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APPLICATION OF MATLAB™ SOFTWARE PACKAGE FOR IDENTIFICATION OF MECHANICAL SYSTEMS MODAL MODELS†

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Application of MATLAB[1] Software Package for identification of mechanical systems modal models is considered as an example of its scientific and engineering use. The main features of MATLAB package and foundations of modal modelling are presented. An algorithm of using MATLAB in modal model identification procedure is formulated. The results of application of the algorithm to identification of the modal model of a simple real system (single link elastic robot arm) are included.

1. Introduction

The main aim of the paper is to present the possibilities of a general purpose software package application for modal analysis of mechanical structures. Though for the sake of simplicity the example of the single elastic robot link is considered in the paper, the described software package may as well be used for other more complicated structures like machinery or vehicles.

1.1. MATLAB Software Package

MATLAB (MATrix LABoratory) is a widely used high performance software package for numeric computation (MATLAB, 1994; PC-MATLAB, 1994; SP Toolbox, 1994). Current range of its application covers a great variety of scientific, engineering and educational problems. At the beginning of the 1980s, when MATLAB was developed, it was designed to be a user friendly interface for matrix computation procedures of LINPACK and EISPACK libraries. Matrix algebra is commonly used in theoretical (modelling, design) and experimental (data processing, identification) analysis of linear mechanical systems. Apart from the computational effectiveness, interactive communication and simple way of programming, the main feature of MATLAB is its open structure easily enabling the users to enlarge its application to new problems by adding a new source code file. Such a file is called M-File and it is a procedure with formal parameters or an interpreter script. This makes it possible to form the programming environment for the particular user and specific problem in a flexible way.

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[1] MATLAB is a trademark of MathWorks, Inc.

From the user's point of view the semantic form of MATLAB language is coherent, clear and it possesses effective syntax structure.

MATLAB provides many standard procedures (toolboxes) like: SIGNAL Processing Toolbox, CONTROL Processing Toolbox, MULTIVARIABLE FREQUENCY DOMAIN Toolbox, μ-ANALYSIS and SYNTHESIS Toolbox, ROBUST CONTROL Toolbox, SYSTEM IDENTIFICATION Toolbox, STATE SPACE IDENTIFICATION Toolbox, and NEURAL NETWORK Toolbox.

The toolboxes mentioned above were developed as M-Files taking advantage of MATLAB's internal expansiveness. It is possible to use MATLAB with some other programming tools equipped with user's interface or cooperating with MATLAB in an integrated programming environment. Another possible solution is to use MATLAB as a system kernel with a shell providing additional external user's interface. It makes it possible to achieve a new function of the package operation, e.g. simulation of non-linear system dynamics. The mentioned features of MATLAB enable us to apply the package to the solution of mechanical system identification problem.

1.2. Modal Modelling of Mechanical Systems

Today's engineers are faced with many complex noise and vibration problems associated with the design and troubleshooting of structures. Present day's structures are typically more complex in terms of design and materials and that is why the structural analyst must design accurate and realistic models and the experimenters must be able to accurately define a dynamic response of structures to specified input forces.

Recently, many tools have been developed to assist the structural engineer in these areas. The analyst now uses sophisticated finite element programs which aid in the understanding and the design of structures. The experimenter has sophisticated digital signal analysis equipment that aids him to quantify and understand a structure's dynamic and input forces. However, there is little communication between the analysts and experimenters. Due to the complexity of today's problems, the need for communication and exchange of ideas is even more critical. The main tool for the analysts and experimenters to communicate could be modal analysis. It is a useful tool for both specialists.

Model Analysis is a technique applied in vibration analysis to describe the dynamic behaviour of mechanical structures. The analytical modal analysis consists in analysis of the structural mathematical model in order to find modal parameters of the structure. Mathematically, it can be considered as the eigenvalue problem. On the other hand, experimental modal analysis consists in synthesis of the modal model on the basis of experimental data. Actually, the mathematical structural model can be obtained.

The modal model can be defined as a set of modal parameters within a given range of frequencies. The modal parameters are: modal frequency, modal damping, and mode shape. The modal parameters of all modes within the frequency range of interest constitute a complete dynamic description of the structure.

The modal models can be applied for the following purposes:

- to understand and communicate how structures behave under dynamic loads (Natke, 1988);
- to use in data reduction and smoothing techniques (Natke, 1989; Sheleton *et al.*, 1982);
- to simulate and predict the response to the assumed external forces (Ewins, 1986);
- to simulate changing dynamic characteristics, due to physical modification, predict the necessary physical modifications required to obtain a desired dynamic property and predict the combined behaviour when two or more structures are coupled together as a unit (Brandon, 1990).
- to update (verify and improve) the theoretical models (finite element models) (Imsegun and Visser, 1990; Larssen and Sas, 1992; Steinwender, 1991).

