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DISTRIBUTED MODEL REFERENCE CONTROL FOR SYNCHRONIZATION OF A VEHICLE PLATOON WITH LIMITED OUTPUT INFORMATION AND SUBJECT TO PERIODICAL INTERMITTENT INFORMATION

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Vehicles involved in platoon formation may experience difficulties in obtaining full-state information that can be exchanged and used for controller synthesis. Therefore, a distributed controller based on a model reference and designed utilizing a cooperative observer is proposed for vehicle platoon synchronization. The proposed controller is composed of three main blocks, namely, the reference model, the cooperative observer and the main controller. The reference model is developed by using a homogeneous vehicle platoon that utilizes cooperative full-state information. The cooperative observer is a state estimator which is constructed based on the cooperative output estimation error. It provides state estimates to be used by the main controller. The main controller is constructed from a nominal control and a synchronization input. The nominal control has the main task of tracking the lead vehicle, while in order to reduce the synchronization error, the synchronization input is added by utilizing the cooperative disagreement error. Stability analysis is focused on the vehicle platoon when it is subjected to completely periodical intermittent information. The condition on the information rate is derived for guaranteeing the synchronization of the platoon. Numerical simulation of a vehicle platoon consisting of one leader and five followers is used to examine the performance of the controller.

Keywords: cooperative observer, distributed model reference control, intermittent information, vehicle platoon.

1. Introduction

With advances in autonomous vehicle technology, future vehicles will lead to connected and automated vehicles as a part of smart transportation systems. There are two keywords for the future vehicle technology, namely, "autonomous" and "connected." An autonomous vehicle can be defined as a vehicle that can move autonomously by taking every action without human intervention (Cavazza *et al.*, 2019). Connected vehicles mean that each vehicle will be equipped with features that allow it to exchange information with other vehicles, cyclists, bikers, pedestrians and road infrastructures. The information can be the position, the velocity or the acceleration of the vehicle; it can also be road traffic information such as the traffic density, the level of congestion, the velocity variation between vehicles (Yan *et al.*, 2013). With this collective information, intelligent road management can be established (Chang *et al.*, 2019) and new vehicle features can be established to increase safety, obtain information about emergency situations and weather conditions, improve environmental quality and increase mobility in driving. The driving mobility can be improved by collaboration between vehicles, one of which is known as the vehicle platoon.

A vehicle platoon consists of several automated vehicles (1-leader and N-followers) that agree to convoy

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together by keeping the distance between vehicles according to the agreement, all followers being able to follow the designated leader vehicle by utilizing the information exchanged between the vehicles. In other words, in a vehicle platoon formation, all follower vehicles will synchronize their inter-vehicle distance, velocity and acceleration to the lead vehicle. The information flow between the vehicles can be realized by using a directed or an undirected topology. Many types of directed topologies have been used for vehicle platoon application such as predecessor following (PF) (Wang *et al.*, 2017), predecessor- leader following (PLF) (Di Bernardo *et al.*, 2014), two-predecessors-following (TPF) (Jond and Yıldız, 2022) and two-predecessor leader following (TPLF) (Zheng *et al.*, 2016). Meanwhile, undirected topologies that are usually employed in vehicle platoon applications are either bidirectional (BD) or bidirectional leader (BDL) as used by Yan *et al.* (2018) and Zheng *et al.* (2015), respectively.

Inter-vehicular distance can be realized by using constant spacing policy (Zheng *et al.*, 2016), constant time heading policy (Abou Harfouch *et al.*, 2017), delay-based spacing policy (Besselink and Johansson, 2017) or nonlinear distance policy (Wijnbergen *et al.*, 2021). Vehicle dynamics can be expressed a through single or double integrator model (Li *et al.*, 2017) or a linearized third-order model as used by Hu *et al.* (2020). A linearized third-order model can better represent the actual dynamics, namely, by representing the time constant of the powertrain dynamics (Li *et al.*, 2017).

The vehicle platoon is one of the examples of the leader-follower problem of multi-agent systems (MASs). Thus, many controllers designed for leader-follower MAS can be implemented for the vehicle platoon, such as distributed cooperative state variable feedback control, model predictive control and slide-mode control as proposed by Zhang *et al.* (2011), Franzè *et al.* (2018) and Long *et al.* (2014), respectively. The controller presented by Hamdi *et al.* (2021) and Kukurowski *et al.* (2022) also has the possibility to be implemented in a vehicle platoon. The development of a distributed controller for a vehicle platoon has also been carried out either specifically for a certain topology or for more general topologies. Abou Harfouch *et al.* (2017) proposed a distributed controller based on model reference adaptive control for a vehicle platoon subjected to uncertainty under PF topology, while Di Bernardo *et al.* (2014) developed a distributed consensus strategy of a vehicle platoon with PFL topology by considering the communication delay. For more general topologies containing a spanning tree, Prayitno and Nilkhamhang (2021) developed a distributed controller based on model reference adaptive control with the aim of reducing the effect of uncertainty on the vehicle follower. Moreover, Prayitno and Nilkhamhang (2022) developed a distributed controller based on model reference for each follower with the aim of synchronizing a vehicle platoon in the case of the leader moves with time-varying velocity. However, the above control schemes (Abou Harfouch *et al.*, 2017; Di Bernardo *et al.*, 2014; Prayitno and Nilkhamhang, 2021; 2022) are designed with the assumption of a continuous information flow from the connected neighbors.

In practice, a continuous information flow between vehicles is difficult to maintain due to sensor limitations, possible sensor and communication device failures, obstructions or packet losses (Xie and Lin, 2020; Liu *et al.*, 2020). Therefore, the information flow between vehicles will experience intermittent characteristics. Intermittent information from the connected neighbors in a vehicle platoon application under directed topology has been numerically observed by Prayitno and Nilkhamhang (2022), with the result that synchronization can still be achieved during an intermittent communication loss as long as the spanning tree condition with the leader acting as a root tree is still fulfilled. However, Prayitno and Nilkhamhang (2022) did not provide the stability analysis of the system under intermittent conditions. The stability analysis of the platoon when the conditions of the spanning tree are met intermittently can be made by assuming that the information flow between vehicles is completely periodic intermittent, i.e., the information flow is periodically ON and OFF over time (Huang *et al.*, 2014). This assumption is used in this study.

There are many works discussing the leader-follower consensus of MASs subjected to intermittent information that can be implemented in the vehicle platoon. Xu *et al.* (2019) designed a distributed controller for leader-follower in which the absence of position information on the followers was circumvented by transmitting the leader's information intermittently to each follower consecutively. In the work of Xie and Lin (2020), a global leader-follower consensus of MASs subjected to actuator saturation is proposed by utilizing intermittent information from the connected neighbors. Meanwhile, Huang *et al.* (2014) and Liu *et al.* (2020) proposed a consensus control algorithm for a second-order nonlinear system based on intermittent neighbors' state information. The research of Huang *et al.* (2014) is further extended (Huang *et al.*, 2015) in order to deal with relatively delayed and periodical intermittent neighbors' information. Jiang *et al.* (2018) proposed an optimal-based controller for the consensus of a leader-following MAS. Xu *et al.* (2021) proposed an adaptive intermittent output consensus for a leader-following linear MAS. However, the above works (Huang *et al.*, 2014; 2015; Xu *et al.*, 2019; 2020; Wang *et al.*, 2019; Jiang *et al.*, 2018) are designed based on the assumptions that the leader has zero input (moves at a constant velocity) and each follower has full-state information.

The assumption of the leader moving at a constant velocity is not practical for vehicle platoon since in reality the lead vehicle should move with a varying velocity, depending on the traffic conditions. Moreover, the assumption of the existence of full-state information may turn out difficult for a follower which has limited output information. A cooperative observer based on neighborhood output estimation error was proposed by Zhang *et al*. (2011) to acquire full-state estimates of the follower. Then, synchronization control is designed by using the cooperative state estimation tracking error to reduce the tracking error to the leader. Xu *et al*. (2020) investigated consensus of a linear MAS that utilized estimated disturbance and local relative information under intermittent communication. However, again, Zhang *et al*. (2011) and Xu *et al*. (2020) assumed that the leader has zero input.

To sum up, three main issues will be investigated in this paper, namely, guaranteeing stability of the platoon when (i) the followers have limited output information, (ii) the leader moves with a time-varying velocity, and (iii) the vehicles are subjected to intermittent information. Consequently, this paper proposed a distributed model reference controller designed by utilizing a cooperative observer for synchronization of the vehicle platoon, where the followers have limited output information and the leader moves with a time-varying velocity. The limitation of the output information is solved by a cooperative observer which produces full-estimated state information which will later be used in controller design. To anticipate the possibility of the leader moving with a time-varying velocity, a reference model is used, where the resulting cooperative tracking error will be employed as a virtual reference which will be followed by the actual tracking error. By using this virtual tracking error reference, every movement of the leader will be translated into a tracking error which will then be relayed quickly to all vehicles so that the vehicles have the ability to anticipate faster. Moreover, the stability analysis of the platoon will be evaluated under completely periodic intermittent information conditions.

