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ADAPTIVE AND ROBUST CONTROL OF FLEXIBLE JOINT ROBOTS IN CONSTRAINED MOTION†

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This paper addresses motion and force control issues of flexible joint robots in constrained motion. A two-stage control scheme, consisting of a constrained motion controller and a joint torque controller, is established in a systematic way for general n-link flexible joint robots. To deal with uncertainties in the parameters of the robotic system, adaptive and robust control algorithms are developed assuming that all system parameters, including the joint flexibility values, are unknown except for some of their bounds. The system stability is analyzed via the Lyapunov stability theory. It is shown that with the proposed control method, the closed-loop system is uniformly stable, and motion and force tracking errors are uniformly ultimately bounded. Simulation results illustrate the effectiveness of the proposed control method.

1. Introduction

In many robotic applications, it is necessary to control both motion of the end-effector and contact force between the end-effector and environment. Numerous approaches have been proposed to deal with this problem. Typical force control schemes are the explicit force control, hybrid position/force control and impedance control (Hogan, 1985; Raibert and Craig, 1981; Whitney, 1987). The underlined control problem, that has received extensive attention in the literature, is control of manipulators in constrained motion. During constrained motion, the end-effector of robot manipulator is assumed to be in contact with rigid frictionless surfaces, which impose kinematic constraints on the robot motion. Solutions to this control problem can be found in (Kankaanranta and Koivo, 1988; McClamroch and Wang, 1988; Mills and Goldenberg, 1989; etc.). In the work (Kankaanranta and Koivo, 1988), a method for reducing the dimension of the dynamic model of constrained motion is presented, and a control method which leads to exact decoupling of position and force controlled directions is proposed. Similar studies have been conducted by McClamroch and Wang, and a general theoretical framework of constrained motion control has been developed. In (Mills and Goldenberg, 1989), descriptor theory is applied to constrained motion control and a linearized feedback controller is developed. Some adaptive control

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algorithms and robust control methods have also been proposed to deal with the case of parametric uncertainties (Su *et al.*, 1990; Zhen and Goldenberg, 1994; etc.).

However, in the work cited above the robot manipulator is modeled as a completely rigid structure. So far, only a few studies have addressed the force control of flexible joint robots. The works (Marino and Nicosia, 1984; Spong, 1987; Sweet and Good, 1984) have pointed out that control of robots based on rigid-body dynamics formulation is inadequate in dealing with more stringent operating conditions. The elasticity of the transmission elements between actuators and links has a significant influence on the robot dynamics. In some cases, joint flexibility can even lead to instability if it is neglected. It is more critical to account for joint flexibility when dealing with a force control problem than with a pure position control problem. Spong (1989) addressed the force control issue of flexible joint robots and derived a control algorithm for both hybrid control and impedance control methods. The results are based on the exact knowledge of the dynamic model. Recently, an adaptive force control scheme for a single-link mechanism with joint flexibility was developed by Lian *et al.* (1991). In their work, an explicit force control approach is used, but constrained motion control is not considered, and the method cannot be easily extended to the general n-link case.

In this paper, we consider the control problem of flexible joint robots in constrained motion using joint torque feedback. First, the constrained dynamic model of flexible joint robots is derived. Then, based on the two-stage control strategy (Lin and Goldenberg, 1995), a control scheme consisting of a constrained motion controller and a joint torque controller is established in a systematic way for the general n-link case. To deal with the uncertainties of the robotic system, adaptive and robust control algorithms are developed assuming that all system parameters, including the joint flexibility values, are unknown except for some of their bounds. The system stability is analyzed via the Lyapunov theory. It is shown that, with the proposed controller, the closed-loop system is uniformly stable, and the tracking errors are uniformly ultimately bounded. The major contribution of this work is the development of a new control method for flexible joint robots in constrained motion. The method provides a systematic approach to motion and force control of flexible joint robots in the general n-link case without requiring the exact knowledge of robotic manipulator parameters.

The paper is organized as follows. In Section 2, the constrained dynamic model of flexible joint robots is derived. The proposed control scheme and convergence analysis are developed in Section 3. Simulation results are presented in Section 4 to illustrate the effectiveness of the proposed control methods. Conclusions are given in Section 5.

2. Constrained Dynamic Model of Flexible Joint Robots

Consider the dynamic equations of a constrained n-link flexible joint robot with joint torque measurements described as follows (Kircanski and Goldenberg, 1997; Spong, 1987):

$$D(q_l)\ddot{q}_l + C(q_l, \dot{q}_l)\dot{q}_l + G(q_l) = \tau_s + f \tag{1}$$

$$I_m \ddot{q}_m + B_m \dot{q}_m + \tau_s = u \tag{2}$$

$$\tau_s = K_s (q_m - q_l) \tag{3}$$

where $D(q_l)$ is the $n \times n$ positive definite, symmetric inertia matrix; $C(q_l, \dot{q}_l)$ is the $n \times 1$ vector containing Coriolis, centrifugal terms; $G(q_l)$ is the $n \times 1$ vector of gravitational terms; I_m is the $n \times n$ constant diagonal matrix with diagonal elements $\gamma_i(\gamma_i + 1)J_{mi}$, J_{mi} stands for the inertia of the rotor/gear; B_m is the $n \times n$ diagonal matrix of damping terms and K_s is an $n \times n$ diagonal matrix of the joint torsional stiffness. Here q_l and q_m are n-dimensional vectors which represent the link angles and rotor angles, respectively. Moreover, u is the $n \times 1$ input torque control vector, τ_s denotes the vector of joint torque measurements and f is the vector of generalized contact force[1] in joint space.

Let $X \in \mathbb{R}^n$ denote the generalized position vector of the end-effector in Cartesian space. The algebraic equation for the constraints can be written as

$$\Phi(X) = 0 \tag{4}$$

where the mapping Φ: $\mathbb{R}^n \to \mathbb{R}^\kappa$ is twice continuously differentiable. Assuming that the vector X can be expressed in joint space by the relation

$$X = H(q_l) \tag{5}$$

then the constraint equation (4) can be expressed in joint space as

$$\Psi(q_l) = \Phi\big(H(q_l)\big) = 0 \tag{6}$$

The Jacobian matrix of the above constraint equation is

$$J_c(q_l) = \frac{\partial \Psi(q_l)}{\partial q_l} = \frac{\partial \Phi}{\partial X} \frac{\partial H(q_l)}{\partial q_l} \tag{7}$$

Since $\Psi(q_l) = 0$ is identically satisfied, it is evident that $J_c(q_l)\dot{q}_l = 0$. When the end-effector is moving along the constrained surface, the constraint force in joint space is then given by (McClamroch and Wang, 1988)

$$f = J_c^T(q_l)\lambda \tag{8}$$

where $\lambda \in \mathbb{R}^\kappa$ is the generalized Lagrange multiplier associated with the constraints, and it represents independent normal contact force components.