Modal analysis found a very wide application in industry because dynamic behaviour of the final product predicted by simulation can be verified on the real object by the experimental modal analysis.

For mathematical formulation some assumptions have to be made. The first assumption is that the structure is a linear system whose dynamics may be represented by a set of linear, second-order, ordinary differential equations. The second assumption is that the structure obeys Maxwell's reciprocity theorem. The third assumption is that the structure during the test can be considered as time-invariant (time-invariant model parameters). The fourth assumption is that the damping is small or the damping is proportional to mass and/or stiffness.

Let us consider a multiple degrees of freedom (DOF) system consisting of mass, spring and damping elements. The equation of motion in a general case has the form

$$M\ddot{x} + C\dot{x} + Kx = F(t) \tag{1}$$

where M, C, K are, respectively, the mass, damping and stiffness matrices; $\ddot{x}$, $\dot{x}$, x – the acceleration, velocity and displacement vectors; F is the force vector.

The above formula constitutes a set of second-order, linear, time-invariant, differential equations. As the equations are coupled, the system must be solved simultaneously. Assumption of proportional damping enables us to decouple the equations. The modal parameters will result as the solution to the homogeneous portion of the differential equations summarized in eqn. (1). We shall consider first the free vibration solution in order to determine modal parameters. In this case $F = 0$, and then vector $x(t)$ can be written in the form

$$x(t) = \sum_{r=1}^{N} x_{0r} \exp(\lambda_r t) + \sum_{r=1}^{N} x_{0r}^{*} \exp(\lambda_r^{*} t) \tag{2}$$

where $(\cdot)^*$ denotes the complex conjugate of $(\cdot)$, $\lambda_r = \sigma_r + j\omega_r$ and σ_r, ω_r denote a damping factor and a damped natural frequency, respectively.

The modal model can be written in the matrix $(N \times N)$ form:

– $[\Omega_r^2]$ is the natural frequency matrix (diagonal), $\Omega_r^2 = \omega_r^2 + \sigma_r^2$;

- $[\delta_r]$ is the modal damping matrix (diagonal), $\delta_r = -\sigma_r/\Omega_r$;
- $[\psi]$ is a modal matrix with the columns which are the mode shapes for given frequency ω_r (relative values of x).

The modal model possesses the orthogonality properties which, concisely stated, are as follows:

$$\begin{aligned} \psi^T M \psi &= [m_r] \\ \psi^T C \psi &= [c_r] \\ \psi^T K \psi &= [k_r] \end{aligned} \tag{3}$$

from which $[\Omega_r^2] = [m_r]^{-1}[k_r]$ and matrices $[m_r]$ and $[k_r]$ are called modal mass and modal stiffness matrices, respectively. For proportional damping modal damping matrix $[\delta_r]$ is introduced. After the mass normalization we can obtain the following:

$$\begin{aligned} \varphi^T M \varphi &= [I_r] \\ \varphi^T C \varphi &= [\delta_r] \\ \varphi^T K \varphi &= [\Omega_r^2] \end{aligned} \tag{4}$$

where $[I_r]$ is an identity matrix.

We can easily notice that after introducing new coordinates into eqn. (1) we shall obtain the decoupled set of second-order differential equations of the form

$$[I_r]\ddot{q} + [\delta_r]\dot{q} + [\omega_r^2]q = \varphi^T F \tag{5}$$

where $x = \varphi q$. The new coordinates are called the modal coordinates and they have no physical meaning. This is a disadvantage of the modal models application. But the form of the set of equations (5) is very useful because each equation (for the r-th mode) can be solved independently.