The challenge of this paper is to derive the condition of the information rate such that the followers can still achieve synchronization to the leader, when the platoon subjected to completely periodical intermittent information. This work focuses on the vehicle platoon with a linearized third-order model, constant spacing policy and directed topology. Constant spacing policy is used in this study since it is velocity independent and it has an advantage in terms of its ability to maximize the road capacity. Moreover, small constant spacing is proven to be able to save fuel significantly in vehicle platoons of heavy-duty trucks (Ozkan and Ma, 2021). However, the constant spacing policy has a weakness in terms of safety, especially when the leader makes a sudden brake

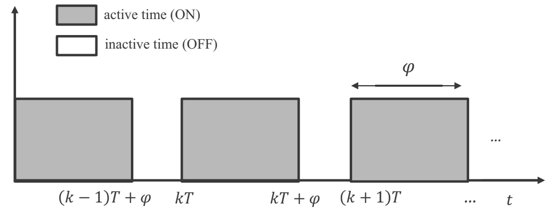

Fig. 1. Illustration of periodically intermittent information.

and the followers do not have direct information from the leader, which may cause a collision with a predecessor vehicle. Fortunately, this problem will also be considered in the proposed controller through the virtual cooperative tracking error concept as described above.

The main contributions of this research can be explained as follows:

- The proposed controller can be implemented in a vehicle platoon, even if the follower has limited output information, which produces satisfactory synchronization performance. Compared with DMRC (Prayitno and Nilkhamhang, 2022) which was designed using full-state information, the performance is relatively equivalent.
- A condition on the information rate is established to ensure the synchronization of the platoon, as claimed in the main result;
- Compared with the results of Zhang *et al*. (2011), the proposed control considers the possibility of leader moves with a time-varying velocity and vehicles subjected to completely periodical intermittent information. The stability of the platoon is achieved as long as a condition on the information rate is satisfied.

The structure of this paper is as follows. The next section provides the problem formulation and then proceeds with the control design along with a detailed explanation. Then, the main result of this research will be presented which is supported by a stability analysis. In the final section, numerical simulations and analysis will be given to verify the performance of the proposed controller, followed by conclusions.

2. Problem formulation

Vehicle dynamics of the platoon as used by Zheng *et al*. (2019) are adopted for a homogeneous vehicle platoon and expressed as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i \in \{0, 1, \ldots, N\}, \tag{1}$$

$$y_i(t) = Cx_i(t), \tag{2}$$

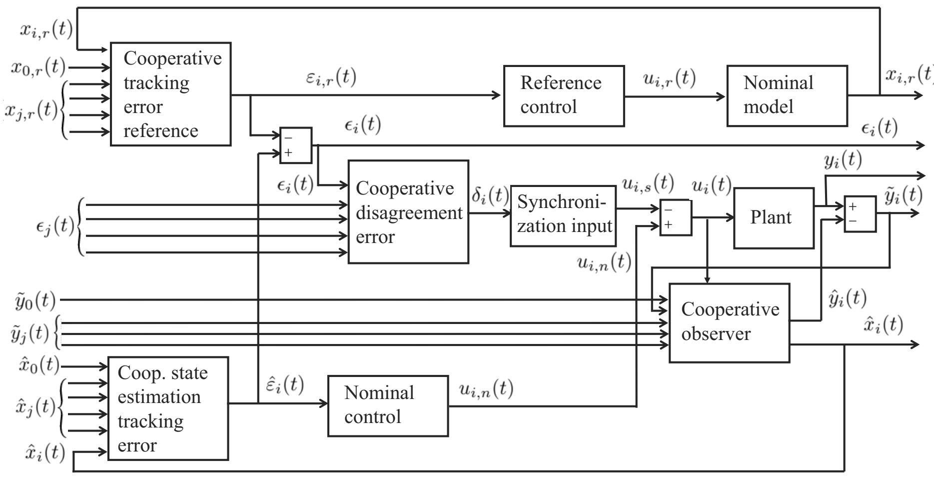

Fig. 2. DMRC-CO block diagram.

where index $i = 0$ signifies the lead vehicle and $i \in \{1, 2, \dots, N\}$ represent the followers. Here, x_i, y_i, u_i signify the state, output, and input of the i-th vehicle respectively. System and control input matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix}, \tag{3}$$

while C is the output matrix. Here, τ is time constant of the powertrain dynamics. In this paper, $x_i = [p_i + i \cdot d_r \quad v_i \quad a_i]^T$, where p_i, v_i, a_i and d_r are the position, velocity, acceleration and desired constant spacing, respectively.

The vehicle-to-vehicle information flow in the platoon is explained by a graph, $\mathcal{G}(\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{n_1, n_2, \dots, n_N\}$ is the set of followers, and $\mathcal{E}$ is a set of information links between the vehicle followers. Denote by $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ the adjacency matrix, where $a_{ij} = 1$ means that information is received by follower i from follower j, otherwise $a_{ij} = 0$. Denote by $\mathcal{B} = \operatorname{diag}\{b_{11}, b_{22}, \dots, b_{NN}\}$ the degree matrix, where $b_{ii} = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix can be computed by $\mathcal{L} = [\ell_{ij}] = \mathcal{B} - \mathcal{A} \in \mathbb{R}^{N \times N}$. A pinning gain matrix, $G = \operatorname{diag}\{g_{11}, g_{22}, \dots, g_{NN}\}$, is used to show information from the leader that can be received directly by the followers. The value of $g_{ii} = 1$ signifies that follower i receives direct information from the lead vehicle. An augmented graph $\tilde{\mathcal{G}}(\tilde{\mathcal{N}}, \tilde{\mathcal{E}})$ is defined as $\tilde{\mathcal{N}}$ which includes the leader as a node and $\tilde{\mathcal{E}} \subseteq \tilde{\mathcal{N}} \times \tilde{\mathcal{N}}$. In order to represent the complete information flow in the platoon, matrix $\mathcal{H} = \mathcal{L} + G$ is used.

Assumption 1. (*Zhang and Lewis, 2012*) The vehicle-to-vehicle communication graph $\tilde{\mathcal{G}}$ is directed and has a spanning tree with the lead vehicle acting as a root node.

Lemma 1. (Zhang and Lewis, 2012; Ou, 2009) *Based on Assumption 1, matrix $\mathcal{H}$ is a nonsingular M-matrix. Then*

$$\begin{aligned} \mathcal{F} &= [f_1, f_2, \cdots, f_N]^T = \mathcal{H}^{-1}\underline{1}, \\ \Pi &= \operatorname{diag}\left\{\frac{1}{f_1}, \frac{1}{f_2}, \cdots, \frac{1}{f_N}\right\}, \\ \Lambda &= \Pi\mathcal{H} + \mathcal{H}^T\Pi, \end{aligned} \tag{4}$$

Here $\underline{1} = \operatorname{col}(1, 1, \dots, 1) \in \mathbb{R}^N$, $\Pi > 0$ *and* $\Lambda > 0$.

Completely periodical intermittent information means that the information signal is periodically ON and OFF are time (Huang *et al.*, 2014), as illustrated in Fig. 1. The time span over which information is ON, is called "active time," and when information is OFF, it is called "inactive time." Each period is composed of "active time" which regards time intervals $[kT, kT + \varphi]$, $k \in \mathbb{N}$ and "inactive time" which pertains to time intervals $[kT + \varphi, (k + 1)T]$, $k \in \mathbb{N}$. Here $T > 0$ is the information period and φ is the minimum information length over one period. The information rate is defined as φ/T.

The aim of this study is to design a distributed model reference controller based on a cooperative observer for each follower with limited-output information and to define a condition on the information rate such that each follower still achieves synchronization with the leader state with a bounded error, when the platoon is subjected to completely periodical intermittent information.

3. Distributed model reference controller based on a cooperative observer

3.1. Overview. The proposed control uses the distributed model reference control (DMRC) structure of Prayitno and Nilkhamhang (2022) that is modified to be designed based on limited-output information using a cooperative observer and to be analyzed under the assumption of completely periodical intermittency. The modified controller is called a distributed model reference control based on the cooperative observer (DMRC-CO). The appropriate block diagram is shown in Fig. 2. It consists of three main blocks, namely, a virtual reference, a cooperative observer and a main controller.

These three blocks have different purposes for solving the problems that have been identified in the Section 1. The virtual reference is designed for a general topology which has the capability to be a reference for all followers, i.e., to obtain information about the time-varying movements of the leader, even when the follower is not directly connected to the leader. This can be achieved by utilizing the cooperative tracking error of the reference model, and that is why it is selected as a virtual reference. The reference model is constructed by using cooperative state variable feedback control (CSVFB) of a homogeneous vehicle platoon and under the assumption that the leader moves at a constant velocity, as proposed by Zhang *et al.* (2011). This controller has a guarantee that the cooperative tracking error goes to zero as time goes to infinity.

In the block diagram, the virtual reference is denoted by $\varepsilon_{i,r}(t)$. The cooperative observer is designed as an estimator of the follower's full state, $\hat{x}_i(t)$, which will later be used for controller synthesis. The observer utilizes both the internal and neighbors' output estimation error, $\{\tilde{y}_i(t), \tilde{y}_0(t), \tilde{y}_j(t)\}$. Meanwhile, the main controller consists of a nominal control, $u_{i,n}(t)$, and a synchronous input, $u_{i,s}(t)$. The nominal control is responsible for tracking the leader's state by utilizing the cooperative state estimation tracking error, $\hat{\varepsilon}_i(t)$, designed using the collective information from the internal state estimation error, $\hat{x}_i(t)$, and the connected neighbors' state estimation error, $\{\hat{x}_0(t), \hat{x}_j(t)\}$.