Since the presence of κ constraints causes the robot to lose κ degrees of freedom, $n - \kappa$ linearly independent coordinates are sufficient to characterize the constrained motion. Let us partition the vector q_l as

$$q_l = \begin{bmatrix} q_l^1 \\ q_l^2 \end{bmatrix} \tag{9}$$

[1] Hereafter, we use 'force' to mean generalized force that could be force and/or torque.

with

$$q_l^1 = \left[q_1^1, \ldots, q_{n-\kappa}^1\right]^T \in \mathbb{R}^{n-\kappa}, \qquad q_l^2 = \left[q_1^2, \ldots, q_\kappa^2\right]^T \in \mathbb{R}^\kappa$$

According to the implicit function theorem, there always exists a function σ which can be obtained from the constraint eqn. (6), such that (McClamroch and Wang, 1988)

$$q_l^2 = \sigma(q_l^1) \tag{10}$$

Thus, the joint space vector is expressed as

$$q_l = \begin{bmatrix} q_l^1 \\ \sigma(q_l^1) \end{bmatrix}$$

Defining

$$L(q_l^1) = \begin{bmatrix} I_{n-\kappa} \\ \dfrac{\partial\sigma(q_l^1)}{\partial q_l^1} \end{bmatrix} \tag{11}$$

we have

$$\dot{q}_l = \begin{bmatrix} \dot{q}_l^1 \\ \dot{q}_l^2 \end{bmatrix} = L(q_l^1)\dot{q}_l^1, \qquad \ddot{q}_l = L(q_l^1)\ddot{q}_l^1 + \dot{L}(q_l^1)\dot{q}_l^1$$

Substituting the above relations into (1), we get

$$D(q_l^1)L(q_l^1)\ddot{q}_l^1 + B(q_l^1, \dot{q}_l^1)\dot{q}_l^1 + G(q_l^1) = \tau_s + J_c^T(q_l^1)\lambda \tag{12}$$

where

$$B(q_l^1, \dot{q}_l^1) = D(q_l^1)\dot{L}(q_l^1) + C(q_l^1, \dot{q}_l^1)L(q_l^1)$$

It can be shown that the constrained dynamic system (12) has the following properties.

Property 1. *The LHS of (12) can be expressed as (Su et al., 1990)*

$$D(q_l^1)L(q_l^1)\ddot{q}_l^1 + B(q_l^1, \dot{q}_l^1)\dot{q}_l^1 + G(q_l^1) = Y^1(q_l^1, \dot{q}_l^1, \ddot{q}_l^1)P \tag{13}$$

where $Y^1(\cdot)$ is an $n \times r$ matrix of known functions, referred to as the regressor, and P is an r-dimensional vector of parameters.

Property 2. *Let $A(q_l^1) = L^T(q_l^1)D(q_l^1)L(q_l^1)$. Then*

$$\dot{A}(q_l^1) - 2L^T(q_l^1)B(q_l^1, \dot{q}_l^1) = L^T(\dot{D} - 2C)L \tag{14}$$

is skew symmetric, as $(\dot{D} - 2C)$ is skew symmetric (Slotine and Li, 1987).

Property 3. *Since* $J_c\dot{q}_l = 0$, *i.e.* $J_c(q_l^1)L(q_l^1)\dot{q}_l^1 = 0$, *and* q_l^1 *is linearly independent, we have*

$$J_c(q_l^1)L(q_l^1) = 0, \quad L^T(q_l^1)J_c^T(q_l^1) = 0$$

Thus, multiplying eqn. (12) from left by L^T yields

$$A(q_l^1)\ddot{q}_l^1 + L^T(q_l^1)B(q_l^1,\dot{q}_l^1)\dot{q}_l^1 + L^T(q_l^1)G(q_l^1) = L^T(q_l^1)\tau_s \tag{15}$$

For controlling the system with joint torque feedback, we assume that the link angle q_l, joint torque τ_s and constraint force λ are available for feedback control. From (3), we have

$$q_m = K_s^{-1}\tau_s + q_l$$

Substituting this equality into (2), we get

$$I_{ts}\ddot{\tau}_s + B_{ts}\dot{\tau}_s + h_{ts}(q_l^1,\dot{q}_l^1,\ddot{q}_l^1,\tau_s) = u \tag{16}$$

where

$$I_{ts} = I_m K_s^{-1}$$

$$B_{ts} = B_m K_s^{-1}$$

$$h_{ts} = I_m\ddot{q}_l + B_m\dot{q}_l + \tau_s = I_m L\ddot{q}_l^1 + (I_m\dot{L} + B_m L)\dot{q}_l^1 + \tau_s$$

Equations (12) and (16) constitute a dynamic representation for the flexible joint robot in constrained motion.

3. Control of Constrained Flexible Joint Robots

The objective of constrained robot control is to determine the input torque necessary to achieve trajectory tracking on the constrained surface with specified constraint forces. It is noticed that the constrained link dynamics (12) has exactly the same formulation as the rigid one if τ_s is viewed as the 'input torque'. To achieve the control objective, a suitable 'input torque', which will be denoted here as a 'desired joint torque' τ_{sd} since joint flexibility exists, should be generated. Based on the two-stage control strategy (Lin and Goldenberg, 1995), a control scheme consisting of a constrained motion controller and a joint torque controller is developed. The constrained motion controller is designed to generate the 'desired joint torque' τ_{sd} and the joint torque controller to regulate the required control torque u, so that the joint torque τ_s tracks the desired joint torque τ_{sd}, and thus the whole system achieves the control objective. The control scheme is shown in Fig. 1.

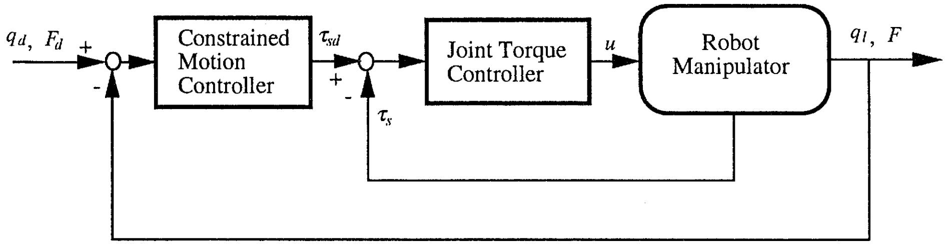

Fig. 1. The two-stage control scheme with joint torque feedback.

3.1. Control Scheme with All Parameters Known

If all dynamic parameters are known exactly, the constrained motion and joint torque controllers are designed under the following assumptions:

(A1) the signals $q_l, \dot{q}_l$, τ_s, $\dot{\tau}_s$, λ and $\dot{\lambda}$ are available for feedback control;

(A2) the desired trajectory $q_d(t) \in C^4$ and the desired force $\lambda_d \in C^2$ are bounded.

3.1.1. Constrained Motion Controller

Let us define the tracking errors:

$$e_l = q_l - q_d$$

$$e_f = \lambda - \lambda_d$$

$$e_t = \tau_s - \tau_{sd}$$

and write

$$v_l^1 = \dot{q}_d^1 - \Lambda_l(q_l^1 - q_d^1)$$

$$r_l = \dot{q}_l^1 - v_l^1 = (\dot{q}_l^1 - \dot{q}_d^1) + \Lambda_l(q_l^1 - q_d^1)$$

$$F_c = J_c^T \lambda_d - J_c^T k_I \int_0^t (\lambda - \lambda_d)\,\mathrm{d}t$$

The desired joint torque τ_{sd} is generated by using the method of Slotine and Li (1987),

$$\tau_{sd} = D(q_l^1)L(q_l^1)\dot{v}_l^1 + B(q_l^1, \dot{q}_l^1)v_l^1 + G(q_l^1) - K_{lD}L(q_l^1)r_l - F_c \tag{17}$$

where K_{lD}, k_I and Λ_l are diagonal matrices of positive gains.

