

## SPREADABILITY AND SPRAY ACTUATORS

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Spreadability of distributed systems has been recently introduced (El Jai and Kassara, 1994; El Jai *et al.*, 1995). The aim of this paper is to propose a new approach to spreadability based on considering actuators as generators of spreadability. This approach is related to ecological and environmental problems, like vegetation dynamics, where the actuators are assumed to be governed by a transport equation while the system is described by a diffusion model.

### 1. Introduction

Nowadays man's activity perturbs significantly the evolution of our planet. Among the effects of these disturbances one can enumerate the reduction of vegetable areas. Consequently, it is essential to create adequate models which describe this evolution and to predict, for example, what areas will be vegetated in the future. Wide literature is focused on this problem, see (El Jai *et al.*, 1995) and the references therein. Most of the models have origins in empirical biogeographical approaches. This is the case of Budyko models (Budyko, 1971; Monserud, 1993), Gap models (Shugart, 1984; Watt, 1947) or cellular automata (Colasanti and Grime, 1993). The spreadability approach (El Jai and Kassara, 1994) is innovative in this field because it is based on finding a distributed-parameter system which describes, on certain time and space scales, a growing area which satisfies a given property. This paper is motivated by the following principle. As the vegetation generates more vegetation, this suggests that one should consider a vegetable area as the support of an actuator (El Jai and Pritchard, 1988). Moreover, based on what ecologists suggest (Solomon and Shugart, 1993), this support is assumed to be embedded in a global diffusion system.

In the next section, we recall some basic results on spreadability. In Section 3, we define spray actuators extending the results of (El Jai and Pritchard, 1988). Section 4 is then focused on modelling the actuator support by a transport model which excites a global diffusion system. Finally, an application in the one-dimensional case is detailed. The results are also illustrated with a one-dimensional example.

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## 2. Preliminaries

Let  $(S)$  be a given distributed system defined on an open bounded domain  $\Omega \subset \mathbb{R}^n$ . Its initial state  $y_0$  is assumed to be known in a given subregion  $\omega_0 \subset \Omega$ . The state of the system will be denoted by  $y(x, t, y_0)$ , where  $x \in \Omega$  and  $t \in I = ]0, T[$  are respectively the space and time variables. Let  $p$  be a given profile in the state space  $L^2(\Omega)$ . We consider the family of sets

$$\omega_t = \left\{ x \in \Omega \mid y(x, t, y_0) = p(x) \right\} \tag{1}$$

which are closed if we assume that  $y$  and  $p$  are real-valued and continuous. At the initial time  $t = 0$ , we have  $\omega_0 = \{x \in \Omega \mid y(x, 0, y_0) = p(x)\}$ .

**Definition 1.** The system  $(S)$  is said to be *p-spreadable* from  $\omega_0$  in the time interval  $I$  if the family  $\{\omega_t\}_{t \in I}$  is increasing, i.e.

$$\omega_t \subset \omega_s \quad \text{if } s \geq t \tag{2}$$

In particular, if  $p = 0$ , then the system is said to be *null-spreadable*.

Spreadability has also been considered from another point of view, by considering the growth of the areas of  $(\omega_t)$  (Bernoussi, 1998). We are also tempted to connect the spreadability to the viability of dynamical systems as developed in (Aubin, 1991). All these ideas seem, in some sense, realistic as regards spreadable environmental systems, but at this moment they do not actually lead to significant results.

**Remark 1.** The notion of *p-spreadability* is closely linked to the choice of the initial state  $y_0$ . For that purpose, if we consider the map  $\varphi_1$  defined by

$$\varphi_1: (y_0, t) \in L^2(\Omega) \times ]0, T[ \longrightarrow y(\cdot, t, y_0) \in L^2(\Omega) \tag{3}$$

which is continuous with respect to  $y_0$ , where  $y$  is the solution of the system  $(S)$  and the map  $\varphi_2$  is such that

$$\varphi_2: y \in L^2(\Omega) \longrightarrow \left\{ x \in \Omega \mid y(x) = p(x) \right\} \in \mathcal{P}(\Omega) \tag{4}$$

$\mathcal{P}(\Omega)$  holding for the parts of  $\Omega$ , then  $\varphi_2 \circ \varphi_1(y_0, t) = \omega_t$ , where  $\omega_t$  is given by (1). The system  $(S)$  is *p-spreadable* if  $\varphi_2 \circ \varphi_1$  is an increasing function with respect to  $t$ .