In order to find a relation between modal model and frequency response of the system we should consider eqn. (1) with the right-hand side not equal to zero and take $F(t) = f \exp(j\omega t)$. The equation of motion then becomes

$$(K + j\omega C - \omega^2 M)x \exp(j\omega t) = f \exp(j\omega t) \tag{6}$$

Rearranging to solve for unknown responses we obtain

$$x = (K + j\omega C - \omega^2 M)^{-1} f \tag{7}$$

which may be also written in the form

$$x = \alpha(\omega) f \tag{8}$$

where $\alpha(\omega)$ is a receptance $(N \times N)$ matrix for the system and constitutes its response model (frequency response function — FRF) whose elements are defined as follows:

$$\alpha_{jk}(\omega) = \left(\frac{x_j(\omega)}{f_k(\omega)}\right) \tag{9}$$

where $f_m \neq 0$ for $m = k$ and $f_m = 0$ for $m \neq k$, $m = 1, ..., N$. Each element $\alpha_{jk}(\omega)$ and represents an individual FRF for the k-th excitation point and j-th measurement point. Formula (7) includes the inversion of a system matrix at each frequency which has several disadvantages, namely:

- it becomes costly for large systems (large N);
- it is inefficient if only a few of the individual frequency response expressions are required;
- it provides no insight into the form of the various frequency response properties.

For these reasons an alternative means of deriving the frequency response parameters are used which make use of the modal properties of the system. Combining eqns. (7) and (9) we obtain

$$(K + j\omega C - \omega^2 M) = \alpha(\omega)^{-1} \tag{10}$$

and after multiplying both sides by φ^T and post multiplying the result by φ we get

$$\varphi^T (K + j\omega C - \omega^2 M)\varphi = \varphi^T \alpha(\omega)^{-1}\varphi \tag{11}$$

which, in turn, implies

$$\alpha(\omega) = \varphi \left[\lambda_r^2 - \omega^2\right]^{-1} \varphi^T \tag{12}$$

Equation (12) enables us to compute an individual frequency response element α_{jk} using the following formula:

$$\alpha_{jk}(\omega) = \sum_{r=1}^{N} \frac{{}_r\varphi_j \, {}_r\varphi_k}{\lambda_r^2 - \omega^2} = \sum_{r=1}^{N} \frac{{}_rA_{jk}}{\lambda_r^2 - \omega^2} \tag{13}$$

where ${}_rA_{jk}$ is a modal constant.

The above formula is a basic formula in the experimental modal analysis. The FRF can be relatively easily measured on a real mechanical system and next the modal parameters should be estimated during identification procedure. The form of the modal model indicates that the modal model parameters estimation methods are essentially based on matrix computations. As it was stressed in the previous section, MATLAB is especially applicable for matrix computations.

2. Modal Model Identification Procedure — M(atlab) File Structure

Modal model identification procedure for an assumed form of Frequency Response Function can be divided into three stages (see Fig. 1). The first stage of the procedure is measurement. It comprises: choice of measurement points, measurement directions in each point, choice of transducers, preamplifiers, and sources of excitation. The second stage (preprocessing) deals with initial processing of the measured data, e.g. filtering (including antialiasing filtering) and A/D conversion. The results of the second stage are usually stored digital data. The second stage completes data acquisition. The third stage of the identification procedure is processing. It consists in a proper analysis of recorded data in order to achieve estimates of parameters of the chosen modal model of the system.

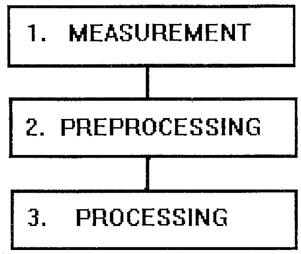


Fig. 1. Stages of identification procedure.

Stages 1 and 2 are performed by dedicated hardware. Stage 3 may be realized with the use of dedicated analyzer, dedicated software package run on PC or workstation computer (e.g. STAR from SMS, I-DEAS-Modal from SDRC Inc., CADA from LMS) or general purpose software package (like MATLAB). In practice dedicated analyzers usually are used, but recently rising computing efficiency (i.e. speed of computing, capacity of external memory, storage and disk speed) followed be availability of proper dedicated software on the market and a decrease in the computers prices caused a wide use of computers for that purpose. As both dedicated analyzers and computer analysing software are rather expensive for laboratory identification as well as for education, it is more convenient to use a universal scientific and engineering calculation software package rather than a dedicated one. The main reason for this is that it may be used for a variety of other purposes by many other users. One of such universal packages is The Mathworks' MATLAB available for DOS and UNIX computers. MATLAB enables us to use e.g. processing functions based on Fast Fourier Transform spectrum analysis, power spectrum estimation, cepstrum, correlation, IIR and FIR filters design. The use of M-FILE technique provides a structural form for the program. Graphic display possibilities of data, mid and final results are powerful and easy to use.

