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OPTIMAL TRAJECTORY DESIGN FOR THE IDENTIFICATION OF ROBOT AND LOAD DYNAMICS†

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In this paper, we discuss the issue of the optimal trajectory design for the purpose of identification of the inertial and load parameters of a robot. Two dynamical models have been introduced: differential and integral; both of them can be used to estimate the robot dynamic parameters in which they appear to be linear. Functions associated with the parameters are highly non-linear which in general involve joint positions, velocities and accelerations. A standard sequential least-squares technique can be used to estimate the unknown parameters. This method requires an information matrix which has to be inverted. For particular joint signals this matrix is singular. In this paper, we propose to optimize the condition number of this matrix in both formulations: differential and integral. This work illustrates the difficulty of maintaining persistent excitation during the experimental identification of robot and load dynamic parameters.

1. Introduction

Experimental identification of robot and load dynamic parameters is an important problem in model-based robot dynamics algorithms. Robot and load dynamic parameters, namely: mass, centre of mass, inertia tensor, and friction parameters for each link robot, appear to be constant coefficients or linear combinations of constant coefficients in the dynamic model of a robot. Due to this property, any least-squares technique can be used for the purpose of identification. Unfortunately, experimental identification is not as simple as it looks at first sight. Robot dynamic equations are functions of joint positions, velocities, and accelerations. The information matrix which appears in the least-squares scheme depends on these quantities and cannot be easily inverted. Its inversion depends on the shape of the joint positions, velocities, and acceleration assuming that the model considered is canonical. Canonical models are expressed in terms of a set of base parameters which are linearly independent. As shown in the paper, in order to design the optimal trajectory for the purpose of identification of the base inertial parameters a systematic approach has to be implemented. A trial and error method of choosing the optimal trajectory does not always give good results. In this paper, we try to overcome this difficulty for differential and integral models for both robot and load dynamic parameters.

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Robot dynamic models can be derived based on the Newton-Euler (Dutkiewicz *et al.*, 1993a) or Lagrangian formalism (Kozłowski, 1992). It is well-known that both approaches lead to the same dynamic model. Generally, as it was mentioned above, we can consider differential or integral models. The first ones are the same as the standard equations of motion for robot dynamics. This model can be written in the following compact form (Kozłowski and Dutkiewicz, 1995):

$$\tau = D(q, \dot{q}, \ddot{q})X + \tau_f \tag{1}$$

where $q, \dot{q}, \ddot{q}$, are the vectors of generalized positions, velocities, and accelerations, respectively. The quantity D is an observation matrix which depends on signals $q, \dot{q}, \ddot{q}$, τ is the vector of actuating torques at the joints, X is the vector of inertial parameters, and τ_f is the vector of friction torques at the joints. The friction torques depend on the generalized positions, velocities, direction of rotation, temperature, and other factors which are very difficult to describe in the analytical form (Dutkiewicz *et al.*, 1993b; Lu *et al.*, 1993; Seeger and Leonhard, 1989; Seeger, 1991).

It is assumed that the model described by eqn. (1) is canonical, which means that the vector of X parameters consists of the minimum number of parameters which are combinations of the link inertial parameters (namely mass, and first and second moments of the individual links). Integral models are derived based on the energy theorem which states that the work of the forces which are applied to the system, and are not derived from a potential, is equal to the change of the total energy of the system, thus (Gautier *et al.*, 1994; 1995; Prüfer *et al.*, 1994; Willems, 1972):

$$\int_{t_1}^{t_2} \tau^T \dot{q}\, \mathrm{d}t = d(q, \dot{q})^T X + \int_{t_1}^{t_2} \tau_f^T \dot{q}\, \mathrm{d}t \tag{2}$$

where $d(q, \dot{q})$ is an observation vector which depends on the generalized positions and velocities, the integral on the left-hand side of eqn. (2) represents the total energy which goes to the system, the integral on the right-hand side represents the total energy lost in friction phenomena, and t_1 and t_2 represent two distinct time moments at which the total energy of the system is calculated. It is also assumed that the model represented by eqn. (2) is canonical. Note that both the differential and integral models have the same set of the minimum number of the inertial parameters which are a combination of the inertial parameters of the individual links. In both representations it is assumed that the friction torques τ_f are represented in the compact form and we do not look for the friction coefficients which appear in the friction model but rather for the friction torques in general.

It is usually difficult to measure the friction torques for both differential and integral models. Seeger (1991) proposed a method to measure the friction torques for a class of geared robots, taking PUMA 560 robot as an example. For the same class of robots but described by an integral model Kozłowski and Dutkiewicz (1995) proposed a method to measure the friction torques.