**Remark 2.** We can easily show that, with the change of variables  $z = y - p$ , the *p-spreadability* of a system  $(S_y)$  with the state  $y$  is equivalent to the null-spreadability of the corresponding system  $(S_z)$  with the state  $z$ .

### 3. Spray Actuators

El Jai and Pritchard (1988) defined various types of actuators in distributed-parameter systems. They showed how important are the choices of actuators which achieve certain concepts in systems analysis. In (El Jai *et al.*, 1995) these definitions were extended to the case of regional analysis. In both the works an actuator is defined by the couple  $(D, g)$ , where  $D$  defines the support of the actuator and  $g$  is a spatial distribution on the support. In this section, we extend these definitions by considering the distribution  $g$  as a given function in the whole domain  $\Omega$ , while the support  $D$  is expanding in time.

**Definition 2.** A *spray* (or *expanding*) *actuator* is defined by any couple  $((\omega_t), g)$ , where:

- $(\omega_t)_{t \geq 0}$  is a family of closed sets defined by:

$$\begin{aligned}
 h : \quad \mathcal{P}(\Omega) \times [0, 1] &\longrightarrow \mathcal{P}(\Omega) \\
 (\omega, t) &\longrightarrow \omega_t
 \end{aligned}
 \tag{5}$$

such that

$$\begin{cases}
 h(\omega, 0) = \omega_0 \\
 h(\omega, s) \supset h(\omega, t) \quad \text{if } s \geq t
 \end{cases}
 \tag{6}$$

$(\omega_t)$  being the support of this moving actuator.

- $g$  is a spatial distribution,  $g \in L^2(\Omega)$ , such that  $\forall t \in [0, 1]$  the couple  $(\omega_t, g)$  is an actuator in the common sense (El Jai and Pritchard, 1988).

**Remark 3.** In some particular situations, we can have  $\omega_0 = \{b\}$  (i.e. the action starts from a point  $b$  or, more generally, from a set  $B$  such that  $\text{meas}(B) = 0$ ). In this case there are some difficulties regarding the transition of the spreadability from  $b$  to a nonempty subdomain  $\omega_t$ . For more details, see (El Yacoubi and El Jai, 1992), where it is shown that a pointwise actuator can be considered as a limit of zone actuators by considering suitable spatial distributions.

**Remark 4.** We can also consider the notion of an *absorbing* actuator when the map  $h$  in (6) satisfies  $h(\omega, t) \supset h(\omega, s)$  for all  $t \leq s$ .

The term *spray actuator* is motivated by the notion of spray control introduced in (El Jai *et al.*, 1997). In this paper, the authors define the spray control as a control which yields the spreadability of a system.

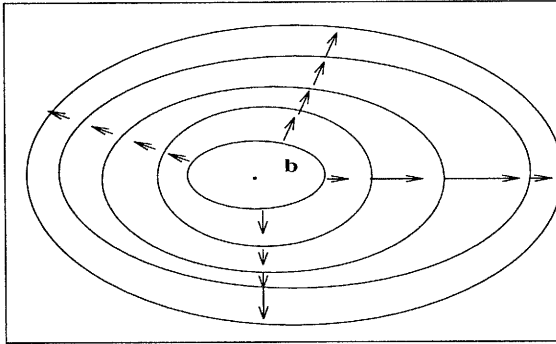


Fig. 1. Expanding actuator supports.