Subtracting (17) from (12) and re-arranging the terms leads to the following equations:

$$\begin{aligned} D(q_l^1)L(q_l^1)\dot{r}_l + B(q_l^1, \dot{q}_l^1)r_l + K_{lD}Lr_l &= e_t + J_c^T\Big[(\lambda - \lambda_d) + k_I \int_0^t (\lambda - \lambda_d)\,\mathrm{d}t\Big] \\ &= e_t + J_c^T\Big[e_f + k_I \int_0^t e_f\,\mathrm{d}t\Big] \end{aligned} \tag{18}$$

and

$$A(q_l^1)\dot{r}_l + L^T B(q_l^1, \dot{q}_l^1) r_l + L^T K_{lD} L r_l = L^T e_t \tag{19}$$

3.1.2. Joint Torque Controller

In order to control the actual joint torque to follow the desired joint torque τ_{sd}, a computed-torque like control law is adopted for (16):

$$u = I_{ts}\dot{v}_t + B_{ts} v_t + h_{ts} - K_{tD} r_t \tag{20}$$

where $v_t = \dot{\tau}_{sd} - \Lambda_t(\tau_s - \tau_{sd}) = \dot{\tau}_{sd} - \Lambda_t e_t$, $r_t = \dot{\tau}_s - v_t$, and K_{tD} and Λ_t are diagonal positive definite gain matrices. Then, from (16) and (20), we have:

$$I_{ts}\dot{r}_t + B_{ts} r_t + K_{tD} r_t = 0 \tag{21}$$

3.1.3. Convergence Analysis

The stability and convergence properties of the closed loop system are analyzed based on the Lyapunov stability theory. First, an important lemma is recalled, then Theorem 1 is proved.

Lemma 1. (Horn and Johnson, 1985) *Suppose that a symmetric matrix Q is partitioned as:*

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}$$

where Q_{11} and Q_{22} are square. The matrix Q is positive definite if and only if Q_{11} is positive definite and $Q_{22} > Q_{12}^T Q_{11}^{-1} Q_{12}$.

Theorem 1. *Consider the robot dynamic system (12) and (16). The constrained motion controller (17) and joint torque controller (20) stabilize the closed loop system and achieve global asymptotic convergence, i.e.*

$$\lim_{t\to\infty} r_l = 0, \qquad \lim_{t\to\infty} r_t = 0, \qquad \textit{and} \qquad \lim_{t\to\infty} e_f = 0$$

Proof. Choosing the Lyapunov function candidate as

$$V = \frac{1}{2} r_l^T A(q_l^1) r_l + \frac{1}{2} r_t^T I_{ts} r_t + e_t^T \Lambda_t^T K_{tD} e_t \ > \ 0 \tag{22}$$

the differentiation of V gives

$$\begin{aligned} \dot{V} &= r_l^T A \dot{r}_l + \frac{1}{2} r_l^T \dot{A} r_l + r_t^T I_{ts} \dot{r}_t + 2 e_t^T \Lambda_t^T K_{tD} \dot{e}_t \\ &= r_l^T A \dot{r}_l + \frac{1}{2} r_l^T \dot{A} r_l - r_t^T B_{ts} r_t - \dot{e}_t^T K_{tD} \dot{e}_t - e_t^T \Lambda_t^T K_{tD} \Lambda_t e_t \end{aligned} \tag{23}$$

From (19) and the fact that $(\dot{A} - 2L^T B)$ is skew symmetric, we have

$$\begin{aligned}\dot{V} &= -r_l^T L^T K_{lD} L r_l + r_l^T L^T e_t - r_t^T B_{ts} r_t - \dot{e}_t^T K_{tD} \dot{e}_t - e_t^T \Lambda_t K_{tD} \Lambda_t e_t \\ &= -\begin{bmatrix} (Lr_l)^T & e_t^T \end{bmatrix} Q \begin{bmatrix} Lr_l \\ e_t \end{bmatrix} - r_t^T B_{ts} r_t - \dot{e}_t^T K_{tD} \dot{e}_t \end{aligned} \tag{24}$$

with

$$Q = \begin{bmatrix} K_{lD} & -\frac{1}{2}I \\ -\frac{1}{2}I & \Lambda_t^T K_{tD} \Lambda_t \end{bmatrix}$$

By Lemma 1, for matrix Q to be a positive definite matrix, the requirement is

$$\Lambda_t^T K_{tD} \Lambda_t > \frac{1}{4} K_{lD}^{-1}$$

Since K_{tD}, K_{lD} and Λ_t are diagonal matrices of controller gains, we can choose them properly to meet the requirement, such that $Q > 0$ and $\dot{V} < 0$.

Thus, the system is globally asymptotically stable in the sense of the Lyapunov, and we have

$$\lim_{t\to\infty} r_l = 0, \qquad \lim_{t\to\infty} r_t = 0$$

Moreover, as $t \to \infty$, the system converges to the equilibrium point and (18) results in

$$e_f + k_I \int_0^t e_f \, \mathrm{d}t = 0$$

Hence, we have $\lim_{t\to\infty} e_f = 0$ as well. This completes the proof of the theorem. ■

Remark 1. From the control design and the convergence analysis, it can be observed that, with the joint torque controller, motion and force control methods developed for the rigid case can be used for controlling flexible joint robots. This will greatly facilitate the control design of robot manipulators.

Remark 2. In much the same way as in (Lin and Goldenberg, 1995), $\ddot{q}_l^1$ can be calculated using τ_s, q_l^1 and $\dot{q}_l^1$ in (15).

3.2. Adaptive and Robust Control with All Parameters Unknown

In practice, there are uncertainties in the manipulator parameters. Here, we assume that all parameters, of both link dynamics and drive system, including the joint flexibility values, are unknown except for some of their bounds. To handle this case, adaptive and robust control algorithms are developed based on the result of the previous section.

3.2.1. Adaptive Constrained Motion Controller

According to Property 1, and (17), the adaptive control law is derived as

$$\begin{aligned}\tau_{sd} &= \hat{D}(q_l^1)L(q_l^1)\dot{v}_l^1 + \hat{B}(q_l^1,\dot{q}_l^1)v_l^1 + \hat{G}(q_l^1) - K_{lD}L(q_l^1)r_l - F_c \\ &= Y^1(q_l^1,\dot{q}_l^1,v_l^1,\dot{v}_l^1)\hat{P} - K_{lD}Lr_l - F_c \end{aligned} \tag{25}$$

Subtracting (25) from (12) and re-arranging the terms, yields

$$\begin{aligned} &D(q_l^1)L(q_l^1)\dot{r}_l + B(q_l^1,\dot{q}_l^1)r_l + K_{lD}Lr_l \\ &= (\hat{D}-D)L\dot{v}_l^1 + (\hat{B}-B)v_l^1 + (\hat{G}-G) + e_t + J_c^T\Big[(\lambda-\lambda_d) + k_I\int_0^t(\lambda-\lambda_d)\,\mathrm{dt}\Big] \\ &= Y^1(q_l^1,\dot{q}_l^1,v_l^1,\dot{v}_l^1)\tilde{P} + e_t + J_c^T\Big[e_f + k_I\int_0^t e_f\,\mathrm{dt}\Big] \end{aligned} \tag{26}$$

with $\tilde{P} = \hat{P} - P$.