Generally speaking, friction models are difficult to identify. This is due to the fact that the friction torque depends on many parameters, e.g. the temperature, direction of rotation, etc. Besides, the friction torque is non-linear with respect to some parameters. Researches usually introduce two friction coefficients: Coulomb and viscous, but in many practical situations we have to deal with more friction parameters. In spite of that, taking corresponding $q, \dot{q}, \ddot{q}$ functions associated with friction coefficients, we have found that they are becoming linearly dependent for a wide class of trajectories. This greatly complicates the situation. Integrating d functions over very short time intervals improves the situation but still the results are prone to be linearly dependent. In such cases, a necessary condition for identification is not satisfied, and we are not dealing with the canonical model with respect to friction coefficients. Therefore, in order to avoid such a situation, we propose to measure the friction torque directly and separate it in eqn. (2) as an individual term.

So far only a few experimental studies on *off-line* robot dynamics estimation have been analysed in the robotics literature (cf. e.g. Atkeson *et al.*, 1986; Caccavale and Chiacchio, 1994a; 1994b; Dutkiewicz *et al.*, 1993b; Lu *et al.*, 1993; Presse and Gautier, 1991; Schaefers *et al.*, 1994; Szynkiewicz *et al.*, 1990; van der Linden and van der Weiden, 1994). This is because most of the industrial robots do not have position, velocity, acceleration, and force and torque sensors which are necessary to perform the identification experiments. Robots usually are equipped with position sensors. In order to get velocity and acceleration signals one can differentiate these signals, but this leads to some numerical errors. Not many papers are devoted to the identification of load parameters (namely: mass, centre of mass, and six parameters of the inertia tensor). Some results can be found in (Atkeson *et al.*, 1986) and (Dutkiewicz *et al.*, 1993a). In order to carry out these experiments, the robot has to be equipped with a force and torque sensor.

Some remarks concerning a comparison of the differential and integral models can be found in the work done by Prüfer *et al.* (1994). In this paper, we extend these results by considering the design of the optimal trajectory for both types of models. Generally, one can notice that the differential model is richer in information since all equations for the generalized torques are present. In the case of the integral model (energy model), we deal only with one scalar equation. Because of that, the optimal trajectory design for the integral model is crucial. It has been noticed that the identification results in the case of the integral model appear to be not very sensitive to filtering measurements because of its natural low-pass filter behaviour. Comparing both models from the measurement point of view one can notice that in the case of the integral model the acceleration signals are not required. In order to avoid the acceleration signals in the differential model one can integrate the differential model but it is not preferred because an integrator is an infinite-gain filter at zero frequency (Lu *et al.*, 1993). This means that large errors can result from small low-frequency errors such as offsets. To overcome this shortcoming, a low-pass filter with unit gain at zero frequency can be applied to the differential model (cf. Lu *et al.*, 1993; Schaefers *et al.*, 1994). In this paper, we rather focus our attention on the optimal trajectory design for both models, under discussion.

Different criteria can be used to optimize the input trajectory for the identification experiment. Exciting signals for single-input single-output linear systems were considered by Marrels *et al.* (1987). They proposed to maximize for comparison a minimum singular value, condition number and determinant of the information matrix.

The same criteria can be used for non-linear systems but one has to be careful with generalization of the results for linear systems. The first considerations on finding exciting trajectories for the identification of the dynamic parameters of a robot were given by Armstrong (1989). He suggested to minimize the condition number (Klema and Laub, 1980) or the reciprocal of the minimum singular value of the information matrix. Vandanjon *et al.* (1995) proposed the minimization of the Frobenius condition number of the information matrix. A comparison of different criteria of exciting trajectories for robot identification were considered by Presse and Gautier (1993), Gautier and Khalil (1992) or Presse and Gautier (1991). They proposed a criterion which takes into account *a priori* information about the measurement vector and *a priori* knowledge of the solution. Most of the authors propose to use minimization of the condition number of the information matrix as a criterion of the exciting trajectories for robot identification. Due to lack of *a priori* information mentioned above we have decided to use the condition number as a criterion. We have implemented this criterion for both integral and differential models. Both models are considered for robot parameter identification. For load identification we have used the differential model with the appropriate optimization scheme. The optimization scheme follows that presented by Armstrong (1989) with the extension to the integral model. Some preliminary results obtained by the authors are presented in Dutkiewicz and Kozłowski (1994).

The paper is organized as follows. In Section 2 we review the least-squares technique for dynamic parameter identification. In the next section we describe the optimization procedure. Design of the exciting trajectories for the IRp-6 robot for the purpose of robot and load dynamic parameters is presented in Section 4. Identification of the inertial parameters with exciting trajectories is discussed in Section 5. The last section presents concluding remarks.

