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SMOOTH FEEDBACK POSTURE STABILIZATION OF A CLASS OF MOBILE ROBOTS

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The problem of smooth state feedback posture stabilization of the posture kinematic models of a class of restricted mobility robots is addressed. Three techniques are presented: the first two methods (output linearization and Lyapunov design) ensure the posture stabilization with internal stability of the system while the third one (time-varying smooth state feedback) ensures the stabilization of the full state.

1. Introduction

Wheeled mobile robots (WMR) constitute a typical example of mechanical systems with non integrable velocity constraints. A general formalism for such WMR is presented in (Campion *et al.*) where it is shown that, from a kinematic point of view, all WMR can be described by five "posture kinematic models" whose generic structures depend on the number of steering wheels. For restricted mobility robots this posture kinematic model is controllable but not stabilizable by smooth time invariant feedback.

In this paper we restrict ourselves to a class of restricted mobility robots characterized by the fact that the number of degrees of freedom is equal to three (as it is the case for most commercial mobile robots). For such robots the state-space vector of the posture kinematic model involves the posture coordinates, characterizing the position of the robot on the plane of motion, but also internal variables, namely the orientation angles of the steering wheels. From a user's point of view, we are only interested in the control of the robot posture, and not in the control of the internal variables, provided the system is internally stable. Our purpose is to describe several smooth feedback control laws ensuring the posture stabilization. The existence of such control laws is not in contradiction with the fact that the posture kinematic model is not stabilizable by a time invariant smooth state feedback : we do not stabilize the full state vector but only a part of it (the posture coordinates). We present in Sections 2 and 3 static time invariant smooth state feedback laws ensuring the stabilization of the posture, with internal stability of the closed loop system. On the other hand, it has been shown (see Coron, 1992) that controllable driftless systems can always be stabilized using a time- varying smooth state feedback control. In Section 4 we present a systematic procedure introduced in (Pomet, 1992) allowing to stabilize the posture kinematic models of all WMR, using time varying smooth state feedback.

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2. Posture Kinematic Models of Wheeled Mobile Robots

We consider ideal wheeled mobile robots (WMR) made up of a rigid frame equipped with undeformable wheels and moving on a horizontal plane. The posture of the robot, i.e. its position on the plane, is characterized by a vector ξ of three generalized coordinates (see Fig. 1)

$$\xi = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \tag{1}$$

where x, y denote the Cartesian coordinates of a reference material point of the frame, and θ is the robot orientation. Obviously, additional generalized coordinates are necessary to describe the full robot configuration: orientation angles for orientable wheels and rotation angles for all the wheels.

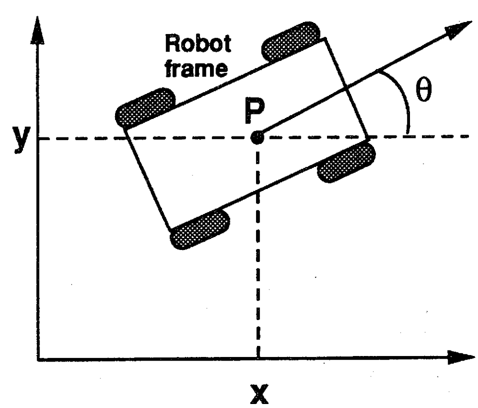

Fig. 1. Posture definition.

It has been shown (see e.g. Campion *et al.*) that, if we are only interested in the kinematic behaviour of the robot posture, it is sufficient to consider the so-called *posture kinematic model*, which takes the following state-space form

$$\begin{aligned} \dot{\xi} &= S(\theta, \beta)u \\ \dot{\beta} &= v \end{aligned} \tag{2}$$

or

$$\begin{pmatrix} \dot{\xi} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} S(\theta,\beta) & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \tag{3}$$

where u represents velocity control inputs, β is an internal state associated to the orientation of the steering wheels (more precisely, β can be defined as the orientation

angles of the wheels that can be steered independently), v represents the inputs associated to the orientation of these independent steering wheels.
The various terms in this model satisfy the following dimensionality requirements:

1. $\dim(u) \stackrel{\Delta}{=} \delta_m$ is the number of degrees of mobility, i.e. the number of degrees of freedom of the posture that can be directly manipulated without reorientation of the steering wheels. Intuitively it corresponds to the number of degrees of freedom the robot could have instantaneously, from its current configuration, without steering any of its wheels, i.e. with frozen β. Obviously $\delta_m \leq 3$.