3.2.2. Robust Joint Torque Controller

To deal with the uncertainty in the rotor subsystem, let us define an $n \times p$ matrix Y_t and a p-dimensional vector of parameters P_t as follows:

$$Y_t(\dot{q}_l,\ddot{q}_l,\tau_s,\dot{\tau}_s,\ddot{\tau}_s) = \Big[\mathrm{diag}\,(\ddot{\tau}_s),\ \mathrm{diag}\,(\dot{\tau}_s),\ \mathrm{diag}\,(\ddot{q}_l),\ \mathrm{diag}\,(\dot{q}_l),\tau_s\Big]$$

$$P_t = \Big[..I_{tsi}..,\ ..B_{tsi}..,\ ..I_{mi}..,\ ..B_{mi}..,\ 1\Big]^T$$

Equation (16) can then be written as

$$Y_t(\dot{q}_l,\ddot{q}_l,\tau_s,\dot{\tau}_s,\ddot{\tau}_s)P_t = u$$

We assume that the uncertainty of the parameters is bounded, i.e. for the available parameters $\hat{P}_t$ there exists $\rho \in \mathbb{R}_+$, such that

$$\|\tilde{P}_t\| = \|\hat{P}_t - P_t\| \le \rho \tag{27}$$

Based on (20), using the control law:

$$u = \hat{I}_{ts}\dot{v}_t + \hat{B}_{ts}v_t + \hat{h}_{ts} - K_{tD}r_t + Y_t\Delta u \tag{28}$$

we have

$$\begin{aligned} I_{ts}\dot{r}_t + B_{ts}r_t + K_{tD}r_t &= (\hat{I}_{ts}-I_{ts})\dot{v}_t + (\hat{B}_{ts}-B_{ts})v_t + (\hat{h}_{ts}-h_{ts}) + Y_t\Delta u \\ &= Y_t(\dot{q}_l,\ddot{q}_l,\tau_s,v_t,\dot{v}_t)(\tilde{P}_t + \Delta u) \end{aligned} \tag{29}$$

where Δu is defined below in (30).

3.2.3. Convergence Analysis

Theorem 2. *Using the controllers (25) and (28) with*

$$\Delta u = \begin{cases} -\rho \dfrac{Y_t^T r_t}{\|Y_t^T r_t\|} & \text{if} \quad \|Y_t^T r_t\| > \epsilon \\ -\dfrac{\rho}{\epsilon} Y_t^T r_t & \text{if} \quad \|Y_t^T r_t\| \leq \epsilon \end{cases} \tag{30}$$

and parameter update law

$$\dot{\hat{P}} = -\Gamma^{-1} {Y^1}^T (q_l^1, \dot{q}_l^1, v_l^1, \dot{v}_l^1) L r_l \tag{31}$$

the closed-loop system is uniformly stable and the tracking errors are uniformly ultimately bounded (u.u.b.).

Proof. Choosing the Lyapunov function candidate as

$$V = \frac{1}{2} r_l^T A(q_l^1) r_l + \frac{1}{2} \tilde{P}^T \Gamma \tilde{P} + \frac{1}{2} r_t^T I_{ts} r_t + e_t^T \Lambda_t^T K_{tD} e_t > 0 \tag{32}$$

through a similar calculation of previous section, the differentiation of V gives

$$\dot{V} = -\begin{bmatrix} (Lr_l)^T & e_t^T \end{bmatrix} Q \begin{bmatrix} Lr_l \\ e_t \end{bmatrix} - r_t^T B_{ts} r_t - \dot{e}_t^T K_{tD} \dot{e}_t$$

$$+ \tilde{P}^T ({Y^1}^T L r_l + \Gamma \dot{\hat{P}}) + (Y_t^T r_t)^T (\tilde{P}_t + \Delta u) \tag{33}$$

With the parameter update law (31) and control Δu (30), we can show that $\dot{V} < 0$. Let us examine the last term in the above. If $\|Y_t^T r_t\| > \epsilon$, then

$$\begin{aligned} (Y_t^T r_t)^T (\tilde{P}_t + \Delta u) &= (Y_t^T r_t)^T \left(\tilde{P}_t - \rho \frac{Y_t^T r_t}{\|Y_t^T r_t\|} \right) \\ &\leq \|Y_t^T r_t\| \left(\|\tilde{P}_t\| - \rho \right) < 0 \end{aligned} \tag{34}$$

If $\|Y_t^T r_t\| \leq \epsilon$.

$$\begin{aligned} (Y_t^T r_t)^T (\tilde{P}_t + \Delta u) &\leq (Y_t^T r_t)^T \left(\rho \frac{Y_t^T r_t}{\|Y_t^T r_t\|} + \Delta u \right) \\ &= (Y_t^T r_t)^T \left(\rho \frac{Y_t^T r_t}{\|Y_t^T r_t\|} - \frac{\rho}{\epsilon} Y_t^T r_t \right) \end{aligned} \tag{35}$$

This last term achieves a maximum value of $\epsilon\rho/4$ when $\|Y_t^T r_t\| = \epsilon/2$. That is,

$$\begin{aligned} \dot{V} &\leq -\begin{bmatrix} (Lr_l)^T & e_t^T \end{bmatrix} Q \begin{bmatrix} Lr_l \\ e_t \end{bmatrix} - r_t^T B_{ts} r_t - \dot{e}_t^T K_{tD} \dot{e}_t + \frac{\epsilon\rho}{4} \\ &\leq -\lambda_{\min}(Q) \|(r_l^T, e_t^T)\|^2 - \min(B_{tsi}) \|r_t\|^2 - \min(K_{tDi}) \|\dot{e}_t\|^2 + \frac{\epsilon\rho}{4} \end{aligned} \tag{36}$$

If $\lambda_{\min}(Q)\|((Lr_l)^T, e_t^T)\|^2 + \min(B_{tsi})\|r_t\|^2 + \min(K_{tDi})\|\dot{e}_t\|^2 > \epsilon\rho/4$, then $\dot{V} < 0$, uniform ultimate boundedness and uniform stability follow using the results of Corless and Leitmann (1981).

The uniform stability of the system and uniform ultimate boundedness of the tracking errors r_l and e_t guarantee the boundedness of e_f. As the errors r_l and e_t can be made arbitrarily small by a suitable selection of the coefficient ϵ, from eqn. (26), force error e_f will be affected mainly by the error in parameter estimation. The integral action in F_c will reduce the error and make e_f tend to zero. ■

Remark 3. It is noticed that using a single bound ρ to measure the parameter uncertainty may lead to overly conservative design and limit the adjustable capability of the controller (Liu and Goldenberg, 1996). To minimize this respect, $Y_t\Delta u$ can be partitioned as follows.