2. Identification Scheme

In this section, we review the least-squares method. It is assumed that differential and integral models can be used. In both situations the standard least-squares method can be used. We try to keep the considerations in this section to be general and applicable to many situations in the process of identification in the area of robotics.

Now recall eqns. (1) and (2). Equation (1) is written in the vector form. For comparison, eqn. (2) is a single one. The quantity D in eqn. (1) is a matrix function which depends on joint positions, velocities, and accelerations and d depends only on joint positions and velocities. These circumstances do not impose restrictions on the proposed identification method. As was mentioned, in both formulations we have the same vector of inertial parameters to be estimated. It is assumed that the number of parameters is minimal, so the model is canonical. These parameters are combinations of the inertial parameters of individual links. Note that in eqn. (1) the parameters X appear in each equation of the joint torque for individual links. It may happen that some functions associated with the elements of the vector X are zero in equations for the joint torques.

From now on, we will assume that we take only one equation from the set of equations for joint torques. For the integral model we have by definition only one equation. Notice that this is not a limitation because in the process of identification we have to take many equations at discrete time constant anyway, in order to minimize the estimation error. We can choose either all the joint torques or only some of them or only one. We have to remember that the functions associated with the vector of parameters are different in each equation for the torque. The i-th equation can be written as follows:

$$\tau_i - \tau_{f_i} = \Phi_i^T \left(q, \dot{q}, \ddot{q}\right) X + w_i \tag{3}$$

In the last equations, Φ_i is a vector of the basic functions for the arbitrary i-th joint torque, which in general depends on joint positions, velocities and accelerations. In the sequel, we will omit the arguments $q, \dot{q}, \ddot{q}$. In eqn. (3), w_i represents the observation error. The quantity τ_{fi} is a precomputed friction torque (cf. Kozłowski and Dutkiewicz, 1995).

The least-squares method applied to eqn. (3) leads to the minimization of the mean-squared observation error

$$\hat{X} = \min_X \left[\tau_i - \tau_{f_i} - \Phi_i^T X\right]^2 \tag{4}$$

The solution to this problem is given by

$$\hat{X} = \left(\Phi_i \Phi_i^T\right)^{-1} \Phi_i \left(\tau_i - \tau_{f_i}\right) \tag{5}$$

Assuming that we collect K equations of type (3) discretized with a sampling time ΔT, we write the solution to eqn. (4) in the recurrence form as follows:

$$\hat{X}_{k+1} = \hat{X}_k + P_{k+1} \Phi_i(k\Delta T) \left[\tau_i(k\Delta T) - \tau_{f_i}(k\Delta T) - \Phi_i^T(k\Delta T)\hat{X}_k\right] \tag{6}$$

$$P_{k+1} = P_k - \left(1 + \Phi_i^T(k\Delta T) P_k \Phi_i(k\Delta T)\right)^{-1} P_k \Phi_i(k\Delta T) \Phi_i^T(k\Delta T) P_k \tag{7}$$

where

$$P_k = \left(\sum_{j=1}^{k} \Phi_i(j\Delta T) \Phi_i(j\Delta T)\right)^{-1} \tag{8}$$

is the information matrix. In the process of calculation this matrix has to be inverted, so it has to be non-singular. The information matrix depends on the measured signals, joint positions, velocities, and accelerations. These signals are usually noisy. It is difficult to extract the noise signals as one additive noise signal in eqn. (3) due to the fact that the basic functions are highly non-linear. Therefore w_i in eqn. (3) represents rather the measurement error associated with the torque measurements. Several simulation experiments were run (cf. Kozłowski, 1992), assuming that different signals were corrupted by Gaussian noise with known characteristics. The simulation results showed that the least-squares method is robust and handles this situation very well. In most cases the estimates of the parameters were unbiased.

Based on these observations one can notice that the trajectory chosen has to be such that the information matrix can be inverted. The requirement for sufficient excitation is well-known from the identification literature and can be formulated as follows:

$$\exists\,\alpha,\beta > 0 \forall k: \quad \alpha I < \sum_{j=k}^{k+p} \Phi_i(j\Delta T)\Phi_i^T(j\Delta T) < \beta I \tag{9}$$

where I is the identity matrix. Equation (9) says that the information matrix is positive definite over each sufficiently long portion of the trajectory. This requirement is stronger: α must be not only positive, it must be reasonable large.