The synchronization input is designed to reduce the disagreement error, $\epsilon_j(t)$, which is the error between the cooperative tracking error reference, $\varepsilon_{i,r}(t)$, and the cooperative state estimation tracking error, $\hat{\varepsilon}_i(t)$. The synchronization input itself utilizes the cooperative disagreement error, $\delta_i(t)$, which is designed using the collective information from the internal and connected neighbors' disagreement errors, $\{\epsilon_i(t), \epsilon_0(t), \epsilon_j(t)\}$.

3.2. Controller design.

3.2.1. Virtual reference. The reference model is a homogeneous vehicle platoon which applied CSVFB. The dynamics of the vehicles are represented by

$$\dot{x}_{i,r}(t) = Ax_{i,r}(t) + Bu_{i,r}(t), \quad i \in \{0, 1, \dots, N\}, \tag{5}$$

where $x_{i,r}(t)$ is the reference state of the i-th vehicle and $u_{i,r}(t)$ is the nominal control input for the i-th reference vehicle. For the reference model, it is assumed that $u_{0,r}(t) = 0$. By considering completely periodical intermittent information, the nominal control for each reference follower is designed as

$$u_{i,r}(t) = \begin{cases} c_1 K \varepsilon_{i,r}(t), & t \in [kT, kT + \varphi], \\ 0, & t \in [kT + \varphi, (k+1)T], \end{cases} \tag{6}$$

where $\varepsilon_{i,r}(t)$ is the cooperative tracking error that is used as a virtual reference, defined as

$$\varepsilon_{i,r}(t) = \sum_{j=1}^{N} a_{ij}(x_{j,r}(t) - x_{i,r}(t)) + g_{ii}(x_{0,r}(t) - x_{i,r}(t)). \tag{7}$$

Here, c_1 is a coupling gain and K is the feedback gain which can be calculated as

$$K = R^{-1} B^T P, \tag{8}$$

where P is a solution to

$$0 = A^T P + PA + Q - PBR^{-1}B^T P, \tag{9}$$

with $Q > 0$ and $R > 0$. Substituting (6) into (5) yields

$$\dot{x}_{i,r}(t) = \begin{cases} Ax_{i,r}(t) + c_1 BK \varepsilon_{i,r}(t), & t \in [kT, kT + \varphi], \\ Ax_{i,r}(t), & t \in [kT + \varphi, (k+1)T]. \end{cases} \tag{10}$$

Remark 1. The control signal in (6) has been shown to be able to make the tracking error approach zero as time goes to infinity; cf. Zhang *et al.* (2011). This implies that the virtual reference approaches zero as time approaches infinity. This is the reason behind choosing the control signal (6) as a nominal control in the reference model. The control signal during information inactivity (OFF), $t \in [kT + \varphi, (k+1)T]$, is zero since all the elements of the adjacency and pinning gain matrices are zero.

3.2.2. Cooperative observer. Assuming that the follower does not have full-state information, a distributed cooperative observer is designed in order to estimate the full-state information of the real follower by utilizing the available connected neighbors' output measurements. Let $\hat{x}_i(t)$ be an estimate of $x_i(t)$, and $\hat{y}_i(t) = C\hat{x}_i(t)$ be an estimate of $y_i(t)$. Then the state and output estimation errors can be expressed as $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ and $\tilde{y}_i(t) = y_i(t) - \hat{y}_i(t)$, respectively. Following Lewis *et al.* (2013), the cooperative output estimation error is defined as

$$\psi_i(t) = \sum_{j=1}^{N} a_{ij}(\tilde{y}_j(t) - \tilde{y}_i(t)) + g_{ii}(\tilde{y}_0(t) - \tilde{y}_i(t)). \tag{11}$$

The cooperative observer is defined as

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) - c_f F\psi_i(t), \tag{12}$$

where $c_f > 0$ and F are the scalar and the observer gain, respectively. F can defined as

$$F = PC^T R^{-1}, \tag{13}$$

where P is a solution to

$$0 = A^T P + PA + Q - PC^T R^{-1} CP, \tag{14}$$

with $Q > 0$ and $R > 0$.

3.2.3. Main controller.

The control signal in each follower vehicle is designed as

$$u_i(t) = \begin{cases} u_{i,n}(t) - u_{i,s}(t), & t \in [kT, kT + \varphi], \\ 0, & t \in [kT + \varphi, (k+1)T], \end{cases} \tag{15}$$

where

$$u_{i,n}(t) = c_1 K\hat{\varepsilon}_i(t), \tag{16}$$
$$u_{i,s}(t) = c_2 K\delta_i(t). \tag{17}$$

Here c_1 and c_2 are the coupling gains which will be determined later, K is the feedback gain as defined in (8), while $\hat{\varepsilon}_i(t)$ and $\delta_i(t)$ are the cooperative state estimation tracking error and the cooperative disagreement error, respectively

$$\hat{\varepsilon}_i(t) = \sum_{j=1}^{N} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + g_{ii}(\hat{x}_0(t) - \hat{x}_i(t)), \tag{18}$$

$$\delta_i(t) = \sum_{j=1}^{N} a_{ij}(\epsilon_j(t) - \epsilon_i(t)) - g_{ii}\epsilon_i(t). \tag{19}$$

Here $\epsilon_i(t)$ is the disagreement error defined as

$$\epsilon_i(t) = \hat{\varepsilon}_i(t) - \varepsilon_{i,r}(t). \tag{20}$$

The closed-loop system and observer of the i-th follower are as follows: For $t \in [kT, kT + \varphi]$

$$\dot{x}_i(t) = Ax_i(t) + c_1 BK\Big\{\sum_{j=1}^{N} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + g_{ii}(\hat{x}_0(t) - \hat{x}_i(t))\Big\} - c_2 BK\Big\{\sum_{j=1}^{N} a_{ij}(\epsilon_j(t) - \epsilon_i(t)) - g_{ii}\epsilon_i(t)\Big\}, \tag{21}$$

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + c_1 BK\Big\{\sum_{j=1}^{N} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + g_{ii}(\hat{x}_0(t) - \hat{x}_i(t))\Big\} - c_2 BK\Big\{\sum_{j=1}^{N} a_{ij}(\epsilon_j(t) - \epsilon_i(t)) - g_{ii}\epsilon_i(t)\Big\} - c_1 F\Big\{\sum_{j=1}^{N} a_{ij}(\tilde{y}_j(t) - \tilde{y}_0(t)) + g_{ii}(\tilde{y}_0(t) - \tilde{y}_i(t))\Big\}, \tag{22}$$

For $t \in [kT + \varphi, (k+1)T]$:

$$\dot{x}_i(t) = Ax_i(t), \tag{23}$$

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) - c_1 F\Big\{\sum_{j=1}^{N} a_{ij}(\tilde{y}_j(t) - \tilde{y}_0(t)) + g_{ii}(\tilde{y}_0(t) - \tilde{y}_i(t))\Big\}. \tag{24}$$

Remark 2. The vehicle followers send and receive information to the connected neighbors that consists of $x_{i,r}(t)$, $\hat{x}_i(t)$, $\epsilon_i(t)$ and $\tilde{y}_i(t)$, Fig. 2. The vehicle leader transmits $x_{0,r}(t)$, $\hat{x}_0(t)$, $\epsilon_0(t)$ and $\tilde{y}_0(t)$ to the connected vehicle followers. It is reasonable to assume that $x_0(t) = \hat{x}_0(t)$; therefore $\epsilon_0(t) = 0$ and $\tilde{y}_0(t) = 0$.

Remark 3. The novelty of the proposed control lies in the use of a cooperative observer as a provider of information used by the nominal control and the cooperative disagreement error for the synchronization input. Moreover, the stability analysis of the proposed control when the platoon is subjected to completely periodical intermittent information is presented in Section 4.

3.3. Global dynamics notation.

3.3.1. Global virtual reference.

The global dynamics of the reference model are

$$\dot{\underline{x}}_{0,r}(t) = (I_N \otimes A)\underline{x}_{0,r}(t), \tag{25}$$

where $\underline{x}_{0,r}(t) = [x_{0,r}(t), x_{0,r}(t), \dots, x_{0,r}(t)]^T$ and $\otimes$ represents the Kronecker product,

$$\begin{aligned}\dot{x}_r(t) &= (I_N \otimes A - c_1\mathcal{H} \otimes BK)x_r(t)\\ &\quad + (c_1\mathcal{H} \otimes BK)\underline{x}_{0,r}(t), \quad t \in [kT, kT+\varphi],\end{aligned} \tag{26}$$

and

$$\dot{x}_r(t) = (I_N \otimes A)x_r(t), \quad t \in [kT+\varphi, (k+1)T], \tag{27}$$

where $x_r(t) = [x_{1,r}(t), x_{2,r}(t), \dots, x_{N,r}(t)]^T$.

The global tracking error dynamics can be represented as

$$\begin{aligned}\dot{e}_r(t) &= (I_N \otimes A - c_1\mathcal{H} \otimes BK)e_r(t),\\ &\qquad t \in [kT, kT+\varphi],\end{aligned} \tag{28}$$

and

$$\dot{e}_r(t) = (I_N \otimes A)e_r(t), \quad t \in [kT+\varphi, (k+1)T], \tag{29}$$

where $e_r(t) = [e_{1,r}(t), e_{2,r}(t), \dots, e_{N,r}(t)]^T = x_r(t) - \underline{x}_{0,r}(t)$ is the global tracking error of the reference model.