Similar to (Liu and Goldenberg, 1996), suppose we have the knowledge of the uncertainty bounds of each parameter component:

$$\|\tilde{P}_{ti}\| \leq \rho_i, \qquad i = 1, 2, \ldots, p \tag{37}$$

Partitioning

$$Y_t P_t = \begin{bmatrix} Y_{t1}, & \ldots, & Y_{tp} \end{bmatrix} \begin{bmatrix} P_{t1} \\ \vdots \\ P_{tp} \end{bmatrix} = \sum_{i=1}^{p} Y_{ti} P_{ti} \tag{38}$$

$$Y_t \Delta u = \begin{bmatrix} Y_{t1}, & \ldots, & Y_{tp} \end{bmatrix} \begin{bmatrix} \Delta u_1 \\ \vdots \\ \Delta u_p \end{bmatrix} = \sum_{i=1}^{p} Y_{ti} \Delta u_i \tag{39}$$

we have

$$\Delta u_i = \begin{cases} -\rho_i \dfrac{Y_{ti}^T r_t}{\|Y_{ti}^T r_t\|} & \text{if} \quad \|Y_{ti}^T r_t\| > \epsilon_i \\ -\dfrac{\rho_i}{\epsilon_i} Y_{ti}^T r_t & \text{if} \quad \|Y_{ti}^T r_t\| \leq \epsilon_i \end{cases} \tag{40}$$

Thus, our knowledge of the parameter uncertainty can be better utilized in control design as there is more freedom to adjust the control gains.

4. Simulation Examples

In order to verify the effectiveness of the control method proposed in this paper, numerical simulations were carried out based on the dynamic model of the IRIS (Institute of Robotic and Intelligent System) robot built in the Robotics and Automation Laboratory at the University of Toronto (Kircanski and Goldenberg, 1997). Consider the robot manipulator with an upper arm and a forearm located in a vertical plane and moving in contact with a circular path constraint, as shown in Fig. 2.

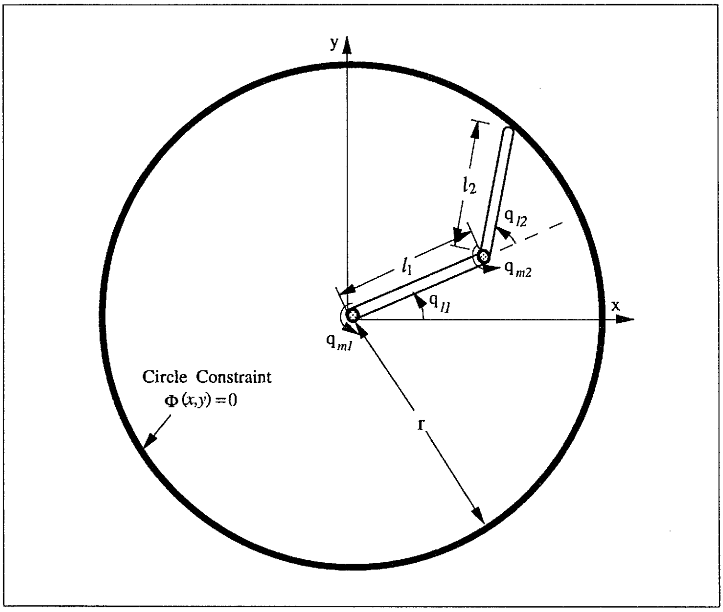

Fig. 2. The schematic diagram of a robot manipulator and the circle constraint.

The dynamic model of the rotor subsystems with joint torque sensors is given as follows:

$$\gamma_1(\gamma_1+1)J_{m1}\ddot{q}_{m1}+\gamma_1(\gamma_1+1)b_{m1}\dot{q}_{m1}+\tau_{s1}=u_1$$

$$\gamma_2(\gamma_2+1)J_{m2}\ddot{q}_{m2}+\gamma_2(\gamma_2+1)b_{m2}\dot{q}_{m2}+\tau_{s2}=u_2$$

and the corresponding link dynamics is:

$$H_{11}\ddot{q}_{l1}+H_{12}\ddot{q}_{l2}-h\dot{q}_{l2}^2-2h\dot{q}_{l1}\dot{q}_{l2}+G_1=\tau_{s1}+f_1$$

$$H_{21}\ddot{q}_{l1}+H_{22}\ddot{q}_{l2}+h\dot{q}_{l1}^2+G_2=\tau_{s2}+f_2$$

where

$$H_{11}=a_1+a_3\cos(q_{l2}),\qquad H_{12}=a_2+a_3\cos(q_{l2})$$

$$H_{21}=H_{12},\qquad H_{22}=a_2,\qquad h=a_3\sin(q_{l2})$$

$$G_1=a_4g\cos(q_{l1})+a_5g\cos(q_{l1}+q_{l2})$$

$$G_2=a_5g\cos(q_{l1}+q_{l2})$$

The values of the parameters are listed in Table 1.

Table 1. The parameter values of the flexible joint robots.

$J_{m1} = 9.0 \times 10^{-6}\,\mathrm{kgm}^2$	$J_{m2} = 8.0 \times 10^{-6}\,\mathrm{kgm}^2$
$b_{m1} = 5.4 \times 10^{-4}\,\mathrm{Nm/(rad/sec)}$	$b_{m2} = 6.4 \times 10^{-4}\,\mathrm{Nm/(rad/sec)}$
$\gamma_1 = 100$	$\gamma_2 = 50$
$K_{s1} = 10$	$K_{s2} = 8.5$
$a_1 = 0.1499$	$a_2 = 0.0311$
$a_3 = 0.0235$	$a_4 = 0.6762$
$a_5 = 0.1288$	

The constraint surface is expressed in the task space $X = [x, y]^T$ as

$$\Phi(X) = x^2 + y^2 - r^2 = 0$$

The transformation from task space to joint space is given by

$$H(q_l) = \begin{bmatrix} l_1 \cos(q_{l1}) + l_2 \cos(q_{l1} + q_{l2}) \\ l_1 \sin(q_{l1}) + l_2 \sin(q_{l1} + q_{l2}) \end{bmatrix}$$

Thus, in terms of joint space coordinates, the constraint can be expressed as

$$\Psi(q_l) = \Phi\big(H(q_l)\big) = l_1^2 + l_2^2 + 2 l_1 l_2 \cos(q_{l2}) - r^2 = 0$$

and the Jacobian matrix (7) is

$$J_c^T(q_l) = \begin{bmatrix} 0 \\ -2 l_1 l_2 \sin(q_{l2}) \end{bmatrix}$$

Since q_{l2} is constant on the constraint surface, let $q_l^1 = q_{l1}$. Then the matrix defined in (11) is

$$L^T(q_l^1) = [1 \;\; 0]$$

The constraint forces are

$$f_1 = 0, \qquad f_2 = -2 l_1 l_2 \sin(q_{l2}) \lambda$$

In simulations, the desired trajectory q_d^1 is generated by a fifth-order polynomial (from $-90°$ to $10°$ in the first 2 seconds, then constant afterwards), and the desired λ_d is chosen as a constant (10 N). The control gains were chosen to be $K_{lD} = 2.0I$, $\Lambda_l = 25I$, $K_{tD} = \mathrm{diag}\,(0.015, 0.012)$ and $\Lambda_t = \mathrm{diag}\,(20, 25)$. The simulation results are presented in Figs. 3 to 8.