3. Optimization Procedure

In this section, we recall results presented by Armstrong (1989) who suggested to minimize the condition number of the information matrix. Some authors call this matrix Persistent Excitation one. This is a typical non-linear path optimization problem (Bryson and Ho, 1975). The cost function, as suggested by Armstrong (1989), is most naturally started at the end of trajectory. This condition cannot be evaluated knowing only the terminal manipulator state; it is necessary to know each of the P_K matrix elements which are products of the basis functions — the elements of the vector Φ_i. The cost function to be optimized is written as

$$J = F(P_K) = \sum_k J(k\Delta T) \tag{10}$$

where F may be the condition number or may be the reciprocal of the minimum singular value of the P_K matrix (cost function), and $J(k\Delta T)$ is a cost function evaluated at the stage k. Now we apply an algorithm similar to that of Bryson and Ho (1975). First we calculate the differential of the cost function J

$$\mathrm{d}J = \sum_m \sum_n \frac{\partial J}{\partial P_{mn}} \sum_k \sum_i \sum_r \frac{\partial P_{mn}}{\partial\, \Phi_{ir}(k\Delta T)} \mathrm{d}\Phi_{ir}(k\Delta T) \tag{11}$$

where P_{mn} stands for the mn-th element of the matrix P_K, i is the current number of the vector of basis functions in general, and r is the current number of basis functions (R is the total number of basis functions). Assuming that the upper index of summation in eqn. (8) is equal to K (which is consistent with the assumption that we calculate the cost function at the end of the trajectory), the partial derivatives of the elements P_{mn} are given by

$$\frac{\partial P_{mn}}{\partial \Phi_{ir}(k\Delta T)} = \begin{cases} 0 & \text{for } r \neq m, n \\ \dfrac{1}{K}\Phi_{in}(k\Delta T) & \text{for } r = m \\ \dfrac{1}{K}\Phi_{im}(k\Delta T) & \text{for } r = n \\ \dfrac{2}{K}\Phi_{im}(k\Delta T) & \text{for } r = m = n \end{cases} \tag{12}$$

Finally, $d\Phi_{ir}(k\Delta T)$ in eqn. (11) stands for the differential of the r-th basis function at the point $k\Delta T$.

The Hamiltonian, which will be minimized by a gradient search, is formed by adjoining the Lagrange multipliers to the cost function $J(k\Delta T)$, giving (Bryson and Ho, 1975)

$$H(k\Delta T) = J(k\Delta T) + \lambda^T\Big[(k+1)\Delta T\Big] f(k\Delta T) \tag{13}$$

where f is a discrete-time function which depends in general on joint positions, velocities and accelerations at the discrete time $(k\Delta T)$. If we denote by $X = [q, \dot{q}]$ the vector of joint position and velocities, then f in the discrete time can be written as follows:

$$f(k\Delta T) = f\Big[X(k\Delta T), \ddot{q}(k\Delta T)\Big] \tag{14}$$

In the optimization, the trajectory is specified as a sequence of accelerations $\ddot{q}$. At the beginning of the optimization procedure an initial trajectory is specified and then the following calculations are performed

$$\begin{aligned} &\lambda(K) = 0 \\ &\lambda^T(k\Delta T) = \frac{\partial J(X,\ddot{q})}{\partial X(k\Delta T)} + \lambda^T\big((k+1)\Delta T\big)\frac{\partial f(X,\ddot{q})}{\partial X(k\Delta T)} \\ &\ddot{q}(k\Delta T)_{m+1} = \ddot{q}(k\Delta T)_m - \mu_{cv}\left(\frac{\partial J}{\partial \ddot{q}(\Delta T)} + \lambda^T(k\Delta T)\frac{\partial f}{\partial \ddot{q}(k\Delta T)}\right)^T \end{aligned} \tag{15}$$

In the above expressions m stands for the number of iterations over the whole trajectory (assuming that one iteration consists of sweeping the trajectory through all discrete time points K).

To complete the solution we have to specify the function f:

$$f\Big[(k+1)\Delta T\Big] = AX(k\Delta T) + B\ddot{q}(k\Delta T) \tag{16}$$

where A and B are matrices which depend on ΔT and ΔT^2 (Dutkiewicz and Kozłowski, 1994). From the last expression the partial derivatives with respect to $X(k\Delta T)$ and $\ddot{q}(k\Delta T)$ can easily be calculated. Coefficients of the gain μ_{cv}, which is a diagonal matrix, are chosen by observing the differential dP_K, which in the calculation procedure is approximated by ΔP_K. All the partial derivatives which cannot be calculated directly are calculated numerically. In the case of the integral model, the state vector represents positions, and the control vector represents velocities.

