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FRACTIONAL TIME–INVARIANT COMPARTMENTAL LINEAR SYSTEMS

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Fractional time-invariant compartmental linear systems are introduced. Controllability and observability of these systems are analyzed. The eigenvalue assignment problem of compartmental linear systems is considered and illustrated with a numerical example.

Keywords: compartmental system, fractional system, linear system, controllability, observability, eigenvalue assignment.

1. Introduction

In positive systems inputs, state variables and outputs take only nonnegative values. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear behavior can be found in engineering, management science, economics, social sciences, biology and medicine. An overview of the state of the art in positive systems theory is given by Farina and Rinaldi (2000) as well as Kaczorek (2002).

The eigenvalue and invariants assignment by state and output feedbacks was investigated by Busłowicz (2008; 2012) as well as Farina and Rinaldi (2000). Positive linear systems with various fractional orders were addressed by Kaczorek (2010; 2011a; 2011b), Kaczorek and Rogowski (2015), as well as Sajewski (2016). Selected problems in the theory of fractional linear systems were investigated in the or monographs by Kaczorek (2011a) as well as Kaczorek and Rogowski (2015). Stability of discrete-time fractional linear systems with delays was investigated by Ruszewski (2019).

In this paper fractional compartmental time-invariant linear systems will be introduced and analyzed.

In Section 2 the basic definitions and theorems concerning fractional and positive linear systems are recalled. The fractional compartmental linear systems are introduced in Section 3. Controllability and observability of fractional compartmental linear systems is analyzed in Section 4 and the eigenvalue assignment problem in Section 5. Concluding remarks are given in Section 6.

The following notation will be used: $\mathbb{R}$, the set of real numbers; $\mathbb{R}^{n\times m}$, the set of $n \times m$ real matrices; $\mathbb{C}$, the field of complex numbers; $\mathbb{Z}_+$, the set of nonnegative integers; $\mathbb{R}_+^{n\times m}$, the set of $n \times m$ matrices with nonnegative entries and $\mathbb{R}_+^n = \mathbb{R}_+^{n\times 1}$; M_n, the set of $n \times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries); I_n, the $n \times n$ identity matrix.

2. Preliminaries

Consider the fractional continuous-time linear system

$$\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = Ax + Bu, \quad 0 < \alpha < 1, \tag{1a}$$

$$y = Cx, \tag{1b}$$

where $x = x(t) \in \mathbb{R}^n$, $u = u(t) \in \mathbb{R}^m$, $y = y(t) \in \mathbb{R}^p$ are the state, input and output vectors, respectively, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and

$$\begin{aligned}\frac{\mathrm{d}^\alpha x(t)}{\mathrm{d}t^\alpha} &= \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{\dot{x}(\tau)}{(t-\tau)^\alpha}\mathrm{d}\tau,\\ \dot{x}(\tau) &= \frac{\mathrm{d}x(\tau)}{\mathrm{d}\tau}\end{aligned} \tag{1c}$$

is the Caputo fractional derivative and

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,\mathrm{d}t, \quad \mathbb{R}(z) > 0 \tag{1d}$$

is the gamma function (Kaczorek, 2011a; Kaczorek and Rogowski, 2015).

Definition 1. (*Kaczorek, 2011a; Kaczorek and Rogowski, 2015*) The fractional continuous-time linear system (1a) and (1b) is called (internally) *positive* if $x(t) \in \mathbb{R}^n_+$, $y(t) \in \mathbb{R}^p_+$, $t \geq 0$ for any initial conditions $x(0) \in \mathbb{R}^n_+$ and all inputs $u(t) \in \mathbb{R}^m_+$, $t \geq 0$.