2. $\dim(\beta) = \dim(v) \stackrel{\Delta}{=} \delta_S$, the number of degrees of steeribility, i.e. the number of wheels that can be steered independently. The non-slipping condition imposes that $\delta_S \leq 2$.

3. $\delta_M = \delta_m + \delta_S$ is the number of degrees of maneuvrability, i.e. the number of degrees of freedom including the orientation angles of the steering wheels.

4. $S(\theta, \beta)$ is a $3 \times \delta_m$ kinematic full rank matrix

According to these definitions it is shown in (Campion *et al.*) that the kinematic posture models of ideal WMR can be classified into five categories, depending on the values of the indices δ_m and δ_S.

The first class is characterized by $\delta_m = 3$, $\delta_S = 0$. Such robots are said to be *omnidirectional.* The model reduces to $\dot{\xi} = S(\theta)u$ where $S(\theta)$ is a square non-singular matrix, which makes the posture control from the inputs u trivial.

The other four classes correspond to the following pairs of values for (δ_m, δ_S): (2,1), (1,2), (2,0), (1,1). Such robots are referred to as *restricted mobility robots.* For such robots $S(\theta, \beta)$ is no more a square matrix, making the posture control problem more difficult. The generic form of the corresponding posture kinematic models is given in (Campion *et al.*).

Several structural properties of the posture kinematic models have been pointed out:

P1. The generic posture kinematic models are *irreducible and strongly accessible.* This property results directly from the fact that the involutive closure of the distribution generated by the columns of the matrix $\begin{pmatrix} S(\theta,\beta) & 0 \\ 0 & I \end{pmatrix}$ involved in (3) is of full rank.

P2. The posture kinematic model of restricted mobility robots with $\delta_m \leq 2$ is *not stabilizable* by a continuous static time-invariant smooth state feedback. This property results from the necessary condition of Brockett (1983).

P3. The posture kinematic model is *stabilizable by a continuous time-varying static state feedback.* This property is a special case of a general stabilizability result for driftless systems (Coron, 1992). In Section 4 we shall use the systematic design procedure proposed by Pomet (Pomet, 1992) to construct such time-varying control laws.

In this paper we address the problem of finding a smooth state feedback control law to drive automatically the robot, from an arbitrary initial posture $\xi(0) \neq 0$ to a final desired posture (without loss of generality) as the origin of the posture space

$$\lim_{t \to \infty} \xi(t) = 0$$

under the following conditions:

- $\dot{\beta}(t)$ and $\beta(t)$ are bounded
- $\lim_{t \to \infty} \dot{\beta}(t) = 0$

For omnidirectional robots the solution to this problem is trivial. For restricted mobility robots (i.e. if $\delta_m \leq 2$) the situation is less favourable. We know, from P3, that there exist *time-varying* smooth feedback posture stabilizing control laws, but we are also interested to achieve the solution using *time-invariant* smooth feedback controls. Clearly, from Property P2, if $\delta_S = 0$, such a control does not exist, because, in this case, the state vector reduces to the posture vector ξ. For robots with $\delta_S \geq 1$ and $\delta_M = 3$, it can be hoped however to achieve posture stabilization, using a time-invariant smooth state feedback, because in this case the posture vector ξ is only a part of the full state vector (ξ, β).

Therefore we restrict ourselves to a subclass of restricted mobility robots characterized by $\delta_M = 3$, i.e. to robots with the pair (δ_m, δ_S) equal either to (2,1) or to (1,2). Notice that most commercial robots on the market belong to the latter class (robots with at least two steering wheels).