4. Numerical Results

In (Dutkiewicz *et al.*, 1993a) we showed experimental results for the dynamic and load parameters identification of the IRp-6 robot. We identified inertial parameters of the robot by making use of the integral model. It was not possible to estimate all the inertial parameters in one experiment (by inputting one test trajectory for all joints). These difficulties were caused by linearly dependent basis functions for certain regions and because it was not possible to move all the joints very fast. Therefore we decided to move one joint at a time while keeping the others idle. Then we moved two joints with constant velocities. Combining the results obtained from different movements it was possible to identify all the inertial parameters. Next we applied the differential model for the purpose of identification. The results were almost the same.

To improve the accuracy of the identified parameters we ran a program which calculats optimal trajectories according to the scheme presented in Section 3. First, we implemented differential model calculations for all the joints of the IRp-6 robot. The initial value of the condition number for a test trajectory was 10^{14}. After 90 iterations the condition number was 10^{13}. The optimization procedure for these iterations took about 20 hours on a PC/486 computer (the programs were written in Pascal). Generally speaking, for three joints of the IRp-6 robot, the optimization procedure did not produce satisfactory results. Next we considered eqn. (3) for $i = 1$. We assumed that the starting trajectories were spline polynomials for the first and second joints and the cosine function for the third joint. The initial value of the condition number was $1.9\,10^5$, and after 88 iterations its value was about 88. The optimal trajectory was calculated at 300 points. Coefficients μ_{cv} which appear in eqn. (15) were chosen as the reciprocal of the maximum value of the Lagrange coefficients λ. The results of the optimization procedure are shown in Figs. 1, 2, and 3.

Next we performed numerical calculations for the integral model. In that case we made use of the Mathematica package (Wolfram, 1992) and ran all programs on a PC. The results for three joints were not satisfactory. Therefore we carried out several experiments moving only one joint while keeping the other joints idle.

For numerical experiments we used eqn. (2). Moving only the second joint while not actuating the first and second joints, and assuming a cosine input trajectory for the second joint we got the initial value of the condition number of about 3000. After about 23 iterations this value changed to 500. The starting and optimal trajectories in this case are shown in Figs. 4, 5 and 6.

Note that in the case of the integral model we did not neglect the friction coefficients in eqn. (2). Most of the authors (e.g. Prüfer *et al.*, 1994) neglect the friction coefficients both in the identification scheme and in the optimization procedure. The friction coefficients cause linear dependence on the parameters which was discussed in Section 3.

Finally, we performed numerical calculations for optimal trajectories for load identification. Here we developed a software package written in C++ language by making use of object-oriented programming techniques. In that case we carried out several numerical experiments by making use of different initial trajectories. We tried spline polynomial trajectories, cosine, and many others. It was more difficult to find a good trajectory due to the fact that for load identification we had to choose five joint positions q_i. The problem is larger in size and takes more computation time. Generally speaking it is more difficult to identify the load parameters due to the

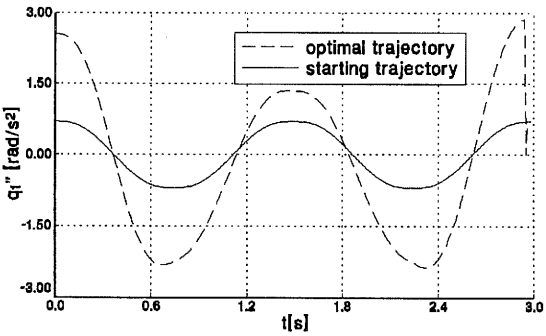

Fig. 1. Starting and optimal trajectories for the first joint; differential model.

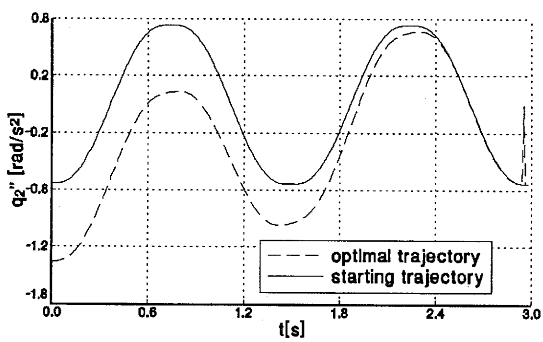

Fig. 2. Starting and optimal trajectories for the second joint; differential model.

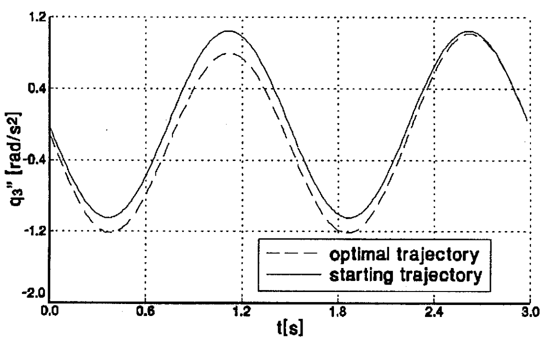

Fig. 3. Starting and optimal trajectories for the third joint; differential model.