#### 4. Study of the Model

Let  $\Omega$  be an open bounded set in  $\mathbb{R}^n$  and  $(S)$  be a distributed system whose state satisfies a given spatial property on a subregion  $\omega_t$  at time  $t$  ( $\omega_t \subset \Omega$ ). For example, in ecology we can consider  $\omega_t$  as an expanding vegetation area. From a mathematical point of view, we may cut  $\omega_t$  and consider the system  $(S)$  only on the residual domain  $\Omega \setminus \omega_t$ . But, when referring to the above example, it is precisely from  $\omega_t$  that the expanding property will be generated. Thus the system will vanish if  $\omega_t$  is split out of the domain. This leads us to considering  $\omega_t$  as an actuator support which grows in time. Inspired by vegetation dynamics problems and various considerations of ecologists, we explore the situation where the system is of the diffusion type and is excited by a moving actuator which is subjected to transport dynamics. The model is then governed by two systems:

1. A diffusion system defined on  $\Omega$  and excited by a control  $u \in L^2(0, T)$  via moving actuator whose support is  $(\omega_t)$ ,  $\omega_t \subset \Omega$ ;
2. A transport system which describes the evolution of the support  $(\omega_t)$ .

This description is suggested in various papers on ecological modelling where the vegetation dynamics is assumed to be of diffusion type and the growing vegetation area is of transport type (Solomon and Shugart, 1993). Finally, we are not concerned here with the choice of the control  $u$  which may be considered as given in  $L^2(0, T)$ .

##### 4.1. System of Distributed Equations

The system can be seen as a diffusion process defined on  $\Omega$  and excited by one source located in  $\omega_t$  ( $\omega_0$  is given at the initial time  $t = 0$ ) and described by the parabolic

equation

$$(S) \begin{cases} \frac{\partial y}{\partial t} = \alpha \Delta y + \chi_{\omega_t} u(t) & \text{on } \Omega \times ]0, T[ \\ y(x, 0) = y_0 & \text{on } \Omega \\ y(\cdot, t) = 0 & \text{on } \partial\Omega \times ]0, T[ \end{cases} \quad (7)$$

where  $\chi_{\omega_t}$  denotes the characteristic function  $\omega_t$  and  $]0, T[$  is a time interval. The system (7) is excited by the actuator  $((\omega_t), 1)$  (El Jai and Pritchard, 1988) to be regarded in the sequel as a spray actuator. We consider the following hypothesis:

$$(H_1) \begin{cases} \bullet y \in \mathcal{D}(-\alpha \Delta) = H^2(\Omega) \cap H_0^1(\Omega), \\ \bullet u \in L^2(0, T) \text{ is given,} \\ \bullet \{\omega_t\}_{t \geq 0} \subset \Omega \text{ with a boundary } \Gamma_t \text{ parameterized by} \\ \Gamma_t = \{(x, t) \in \Omega \times ]0, T[ \mid x = a(t)\} \\ \text{with } a \in C^1(0, T), a(t) \in \Omega \forall t \in ]0, T[. \end{cases}$$

On these assumptions the system (S) has a unique solution depending on the family  $(\omega_t)_{t \in I}$ , where  $(\omega_t)$  signifies the support of the moving actuator defined by (5) and such that the solution of the advection system described by

$$(P_1) \begin{cases} \frac{\partial f}{\partial t} + v \nabla f = 0 & \text{on } Q_T \\ f(\cdot, 0) = f_0 & \text{on } \omega_0 \\ f|_{\Sigma_-} = 0 & \text{on } ]0, T[ \end{cases} \quad (8)$$

where

$$Q_T = \{(x, t) \in \Omega \times ]0, T[ \mid x \in \omega_t\} \quad (9)$$

allows for an expansion of the supports  $(\omega_t)$ . Figure 2 shows, in the one-dimensional case, what the domain  $Q_T$  is in case  $(\omega_t)$  are increasing.

Moreover, we consider the following hypothesis:

$$(H_2) \begin{cases} \bullet v \text{ denotes a velocity field } v \in V \subset \mathbb{R}^n, \\ \bullet \Sigma_- = \{(x, v, t) \in \partial\Omega \times V \times ]0, T[ \mid x \in \Gamma_t \text{ and } v \cdot \nu(x) < 0\} \\ \text{where } \nu(x) \text{ is the outward normal to } \omega_t \text{ at } x \in \Gamma_t, \\ \bullet f \in \mathcal{D}(-v \cdot \nabla) = \left\{ f \in L^2(V; L^2(Q_T)) \mid \right. \\ \left. -v \cdot \nabla f \in L^2(V; L^2(Q_T)), f|_{\Sigma_-} = 0 \right\}, \\ \bullet f_0 \in L^2(\omega_0) \text{ with } f_0 = y_0|_{\omega_0}. \end{cases}$$