In the case of the identification procedure the following five-step algorithm (see Fig. 2) was formulated. Step A of the algorithm includes scaling stored signals values in physical units (transformation: volts to physical units), non-zero mean cancellation, band pass FIR filtration (in order to decrease the influence of vibration modes out of the analysed frequency range). During Step B Fast Fourier Transform of measured signals is performed. Step C includes Frequency Response Function values calculation (Receptance, Mobility or Inertance Function). Step D deals with the estimation of modal model parameters: natural frequencies, modal dampings and modal shapes. Step E consists in graphic display of identification results.

On the basis of the formulated algorithm an M-File *modal.m* was prepared. Each step of the algorithm corresponds to a subroutine of the program which is also M-File and consists of Matlab statements, mathematical functions (e.g. 'abs' for absolute value calculations), signal processing functions (e.g. 'fft' performing Fast Fourier Transform), graphic functions (e.g. 'plot' for drawing linear plots) and user defined functions (M-Files). For better performance each part of the program is followed by the mid-result graphic display which gives some insight into the identification procedure and is very useful both during research and education activity.

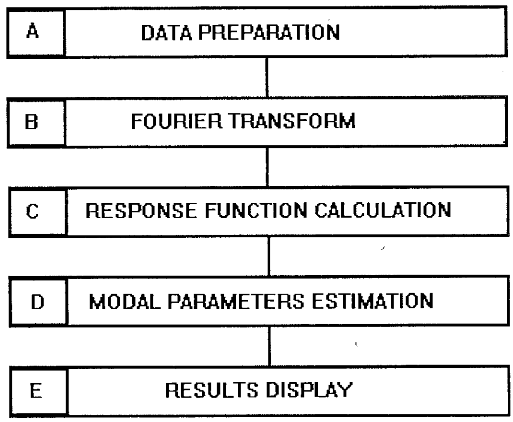


Fig. 2. Identification procedure algorithm — M-File structure.

3. Example of Modal Model Identification of a Single Elastic Robot Link

The formulated algorithm of modal model identification was applied to a real mechanical system. A single elastic robot link modelled as prismatic $(l \times b \times g)$, steel cantilever beam was used in the test.

Perfectly rigid attachment of the link and proportional hysterical structural damping were assumed. For identification the mobility function was chosen in the form

$$Y_{jk}(\omega) = i\omega \sum_{r=1}^{N} \frac{{}_r\varphi_j \; {}_r\varphi_k}{(\omega_r^2 - \omega^2) + j\eta_r \omega_r^2} \tag{14}$$

where $\eta_r = 0.5\delta_r$ is a damping loss factor.

The analysed frequency range was limited to 3–110 Hz (i.e. to three lowest bending vibration modes). Two experiments: sine and impact excitation tests were performed. The acceleration of 10 points equally located along the link was simultaneously measured at sampling frequency: 1 kHz for the sine test and 2 kHz for impact test. For each measurement a sample of 1.024 s was recorded which gave the spectrum resolution of 0.976 Hz (sine test was performed for 24 sine excitation frequencies). Experimental setup is presented in Fig. 3.

In Fig. 4 system input and response for sine test at frequency of 27.34 Hz are presented (both time history and amplitude spectrum). In Fig. 5 the same characteristics for the impact test are presented. The dashed line in the fourth quarter of the figure corresponds to the filtered response signal.

As the method of modal model parameters estimation consists in simple numerical calculations like local maxima searching and other even simpler ones, the use of MATLAB for that purpose is very convenient. Estimated values of natural frequencies and modal dampings (half power points method) are listed in Table 1. Natural mode shapes are presented in Fig. 6.