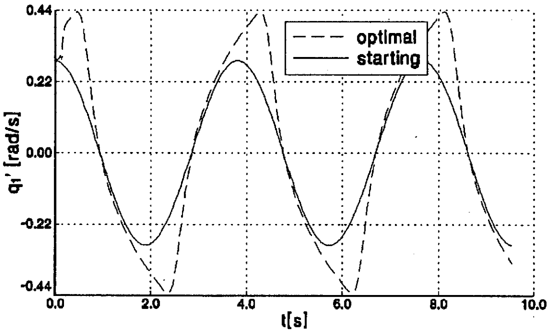

Fig. 4. Starting and optimal trajectories for the first joint; integral model.

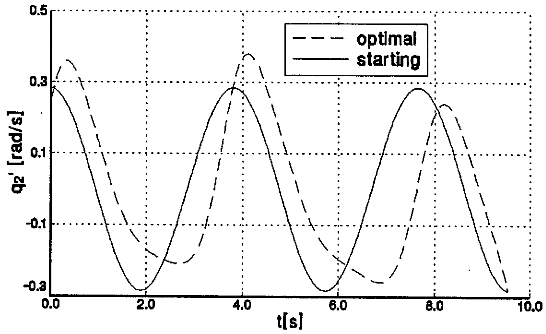

Fig. 5. Starting and optimal trajectories for the second joint; integral model.

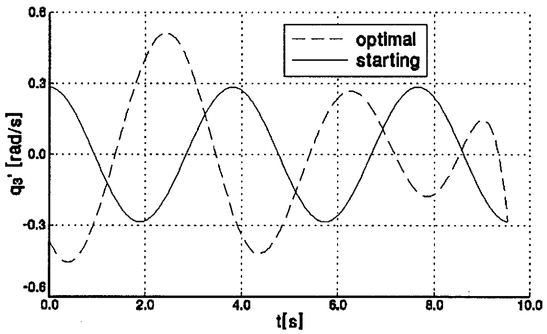

Fig. 6. Starting and optimal trajectories for the third joint; integral model.

fact that joint positions, velocities, and accelerations are very much limited for the IRp-6 robot. The best results were obtained by using the following joint positions (from which joint accelerations required as the input trajectories for the optimization procedure can easily be calculated, cf. eqn. (15)):

$$q_1 = 12 - 0.7\cos(0.4\pi + 1.7\alpha) + 0.8\sin(0.1\pi + 0.8\alpha)$$

$$q_2 = 4 + 1.8\cos(0.4\pi + 1.7\alpha) + 1.3\cos(0.1\pi + 0.3\alpha)$$

$$q_3 = 8 + \cos(0.45\pi + 1.4\alpha) + 0.4\sin(0.4\pi + 0.4\alpha) + 0.7\sin(0.8\pi + 0.6\alpha)$$

$$q_4 = -13 - 1.2\sin(0.45\pi + 1.7\alpha) + 1.8\cos(0.15\pi + 0.7\alpha)$$

$$q_5 = -9 + 2.6\cos(0.15\pi + 2\alpha) - 1.1\sin(0.45\pi + 0.2\alpha)$$

where $\alpha = 2\pi i/(K-1)$, $K = 200$, $i = 0, 1, \ldots, K-1$.

The initial value of the condition number for the above set of trajectories was 90830 and after 40 iterations it fell to about 3000. Starting from this point it was not possible to improve the condition number. For each trajectory we calculated 200 points with the sampling interval $\Delta t = 32\,\text{ms}$. The optimization procedure took about 60 hours of computation time on a PC/486 computer. The numerical results are shown in Figs. 7, 8, 9, 10, and 11.

Finally, we can say that the optimization procedure gave good results. The exciting trajectories were implemented for the identification of robot inertial parameters which is shown in the next section.

5. Experimental Results

In this section, we present experimental results of an identification of the inertial parameters of the IRp-6 robot. These results were reported by Dutkiewicz *et al.* (1993a). The trajectories were chosen in an intuitive way. Nevertheless the results were good in the sense that the computed and predicted torques matched well. We implemented exciting trajectories obtained in Section 4 for the purpose of identification. We tried both differential and integral models. Here we present only the results for one joint for the integral model. The integral model is of interest particularly because of its friction coefficients. Some authors (Prüfer *et al.*, 1994) claim that it is difficult to identify the friction coefficients due to the fact that they easily become linearly dependent in the identification process. We observed that this was not necessarily the case. Using eqn. (2) and moving only the first joint we got the following aggregated parameters and the corresponding d functions