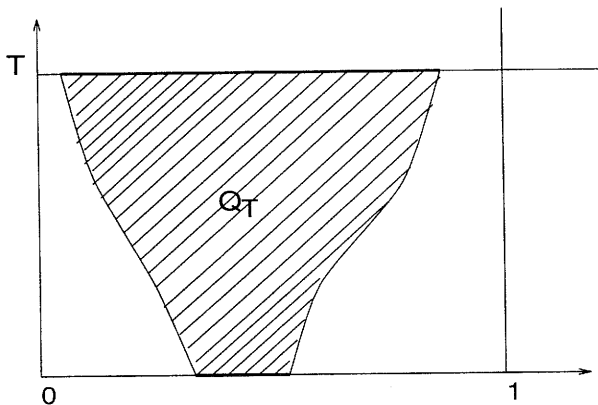


Fig. 2. Subset  $Q_T$  in the one-dimensional case with increasing  $(\omega_t)$ .

**Remark 5.** In (8) the boundary condition means that, for every  $t \in ]0, T[$ ,  $\omega_t$  is a bounded domain,  $\omega_t \subset \Omega \subset \mathbb{R}^n$ , which is surrounded by a total absorber in such a manner that nothing enters  $\omega_t$  from the outside.

Note that the advection equation (8) is defined on a moving domain  $Q_T$ . General situations for this class of equations are developed in (Bardos, 1970) where we can consider a time-varying operator given by

$$A_t = \frac{\partial}{\partial t} + v \cdot \nabla$$

which generates a strongly continuous semi-group under a convenient hypothesis and then the problem  $(P_1)$  has a unique solution in  $L^2(V \times L^2(Q_T))$ . In this paper, inspired by the results of (Conrad *et al.*, 1990), we consider the above problem as a free-boundary one in which the boundary is parameterized by a function  $a$  which satisfies the differential equation

$$(P_2) \begin{cases} a'(t) = H(a(t), f(a(t), t)) & \text{in } ]0, T[ \\ a_{(0)} = a_0 \end{cases} \tag{10}$$

where  $H \in C^{0,1}(\Omega \times \mathbb{R})$  may be unknown.

For the moving boundary problem, we consider a variable change which transforms the domain  $Q_T$  into a fixed domain denoted by  $\hat{Q}_T$ . Accordingly, the problems  $(P_1)$  and  $(P_2)$  are transformed into the problems defined in  $\hat{Q}_T$  which are denoted by  $(\hat{P}_1)$  and  $(\hat{P}_2)$ , respectively. In the next section, we show that the problem  $(\hat{P}_1)$  has a unique solution in the case of one-dimensional systems.

**Remark 6.** Note that the boundary may be defined by a more general equation of the form

$$\Gamma_t = \left\{ (x, t) \in \Omega \times ]0, T[ \mid \varphi(x, t) = 0 \right\}$$

**4.2. One-Dimensional Case**

In the one-dimensional case, we set  $\Omega = ]0, 1[$ ,  $\omega_0 = [a_0, b_0]$  with  $a_0 < b_0$  and  $\omega_t = [a(t), b(t)] \subset \Omega$  for all  $t \in ]0, T[$ . The system (S) becomes then

$$(S_1) \begin{cases} \frac{\partial y}{\partial t} = \alpha \frac{\partial^2 y}{\partial x^2} + \chi_{\omega_t} u(t) & \text{in } ]0, 1[ \times ]0, T[ \\ y(x, 0) = y_0 & \text{in } ]0, 1[ \\ y(0, t) = y(1, t) = 0 & \text{in } ]0, T[ \end{cases}$$

where the support of the actuator ( $\omega_t$ ) may be connected or not. For  $v \in V = [-1, 1]$ , the problem (P<sub>1</sub>) can be rewritten as follows:

$$(P_1) \begin{cases} \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} = 0 & \text{in } Q_T \\ f(\cdot, 0) = f_0 & \text{in } \omega_0 \\ f(a(t), t) = 0 & \text{if } v > 0 \\ f(b(t), t) = 0 & \text{if } v < 0 \end{cases}$$

with  $Q_T = \{(x, t) \in ]0, 1[ \times ]0, T[ \mid a(t) < x < b(t)\}$ . In this case,  $\Gamma_t = \{a(t), b(t)\}$  and

$$\Sigma_- = \begin{cases} \{(a(t), v, t)\} & \text{if } v > 0 \\ \{(b(t), v, t)\} & \text{if } v < 0 \end{cases} \tag{11}$$