A typical example of a (2,1) robot is a robot equipped with one steering wheel and two castor wheels (see Fig. 2). The generic posture kinematic model takes the following form

$$\begin{aligned} \dot{\xi} &= R^T(\theta)\Sigma(\beta)u \\ \dot{\beta} &= v \end{aligned} \tag{4}$$

where $R(\theta)$ is the rotation matrix

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$\Sigma(\beta) = \begin{pmatrix} -\sin\beta & 0 \\ \cos\beta & 0 \\ 0 & 1 \end{pmatrix}, \quad u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{5}$$

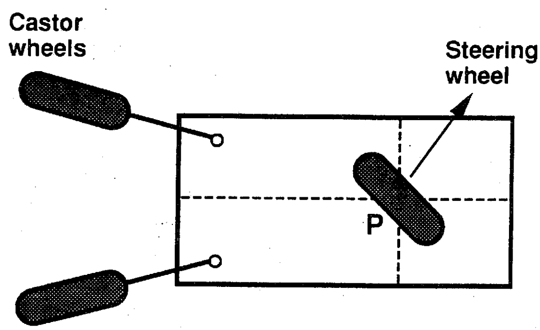

Fig. 2. Example of Type (2,1) robot.

or, explicitly,

$$\begin{aligned} \dot{x} &= -\sin(\theta+\beta)u_1 \\ \dot{y} &= \cos(\theta+\beta)u_1 \\ \dot{\theta} &= u_2 \\ \dot{\beta} &= v \end{aligned} \tag{6}$$

A typical example of a (1,2) robot is a robot equipped with two steering wheels (whose orientations are denoted β_1 and β_2), and a castor wheel (see Fig. 3). The generic posture kinematic model takes the following form

$$\dot{\xi} = R^T(\theta)\Sigma(\beta_1,\beta_2)u \tag{7}$$

$$\dot{\beta}_1 = v_1 \tag{8}$$

$$\dot{\beta}_2 = v_2 \tag{9}$$

where

$$\Sigma(\beta_1,\beta_2) = \begin{pmatrix} -2L\sin\beta_1\sin\beta_2 \\ L\sin(\beta_1+\beta_2) \\ \sin(\beta_2-\beta_1) \end{pmatrix} \tag{10}$$

$2L$ is the distance between the centres of the two steering wheels. More explicitly,

$$\begin{aligned} \dot{x} &= \sigma_1(\theta,\beta_1,\beta_2)u \\ \dot{y} &= \sigma_2(\theta,\beta_1,\beta_2)u \\ \dot{\theta} &= \sigma_3(\beta_1,\beta_2)u \\ \dot{\beta}_1 &= v_1 \\ \dot{\beta}_2 &= v_2 \end{aligned} \tag{11}$$

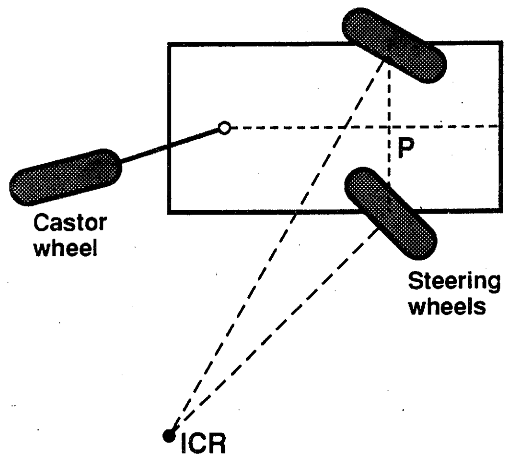

Fig. 3. Example of Type (1,2) robot.

where

$$\sigma_1(\theta, \beta_1, \beta_2) = -L\Big(\sin\beta_1 \sin(\theta + \beta_2) + \sin\beta_2 \sin(\theta + \beta_1)\Big) \tag{12}$$

$$\sigma_2(\theta, \beta_1, \beta_2) = L\Big(\sin\beta_1 \cos(\theta + \beta_2) + \sin\beta_2 \cos(\theta + \beta_1)\Big) \tag{13}$$

$$\sigma_3(\beta_1, \beta_2) = \sin(\beta_2 - \beta_1) \tag{14}$$

We present three solutions to achieve the posture stabilization by smooth static state feedback with internal stability:

- output linearizing feedback (Section 3),
- Lyapunov design (Section 4),
- time-varying smooth feedback (Section 5).

3. Output Linearising Feedback Control

As shown in (d'Andréa-Novel *et al.*) dynamic feedback control allows to fully linearize the system, but the corresponding control becomes singular when the input u_1 (for the (2,1) robot), or u (for the (1,2) robot) is equal to zero, i.e. when the longitudinal velocity of the robot vanishes. This singularity prevents the use of dynamic feedback linearizing control for posture stabilization purpose.