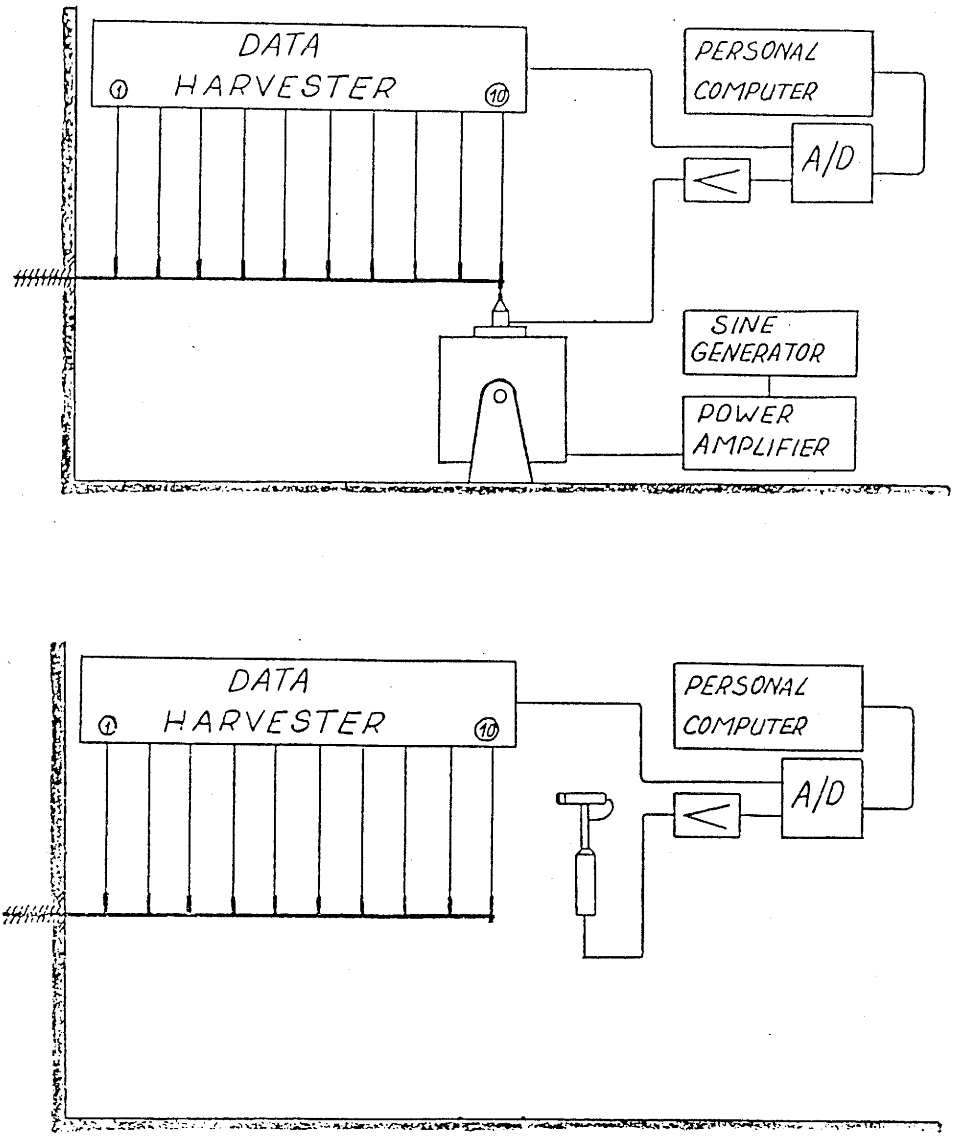


Fig. 3. Experimental setup:

a) for sine test — sine wave generator, power amplifier, exciter, force transducer, conditioning preamplifier, piezoresistive accelerometers, data harvester, A/D converter, personal computer;

b) for impact model test — impact hammer with force transducer, conditioning preamplifier, piezoresistive accelerometers, data harvester, A/D converter, personal computer.

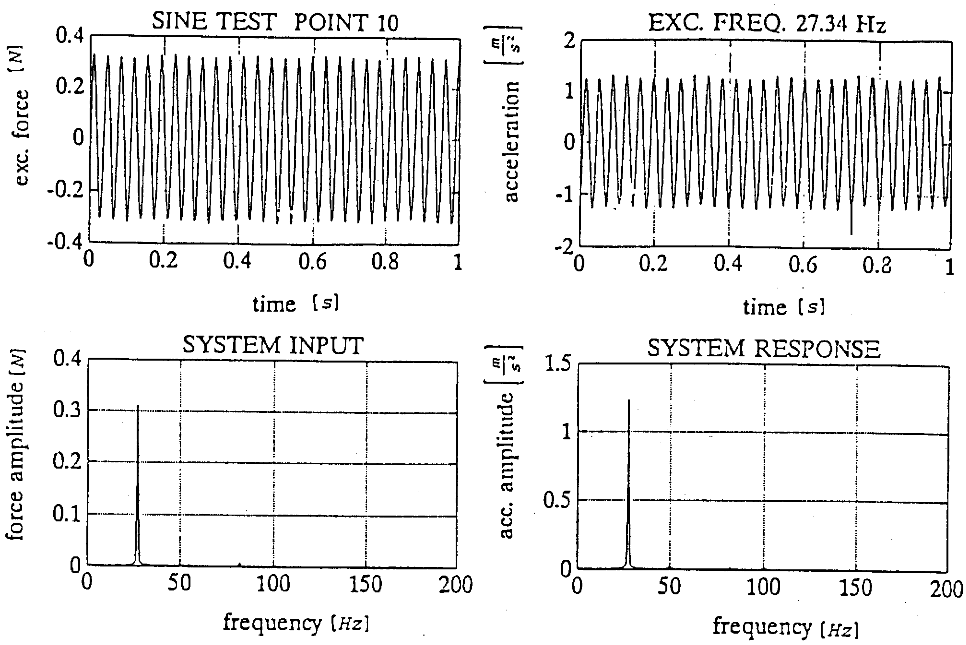


Fig. 4. System input and response for sine test.