The boundary variation is governed by

$$(P_2) \begin{cases} a'(t) = h(a(t), f(a(t), t)), & a(0) = a_0 \\ b'(t) = j(b(t), f(b(t), t)), & b(0) = b_0 \end{cases}$$

**4.2.1. Solution of the System (P<sub>1</sub>)**

At first, it is convenient to reformulate the problem (P<sub>1</sub>) in order to work on a fixed one-dimensional domain. Thus we make the change of variables

$$x \in ]a(t), b(t)[ \longrightarrow r = \frac{x - a(t)}{b(t) - a(t)} \in ]0, 1[ \tag{12}$$

setting

$$\hat{f}(r, t) = f(x, t) \tag{13}$$

So the problem is now defined on the fixed domain  $\hat{Q}_T = ]0, 1[ \times ]0, T[$  and denoted by  $(\hat{P}_1)$ :

$$(\hat{P}_1) \begin{cases} \frac{\partial \hat{f}}{\partial t} + F(r, t) \frac{\partial \hat{f}}{\partial r} = 0 & \text{in } \hat{Q}_T \\ \hat{f}(r, 0) = \hat{f}_0(r) & \text{in } ]0, 1[ \\ \hat{f}(0, t) = 0 & \text{if } v > 0 \text{ in } ]0, T[ \\ \hat{f}(1, t) = 0 & \text{if } v < 0 \text{ in } ]0, T[ \end{cases}$$

On account of (13), we have

$$F(r, t) = -\frac{r(b'(t) - a'(t)) - v + a'(t)}{b(t) - a(t)} \tag{14}$$

and

$$\hat{f}_0(r) = f_0(a_0 + r(b_0 - a_0)) \tag{15}$$

Thus the free-boundary problem links the above relation with the problem

$$(\hat{P}_2) \begin{cases} a'(t) = h(a(t), \hat{f}(0, t)), & a(0) = a_0 \\ b'(t) = j(b(t), \hat{f}(1, t)), & b(0) = b_0 \end{cases}$$

and we have the following result:

**Proposition 1.** *Assume that  $h$  and  $j$  are suitable functions in  $C^{0,1}((0, 1) \times \mathbb{R})$  satisfying the inequalities  $h(0, \cdot) \leq 0$  and  $j(1, \cdot) \geq 0$  (in order to ensure the growth of the family  $(\omega_t)_{t \geq 0}$ ) and let  $0 < a_0 < b_0 < 1$  and  $\hat{f}_0 \in L^2(0, 1)$ . Then there exists a unique solution to the problems  $(\hat{P}_1)$  and  $(\hat{P}_2)$  given by the pair  $(\hat{f}; (a, b))$ , where*

- $(a, b) \in C^1(0, T) \times C^1(0, T)$ , with  $0 < a < b < 1$  on  $]0, T[$  and  $(a, b)$  is a solution to  $(\hat{P}_2)$ ,
- $\hat{f}$  is a unique solution to a linear advection system with a variable velocity field  $F$ .

*Proof.* It is based on the study of the maps  $T_1$  and  $T_2$  defined by the following scheme:

1. First, we consider a function  $w$  given on  $]0, T[$  and solve the initial-value problem

$$\begin{cases} a'(t) = h(a(t), w), & a(0) = a_0 \\ b'(t) = j(b(t), w), & b(0) = b_0 \end{cases}$$



2. We solve the problem  $(\hat{P}_1)$  which arises as a linear advection system with a variable velocity field  $F$  given by (14). In (Bardos, 1970; Dautray and Lions, 1988) it is shown that the solution  $\hat{f}$  is a constant function along the characteristic graph described by the equation

$$\begin{cases} \dot{\xi} = F(\xi, t) \\ \xi(0) = r \end{cases} \tag{16}$$

Then we have  $\partial \hat{f}(\xi(t), t) / \partial t = 0$ .