Figure 3 is the response of the system in the case that all parameters are known precisely. It is illustrated that the tracking errors converge to zero. In the presence of parameter uncertainty, the adaptive and robust control scheme is compared with the non-adaptive-robust case. The results are plotted in Fig. 4 and 6, respectively. It

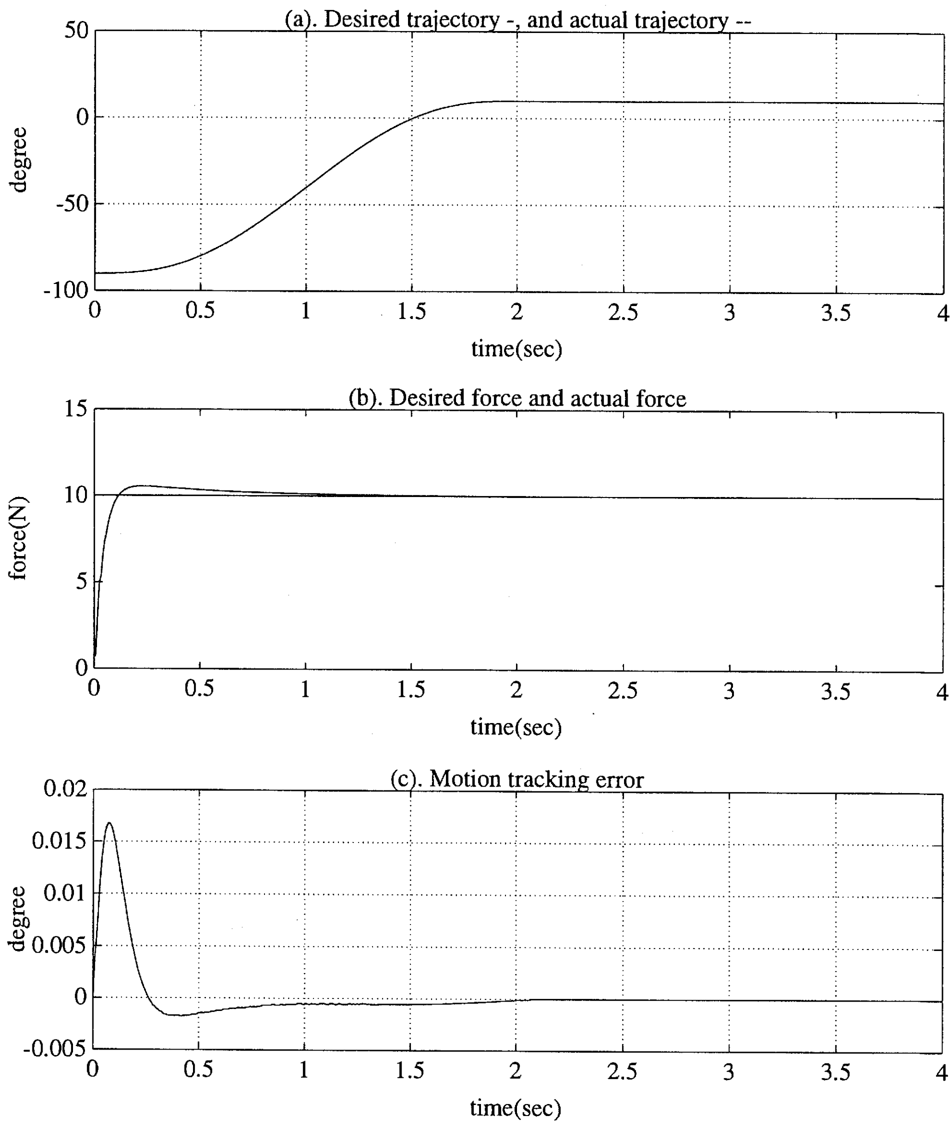

Fig. 3. The response of the system with parameters known precisely.

is shown that the manipulator using the adaptive/robust control scheme can track the desired trajectories closely. The parameter estimation curves are plotted in Fig. 5. It is noticed that parameters do not all converge to their true values; that is the similar phenomenon observed in (Slotine and Li, 1987).

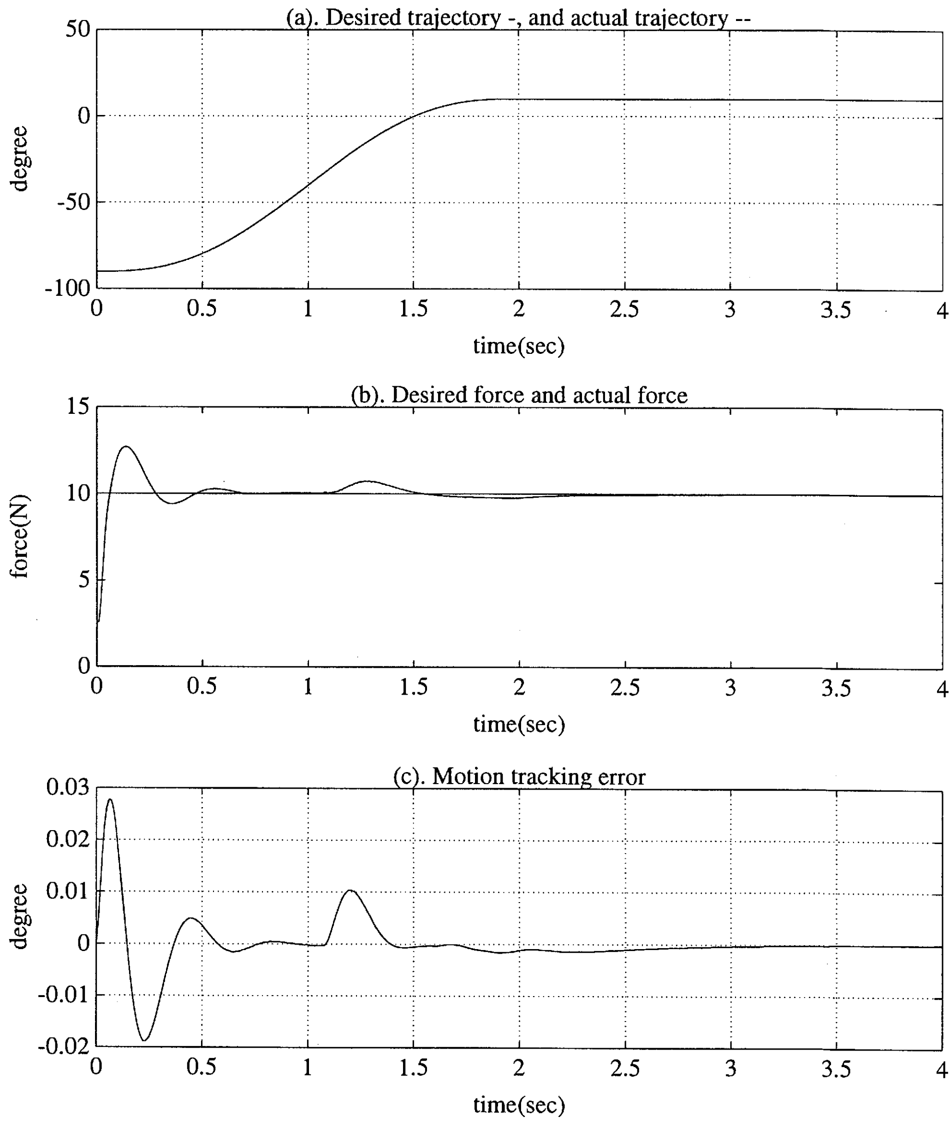

Fig. 4. The response of the adaptive and robust control system: trajectory tracking curves.