Theorem 1. (Kaczorek, 2011; Kaczorek and Rogowski 2015) *The fractional continuous-time linear system (1a) and (1b) is positive if and only if*

$$A \in M_n, \quad B \in \mathbb{R}^{n\times m}_+, \quad C \in \mathbb{R}^{p\times n}_+. \tag{2}$$

Definition 2. (*Kaczorek, 2011a; Kaczorek and Rogowski, 2015*) The positive fractional continuous-time system (1a) and (1b) for $u(t) = 0$ is called *asymptotically stable* if

$$\lim_{t\to\infty} x(t) = 0 \quad \text{for any } x(0) \in \mathbb{R}^n_+. \tag{3}$$

Theorem 2. (Kaczorek, 2011; Kaczorek and Rogowski 2015) *The fractional positive continuous-time linear system (1a) and (1b) for $u(t) = 0$ is asymptotically stable if and only if one of the following equivalent conditions is satisfied:*

1. *All coefficients of the characteristic polynomial*

$$\begin{aligned} p_n(s) &= \det[I_n s - A] \\ &= s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0 \end{aligned} \tag{4}$$

are positive, i.e., $a_i > 0$ for $i = 0, 1, \dots, n-1$.

2. *There exists a strictly positive vector $\lambda^T = [\ \lambda_1 \ \cdots \ \lambda_n]^T$, $\lambda_k > 0$, $k = 1, \dots, n$ such that*

$$A\lambda < 0 \quad \text{or} \quad \lambda^T A < 0. \tag{5}$$

Theorem 3. *The fractional positive system (1a) and (1b) is asymptotically stable if the sum of the entries of each column (row) of the matrix A is negative.*

Proof. Using (5), we obtain

$$\begin{aligned} A\lambda &= \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix} \\ &= \begin{bmatrix} a_{11} \\ \vdots \\ a_{1n} \end{bmatrix} \lambda_1 + \dots + \begin{bmatrix} a_{n1} \\ \vdots \\ a_{nn} \end{bmatrix} \lambda_n < \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \end{aligned} \tag{6}$$

and the sum of the entries of each column of the matrix A is negative since $\lambda_k > 0$, $k = 1, \dots, n$. The proof for the rows is similar. ∎

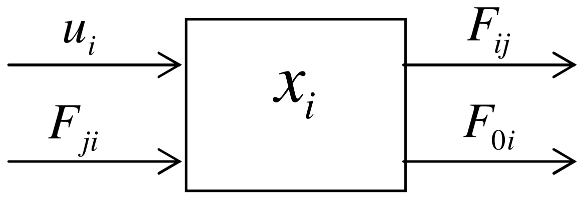

Fig. 1. Compartmental system: i-th subsystem.

3. State equations of fractional linear compartmental systems

Consider the compartmental continuous-time system consisting of n compartments (cf. Fig. 1). Let $x_i = x_i(t)$, $i = 1, \dots, n$ be the amount of a material of the *i*-th compartment, $F_{ij} \geq 0$ be the output flow of the material from the *j*-th to the *i*-th compartment ($i \neq j$), $F_{0i} \geq 0$ be the output of the material from the *i*-th compartment to the environment $u_i = u_i(t)$, be the input flow of the material from environment to the *i*-th compartment. It is assumed that the input material is mixed immediately with the material being in the compartment.

From the material balance of the *i*-th compartment we have the following fractional differential equations:

$$\frac{\mathrm{d}^\alpha x(t)}{\mathrm{d}t^\alpha} = \sum_{\substack{j=1\\ j\neq i}}^{n} (F_{ij} - F_{ji}) - F_{0i} + u_j \tag{7}$$

for $i = 1, \dots, n$, where $\mathrm{d}^\alpha x(t)/\mathrm{d}t^\alpha$ is the fractional derivative of x_i of order α.

It is assumed that the flow F_{ij} depends linearly on x_j, i.e.,

$$F_{ij} = f_{ij}x_j \quad \text{for } i \neq j; \quad i,j = 1, \dots, n, \tag{8}$$

where f_{ij} is a coefficient depending (in a general case) on x_j and the time instant t.

The compartmental system is linear if f_{ij} is independent of x_j and it is additionally time-invariant if f_{ij} is independent of t.