3. Finally, we set, for  $t \in ]0, T[$ ,

$$(T_1 w)(t) = \hat{f}(0, t), \quad (T_2 w)(t) = \hat{f}(1, t)$$

By the compactness arguments, this shows that the maps  $T_1$  and  $T_2$  are strict contractions when they are defined on suitable spaces so that the problems  $(\hat{P}_1)$  and  $(\hat{P}_2)$  have a unique solution in  $]0, T[$ . For more details, see (Seidman, 1987). ■

In (Conrad *et al.*, 1990) it is stated that the pair  $(f, (a, b)) \in L^2(Q_T) \times (C^1(0, T))^2$  is a solution to the problems  $(P_1)$  and  $(P_2)$  on  $]0, T[$  if the pair  $(\hat{f}, (a, b))$  is a solution to the problems  $(\hat{P}_1)$  and  $(\hat{P}_2)$ .

For simplicity, we can consider, in examples, the functions  $h$  and  $j$  independent of  $\hat{f}(0, \cdot)$  and  $\hat{f}(1, \cdot)$ . Then we obtain an explicit solution to the problem  $(\hat{P}_1)$  as follows. Consider the restriction of  $\hat{f}$  to the characteristic graph  $C$  defined by the differential equation

$$\begin{cases} \frac{dR}{dt} = F(R, t) \\ R(0) = r \end{cases} \tag{17}$$

With  $\hat{f}(R(t), t) = \text{const}$ , if  $R(t) \in C$ , we have

$$\frac{\partial \hat{f}(R(t), t)}{\partial t} = \frac{\partial \hat{f}}{\partial r} \frac{dR(t)}{dt} + \frac{\partial \hat{f}}{\partial t} = 0$$

Then we obtain the first equation of  $(\hat{P}_1)$  if we replace  $dR(t)/dt$  by  $F(r, t)$  and set

$$\hat{f}(r, t) = G(t)\hat{f}_0(r) \tag{18}$$

where  $\{G(t)\}_{t \geq 0}$  denotes the semi-group generated by the operator

$$A = -F(r, t) \frac{\partial}{\partial r} \tag{19}$$

and given by

$$(G(t)\varphi)(r) = \begin{cases} \varphi(R(t)) & \text{if } R(t) \in ]0, 1[ \\ 0 & \text{if } R(t) \notin ]0, 1[ \end{cases} \tag{20}$$

Solving the differential equation, we easily obtain

$$R(t) = \frac{vt - a(t) + a_0 + r(b_0 - a_0)}{b(t) - a(t)} \tag{21}$$

Now, let us introduce the time  $t^*$  defined by

$$t^* = \sup \left\{ t \in ]0, T[ \mid R(s) \in ]0, 1[ \text{ for } 0 \leq s \leq t \right\} \tag{22}$$

Then the solution  $\hat{f}$  to the problem  $(\hat{P}_1)$  is given by

$$\hat{f}(r, t) = \begin{cases} \hat{f}_0 \left[ \frac{vt - a(t) + a_0 + r(b_0 - a_0)}{b(t) - a(t)} \right] & \text{if } t < t^* \\ 0 & \text{if } t \geq t^* \end{cases} \tag{23}$$

Notice that  $R(t) \in ]0, 1[$  is equivalent to  $a_0 + R(t)(b_0 - a_0) \in \omega_0$  and, based on (15), we can write

$$\hat{f}(r, t) = \begin{cases} f_0 \left[ a_0 + \frac{vt - a(t) + a_0 + r(b_0 - a_0)}{b(t) - a(t)} (b_0 - a_0) \right] & \text{if } t < t^* \\ 0 & \text{if } t \geq t^* \end{cases} \tag{24}$$

From the above result, (12), (13) and the fact that  $f_0 = y_0$  in  $\omega_0$ , we obtain the solution to the problem  $(P_1)$  in the form

$$f(x, t) = \begin{cases} y_0 \left[ a_0 + (b_0 - a_0) \times \frac{(x - a(t))(b_0 - a_0) + (vt - a(t) + a_0)(b(t) - a(t))}{(b(t) - a(t))^2} \right] & \text{if } t < t^* \\ 0 & \text{if } t \geq t^* \end{cases} \tag{25}$$

Proposition 1 and the underlying assumptions force that the couple  $(([a(t), b(t)]), 1)$  is a spray actuator for the system  $(S_1)$ .