The situation that the joint flexibility is neglected in the control design is also studied. Assuming that the parameters are known, by neglecting the joint flexibility, the dynamic model for control design is given as (Spong, 1987),

$$\bar{D}(q)\ddot{q} + \bar{C}(q, \dot{q})\dot{q} + G(q) = u + f$$

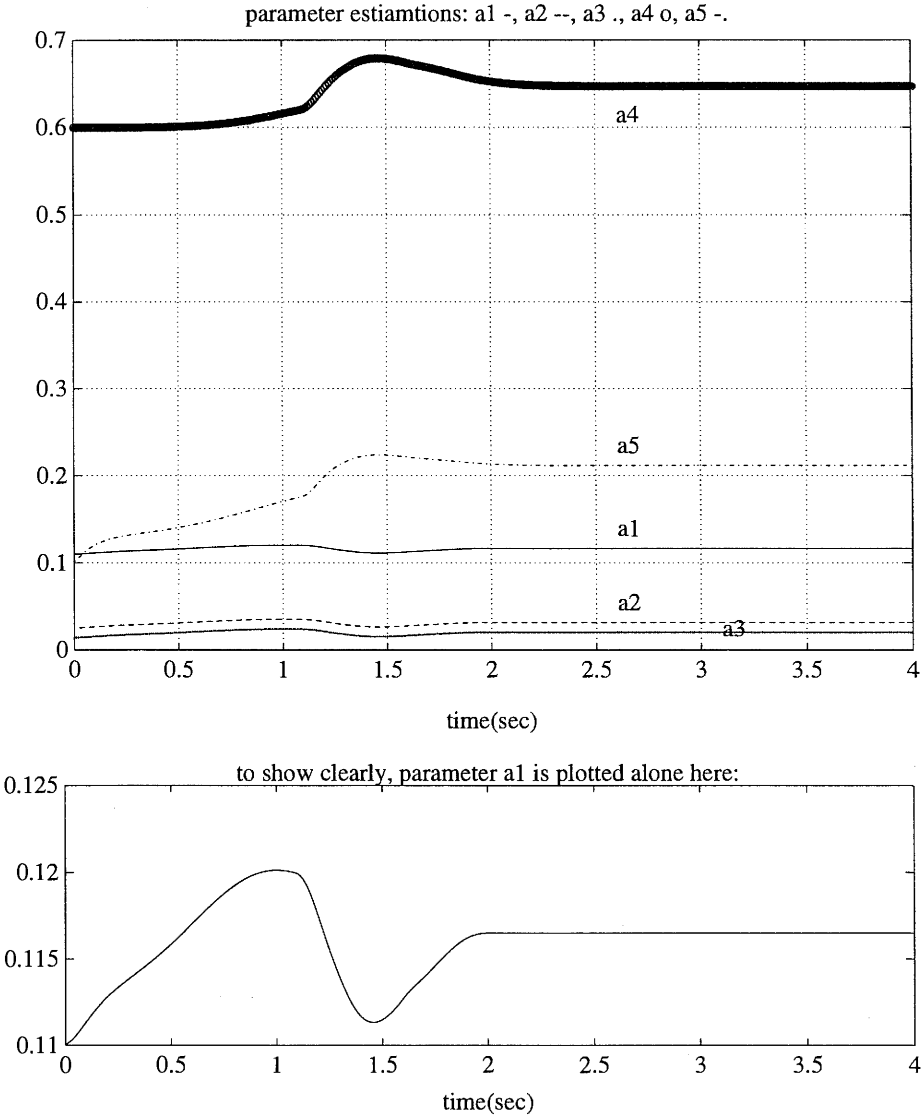

Fig. 5. The response of the adaptive and robust control system: parameter estimation curves.

where $\bar{D} = [D(q) + I_m]$, $\bar{C} = [C(q, \dot{q}) + B_m]$ and $q = q_l = q_m$ The control law as derived in (17) becomes:

$$u = \tau_{sd} = \bar{D}(q^1)L(q^1)\dot{v}^1 + \bar{B}(q^1, \dot{q}^1)v^1 + G(q^1) - K_{lD}L(q^1)r - F_c$$

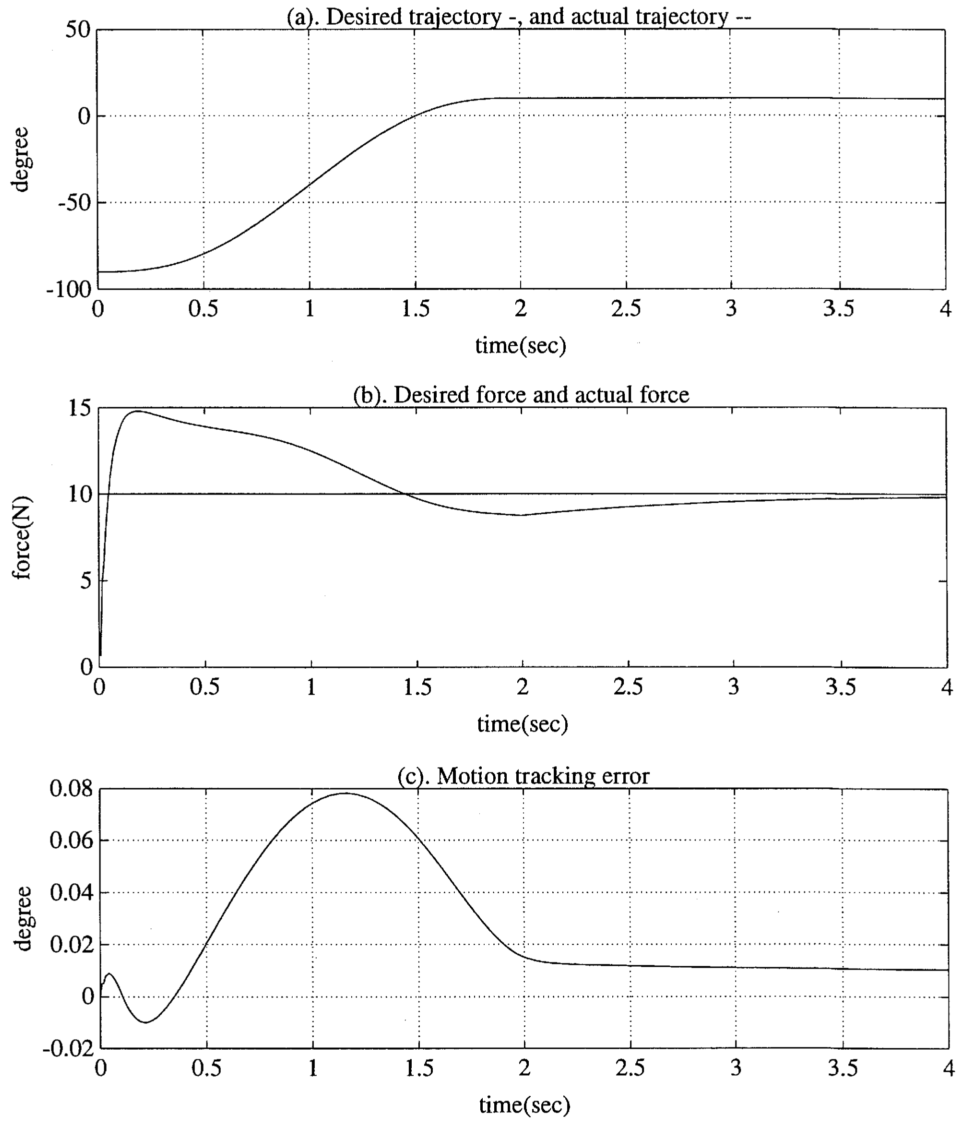

Fig. 6. The response of the non-adaptive-robust control system.