The global virtual reference is $\varepsilon_r(t) = -(\mathcal{H} \otimes I_n)e_r(t)$, which results in dynamics

$$\begin{aligned}\dot{\varepsilon}_r(t) &= (I_N \otimes A - c_1\mathcal{H} \otimes BK)\varepsilon_r(t),\\ &\qquad t \in [kT, kT+\varphi],\end{aligned} \tag{30}$$

and

$$\dot{\varepsilon}_r(t) = (I_N \otimes A)\varepsilon_r(t), \quad t \in [kT+\varphi, (k+1)T], \tag{31}$$

where $\varepsilon_r(t) = [\varepsilon_{1,r}(t), \varepsilon_{2,r}(t), \dots, \varepsilon_{N,r}(t)]^T$.

3.3.2. Global observer and the main control system.

The global dynamics of the leader and the estimated leader can be represented as

$$\dot{\underline{x}}_0(t) = (I_N \otimes A)\underline{x}_0(t) + (I_N \otimes B)\underline{u}_0(t), \tag{32}$$

$$\dot{\underline{\hat{x}}}_0(t) = (I_N \otimes A)\underline{\hat{x}}_0(t) + (I_N \otimes B)\underline{u}_0(t), \tag{33}$$

respectively, where

$$\begin{aligned}\underline{x}_0(t) &= [x_0(t), x_0(t), \dots, x_0(t)]^T,\\ \underline{\hat{x}}_0(t) &= [\hat{x}_0(t), \hat{x}_0(t), \dots, \hat{x}_0(t)]^T,\\ \underline{u}_0(t) &= [u_0(t), u_0(t), \dots, u_0(t)]^T.\end{aligned}$$

The global closed-loop system can be described as

$$\begin{aligned}\dot{x}(t) &= (I_N \otimes A)x(t) - (c_1\mathcal{H} \otimes BK)\hat{x}(t)\\ &\quad + (c_1\mathcal{H} \otimes BK)\underline{\hat{x}}_0(t) - (c_2\mathcal{H}^2 \otimes BK)\hat{x}(t)\\ &\quad + (c_2\mathcal{H}^2 \otimes BK)\underline{\hat{x}}_0(t) + (c_2\mathcal{H}^2 \otimes BK)x_r(t)\\ &\quad - (c_2\mathcal{H}^2 \otimes BK)\underline{x}_{0,r}(t), \quad t \in [kT, kT+\varphi],\end{aligned} \tag{34}$$

and

$$\dot{x}(t) = (I_N \otimes A)x(t), \quad t \in [kT+\varphi, (k+1)T], \tag{35}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T$.

The global observer can be described as

$$\begin{aligned}\dot{\hat{x}}(t) &= (I_N \otimes A - c_1\mathcal{H} \otimes BK)\hat{x}(t)\\ &\quad + (c_1\mathcal{H} \otimes BK)\underline{\hat{x}}_0(t) - (c_2\mathcal{H}^2 \otimes BK)\hat{x}(t)\\ &\quad + (c_2\mathcal{H}^2 \otimes BK)\underline{\hat{x}}_0(t)\\ &\quad + (c_2\mathcal{H}^2 \otimes BK)x_r(t) - (c_2\mathcal{H}^2 \otimes BK)\underline{x}_{0,r}(t)\\ &\quad + [c_1\mathcal{H} \otimes F]y(t) - [c_1\mathcal{H} \otimes FC]\hat{x}(t),\\ &\qquad t \in [kT, kT+\varphi],\end{aligned} \tag{36}$$

and

$$\begin{aligned}\dot{\hat{x}}(t) &= (I_N \otimes A)\hat{x}(t) + [c_1\mathcal{H} \otimes F]y(t)\\ &\quad - [c_1\mathcal{H} \otimes FC]\hat{x}(t), \quad t \in [kT+\varphi, (k+1)T],\end{aligned} \tag{37}$$

where $\hat{x}(t) = [\hat{x}_1(t), \hat{x}_2(t), \dots, \hat{x}_N(t)]^T$, and $y(t) = [y_1(t), y_2(t), \dots, y_N(t)]^T$.

Denote by $\tilde{x} = [\tilde{x}_1(t), \tilde{x}_2(t), \cdots, \tilde{x}_N(t)]^T = x(t) - \hat{x}(t)$ the global state estimation error. Then the global dynamics of the state estimation error becomes

$$\dot{\tilde{x}}(t) = [(I_N \otimes A) - (c_1\mathcal{H} \otimes FC)]\tilde{x}(t), \tag{38}$$

for all time instants. According to Lemma 3.5 of Lewis *et al.* (2013), matrix $[(I_N \otimes A) - (c_1\mathcal{H} \otimes FC)]$ is Hurwitz. Therefore, the global tracking error dynamics of the estimated vehicle followers with respect to the vehicle leader can be expressed as

$$\begin{aligned}\dot{\hat{e}}(t) &= (I_N \otimes A - (c_1\mathcal{H} \otimes BK))\hat{e}(t)\\ &\quad - (c_2\mathcal{H}^2 \otimes BK)\hat{e}(t) + (c_2\mathcal{H}^2 \otimes BK)e_r(t)\\ &\quad + (c_1\mathcal{H} \otimes FC)\tilde{x}(t) - (I_N \otimes B)u_0(t),\\ &\qquad t \in [kT, kT+\varphi],\end{aligned} \tag{39}$$

and

$$\dot{\hat{e}}(t) = (I_N \otimes A)\hat{e}(t) + [c_1\mathcal{H} \otimes FC]\tilde{x}(t) - (I_N \otimes B)u_0(t), \quad t \in [kT, kT + \varphi], \tag{40}$$

where $\hat{e}(t) = [\hat{e}_1(t), \hat{e}_2(t), \dots, \hat{e}_N(t)]^T = \hat{x}(t) - \underline{\hat{x}}_0(t)$. The global cooperative state estimation tracking error is defined as $\hat{\varepsilon}(t) = -(\mathcal{H} \otimes I_n)\hat{e}(t)$, which yields the following dynamics:

$$\begin{aligned}\dot{\hat{\varepsilon}}(t) &= (I_N \otimes A - c_1\mathcal{H} \otimes BK)\hat{\varepsilon}(t) \\ &\quad - (c_2\mathcal{H}^2 \otimes BK)\hat{\varepsilon}(t) + (c_2\mathcal{H}^2 \otimes BK)\varepsilon_r(t) \\ &\quad - (c_1\mathcal{H}^2 \otimes FC)\tilde{x}(t) + (\mathcal{H} \otimes B)\underline{u}_0(t), \\ &\qquad t \in [kT, kT + \varphi],\end{aligned} \tag{41}$$

and

$$\dot{\hat{\varepsilon}}(t) = (I_N \otimes A)\hat{\varepsilon}(t) - (c_1\mathcal{H}^2 \otimes FC)\tilde{x}(t) + (\mathcal{H} \otimes B)\underline{u}_0(t), \quad t \in [kT + \varphi, (k+1)T], \tag{42}$$

where $\hat{\varepsilon}(t) = [\hat{\varepsilon}_1(t), \hat{\varepsilon}_2(t), \dots, \hat{\varepsilon}_N(t)]^T$.

Similarly, denote by

$$\begin{aligned}\epsilon(t) &= [\epsilon_1(t), \epsilon_2(t), \cdots, \epsilon_N(t)]^T \\ &= \hat{\varepsilon}(t) - \varepsilon_r(t),\end{aligned}$$

the global disagreement error. Then

$$\begin{aligned}\dot{\epsilon}(t) &= (I_N \otimes A - c_1\mathcal{H} \otimes BK)\epsilon(t) \\ &\quad - (c_2\mathcal{H}^2 \otimes BK)\epsilon(t) - (c_1\mathcal{H}^2 \otimes FC)\tilde{x}(t) \\ &\quad + (\mathcal{H} \otimes B)\underline{u}_0(t), \qquad t \in [kT, kT + \varphi],\end{aligned} \tag{43}$$

and

$$\dot{\epsilon}(t) = (I_N \otimes A)\hat{\varepsilon}(t) - (c_1\mathcal{H}^2 \otimes FC)\tilde{x}(t) + (\mathcal{H} \otimes B)\underline{u}_0(t), \qquad t \in [kT + \varphi, (k+1)T]. \tag{44}$$

The global cooperative disagreement error is $\Delta(t) = -(\mathcal{H} \otimes I_n)\epsilon(t)$. Then

$$\begin{aligned}\dot{\Delta}(t) &= (I_N \otimes A - c_1\mathcal{H} \otimes BK)\Delta(t) \\ &\quad - (c_2\mathcal{H}^2 \otimes BK)\Delta(t) + (c_1\mathcal{H}^3 \otimes FC)\tilde{x}(t) \\ &\quad - (\mathcal{H}^2 \otimes B)\underline{u}_0(t), \quad t \in [kT, kT + \varphi],\end{aligned} \tag{45}$$

and

$$\dot{\Delta}(t) = (I_N \otimes A)\Delta(t) + (c_1\mathcal{H}^3 \otimes FC)\tilde{x}(t) - (\mathcal{H}^2 \otimes B)\underline{u}_0(t), \quad t \in [kT, kT + \varphi], \tag{46}$$

where $\Delta(t) = [\delta_1(t), \delta_2(t), \cdots, \delta_N(t)]^T$.

