Int. J. Appl. Math. Comput. Sci., 2002, Vol.12, No.4, 523–531

CONTROL OF AN INDUCTION MOTOR USING SLIDING MODE LINEARIZATION

ERIK ETIEN*, SÉBASTIEN CAUET
LAURENT RAMBAULT, GÉRARD CHAMPENOIS

* Laboratoire d'Automatique et d'Informatique Industrielle, 60 avenue du recteur Pineau, 86022 Poitiers, France
e-mail: etien@esip.univ-poitiers.fr

Nonlinear control of the squirrel induction motor is designed using sliding mode theory. The developed approach leads to the design of a sliding mode controller in order to linearize the behaviour of an induction motor. The second problem described in the paper is decoupling between two physical outputs: the rotor speed and the rotor flux modulus. The sliding mode tools allow us to separate the control from these two outputs. To take account of parametric variations, a model-based approach is used to improve the robustness of the control law despite these perturbations. Experimental results obtained with a laboratory setup illustrate the good performance of this technique.

Keywords: induction motor, sliding mode control, linearization

1. Introduction

In variable speed domain, many applications need high performances in terms of torques and accuracy. To obtain high performances, several control methods have been developed in the last few years. Sliding mode theory, stemmed from the variable-structure control family, has been used for the Induction Motor (IM) drive for a long time (Utkin, 1999). Introduced as a relatively easier control design, the sliding mode using switched controls produces chattering phenomena and torque perturbations. Many solutions try to limit those drawbacks by using, e.g., smoothed nonlinearities, but at present the main stream of the sliding mode in IM control is the design of flux observers (Edwards and Spurgeon, 1994).

On the other hand, many methods of nonlinear system control have been developed, such as exact input-output linearization or backstepping (Chiasson, 1996; Taylor, 1994). It is well known that the use of these methods in practical applications needs adaptive solutions to deal with robustness problems (Marino and Valigi, 1991; 1993; Von Raumer *et al.*, 1993a; 1993b).

At first, this paper employs sliding mode theory, well known by speed drive conceptors, in order to linearize IM behaviour. The choice of a particular sliding surface permits to create a link between sliding mode theory and input-output linearization.

In the second part, the sliding mode in the tracking problem applied to an IM is considered. In the presence of some particular reference input signal, the tracking yields unstable behaviour. To show that the stability depends on the input signals, in Section 4 a stability analysis based on the Lyapunov theory is presented. A model-based approach is proposed to solve the stability problem and to improve the robustness of stabilization (Edwards and Spurgeon, 1998).

Section 2 presents an IM model in the concordia frame. The third section develops sliding mode theory and its application to linearization. Section 4 introduces a reference model to improve the robustness of linearization. Some experimental results are presented in Section 5.

2. Description of the Electrical Motor

In order to design sliding mode control, we use the IM model with respect to a fixed stator reference frame $\{\alpha, \beta\}$ (see Appendix). The main reason behind this choice is the improvement of performances concerning the numerical precision. In fact, by using the fixed stator reference frame, it is not necessary to implement a rotor position measurement (Barbot *et al.*, 1992).

The objective is to control the following two physical quantities: the rotor speed ω and the magnitude of the rotor flux $|\Phi|^2 = \Psi_{r\alpha}^2 + \Psi_{r\beta}^2$.

We define a new state space representation:

$$\begin{cases} \dot{\underline{x}} = f(\underline{x}, t) + B(\underline{x}, t)\underline{u}, \\ \underline{y} = C(\underline{x}, t) \end{cases} \tag{1}$$

with $\underline{x}^T = [\omega \quad |\phi| \quad \dot{\omega} \quad |\dot{\phi}|]$. Here

$$f(\underline{x},t) = \begin{bmatrix} \begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \underline{x} \\ f_3(\underline{x}) \\ f_4(\underline{x}) \end{bmatrix} \tag{2}$$

with

$$A_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{3}$$

With no loss of generality, the state vector $\underline{x}$ can be written as follows:

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} \tag{4}$$

with

$$\underline{x}_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} \omega \\ |\phi| \end{bmatrix}$$

and

$$\underline{x}_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = \begin{bmatrix} \dot{\omega} \\ |\dot{\phi}| \end{bmatrix}. \tag{5}$$

Then

$$\dot{\underline{x}}_1 = \underline{x}_2. \tag{6}$$

We find the following functions:

$$\begin{aligned} f_3(\underline{x}) &= -\mu p\beta x_{11}x_{12} - (\alpha+\gamma)x_{21} \\ &\quad - \frac{\mu p\beta x_{11}x_{22}}{2\alpha M_{sr}} - \frac{\mu p x_{11}x_{12}}{M_{sr}}, \end{aligned} \tag{7}$$

$$\begin{aligned} f_4(\underline{x}) &= (4\alpha^2 + 2\alpha^2 M_{sr}\beta)x_{12} + \frac{2\alpha p M_{sr} x_{11} x_{21}}{\mu} \\ &\quad - (3\alpha+\gamma)x_{22} \\ &\quad + \frac{2\alpha^2 M_{sr}^2}{x_{12}} \left[\left(\frac{x_{22} + 2\alpha x_{12}}{2\alpha M_{sr}} \right)^2 + \frac{x_{21}^2}{\mu^2} \right]. \end{aligned} \tag{8}$$

