COMMENT ON THE REMARK BY G.A. KURINA ON THE PAPER (Müller, 1998)

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The author of the paper (Müller, 1998) thanks very much for the valuable remark by G.A. Kurina (see this issue). Her result (Kurina, 1993) simplifies the design of linear-quadratic optimal control, particularly for non-proper linear time-invariant descriptor systems.

The result in (Müller, 1998) is based on a more general approach to adapting the calculus of variations and Pontryagin's maximum principle to descriptor systems, even for nonlinear problems. There, the distinction between proper and non-proper system behaviour is required to formulate correctly the optimization problem with respect to higher-order time derivatives of control inputs which appear in the solution of differential-algebraic equations in the non-proper case (let us remark that in (Müller, 1998) the notions of causal and non-causal system behaviour were used. This is correct for discrete-time systems but not for continuous-time systems. Therefore, it is better to use the notions of proper and non-proper systems instead of causal and non-causal ones). From this point of view, in the second approach in (Müller, 1998) it was necessary to introduce the extension (87) to handle correctly time-derivatives to the control input u.

The result (91) of (Müller, 1998) fully coincides with the result (11) of (Kurina, 2002) for the assumed parameters (10). Equation (91) of (Müller, 1998) leads to

$$\ddot{u} = -k_1\dot{u} - (1+k_2)\,u, \tag{1}$$

using k_1, and k_2 of (Kurina, 2002). Since in this example we have the relations

$$u = -x_3, \quad \dot{u} = -x_2, \quad \ddot{u} = -u - x_1 = x_3 - x_1, \tag{2}$$

equation (1) can be rewritten as

$$u = x_1 + k_1x_2 + (k_2 - 1)\,x_3, \tag{3}$$

which coincides with (11) from (Kurina, 2002) for $k_4 = -1$.

Additionally, (1) and (2) lead to the requirement

$$0 = x_1 + k_1x_2 + k_2x_3 \tag{4}$$

such that by subtracting (4) twice from (3),

$$u = -x_1 - k_1x_2 - (k_2 + 1)\,x_3 \tag{5}$$

is obtained, which coincides with (11) from (Kurina, 2002) for $k_4 = 1$.

Because of (4), the static state feedback (3) or (5) is not unique. For each m, the feedback

$$u = (1+m)\,x_1 + (1+m)\,k_1x_2 + \big[(1+m)\,k_2 - 1\big]x_3 \tag{6}$$

is feasible. But the filter (1) with the initial conditions $u(0) = -x_{30}$ and $\dot{u}(0) = -x_{20}$ defines a unique dynamic feedback for the descriptor system. The consistent initial condition x_{10} has to satisfy (4) for $t = 0$. Therefore, the author prefers the realization of feedback control by the filter (1).

The last remark corresponds to a correction in (Müller, 1998). The formula (92) has to be corrected into

$$P_{12} = -q_1b_1b_3 + \sqrt{(q_1b_3^2 + r_3)\,[q_1b_1^2 + q_2b_2^2 + q_3b_3^2 + r_1]}. \tag{7}$$

References

Kurina G.A. (1993): *On regulating by descriptor systems in an infinite interval*. — Izvestija RAN, Tekhnicheskaja Kibernetika, No. 6, pp. 33–38 (in Russian).

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