The simulation result plotted in Fig. 7 shows that the flexible joint robot system using the rigid control method is unstable if the link angle is used in the feedback control. If the rotor angle is used instead, the system is stable, but it can be seen from Fig. 8 that oscillations occur and the system performance is degraded.

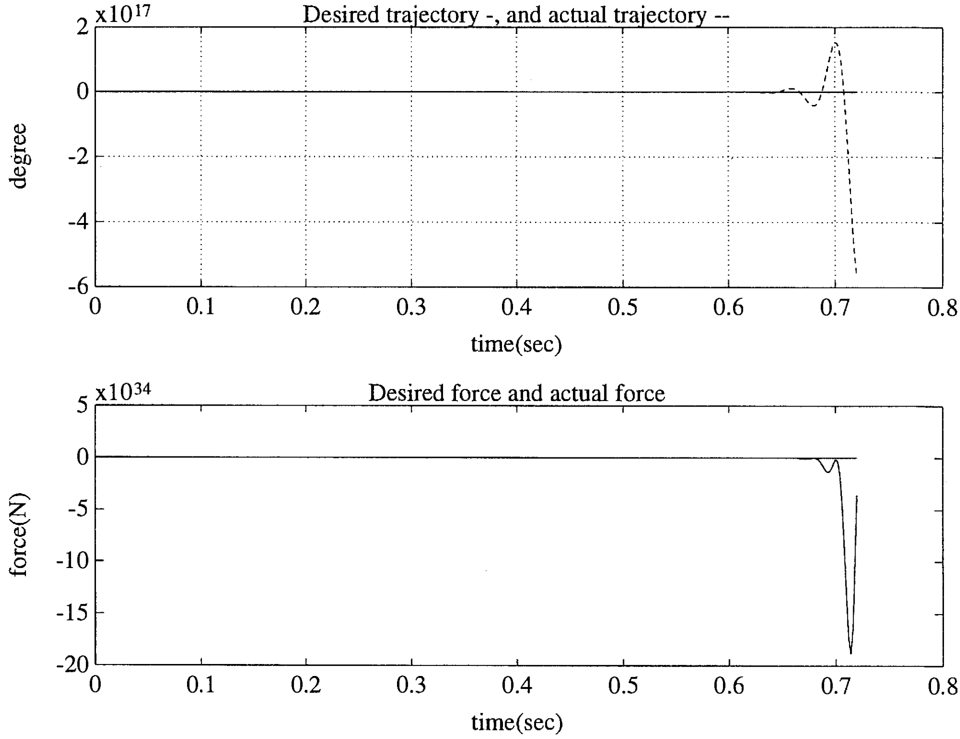

Fig. 7. The response of the system using rigid control with link angle feedback.

5. Conclusion

In this paper, motion and force control of flexible joint robots in constrained motion is considered. A two-stage control scheme, consisting of a constrained motion controller and a joint torque controller, is established in a systematic way for the general n-link case. To deal with uncertainties in the robotic system, adaptive and robust control algorithms are developed assuming that all the system parameters, including the joint flexibility values, are unknown except for some of their bounds. Moreover, the uncertainty bounds needed to derive the robust control law depend only on the parameters of the drive system. It is also easy to partition $Y_t \Delta u$ into different types of components. Thus, our knowledge of the parameter uncertainty can be better utilized in the control design. The proposed controller includes a PI type force feedback control structure which enhances the force tracking performance. The system stability is analyzed via the Lyapunov theory. It is shown that with the proposed controller, the closed-loop system is uniformly stable, and the tracking errors are uniformly ultimately bounded. The major contribution of this work is the development of a new control method for flexible joint robots in constrained motion. The method provides a systematic approach to motion and force control of flexible joint robots in

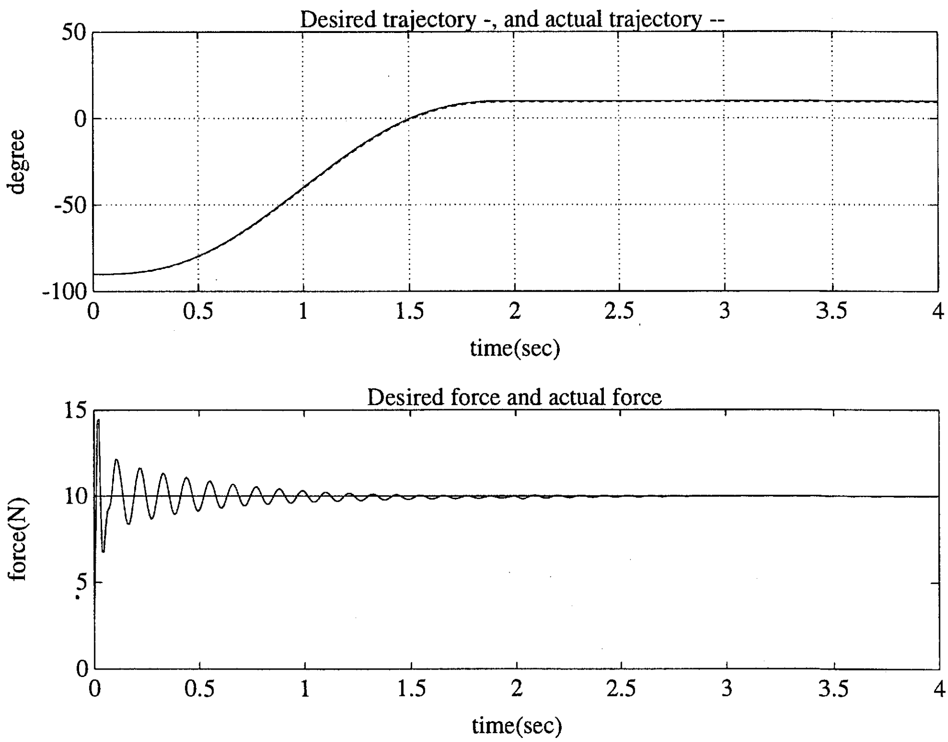

Fig. 8. The response of the system using rigid control with rotor angle feedback.

the general n-link case without requiring the exact knowledge of robotic manipulator parameters.

The simulation results show the effectiveness of the proposed control scheme.

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