$$X_1 = I_{1zz} + I_{a1}n_1^2 + I_{2xx} + I_{3yy}, \qquad d_1 = \frac{1}{2}\dot{q}_1^2$$

$$X_2 = F_{1c}, \qquad d_2 = \int |\dot{q}_1| \mathrm{d}t$$

$$X_3 = F_{1v}, \qquad d_3 = \int \dot{q}_1^2 \mathrm{d}t$$

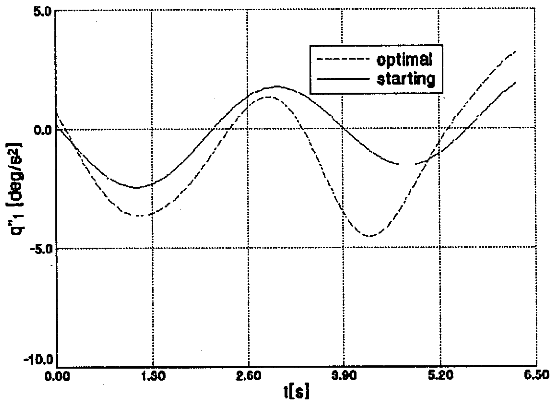

Fig. 7. Starting and optimal trajectories for the first joint; load identification.

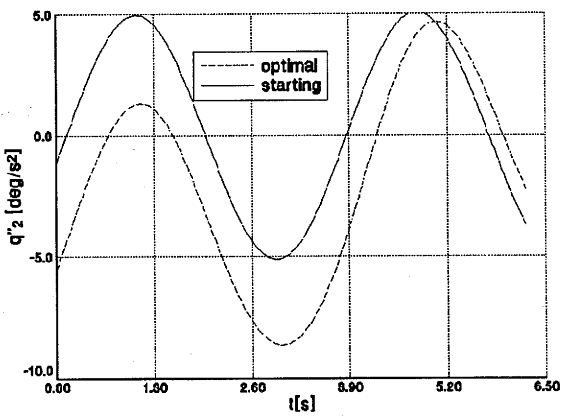

Fig. 8. Starting and optimal trajectories for the second joint; load identification.

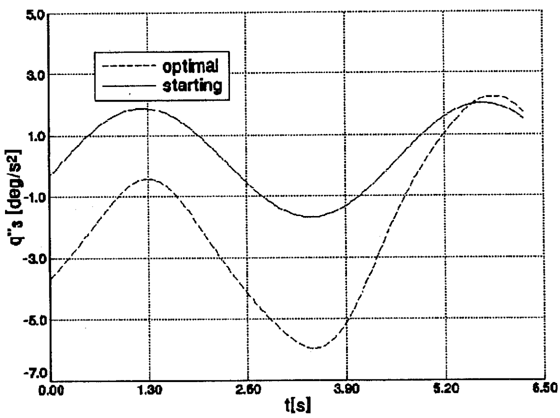

Fig. 9. Starting and optimal trajectories for the third joint; load identification.

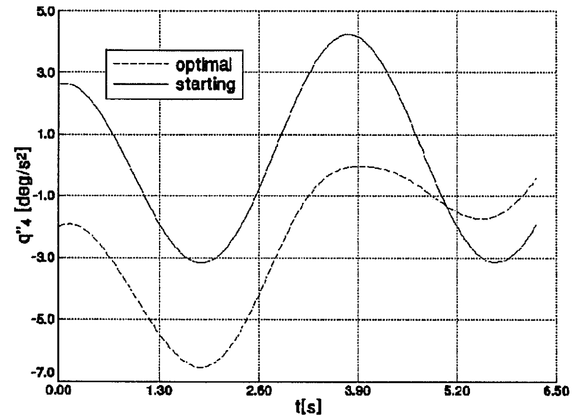

Fig. 10. Starting and optimal trajectories for the fourth joint; load identification.

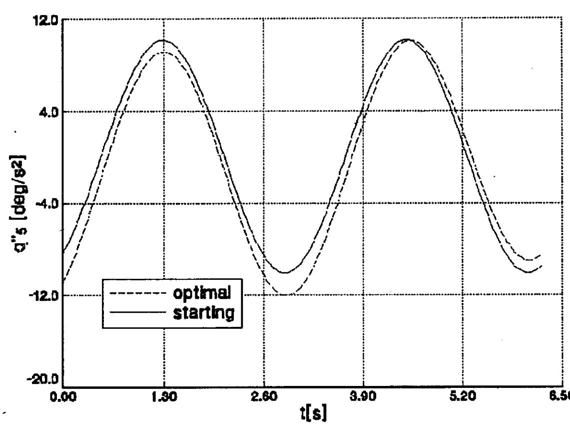

Fig. 11. Starting and optimal trajectories for the fifth joint; load identification.

where F_{1c} and F_{1v} denote the Coulomb and velocity-dependent friction coefficients, respectively. Here we did not precompute the friction torque, because we wanted to check the quality of the identification with exciting trajectories. Fortunately, by moving different IRp-6 joints, we were able to identity all the mass and friction parameters and none of them was involved in two separate movements. This made it possible to avoid the accumulated errors during the identification sequential process (cf. Gautier and Presse, 1991). The identification results are respectively shown in Figs. 12, 13, and 14.