4. Main result

Theorem 1. *Consider a vehicle platoon where the vehicle-to-vehicle communication topology satisfies Assumption 1, the vehicle dynamics are described by (1) and (2) and the virtual reference as in (7). The observer is designed as in (12) with F as in (13). The main controller is designed as in (15) with K as in (8), by selecting*

$$c_1 \geq \frac{1}{\min_{i=1,\dots,N} f_i\lambda_i}, \tag{47}$$

and $c_2 \geq 0$, and if the information rate satisfies

$$\frac{\varphi}{T} > \frac{c}{c+a}, \tag{48}$$

where

$$c = \frac{\bar{\sigma}_{(PA+A^TP)}}{\bar{\sigma}(P)}, \tag{49}$$

and

$$a = \frac{\min_{i=1,\dots,N}(\pi_i)\underline{\sigma}(Q)}{\bar{\sigma}(\Pi)\bar{\sigma}(P)}, \tag{50}$$

then $\Delta(t)$ is bounded such that

$$\lim_{t\to\infty} \|\hat{e}(t)\| \leq \alpha \in \mathbb{R}^+. \tag{51}$$

Here, λ_i is the i-th eigenvalue of Λ, and f_i is the i-th element of $\mathcal{F}$ defined in (4).

Proof. Choose the Lyapunov candidate function as

$$V(t) = \Delta^T(t)(\Pi \otimes P)\Delta(t), \tag{52}$$

which has the following lower and upper bounds:

$$\underline{\sigma}(\Pi)\underline{\sigma}(P)\|\Delta(t)\|^2 \leq V(t) \leq \bar{\sigma}(\Pi)\bar{\sigma}(P)\|\Delta(t)\|^2, \tag{53}$$

where $\underline{\sigma}(\cdot)$ is the minimum singular value and $\bar{\sigma}(\cdot)$ is the maximum singular value.

Inspired by Xu *et al*. (2020) as well as Prayitno and Nilkhamhang (2022), the stability analysis of the proposed control under completely periodical intermittent information is investigated at both time intervals, i.e., "active time" and "inactive time."

For "active time", $t \in [kT, kT + \varphi]$. Taking the time derivative of $V(t)$ along (45), we get

$$\begin{aligned}\dot{V}(t) &= \Delta^T(t)[(\Pi \otimes PA + A^TP) - c_1(\Pi\mathcal{H} \\ &\quad + \mathcal{H}^T\Pi) \otimes PBK]\Delta(t) \\ &\quad - 2\Delta^T(t)[c_2\Pi\mathcal{H}^2 \otimes PBK]\Delta(t) \\ &\quad + 2\Delta^T(t)(c_1\Pi\mathcal{H}^3 \otimes PFC)\tilde{x}(t) \\ &\quad - 2\Delta^T(t)(\Pi\mathcal{H}^2 \otimes PB)\underline{u}_0(t).\end{aligned} \tag{54}$$

Considering Λ in Lemma 1 and (8), Eqn. (54) becomes

$$\begin{aligned}\dot V(t) &= \Delta^T(t)[(\Pi\otimes PA + A^TP)\\&\quad - c_1\Lambda\otimes PBR^{-1}B^TP]\Delta(t)\\&\quad - 2c_2\Delta^T(t)[\Pi\mathcal{H}^2\otimes PBR^{-1}B^TP]\Delta(t)\\&\quad + 2c_1\Delta^T(t)(\Pi\mathcal{H}^3\otimes PFC)\tilde x(t)\\&\quad - 2\Delta^T(t)(\Pi\mathcal{H}^2\otimes PB)\underline{u}_0(t).\end{aligned}\tag{55}$$

Since $\Lambda > 0$, there is a unitary matrix, U, satisfying $U^T\Lambda U = \operatorname{diag}\{\lambda_1, \lambda_2, \dots, \lambda_N\}$. Therefore, (55) can be re-expressed as

$$\begin{aligned}\dot V(t) &= \Big\{\sum_{i=1}^{N}\pi_i\Delta_i^T(t)[PA + A^TP\\&\quad - c_1 f_i\lambda_i PBR^{-1}B^TP]\Delta_i(t)\Big\}\\&\quad - 2c_2\Delta^T(t)[\Pi\mathcal{H}^2\otimes PBR^{-1}B^TP]\Delta(t)\\&\quad + 2c_1\Delta^T(t)(\Pi\mathcal{H}^3\otimes PFC)\tilde x(t)\\&\quad - 2\Delta^T(t)(\Pi\mathcal{H}^2\otimes PB)\underline{u}_0(t),\end{aligned}\tag{56}$$

where π_i is the i-th diagonal element of matrix Π. Since $\tilde x(t)\to 0$, by selecting c_1 as in (47) and $c_2 \ge 0$,

$$\begin{aligned}\dot V &\le \sum_{i=1}^{N}\pi_i\Delta_i^T(t)[PA + A^TP - PBR^{-1}B^TP]\Delta_i(t)\\&\quad - 2\Delta^T(t)(\Pi\mathcal{H}^2\otimes PB)\underline{u}_0(t).\end{aligned}\tag{57}$$

As the leader input $u_0(t)$ is bounded with $\|\underline{u}_0(t)\| \le \beta \in \mathbb{R}$, from (9) it follows that (57) can be expressed as

$$\begin{aligned}\dot V &\le -\min_{i=1,\dots,N}(\pi_i)\underline{\sigma}(Q)\|\Delta(t)\|^2\\&\quad + 2\bar\sigma(\Pi)\bar\sigma(\mathcal{H}^2)\bar\sigma(PB)\beta\|\Delta(t)\|.\end{aligned}\tag{58}$$

Defining a as in (50) and

$$b = \frac{2\bar\sigma(\Pi)\bar\sigma(\mathcal{H}^2)\bar\sigma(PB)\beta}{\sqrt{\bar\sigma(\Pi)\bar\sigma(P)}},$$

(58) can be written down as

$$\dot V(t) \le -aV(t) + b\sqrt{V(t)}.\tag{59}$$

Let

$$z(t) = \sqrt{V(t)}.\tag{60}$$

Then

$$\dot z(t) \le -\frac{a}{2}z(t) + \frac{b}{2}.\tag{61}$$

The solution $z(t)$ in the time interval $t \in [kT, kT+\varphi]$ satisfies

$$z(t) \le \frac{b}{a} + e^{-\frac{a(t-kT)}{2}}\Big(z(kT) - \frac{b}{a}\Big).\tag{62}$$

Therefore,

$$z(kT+\varphi) \le \frac{b}{a} + e^{-\frac{a\varphi}{2}}\Big(z(kT) - \frac{b}{a}\Big).\tag{63}$$

For "inactive time", $t \in [kT+\varphi, (k+1)T]$. The time derivative of (52) along (46) is

$$\begin{aligned}\dot V(t) &= \Delta^T(t)[(\Pi\otimes PA + A^TP)]\Delta(t)\\&\quad + 2\Delta^T(t)(c_1\Pi\mathcal{H}^3\otimes PFC)\tilde x(t)\\&\quad - 2\Delta^T(t)(\Pi\mathcal{H}^2\otimes PB)\underline{u}_0(t),\end{aligned}\tag{64}$$

Since $\tilde x(t)\to 0, \dot V(t)$ can be described as

$$\begin{aligned}\dot V(t) &\le \bar\sigma(\Pi)\bar\sigma(PA + A^TP)\|\Delta(t)\|^2\\&\quad + 2\bar\sigma(\Pi)\bar\sigma(\mathcal{H}^2)\bar\sigma(PB)\beta\|\Delta(t)\|.\end{aligned}\tag{65}$$

By defining c as in (49) and

$$d = \frac{2\bar\sigma(\pi)\bar\sigma(\mathcal{H}^2)\bar\sigma(PB)\beta}{\sqrt{\bar\sigma(\Pi)\bar\sigma(P)}},$$

(65) can be expressed as

$$\dot V(t) \le cV(t) + d\sqrt{V(t)}.\tag{66}$$

Following a similar procedure to (59), by letting $z(t) = \sqrt{V(t)}$, it can easily be obtained that the solution of $z(t)$ in the interval $t \in [kT+\varphi, (k+1)T]$ satisfies

$$z(t) \le -\frac{d}{c} + e^{\frac{c(t-(kT+\varphi))}{2}}\Big(z(kT+\varphi) + \frac{d}{c}\Big),\tag{67}$$

and

$$z((k+1)T) \le -\frac{d}{c} + e^{\frac{1}{2}c(T-\varphi)}\Big(z(kT+\varphi) + \frac{d}{c}\Big).\tag{68}$$

Substituting (63) into (68) yields

$$\begin{aligned}z((k+1)T) &\le -\frac{d}{c} + \Big(\frac{b}{a} + \frac{d}{c}\Big)e^{\frac{1}{2}c(T-\varphi)}\\&\quad + e^{\frac{1}{2}(cT-\varphi(c+a))}\Big(z(kT) - \frac{b}{a}\Big).\end{aligned}\tag{69}$$