Recently a procedure has been proposed in (Mahony *et al.*, 1995) allowing construction of a smooth static output linearizing feedback control which can nevertheless solve our posture control problem.

We first detail this control algorithm for the Type (2,1) robot whose posture kinematic model is given by (4).

Control algorithm

There are two steps in the algorithm.

Step 1: Choose u_1 and u_2 in order to assign stable linear dynamics for two components of the posture vector, namely y and θ. More precisely, the choice

$$u_1 = \frac{-ay}{\cos(\theta+\beta)} \quad \text{and} \quad u_2 = -a\theta \tag{15}$$

leads to the following dynamics

$$\begin{aligned} \dot{x} &= ay\tan(\theta+\beta) \\ \dot{y} &= -ay \\ \dot{\theta} &= -a\theta \\ \dot{\beta} &= v \end{aligned} \tag{16}$$

This control exists as long as $\cos(\theta+\beta) \neq 0$.

Step 2: Choose v in order to assign the following dynamics to the third component of the posture vector, i.e. to x

$$\ddot{x} + 2b\dot{x} + b^2 x \quad \text{with} \quad b > a > 0 \tag{17}$$

This is achieved by the following choice of v:

$$v = a\theta - (a-2b)\sin(\theta+\beta)\cos(\theta+\beta) - 4\frac{b^2}{a}\frac{x}{y}\cos^2(\theta+\beta) \tag{18}$$

This control v exists as long as $y \neq 0$.

This means that, provided $y \neq 0$ and $\cos(\theta+\beta) \neq 0$, the smooth feedback control defined by (26) and (18) assigns the following closed loop behaviour

$$\ddot{x} + 2b\dot{x} + b^2 x = 0 \tag{19}$$

$$\dot{y} + ay = 0 \tag{20}$$

$$\dot{\theta} + a\theta = 0 \tag{21}$$

$$\dot{\beta} = a\theta + (a-2b)\sin(\theta+\beta)\cos(\theta+\beta) - 4\frac{b^2}{a}\frac{x}{y}\cos^2(\theta+\beta) \tag{22}$$

To simplify the calculations we take $a = 1$ and $b = 2$. This implies that the posture vector trajectory is given by

$$x(t) = \Big[x(0) + (2x(0) + \dot{x}(0))t\Big]e^{-2t} \tag{23}$$

$$y(t) = y(0)e^{-t} \tag{24}$$

$$\theta(t) = \theta(0)e^{-t} \tag{25}$$

The following theorem proves of the convergence of the posture as well as the internal stability of the closed loop.

Theorem 1. *Under the following assumptions:*

$$y(0) \neq 0$$

$$\frac{\pi}{2} + q\pi < \theta(0) + \beta(0) < \frac{\pi}{2} + (q+1)\pi, \ \ i.e. \ \cos(\theta(0) + \beta(0)) \neq 0$$

the feedback control defined by (15)-(18) ensures that:

1. $y(t) \neq 0$ *and* $\cos(\theta(t) + \beta(t)) \neq 0 \ \ \forall t$
2. $\xi(t)$ *converges to zero according to (25)*
3. $\beta(t)$ *converges to* $(q+1)\pi$.

Proof.