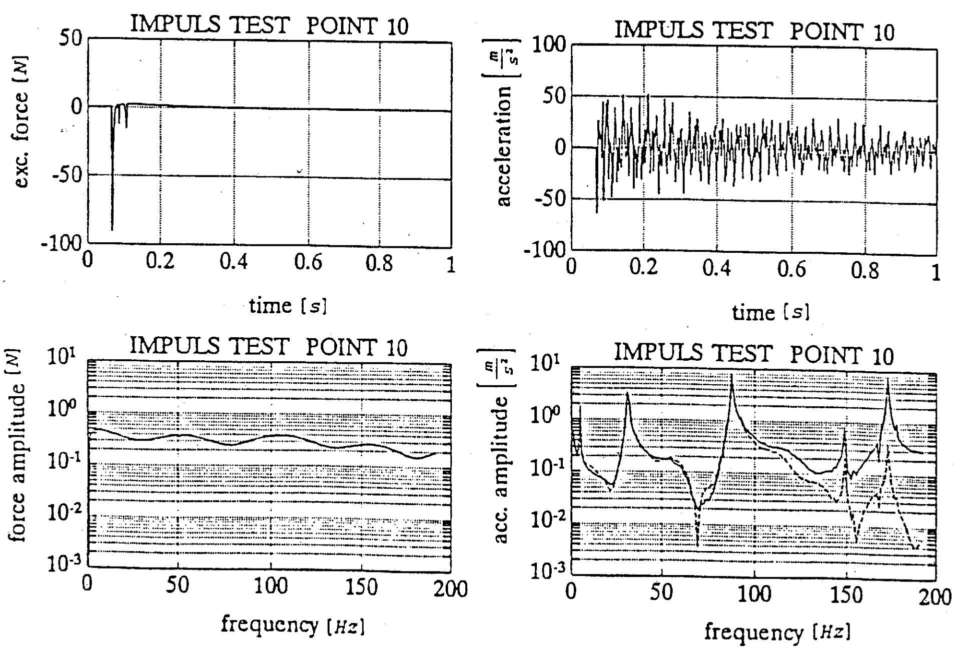


Fig. 5. System input and response for impact test.

Tab. 1. Estimated modal model parameters.

# of mode	natural frequency [Hz]	modal damping [%]
1	4.8	12.50
2	30.9	3.85
3	87.8	1.03

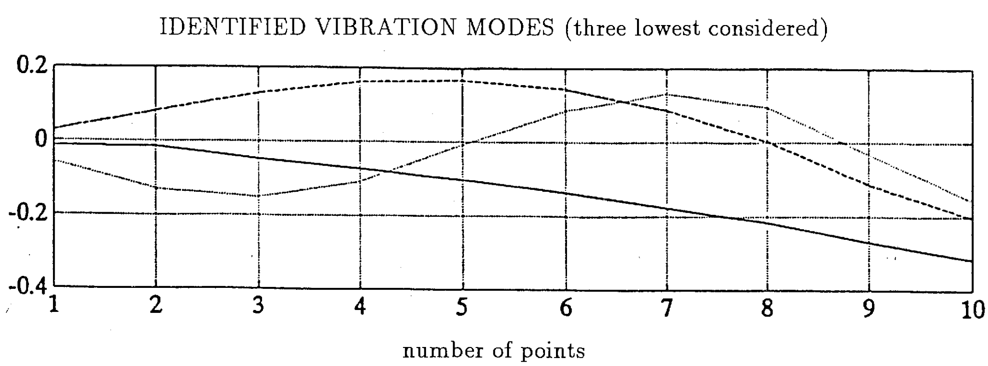


Fig. 6. Natural mode shapes for the considered structure.

Model verification consists in comparison of mobility functions measured and obtained during identification for all measurement points in the frequency range under consideration (see Fig. 7). The comparison for the 10-th measurement point (the free end of the link) is presented in Fig. 8.