#### 4.2.2. Spreadability and Actuators

In this section, we give a result related to the null-spreadability of the system  $(S_1)$  taken in conjunction with the actuator choice.

**Proposition 2.** *The actuator  $(([a(t), b(t)]), 1)$ , where the couple  $(a, b)$  given by  $(\hat{P}_2)$  defines the function  $f$  in (25), is a spray actuator for the system  $(S_1)$ .*

*Proof.* Assume that  $y_0 = 0$  in  $\omega_0$ . Then

$$\omega_0 = \left\{ x \in ]0, 1[ \mid y_0(x) = 0 \right\}$$

From (25) we deduce that

$$\omega_t = \left\{ x \in ]0, 1[ \mid f(x, t) = 0 \right\}$$

Moreover, the functions  $a$  and  $b$  are constructed in such a manner that the family  $(\omega_t) = ([a(t), b(t)])$  satisfies  $\omega_t \subset \omega_s$  when  $s > t$ . Therefore the transport system  $(P_1)$  is null-spreadable and then the system  $(S_1)$  is spreadable via the spray actuator  $((\omega_t), 1)$ . ■

### 4.2.3. Simulation

Consider the one-dimensional case with  $\Omega = ]0, 1[$ ,  $T = 2$ ,  $p = 0$  (null-spreadability). The diffusion system  $(S_1)$  is considered with  $\alpha = 1$  while the transport model is considered with  $v = 1$ . The boundary conditions are homogeneous and of Dirichlet type. The initial state is given by

$$y_0 = \begin{cases} 0 & \text{in } \omega_0 \\ C (\neq 0) & \text{in } \omega_0^c \end{cases}$$

The simulations are performed using a finite-difference method. The system is excited continuously in time via an actuator whose support is described by a transport equation. Figure 3 shows the evolution of the  $(\omega_t)$ 's and one can see how the state is spread over  $\Omega$ . The initial subdomain  $\omega_0 = [0.35, 0.65]$  is centred and we can see the increasing supports  $(\omega_t)$  of the spray actuator in the central part of the domain. The lateral zones coincide with the null-spreadability derived from the Dirichlet homogeneous boundary conditions. We can notice that, at time  $t \simeq 0.5$ , the property is spread over the whole domain  $\Omega = ]0, 1[$ .

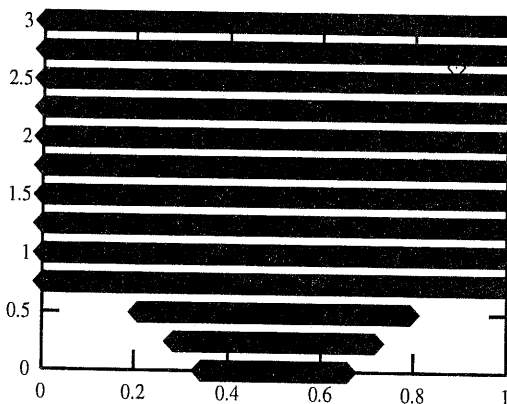


Fig. 3. Evolution of the growing actuator supports  $\omega_t$ .

## 5. Conclusion

This work essentially deals with a possible connection between spreadability and actuators. It comes from realistic assumptions to consider the spreadability zones as actuator supports (a spray actuator). We have considered the case of a diffusion system excited by an actuator whose support is to expand over the space domain. In the case of one-dimensional systems, inspired by free-boundary techniques, we have developed a simple way for making the actuator support expanding with a transport dynamics. The results may be applied to the case of two-dimensional systems, but one has to be more careful as regards the choice of the dynamics of the support boundary (problem  $(P_2)$ ). An illustrative example has been given via a very heavy and difficult simulation. A possible extension of this approach may concern the case of other dynamics or a link which can be made with the choice of the control  $u$ .

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