For simplification of (69), let

$$\begin{aligned}\eta &= -\frac{d}{c} + \Big(\frac{b}{a} + \frac{d}{c}\Big)e^{\frac{1}{2}c(T-\varphi)},\\\delta &= \frac{1}{2}(cT - \varphi(c+a)),\\\rho &= \frac{b}{a}.\end{aligned}$$

Then (68) can be represented as

$$z((k+1)T) \le \eta + e^{\delta}(z(kT) - \rho).\tag{70}$$

From the structure of (70), it follows that

$$\begin{aligned} z(kT) &\le \eta + e^{\delta}(z((k-1)T) - \rho) \\ &\le \eta + \eta e^{\delta} - \rho e^{\delta} + e^{2\delta}(z((k-2)T) - \rho) \\ &\le \eta + \eta(e^{\delta} + e^{2\delta}) - \rho(e^{\delta} + e^{2\delta}) \\ &\quad + e^{3\delta}(z((k-3)T) - \rho) \\ &\le \cdots \le \eta + (\eta - \rho)\sum_{r=1}^{k-1} e^{r\delta} + e^{k\gamma}(z(0) - \rho). \end{aligned} \tag{71}$$

According to (48), $cT - \varphi(c + a) < 0$ implies that $\delta < 0$. For $t \in [kT, +\infty]$ the following holds:

$$\begin{aligned} z(t) &\le z(kT) \\ &\le \eta + (\eta - \rho)\sum_{r=1}^{k-1} e^{r\delta} + e^{k\delta}(z(0) - \rho). \end{aligned} \tag{72}$$

Then

$$\begin{aligned} &\lim_{t\to\infty} z(t) \\ &\le \lim_{k\to\infty} [\eta + (\eta - \rho)\sum_{r=1}^{k-1} e^{r\delta} + e^{k\delta}(z(0) - \rho)]. \end{aligned} \tag{73}$$

But

$$\sum_{r=1}^{+\infty} e^{r\delta} = \frac{1}{e^{-\delta} - 1}.$$

Thus (73) is bounded by $\eta + (\eta - \rho)/(e^{-\delta} - 1)$ as $t \to \infty$. As for (60),

$$\lim_{t\to\infty} \|\Delta(t)\| \le \frac{\eta + (\eta - \rho)(\frac{1}{e^{-\delta}-1})}{\sqrt{\bar{\sigma}(\Pi)\bar{\sigma}(P)}}. \tag{74}$$

Then $\|\epsilon(t)\|$ can be obtained as follows:

$$\|\epsilon(t)\| = \left\|(\mathcal{H} \otimes I_n)^{-1}\Delta(t)\right\| \le \frac{\|\Delta(t)\|}{\underline{\sigma}(\mathcal{H})}. \tag{75}$$

Alternatively,

$$\lim_{t\to\infty} \|\epsilon(t)\| \le \frac{\eta + (\eta - \rho)(\frac{1}{e^{-\delta}-1})}{\underline{\sigma}(\mathcal{H})\sqrt{\bar{\sigma}(\Pi)\bar{\sigma}(P)}}, \tag{76}$$

From the reference model and according to Zhang *et al.* (2011), synchronization of all followers to the leader state is guaranteed. This yields $\varepsilon_r(t) = 0$, and therefore $\hat{\varepsilon}(t) = \epsilon(t)$ as $t \to \infty$ and the relationship between $\|\hat{\varepsilon}(t)\|$ and $\|\hat{e}(t)\|$ becomes

$$\|\hat{e}(t)\| = \left\|(\mathcal{H} \otimes I_n)^{-1}\hat{\varepsilon}(t)\right\| \le \frac{\|\hat{\varepsilon}(t)\|}{\underline{\sigma}(\mathcal{H})}. \tag{77}$$

Thus,

$$\lim_{t\to\infty} \|\hat{e}(t)\| \le \frac{\eta + (\eta - \rho)(\frac{1}{e^{-\delta}-1})}{(\underline{\sigma}(\mathcal{H}))^2\sqrt{\bar{\sigma}(\Pi)\bar{\sigma}(P)}} = \alpha. \tag{78}$$

This completes the proof. ∎

5. Numerical simulation

Simulation results will be presented in two sections. In the first section, the effectiveness of the proposed control under continuous information will be discussed. Then, the performance of the proposed control under completely periodical intermittent information will be presented in the next section.

A group of vehicles (one leader and five followers) forms a platoon based on CSP with $d_r = 5$ used for this simulation and analysis. The TPFL topology is applied in the platoon, Fig. 3, which can be represented by the Laplacian and pinning matrices of Table 1. All vehicles have identical nominal models, represented by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}. \tag{79}$$

It is assumed that each vehicle only obtained information about the position. Therefore,

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \tag{80}$$

The observer and controller are designed by selecting $Q = I_{3\times 3}$ and $R = 1$, which results in

$$F = \begin{bmatrix} 2.1211 & 1.7494 & 0.2500 \end{bmatrix}^T, \tag{81}$$

$$K = \begin{bmatrix} 1 & 2.1211 & 0.7494 \end{bmatrix}. \tag{82}$$

The control protocols (6) and (15) are implemented by selecting coupling gains $c_1 = 1.5$, and $c_2 = 100$. The initial conditions of all vehicles are shown in Table 2. In order to simulate the time-varying velocity of the leader, the bounded nonzero input leader as shown in Fig. 4 is used. The bounded, nonzero input in Fig. 4 is designed to represent the traffic conditions that include constant velocity, acceleration and deceleration which are designed by considering the naturalistic driving data as used by Song *et al.* (2020).

5.1. Continuous information. The aim of this subsection is to show the performance and the effectiveness of the proposed control, (DMRC-OC), which is designed based on limited-output information, under continuous information. Continuous information means that the information is always ON over time, which implies that information is always received by each follower. Continuous information can be implemented in the simulation by setting the information rate $\varphi/T = 1$. The performance of DMRC-OC will be compared with the existing DMRC which is designed based on full-state information, proposed by Prayitno and Nilkhamhang (2022) and CSVFB, proposed by Zhang *et al.* (2011). Figures 5 and 6 along with Table 3 show

Table 1. $\mathcal{L}$ and G matrices of TPFL topology.

$\mathcal{L}$	G
$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ 0 & -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Table 2. Initial conditions of individual vehicles.

(i)	$p_i(0)$	$\hat{p}_i(0)$	$v_i(0)$	$\hat{v}_i(0)$	$a_i(0)$	$\hat{a}_i(0)$
0	60	–	20	–	0	–
1	40	38	18	17	0	0
2	25	27	19	18	0	0
3	17	16	22	23	0	0
4	10	12	21	22	0	0
5	0	2	17	16	0	0

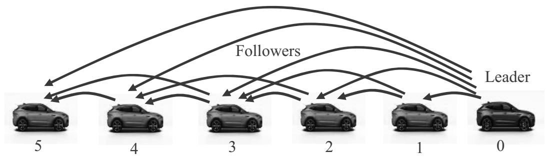

Fig. 3. TPFL topology.

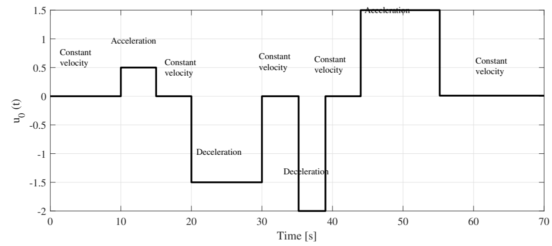

Fig. 4. Nonzero input leader.

the comparison results. It can be seen that DMRC-OC exhibits similar performances to DMRC, although it uses limited-output information, while CSVFB exhibits a significant inter-vehicular distance error when the leader moves with a time-varying velocity. The comparison results show the effectiveness of the proposed control.

The results in Fig. 6 show that all followers with limited-output information are able to maintain the inter-vehicular distance. This means that all followers achieve synchronization with the leader that moves with a time-varying velocity. Moreover, the results in Fig. 7 show that there is a very small residual bounded tracking error in the ranges values shown in Table 3. Since $\tilde{x}(t) \to 0$, synchronization of all followers with the leader is achieved with a small residual bounded error. It is worth noting that even if only position information of the vehicle can be measured , the proposed control is able to make all followers achieve consensus tracking with results similar to DMRC, see Table 3 for the comparison results.

5.2. Complete periodical intermittent information. The aim of this section is to verify the performance and stability of the proposed control (DMRC-CO) when the platoon is subjected to completely periodical intermittent information. From the stability analysis, we deduce that $c = 1.0681$, $a = 0.2110$, $b = d = 0.3133$, $\eta = 2.4323$ and $\rho = 1.4845$. According to (48), we deduce that, to guarantee the synchronization, the condition of the information rate of the vehicle platoon with TPFL topology is $\varphi/T > 0.835$. In order to check the performance of the proposed controller, the information rate $\varphi/T = 0.84$ and the period $T = 5$ s are selected, which yields $\varphi = 4.2$ s and $\alpha = 4.30$.

The performance of the system is shown in Fig. 8. It is clear that all followers are still able to synchronize with the leader by keeping the inter-vehicular distance. However, oscillations occurred in the first follower in the periods of inactive information. This is because the follower is desynchronized when the information is inactive. But, when the information is reestablished, the synchronization of the follower will be achieved again. The longer the inactive time, the bigger the oscillation, which may tend to cause a collision between vehicles and yield instability of the platoon. Interestingly, in the TPFL topology, a significant effect of the intermittent information happens only in the first follower while for the remaining followers, the effects are attenuated. From Fig. 9, it is seen that the inter-vehicular distance error is bounded between -0.52 and 0.52 meters.