As far as the modal modelling aspect of this paper is concerned one can conclude that a good agreement was obtained taking into consideration that the simplest available identification methods were applied. The differences between measured and regenerated mobility functions might arise as a result of the measurement technique applied (proper vibration acceleration measurements for low frequencies is difficult) and inconsistency of the chosen form of modal model with properties of the tested real systems.

4. Remarks

General considerations and the example given here indicate that MATLAB software package can be successfully applied to the modal model identification of mechanical systems. Some of MATLAB features like interactive communication with a user, open structure, computational effectiveness (especially for matrix computations) and easy-to-use graphic display render it a powerful tool for engineering, scientific and educational activities. Finally, the universality and smaller cost than that of dedicated software or hardware should be stressed.

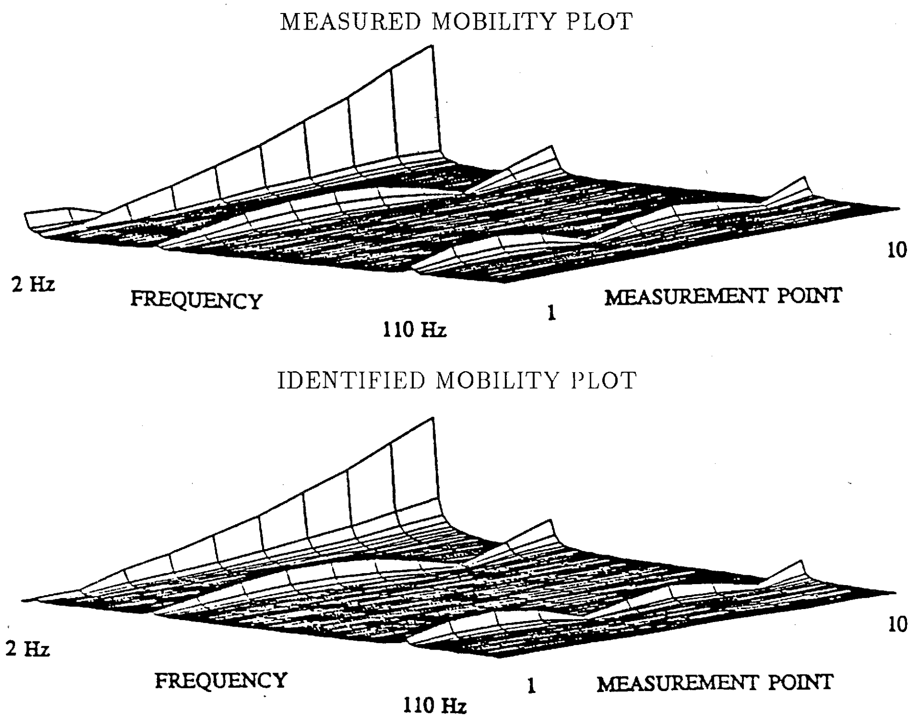
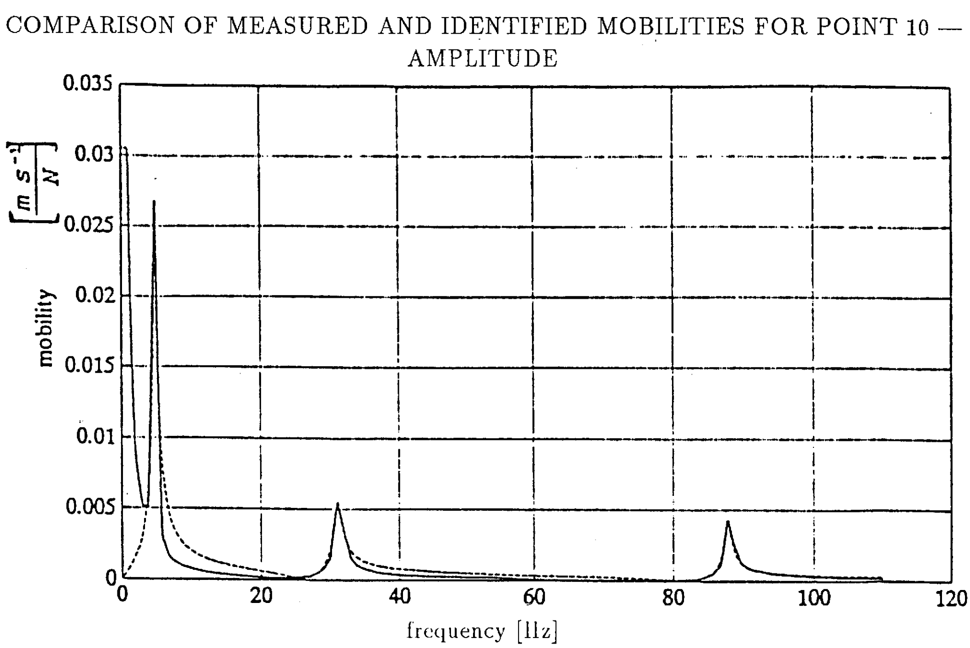


Fig. 7. Comparison of measured and identified mobility functions (amplitude).