1. We first proove by contradiction that the closed loop avoids the singularities $y(t) = 0$ and $\cos(\theta + \beta) = 0$. Suppose that there exists t_1 such that either $y(t_1) = 0$ or $\cos(\theta(t_1) + \beta(t_1)) = 0$, and that the singularities are avoided $\forall t < t_1$, i.e. $\forall t < t_1$ the system evolves according to (22) and (25). Assume first that $y(t_1) = 0$. Then, by continuity, $\forall \varepsilon > 0$, there exists $t_2 < t_1$ such that $y(t_2) = \varepsilon$. Choose $\varepsilon = y(t_0)e^{-t_1} (< y(t_0)e^{-t_2})$, and the contradiction follows immediately from the fact that $y(t_2) = y(t_0)e^{-t_2}$. Assume now that $\cos(\theta(t_1) + \beta(t_1)) = 0$. Without loss of generality we can assume that this singularity corresponds to $\tan(\beta(t_1) + \theta(t_1)) = +\infty$. This means that
$$\forall M > 0, \exists t_2 < t_1 \colon \ \tan(\theta(t) + \beta(t)) > M, \forall t \geq t_2$$
Define $k = \max_{t \leq t_1} \left| \frac{x(0) + (2x(0) + \dot{x}(0))t}{y(0)} e^{-t} \right|$. Then from (22), for $t < t_1$ $\frac{\mathrm{d}}{\mathrm{d}t}(\tan(\theta + \beta)) \leq -3tg(\theta + \beta) + 4k$. Choose $M > \frac{4}{3}k$, and define t_2 such that $\tan(\theta(t_2) + \beta(t_2)) = M$. Then $\forall t (t_2 \leq t < t_1)$, $\frac{\mathrm{d}}{\mathrm{d}t}(\tan(\theta + \beta)) < 0$ which contradicts the fact that $\tan(\theta(t_1) + \beta(t_1)) = +\infty$.
2. This results immediately from (25) because the closed loop avoids the singularities.
3. It results from (22) and (25) that for all t
$$\dot{\theta} + \dot{\beta} = \frac{3}{2} \sin 2(\theta + \beta) - 4\cos^2(\theta + \beta) \frac{x(0) + (2x(0) + \dot{x}(0))t}{y(0)} e^{-t}$$
For $t \to \infty$, there are three stationary points for $(\theta + \beta) : \frac{\pi}{2} + q\pi, (q+1)\pi$ and $\frac{\pi}{2} + (q+1)\pi$. From part 1 we can conclude that the only stable equilibrium is $(\theta + \beta) = (q+1)\pi$. Since θ converges to zero, we conclude that β converges to $(q+1)\pi$.

■

If $y(0)$ is equal to zero, then we use the symmetric algorithm, linearizing in the first step x and θ, and assigning in the second step the dynamics of y.

The control takes the following form:

$$u_1 = \frac{x}{\sin(\theta+\beta)}$$

$$u_2 = -\theta$$

$$v = \theta - 3\cos(\theta+\beta)\sin(\theta+\beta) + 4\frac{y}{x}\sin^2(\theta+\beta)$$

This control exists provided that $x \neq 0$ and $\sin(\theta+\beta) \neq 0$.
Here again, provided that the initial conditions are non-singular (i.e. $x(0) \neq 0$ and $\sin(\theta(0)+\beta(0)) \neq 0$) the closed loop is internally stable and the posture converges to zero.

The only initial conditions at which neither of the above control laws is valid are ($x(0) = 0$ and $\cos(\theta(0)+\beta(0) = 0)$ and ($y(0) = 0$ and $\sin(\theta(0)+\beta(0) = 0)$. These two cases correspond to the situations where the reference point of the robot is located on either axis, with the steering wheel perpendicular to this axis. It is not a real problem: if you start at one of these initial conditions, you have just to modify the wheel orientation before applying one of the two above control laws!

For the **Type (1,2) robot**, the same procedure is applicable.

Step 1: Choose u in order to assign stable linear dynamics to one of the three posture coordinates, say θ:

$$u = \frac{\theta}{\sigma_3} \Longrightarrow \dot{\theta} = -\theta \tag{26}$$

This control exists provided $\quad \sigma_3 \neq 0, \quad$ i.e. $\beta_2 - \beta_1 \neq q\pi$ (27)

Step 2: Choose v_1 and v_2 in order to assign the following dynamics to the remaining two posture coordinates, x and y:

$$\ddot{x} + 4\dot{x} + 4x = 0$$

$$\ddot{y} + 4\dot{y} + 4y = 0$$

The inputs v_1 and v_2 have to satisfy the following conditions:

$$\begin{pmatrix} \sigma_3\frac{\partial\sigma_1}{\partial\beta_1} - \sigma_1\frac{\partial\sigma_3}{\partial\beta_1} & \sigma_3\frac{\partial\sigma_1}{\partial\beta_2} - \sigma_1\frac{\partial\sigma_3}{\partial\beta_2} \\ \sigma_3\frac{\partial\sigma_2}{\partial\beta_1} - \sigma_2\frac{\partial\sigma_3}{\partial\beta_1} & \sigma_3\frac{\partial\sigma_2}{\partial\beta_2} - \sigma_2\frac{\partial\sigma_3}{\partial\beta_2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \frac{\sigma_3^2}{\theta} \begin{pmatrix} -4x - 3L\frac{\sigma_1}{\sigma_3}\theta \\ -4y - 3L\frac{\sigma_2}{\sigma_3}\theta \end{pmatrix} \tag{28}$$

This control exists provided $\theta \neq 0$, and that the matrix involved in (28) is non-singular. This last condition is satisfied if $(\beta_1, \beta_2) \neq (0, 0)$ which is a particular case of condition (27). It is then possible to show, similarly to Theorem 1, that, provided the initial conditions are not singular, the closed loop system will avoid the singularities. This ensures the convergence of the posture to the origin, as well as the internal stability of the closed loop.