From the curves presented in Figs. 12, 13, and 14 one can notice that the estimates are stable by making use of the exciting trajectories. The values of the friction coefficients are very close to the precomputed friction coefficients obtained by Kozłowski and Dutkiewicz (1995). This validates the exciting trajectories for the purpose of identification of inertial parameters. The results for the other joints were of a similar nature.

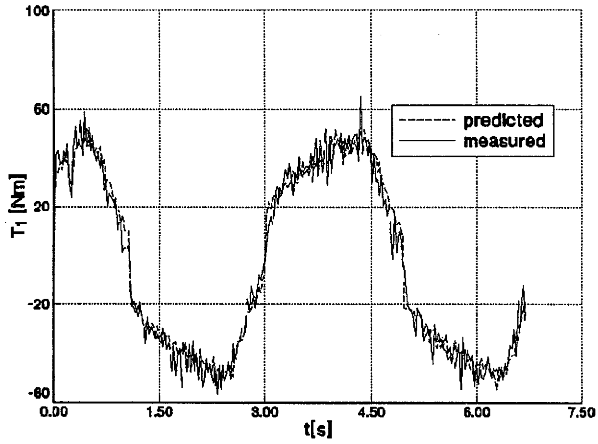

Fig. 12. Mass parameter identification with optimal and non-optimal trajectories for the first joint.

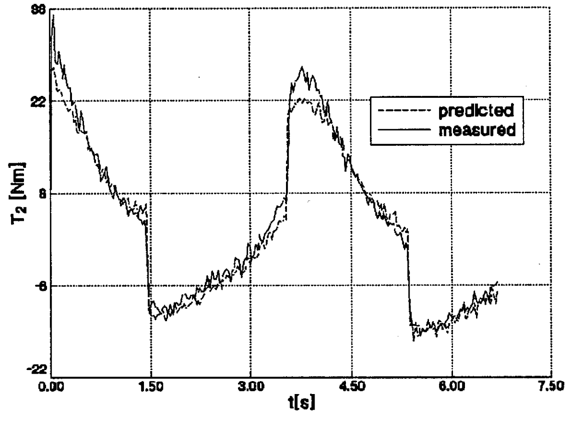

Fig. 13. Coulomb parameter identification with optimal and non-optimal trajectories for the first joint.

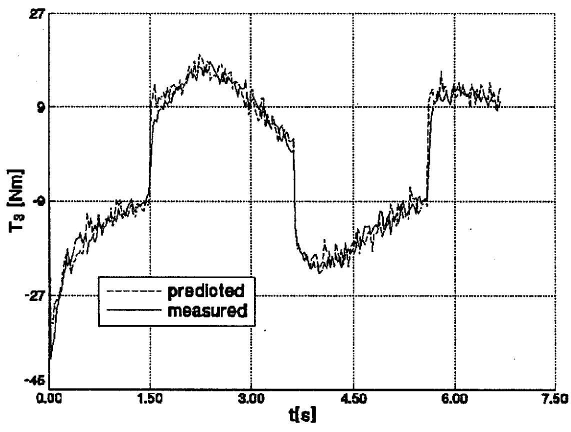

Fig. 14. Velocity friction parameter identification with optimal and non-optimal trajectories for the first joint.

6. Concluding Remarks

In this study, we presented exciting trajectories for dynamic robot and load parameters identification experiments. As a criterion we used minimization of the condition number of the information matrix. We applied this criterion to several identification problems in robotics. We used it in two models: differential and integral. We compared the results with those existing in the robotics literature. For the integral model we tried the short and long integrals discussed by Prüfer *et al.* (1994). Exciting trajectories are very important for the integral model due to a loss of information in this model in comparison with the differential one. Therefore we performed several numerical experiments particularly for this model for both robot and load dynamic parameters identification. A trial and error method in choosing an exciting trajectory does not always give good results. This phenomenon was observed in the process of load identification. Some initial trajectories were chosen close to the optimal ones; for example, we chose some trajectories with the initial condition number as low as 181 and it was not possible to improve it by the optimization procedure. The numerical results were successfully verified by the experimental results with an IRp-6 industrial robot.

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