From (7) and (8) for $i = 1, \dots, n$ we obtain the state equation of the linear compartmental system in the form

$$\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = Fx + Bu, \tag{9}$$

where

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n, \quad u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} \in \mathbb{R}^n,$$

$$F = \begin{bmatrix} f_{11} & \dots & f_{1n} \\ \vdots & \dots & \vdots \\ f_{n1} & \dots & f_{nn} \end{bmatrix} \in \mathbb{R}_+^{n\times n}, \quad B = [I_n] \in \mathbb{R}_+^{n\times n},$$

$$-f_{jj} \geq \sum_{\substack{j=1 \\ j\neq i}}^{n} f_{ij} \geq f_{ij} \geq 0$$

$$\text{for } i \neq j, l, j = 1, \dots, n.$$

Note the following:

1. at each time instant the output flow of a compartment cannot be greater than the whole mass of material inside the compartment;
2. the sum of the entries of every column of the matrix F is not positive;
3. the matrix F is a particular case of the Metzler matrix, $F \in M_n$, since $f_{ij} \geq 0$ for $i \neq j$ and $i, j = 1, \dots, n$.

Note that, if $F_{0i} > 0$,$i = 1, \dots, n$, then from (9) it follows that the sum of entries of every column of the matrix F is negative and by Theorem 3 the fractional compartmental linear system is asymptotically stable. Therefore, the following result has been demonstrated.

Theorem 4. *The fractional compartmental linear system (9) with $F_{0i} > 0$, $i = 1, \dots, n$, is asymptotically stable.*

The output equation of the compartmental system has the form

$$y = Cx, \quad C \in \mathbb{R}^{p\times n}. \tag{10}$$

From (9) and (10) it follows that the compartmental system is a particular case of the positive fractional continuous-time linear system.

4. Controllability and observability of standard and compartmental linear systems

Consider the linear system described by the equations

$$\dot{x} = Ax + Bu, \tag{11a}$$

$$y = Cx, \tag{11b}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ are the state, input and output vectors, respectively, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$.

Definition 3. The linear system (11a) and (11b) (or the pair $(A,\ B)$) is called *controllable* in time $[0,\ t_1]$ if there exists an input $u(t)$ for $t \in [0,\ t_1]$ which steers the state of the system from any initial state $x(0) \in \mathbb{R}^n$ to any given final state x_f, i.e., $x(t_1) = x_f$.

Theorem 5. (Kaczorek, 2011; Kaczorek and Rogowski, 2015) *The linear system (11a) and (11b) is controllable if and only if one of the following conditions is satisfied:*

$$\operatorname{rank}[\ B \quad AB \quad \dots \quad A^{n-1}B\ ] = n \tag{12}$$

or

$$\operatorname{rank}[\ I_n s - A \quad B\ ] = n \tag{13}$$

for all $s \in \mathbb{C}$.

Definition 4. (*Kaczorek, 2011a; Kaczorek and Rogowski, 2015*) The linear system (11a) and (11b) (or the pair $(A,\ C)$) is called *observable* if it possible to find a unique initial value $x(0)$ knowing the output $y(t)$ and input $u(t)$ of the system.

Theorem 6. *The linear system (11a) and (11b) is observable if and only if one of the following conditions is satisfied:*

$$\operatorname{rank} \begin{bmatrix} \mathrm{C} \\ \mathrm{CA} \\ \vdots \\ CA^{n-1} \end{bmatrix} = n, \tag{14}$$

$$\operatorname{rank} \begin{bmatrix} I_n s - A \\ C \end{bmatrix} = n, \tag{15}$$

for all $s \in \mathbb{C}$.

Now let us consider the fractional compartmental linear system (1a) and (1b).

Definition 5. (*Kaczorek, 2011a; Kaczorek and Rogowski, 2015*) The fractional compartmental linear system (1a) and (1b) (or the pair $(A,\ B)$) is called *reachable* in time $[0,\ t_f]$ if the exists an input $u(t)$ for $t \in [0,\ t_f]$ which steers the state of the system from zero initial conditions to the given final state x_f, i.e., $x(\ t_f) = x_f$.

A matrix $A \in \mathbb{R}^{n\times n}$ is called *monomial* if in each its row and in each its column only one entry is positive and the remaining entries are zero.