It must be pointed out that, strictly speaking, this is not a globally smooth feedback control: the feedback law is not smooth in the whole state space but only in regions separated by singularity surfaces. But the analysis shows that, starting in one of these regions, you remain inside this region and you never switch from one control law to another.

4. Lyapunov Design

In this Section we construct globally smooth static state feedback control laws, stabilizing the posture and ensuring the internal stability of the closed loop system.
We consider only Type (2,1) robots. With the following coordinate transformation:

$$\begin{aligned} z_1 &= x\cos(\theta+\beta) + y\sin(\theta+\beta) \\ z_2 &= -x\sin(\theta+\beta) + y\cos(\theta+\beta) \end{aligned} \tag{29}$$

the system equations can be rewritten as follows:

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{\theta} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} 0 & z_2 & z_2 \\ 1 & -z_1 & -z_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ v \end{pmatrix} \tag{30}$$

The coordinates z_1 and z_2 are the components of the position vector $\vec{OP}$ in a frame attached to the steering wheel. Stabilizing the posture is therefore equivalent to stabilizing (z_1, z_2, θ).

Theorem 2. *The feedback control law*

$$u_1 = -k_1 z_2$$

$$u_2 = -k_2\theta$$

$$v = k_3 z_1 + k_2\theta$$

with $k_1, k_2 > 0$ *and* $k_3 \neq 0$ *ensures that*

a) z_1, z_2, θ *are bounded,*

b) *the vector* (z_1, z_2, θ) *converges to the origin,*

c) $z_1^2 + z_2^2$ *is non-increasing,*

d) $\dot{\beta}$ *is bounded and* $\lim_{t\to\infty} \dot{\beta}(t) = 0$.

Proof. Define the candidate Lyapunov function as follows

$$V(z_1, z_2, \theta) = \frac{1}{2}(z_1^2 + z_2^2 + \theta^2) \tag{31}$$

Then, for the closed loop we have

$$\dot{V} = k_1 z_2^2 - k_2\theta^2 \leq 0 \tag{32}$$

This ensures that z_1, z_2 and θ are bounded and that the system converges to the invariant set defined by $z_2 = 0$ and $\theta = 0$. In this invariant, since $k_3 \neq 0, z_1$ is also equal to zero. This implies a) and b).

On the other hand

$$\frac{\mathrm{d}}{\mathrm{d}t}(z_1^2 + z_2^2) = -k_1 z_2^2 \leq 0$$

This implies that $(z_1^2 + z_2^2)$, and therefore $(x^2 + y^2)$, is not increasing (part c). The result d) follows immediately. ■

5. Time-Varying Smooth Static Feedback

The existence of time-varying smooth static feedback laws stabilizing reachable systems has been proved in (Coron, 1992). Such stabilizing control laws have been used first in (Samson, 1990a; 1990b) for a class of WMR, and then propagated widely in the literature with various design techniques (see e.g. Murray and Sastry, 1993).

We present here the constructive procedure of (Pomet, 1992) which is systematic and applicable to the posture models of all WMR. The method can be summarized as follows.

Consider a driftless system

$$\dot{q} = \sum_{i=1}^{m} f_i(q)u_i \quad \text{with} \quad q \in \mathbb{R}^n \tag{33}$$

Assume that the system is strongly accessible and that $f_1(q) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$.