Table 4 compares the bounded tracking error between DMRC-OC under continuous information and under completely periodical intermittent information, where, in general, the tracking error is bounded between -2.64 and 2.69 and lies inside $\alpha = 4.30$. It is very clear that intermittent information reduces the performance of the controller under continuous information as indicated by the increase in the bounded residual error.

Remark 4. Increasing the information rate above the threshold in the condition mentioned will improve the performance and stability of the system, while reducing the information rate will increase the overshoot of the follower during the "inactive" time interval. However, when the information rate does not satisfy the above condition, the system becomes unstable.

Remark 5. For other topologies, i.e., PFL, TPF, and PF, the conditions of the information rate are $\varphi/T > 0.835$, $\varphi/T > 0.915$ and $\varphi/T > 0.962$, respectively. The more complex the topology, the smaller the condition value of the information rate, which results in a more robust

Table 3. Tracking error comparison: CSVFB (Zhang *et al.*, 2011) vs. DMRC (Prayitno and Nilkhamhang, 2022) vs. the proposed DMRC-OC.

Tracking error	CSVFB ($e_{..}$)		DMRC ($e_{..}$)		DMRC-OC ($\hat{e}_{..}$)	
	min	max	min	max	min	max
p [m]	-1.19	1.00	-0.02	0.02	-0.02	0.02
v [m /s]	-0.47	0.43	-0.01	0.01	-0.01	0.01
a [m / s^2]	-0.73	0.77	-0.03	0.03	-0.03	0.02

Table 4. Tracking error comparison: continuous vs. intermittent information.

$\hat{e}_i$	Continuous		Intermittent		Boundary, α
	min	max	min	max	
$\hat{e}_{i,p}$ [m]	-0.02	0.02	-0.52	0.52	
$\hat{e}_{i,v}$ [m/s]	-0.01	0.01	-0.84	0.84	4.30
$\hat{e}_{i,a}$ [m/s^2]	-0.03	0.02	-2.64	2.69	

Table 5. Residual bounded error for various directed topologies, at $t > 10$ s.

Topologies	Selected φ/T		$\hat{e}_{i,p}$ [m]	$\hat{e}_{i,v}$ [m/s]	$\hat{e}_{i,a}$ [m/s^2]
PFL	0.84	min	-0.50	-0.84	-2.58
		max	0.50	0.8	2.53
TPF	0.92	min	-0.17	-0.32	-2.27
		max	0.17	0.32	2.27
PF	0.97	min	-0.34	-0.37	-4.21
		max	0.37	0.37	3.98

system to intermittent information. The resulting residual bounded errors are shown in Table 5.

6. Conclusion

A DMRC-CO is designed for synchronization of the vehicle platoon with limited-output information. Moreover, through detail analysis, when the platoon is subjected to completely periodical intermittent information, a condition on the information rate has been derived to guarantee the platoon synchronization. The proposed controller is applicable for vehicle platoons with any directed topologies which have a spanning tree with the leader acting as a root node and moving with a time-varying velocity. Through numerical simulations, it is shown that followers synchronize with the leader with a bounded error, both under continuous information and completely periodical intermittent information as long as the derived condition on the information rate is fulfilled. It is also shown that the more complex the topology, the more robust the system to intermittent information, but this is accompanied by an increase in the residual bounded error.

The proposed control still assumes that all followers have nominal models and are subject to periodical intermittent information. Therefore, future works may focus on considering some non linearized factors and aperiodical intermittent information. Moreover, working on a real-world experiment or verifying the controlled system by using a physics-based simulation platform such as PreScan or CarSim can also be prospective research directions.

References

Abou Harfouch, Y., Yuan, S. and Baldi, S. (2017). An adaptive switched control approach to heterogeneous platooning with intervehicle communication losses, *IEEE Transactions on Control of Network Systems* **5**(3): 1434–1444, DOI: 10.1109/TCNS.2017.2718359.

Besselink, B. and Johansson, K.H. (2017). String stability and a delay-based spacing policy for vehicle platoons subject

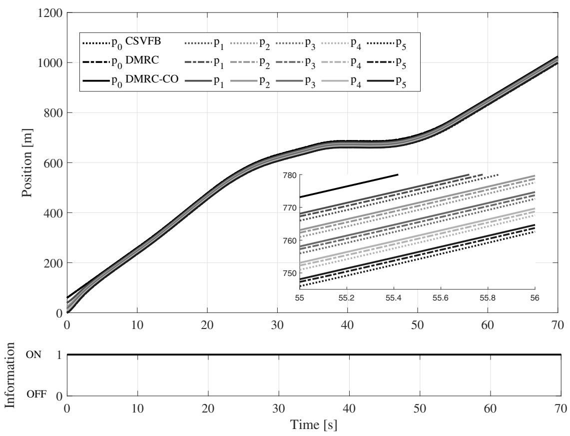

Fig. 5. Synchronization of position with CSP: CSVFB (Zhang *et al.*, 2011) vs. the DMRC (Prayitno and Nilkhamhang, 2022) vs. DMRC-OC.

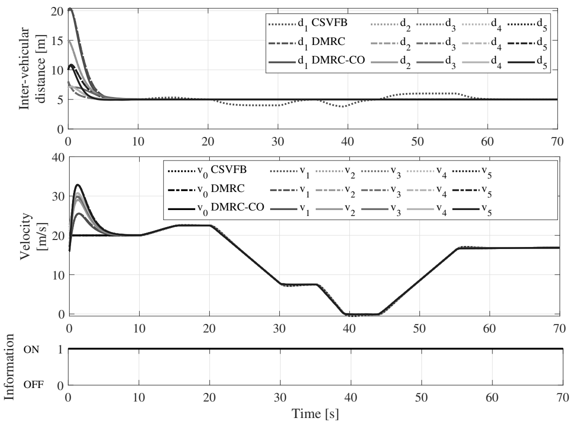

Fig. 6. Synchronization of the position, inter-vehicular distance and estimated velocity under continuous information: CSVFB (Zhang *et al.*, 2011) vs. DMRC (Prayitno and Nilkhamhang, 2022) vs. DMRC-OC.

to disturbances, *IEEE Transactions on Automatic Control* **62**(9): 4376–4391, DOI: 10.1109/TAC.2017.2682421.

Cavazza, B.H., Gandia, R.M., Antonialli, F., Zambalde, A.L., Nicolaï, I., Sugano, J.Y. and Neto, A. D.M. (2019). Management and business of autonomous vehicles: A systematic integrative bibliographic review, *International Journal of Automotive Technology and Management* **19**(1-2): 31–54, DOI: 10.1504/IJATM.2019.098509.

Chang, B.-J., Hwang, R.-H., Tsai, Y.-L., Yu, B.-H. and Liang, Y.-H. (2019). Cooperative adaptive driving for platooning autonomous self driving based on edge computing, *International Journal of Applied Mathematics and Computer Science* **29**(2): 213–225, DOI: 10.2478/amcs-2019-0016.

Di Bernardo, M., Salvi, A. and Santini, S. (2014). Distributed consensus strategy for platooning of vehicles in the presence of time-varying heterogeneous communication delays, *IEEE Transactions on Intelligent Transportation Systems* **16**(1): 102–112, DOI: 10.1109/TITS.2014.2328439.

Franzè, G., Lucia, W. and Tedesco, F. (2018). A distributed model predictive control scheme for leader-follower multi-agent systems, *International Journal of Control* **91**(2): 369–382, DOI: 10.1080/00207179.2017.1282178.

Hamdi, H., Rodrigues, M., Rabaoui, B. and Braiek, N.B. (2021). A fault estimation and fault-tolerant control based sliding mode observer for LPV descriptor systems with time delay, *International Journal of Applied Mathematics and Computer Science* **31**(2): 247–258, DOI: 10.34768/amcs-2021-0017.

Hu, J., Bhowmick, P., Arvin, F., Lanzon, A. and Lennox, B. (2020). Cooperative control of heterogeneous connected vehicle platoons: An adaptive leader-following approach, *IEEE Robotics and Automation Letters* **5**(2): 977–984, DOI: 10.1109/LRA.2020.2966412.

Huang, N., Duan, Z. and Zhao, Y. (2014). Leader-following consensus of second-order non-linear multi-agent systems with directed intermittent communication, *IET Control Theory & Applications* **8**(10): 782–795, DOI: 10.1049/iet-cta.2013.0565.

Huang, N., Duan, Z. and Zhao, Y. (2015). Consensus of multi-agent systems via delayed and intermittent communications, *IET Control Theory & Applications* **9**(1): 62–73, DOI: 10.1049/iet-cta.2014.0729.

Jiang, Y., Zhang, Y. and Wang, S. (2018). Distributed leader-following consensus control based optimal design for multi-agent systems with intermittent communications, *2018 Chinese Control and Decision Conference (CCDC), Shenyang, China*, pp. 5341–5345, DOI: 10.1109/CCDC.2018.8408060.

Jond, H.B. and Yıldız, A. (2022). Connected and automated vehicle platoon formation control via differential games, *IET Intelligent Transport Systems* **17**(2): 312–326, DOI: 10.1049/itr2.12260.