Theorem 7. *The fractional compartmental linear system (1a) and (1b) is reachable if the matrix*

$$R_f = \int_0^{t_f} e^{A\tau} e^{A^T\tau} \mathrm{d}\tau \tag{16}$$

is monomial. The input which steers the state of the system to $x_f = x(t_f)$ is given by

$$u(t) = e^{A^T(t_f - t)} R_f^{-1} x_f \in \mathbb{R}_+^n, \quad 0 \leq t \leq t_f. \tag{17}$$

Proof. If the matrix (16) is monomial then its inverse matrix $R_f^{-1} \in \mathbb{R}_+^{n\times n}$and the input (17) is nonnegative.

Taking into account that $x(0) = 0$ and using (17), we obtain

$$\begin{aligned} x(t_f) &= \int_0^{t_f} e^{A(t_f-t)} Bu(\tau)\,\mathrm{d}\tau \\ &= \int_0^{t_f} e^{A(t_f-\tau)} e^{A^T(t_f-\tau)}\,\mathrm{d}\tau\; R_f^{-1} x_f = x_f \end{aligned} \tag{18}$$

since $B = I_n$.

Therefore, the input (17) steers the state of the system from $x(0)$ to $x_f = x(\,t_f)$. ■

Theorem 8. *The fractional compartmental positive linear system (1a) and (1b) is reachable in time* $[0,\ t_f]$ *if and only if the matrix* $A \in M_n$ *is monomial.*

Proof.
(Sufficiency) If $A \in M_n$is monomial then $e^{At} \in \mathbb{R}_+^{n\times n}$is also monomial. In this case the matrix

$$R_f = \int_0^{t_f} e^{A(t_f-\tau)} e^{A^T(t_f-\tau)}\,\mathrm{d}\tau = \int_0^{t_f} e^{A\tau} e^{A^T\tau}\,\mathrm{d}\tau \tag{19}$$

is monomial.

(Necessity) From the Cayley–Hamilton theorem (Kaczorek, 2011a; Kaczorek and Rogowski, 2015) we have

$$e^{A\tau} = \sum_{i=0}^{n-1} c_i(t) A^i, \tag{20}$$

where $c_i(t), i = 0, 1, \dots, n-1$ are some nonzero functions of time depending on the matrix A.

Using (20) we obtain

$$x_f = \begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix} \begin{bmatrix} v_0(t_f) \\ v_1(t_f) \\ \vdots \\ v_{n-1}(t_f) \end{bmatrix}, \tag{21a}$$

where

$$v_i(t_f) = \int_0^{t_f} c_i(\tau) u(t_f-\tau)\,\mathrm{d}\tau, \quad i = 0, 1, \dots, n-1. \tag{21b}$$

Therefore, for given $x_f \in \mathbb{R}_+^n$ it is possible to find a nonnegative $v_i(t_f)$ for $i = 0,\ 1, \dots, n-1$ if and only if

$$\operatorname{rank}[\ B \quad AB \quad \dots \quad A^{n-1}B\ ] = n. \tag{22}$$

Note that for the nonnegative (21b) it is possible to find a nonnegative $u(t) \in \mathbb{R}_+^m$. This completes the proof. ■

Observability of fractional positive compartmental linear systems is defined in a similar way as for standard positive liner systems. It depends only on the matrices A and C (it is independent of the matrix B). Therefore, instead of system (1), we shall consider the fractional positive compartmental linear system

$$\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = Ax, \quad 0 < \alpha < 1, \tag{23a}$$

$$y = Cx \tag{23b}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$ and $A \in M_n$, $C \in \mathbb{R}_+^{p\times n}$.

The solution to (23a) has the form

$$x(t) = \Phi_0(t) x_0, \tag{24a}$$

where

$$\Phi_0(t) = \sum_{k=0}^{\infty} \frac{A^k t^{k\alpha}}{\Gamma(k\alpha+1)} \tag{24b}$$

is a Mittag-Leffler matrix (Kaczorek, 2011a; Kaczorek and Rogowski, 2015).