Construct the following Lyapunov function

$$V(t,q) = \frac{1}{2}\left[q_n^2 + \cos t(q_1^2 + \ldots + q_{n-1}^2)\right]^2 + \frac{1}{2}\left(q_1^2 + \ldots + q_{n-1}^2\right)$$

Then the following feedback control

$$u_n(q,t) = \sin t(q_1^2 + \ldots + q_{n-1}^2) - \frac{\partial V}{\partial q_n}(q,t)$$

$$u_i(q,t) = -\frac{\partial V}{\partial q} f_i(q,t) \quad i = 1, \ldots, n-1$$

ensures that

a) $V(q,t)$ is non-increasing along the closed loop trajectories,

b) the closed loop system is uniformly asymptotically stable.

The application of this procedure to our mobile robots is straightforward and ensures that the state vector (ξ, β) converges to zero.

Type (2,1) robot

The column of the input matrix associated with the input v is equal to $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$. We then define

$$V(t,x,y,\theta,\beta) = \frac{1}{2}\left(\beta + (x^2+y^2+\theta^2)\cos t\right)^2 + \frac{1}{2}(x^2+y^2+\theta^2)$$

The feedback control is given by

$$\begin{aligned} u_1(t,x,y,\theta,\beta) &= \frac{\partial V}{\partial x}\sin(\theta+\beta) - \frac{\partial V}{\partial y}\cos(\theta+\beta) \\ &= \left[1 + 2\cos t(\beta + (x^2+y^2+\theta^2)\cos t)\right] \\ &\times \left[x\sin(\theta+\beta) - y\cos(\theta+\beta)\right] \end{aligned}$$

$$u_2(t,x,y,\theta,\beta) = -\frac{\partial V}{\partial \theta} = -\theta[1 + 2\cos t(\beta + (x^2+y^2+\theta^2)\cos t)]$$

$$\begin{aligned} v(t,x,y,\theta,\beta) &= \sin t(x^2+y^2+\theta^2) - \frac{\partial V}{\partial \beta} \\ &= (x^2+y^2+\theta^2)(\sin t - \cos t) - \beta \end{aligned}$$

Type (1,2) robot

In the input matrix, the two columns associated with the inputs v_1 and v_2 have the required form and can therefore be selected to play the role of the vector field f_1 in the procedure. Select e.g. the column associated with v_1.

The corresponding Lyapunov function reduces to

$$\begin{aligned} V(t,x,y,\theta,\beta_1,\beta_2) = &\frac{1}{2}\left[\beta_1^2 + (x^2+y^2+\theta^2+\beta_2^2)\cos t\right]^2 \\ &+\frac{1}{2}(x^2+y^2+\theta^2+\beta_2^2) \end{aligned} \tag{34}$$

and the feedback control is given by

$$\begin{aligned} u(t,x,y,\theta,\beta_1,\beta_2) &= -\left(\frac{\partial V}{\partial x}\sigma_1 + \frac{\partial V}{\partial y}\sigma_2 + \frac{\partial V}{\partial \theta}\sigma_3\right) \\ &= -(1+2\cos t)(x\sigma_1 + y\sigma_2 + \theta\sigma_3) \\ &\times \left[\beta_1^2 + (x^2+y^2+\theta^2+\beta_2^2)\cos t\right] \end{aligned} \tag{35}$$

$$\begin{aligned} v_1(t,x,y,\theta,\beta_1,\beta_2) &= \sin t(x^2+y^2+\theta^2+\beta_2^2) - \frac{\partial V}{\partial \beta_1} \\ &= \sin t(x^2+y^2+\theta^2+\beta_2^2) \\ &- 2\beta_1\left[\beta_1^2 + (x^2+y^2+\theta^2+\beta_2^2)\cos t)\right] \end{aligned} \tag{36}$$

$$\begin{aligned} v_2(t,x,y,\theta,\beta_1,\beta_2) &= -\frac{\partial V}{\partial \beta_2} = -\beta_2\left[1 + 2\cos t\beta_1^2\right. \\ &+ \left. 2(x^2+y^2+\theta^2+\beta_2^2)\cos^2 t\right] \end{aligned} \tag{37}$$

where σ_1, σ_2 and σ_3 are defined in (12), (13), (14).

It must be noticed that these control laws ensure the stabilization of the full state and not only of the posture.

6. Conclusion

Most commercial mobile robots have three degrees of freedom. For such robots, we have described in this paper three different methods for the feedback control of the posture with internal stability when the objective is to automatically drive the robot from an arbitrary initial posture to a given target posture.

We have shown in particular how this posture control problem can be solved by a static time invariant smooth state feedback.

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