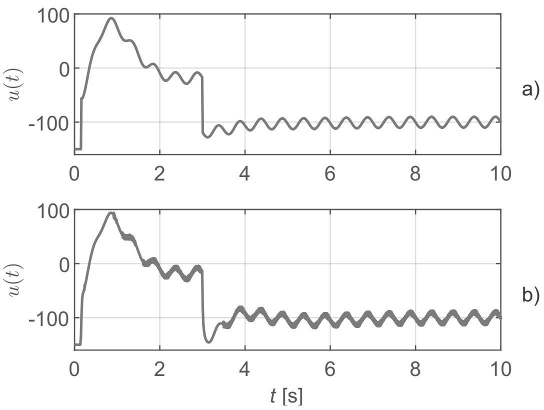

Fig. 7. Control signals: the proposed controller (21) (a), the ST controller (20) (b).

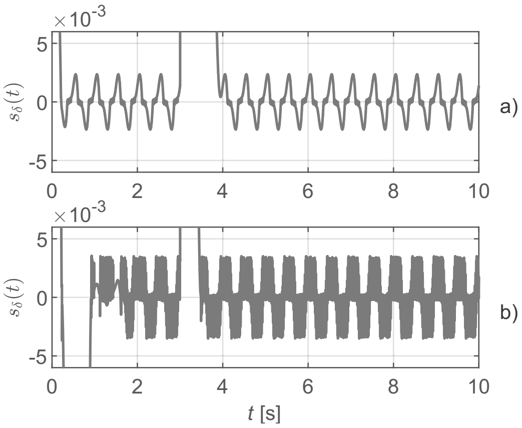

Fig. 8. Sliding variables (enlarged scale): the proposed controller (21) (a), the ST controller (20) (b).

smaller chattering than in the case of DT STC.

The proposed DT SM controller left the saturation mode in finite time as predicted. The theoretically proved stability and the obtained convergence bounds are confirmed by the presented simulation results, which speak in favor to the proposed controller.

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