Definition 6. The fractional positive compartmental linear system (23) is called *observable* on the interval $(0,\ t_f)$ if knowing the output $y(t)$ on the interval $[0,\ t_f]$ it is possible to find (compute) uniquely the initial condition $x_0 = x(0)$.

Theorem 9. *The fractional positive compartmental linear system (23) is observable on the interval* $(0,\ t_f)$ *if and only if the matrix*

$$\Phi_0{}^T(t) C^T C \Phi_0(t) \in \mathbb{R}_+^{n\times n} \tag{25}$$

is monomial.

Proof. Substituting (24a) into (23b), we obtain

$$y(t) = C\Phi_0(t) x_0. \tag{26}$$

Note that $[\Phi_0{}^T(t) C^T C \Phi_0(t)]^{-1} \in \mathbb{R}_+^{n\times n}$ if and only if the matrix (25) is monomial. In this case, from (26) we obtain

$$x_0 = [\Phi_0{}^T(t) C^T C \Phi_0(t)]^{-1} \Phi_0{}^T(t) C^T y(t) \in \mathbb{R}_+^n \tag{27}$$

since $\Phi_0{}^T(t) C^T y(t) \in \mathbb{R}_+^{n\times n}$ for $y(t) \in \mathbb{R}_+^p$. ■

5. Eigenvalue assignment in the fractional compartmental linear systems

Consider the fractional compartmental system (9a) with the state feedback

$$u = Kx, \tag{28}$$

where $K \in \mathbb{R}^{n\times n}$.

Taking into account that $B = I_n$ and using (28), we obtain

$$\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = A_c x, \tag{29}$$

where

$$A_c = A - K. \tag{30}$$

For a given matrix A and the desired closed-loop matrix A_c from (30) we obtain

$$K = A - A_c. \tag{31}$$

Therefore, we have the following result.

Theorem 10. *For the fractional compartmental system (9a) there always exists a state feedback (28) such that the matrix A_c of the closed-loop system has the set of desired eigenvalues.*

Example 1. The system matrix A of the fractional compartmental linear system has the form

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix} \tag{32}$$

and its eigenvalues are $s_1 = s_2 = -1$, $s_3 = -2$, since

$$\begin{aligned} \det[I_3 s - A] &= \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 2 & 5 & s+4 \end{vmatrix} \\ &= s^3 + 4s^2 + 5s + 2 \\ &= (s+1)^2(s+2). \end{aligned} \tag{33}$$

Compute the feedback matrix $K \in \mathbb{R}^{3\times 3}$ such that the eigenvalues of the closed-loop system matrix A are $\bar{s}_1 = -3$, $\bar{s}_2 = -4$, $\bar{s}_3 = -5$.

Note that the desired matrix A_c can be chosen in different forms.

Case 1. The matrix A_c has the same Frobenius canonical form as the matrix (32),

$$A_{c1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -60 & -47 & -12 \end{bmatrix} \tag{34}$$

since $(s+3)(s+4)(s+5) = s^3 + 12s^2 + 47s + 60$.

In this case, using (31)–(33), we obtain

$$\begin{aligned} K_1 &= A - A_{c1} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -60 & -47 & -12 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 58 & 42 & 8 \end{bmatrix}. \end{aligned} \tag{35}$$

Case 2. The matrix A_c has the diagonal form

$$A_{c2} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{bmatrix}. \tag{36}$$

In this case we have

$$\begin{aligned} K_2 &= A - A_{c2} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 1 & 0 \\ 0 & 4 & 1 \\ -2 & -5 & 1 \end{bmatrix}. \end{aligned} \tag{37}$$

Note that the above considerations can be extended to the output feedbacks. ♦

6. Concluding remarks

Fractional compartmental time-invariant linear systems have been analyzed. Basic definitions and theorems concerning fractional time-invariant standard and positive linear systems have been recalled. Fractional compartmental linear systems were introduced and analyzed in Section 3. Controllability and observability of standard and compartmental linear systems were considered in Section 4 and the eigenvalue assignment problem of compartmental linear systems in Section 5. Concluding remarks were given in Section 6. The considerations can be extended to the discrete-time fractional linear systems.

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