Kukurowski, N., Mrugalski, M., Pazera, M. and Witczak, M. (2022). Fault-tolerant tracking control for a non-linear twin-rotor system under ellipsoidal bounding, *International Journal of Applied Mathematics and Computer Science* **32**(2): 171–183, DOI: 10.34768/amcs-2022-0013.

Lewis, F.L., Zhang, H., Hengster-Movric, K. and Das, A. (2013). *Cooperative Control of Multi-Agent Systems: Optimal and Adaptive Design Approaches*, Springer Science, London, DOI: 10.1007/978-1-4471-5574-4.

Li, S.E., Zheng, Y., Li, K., Wu, Y., Hedrick, J.K., Gao, F. and Zhang, H. (2017). Dynamical modeling and distributed control of connected and automated vehicles: Challenges and opportunities, *IEEE Intelligent Transportation Systems Magazine* **9**(3): 46–58, DOI: 10.1109/MITS.2017.2709781.

Liu, Y., Xie, D. and Shi, L. (2020). Consensus of general linear multi-agent systems with intermittent communications, *International Journal of Systems Science* **51**(12): 2293–2305, DOI: 10.1080/00207721.2020.1793236.

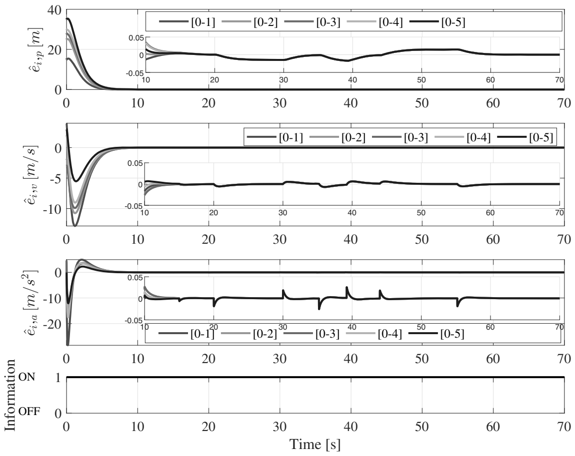

Fig. 7. Tracking error of the estimated followers with respect to the leader of DMRC-OC under continuous information.

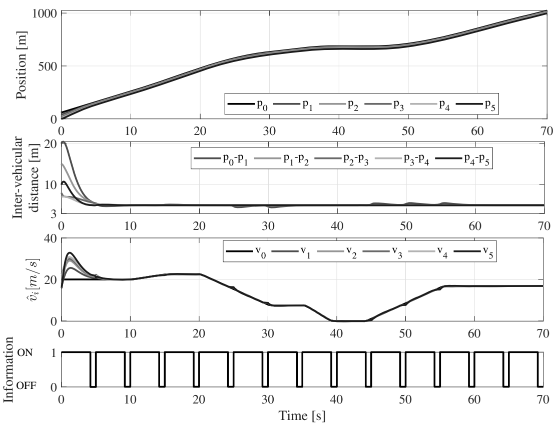

Fig. 8. Synchronization of the position, inter-vehicular distance and estimated velocity with $\varphi = 4.2$ s, and $T = 5$ s.

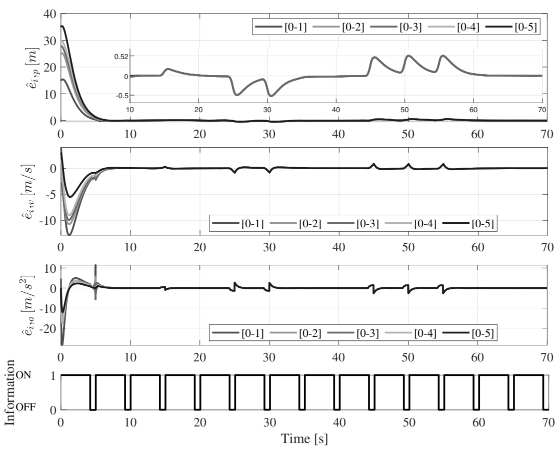

Fig. 9. Tracking error of the estimated followers with respect to the leader with $\varphi = 4.2$ s, and $T = 5$ s.

Long, X., Yu, S., Wang, Y. and Jin, L. (2014). Leader-follower consensus of multi-agent system with external disturbance based on integral sliding mode control, *Proceedings of the 33rd Chinese Control Conference, Nanjing, China*, pp. 1740–1745, DOI: 10.1109/ChiCC.2014.6896891.

Ozkan, M.F. and Ma, Y. (2021). Fuel-economical distributed model predictive control for heavy-duty truck platoon, *2021 IEEE International Intelligent Transportation Systems Conference (ITSC), Indianapolis, USA*, pp. 1919–1926.

Prayitno, A. and Nilkhamhang, I. (2021). Distributed model reference adaptive control for vehicle platoons with uncertain dynamics, *Engineering Journal* **25**(8): 173–185, DOI: 10.4186/ej.2021.25.8.173.

Prayitno, A. and Nilkhamhang, I. (2022). Distributed model reference control for cooperative tracking of vehicle platoons subjected to external disturbances and bounded leader input, *International Journal of Control, Automation and Systems* **20**(6): 2067–2080, DOI: 10.1007/s12555-021-0171-4.

Qu, Z. (2009). *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*, Springer, London, DOI: 10.1007/978-1-84882-325-9.

Song, K., Liu, F., Wang, C., Wang, P. and Min, G. (2020). Driving stability analysis using naturalistic driving data with random matrix theory, *IEEE Access* **8**: 175521–175534.

Wang, F., Liu, Z. and Chen, Z. (2019). Leader-following consensus of second-order nonlinear multi-agent systems with intermittent position measurements, *Science China Information Sciences* **62**(10): 1–16, DOI: 10.1007/s11432-018-9732-7.

Wang, Z., Wu, G. and Barth, M.J. (2017). Developing a distributed consensus-based cooperative adaptive cruise control system for heterogeneous vehicles with predecessor following topology, *Journal of Advanced Transportation* **2017**: 1–16, DOI: 10.1155/2017/1023654.

Wijnbergen, P., Jeeninga, M. and Besselink, B. (2021). Nonlinear spacing policies for vehicle platoons: A geometric approach to decentralized control, *Systems & Control Letters* **153**: 104954, DOI: 10.1016/j.sysconle.2021.104954.

Xie, Y. and Lin, Z. (2020). Global consensus of multi-agent systems with intermittent directed communication in the presence of actuator saturation, *International Journal of Robust and Nonlinear Control* **30**(18): 8469–8484, DOI: 10.1002/rnc.5255.

Xu, C., Xu, H., Su, H. and Liu, C. (2020). Disturbance-observer based consensus of linear multi-agent systems with exogenous disturbance under intermittent communication, *Neurocomputing* **404**: 26–33, DOI: 10.1016/j.neucom.2020.04.051.

Xu, H., Zeng, W. and Xu, C. (2021). Output consensus of multi-agent systems with linear dynamics via asynchronous intermittent control, *2021 40th Chinese Control Conference (CCC),* *Shanghai, China*, pp. 5553–5558, DOI: 10.23919/CCC52363.2021.9549806.

Xu, Z., Zegers, F.M., Wu, B., Dixon, W. and Topcu, U. (2019). Controller synthesis for multi-agent systems with intermittent communication. a metric temporal logic approach, *2019 57th Annual Allerton Conference on Communication, Control, and Computing (Allerton), Monticello, USA*, pp. 1015–1022, DOI: 10.1109/ALLERTON.2019.8919727.

Yan, F., Dridi, M. and El Moudni, A. (2013). An autonomous vehicle sequencing problem at intersections: A genetic algorithm approach, *International Journal of Applied Mathematics and Computer Science* **23**(1): 183–200, DOI: 10.2478/amcs-2013-0015.

Yan, M., Song, J., Yang, P. and Zuo, L. (2018). Neural adaptive sliding-mode control of a bidirectional vehicle platoon with velocity constraints and input saturation, *Complexity* **2018**: 1–11, DOI: 10.1155/2018/1696851.

Zhang, H. and Lewis, F.L. (2012). Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics, *Automatica* **48**(7): 1432–1439, DOI: 10.1016/j.automatica.2012.05.008.

Zhang, H., Lewis, F.L. and Das, A. (2011). Optimal design for synchronization of cooperative systems: State feedback, observer and output feedback, *IEEE Transactions on Automatic Control* **56**(8): 1948–1952, DOI: 10.1109/TAC.2011.2139510.

Zheng, Y., Bian, Y., Li, S. and Li, S.E. (2019). Cooperative control of heterogeneous connected vehicles with directed acyclic interactions, *IEEE Intelligent Transportation Systems Magazine* **13**(2): 127–141, DOI: 10.1109/MITS.2018.2889654.

Zheng, Y., Li, S.E., Li, K., Borrelli, F. and Hedrick, J.K. (2016). Distributed model predictive control for heterogeneous vehicle platoons under unidirectional topologies, *IEEE Transactions on Control Systems Technology* **25**(3): 899–910, DOI: 10.1109/TCST.2016.2594588.

Zheng, Y., Li, S. E., Wang, J., Cao, D. and Li, K. (2015). Stability and scalability of homogeneous vehicular platoon: Study on the influence of information flow topologies, *IEEE Transactions on Intelligent Transportation Systems* **17**(1): 14–26, DOI: 10.1109/TITS.2015.2402153.

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