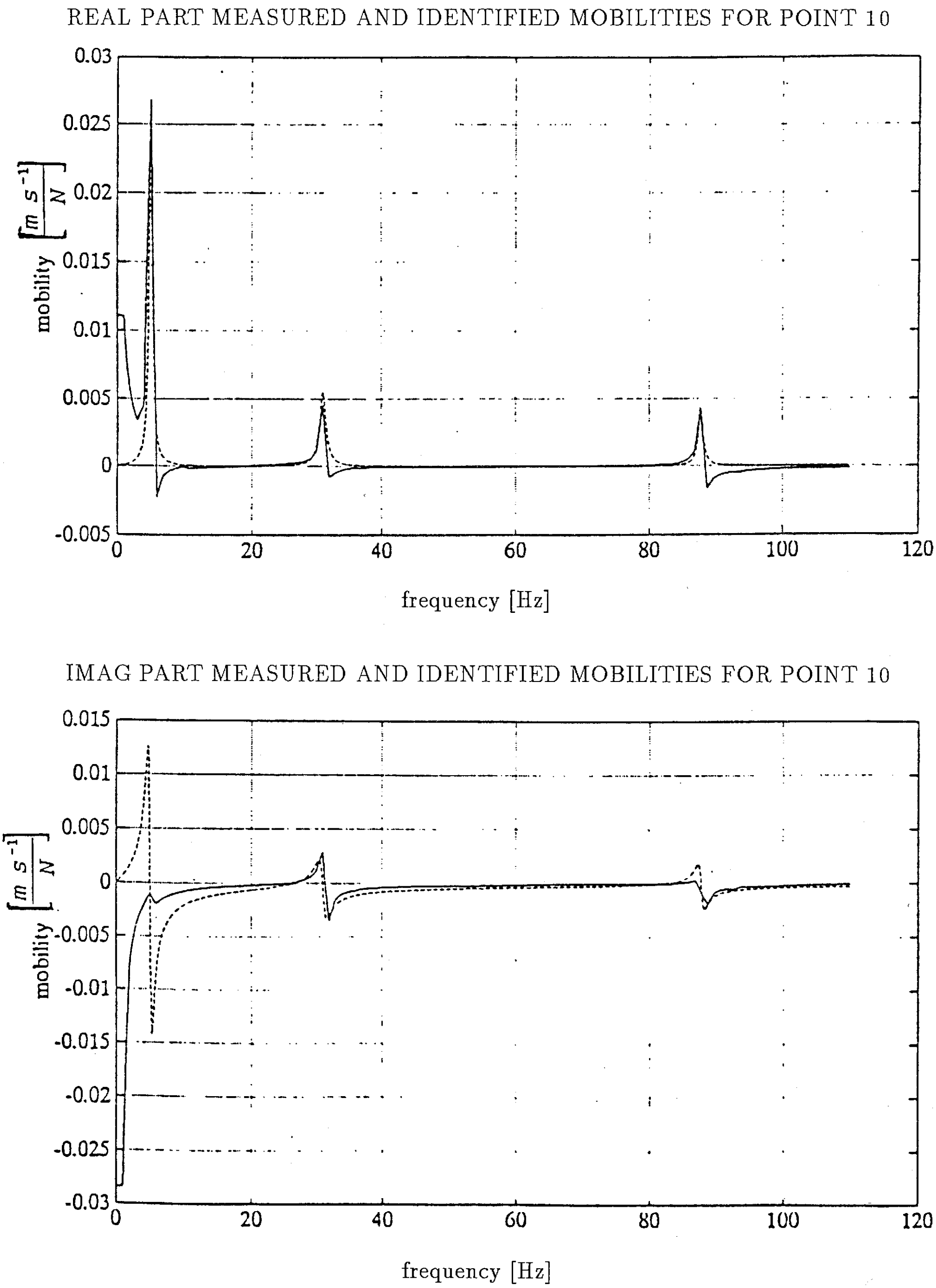


Fig. 8. Comparison of measured and identified mobility functions for the 10-th measurement point (amplitude, real part and imaginary part).

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