This choice of the state space representation provides a particular form for the matrix $f(\underline{x},t)$, which is divided into two blocks: a linear one and a nonlinear one. This particular form, known as the regular form, can be derived from the classical IM model via mathematical transformations (Edwards and Spurgeon, 1998).

In order to take account of the parametric variations, we introduce unknown deviations of the resistance (δR) and the inductance (δL). The IM physical parameters can also be defined as follows:

$$\begin{cases} R_r = R_{rn}(1+\delta R), \quad R_s = R_{sn}(1+\delta R), \\ L_r = L_{rn}(1+\delta L), \end{cases} \tag{9}$$

$$L_s = L_{sn}(1+\delta L), \quad M_{sr} = M_{sn}(1+\delta L). \tag{10}$$

In the following part, the controller is designed in two steps in order to control the output vector $\underline{y}^T = [\ \omega \quad |\phi|\ ]$. The first step consists in linearizing the behaviour of the nonlinear system (1). The other improves the robustness of linearization and the tracking problem.

3. Sliding Mode Linearization

3.1. Sliding Mode Theory

In this subsection, sliding mode theory is summarized. The reader is referred to (Edwards and Spurgeon, 1998; Utkin, 1992) for details. Let us consider the nonlinear system

$$\begin{cases} \dot{\underline{x}} = f(\underline{x},t) + B(\underline{x},t)\underline{u}, \\ \underline{y} = C(\underline{x},t), \end{cases} \tag{11}$$

where $\underline{x}(t) \in \mathbb{R}^n$, $\underline{u}(t) \in \mathbb{R}^m$ and $B(\underline{x},t) \in \mathbb{R}^{n\times m}$.

From the system (11), it is possible to define a set S of the state trajectories $\underline{x}$ such as

$$S = \{\underline{x}(t)\,|\sigma(\underline{x},t) = 0\ \}, \tag{12}$$

where

$$\sigma(\underline{x},t) = \left[\sigma_1(\underline{x},t), \dots, \sigma_m(\underline{x},t)\right]^T = 0 \tag{13}$$

and $[\cdot]^T$ denotes the transposed vector.

S is called the "sliding surface" and the system is said to be in the sliding mode when the state trajectory $\underline{x}$ of the controller plant satisfies $\sigma(\underline{x}(t),t) = 0$ at every $t \geq t_1$ for some t_1. The surfaces are designed so that the state trajectory, restricted to $\sigma(\underline{x}(t),t) = 0$, shows some desired behaviour such as stability or tracking. Commonly, in IM control using sliding mode theory, the surfaces are chosen as functions of the error between the reference input signals and the measured signals (Utkin, 1993).

After this step, the objective is to determine a control law which drives the state trajectories along the surface (12). The following part shows how to use sliding mode theory for linearization behaviour.

In most applications this technique is implemented by using switched controllers in order to improve some performances. The chattering phenomena and the torque perturbations are reduced by adding and designing switch components with hysteresis. The main contribution of this work concerns the application of the sliding mode to linearize a nonlinear system. The next section outlines a method to tune the parameters of the sliding mode controller.

3.2. Application to Linearization

The objective of the first step of the design procedure is to convert the nonlinear system (1) to a linear one defined by the following state space representation:

$$\begin{cases} \underline{\dot{x}} = A\underline{x} + B\underline{v}, \\ \underline{y} = C\underline{x}, \end{cases} \tag{14}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -s_1 s_1' & 0 & (s_1 + s_1') & 0 \\ 0 & -s_2 s_2' & 0 & (s_2 + s_2') \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{15}$$

with

$$\underline{x}^T = \begin{bmatrix} \omega & |\phi| & \dot{\omega} & |\dot{\phi}| \end{bmatrix},$$
$$\underline{v}^T = \begin{bmatrix} v_1 & v_2 \end{bmatrix}, \quad \underline{y}^T = \begin{bmatrix} \omega & |\phi| \end{bmatrix}. \tag{16}$$

The linearized system is equivalent to two independent subsystems. The first one represents a transfer from the new input, v_1, to the speed of the rotor. The second one is also a linear transfer from an input v_2 to the rotor flux modulus. Each subsystem is characterized by two poles: $\{s_1, s_1'\}$ for the first one and $\{s_2, s_2'\}$ for the second one.

Thus the linear system can be written using the transfer matrix

$$G(s) = \begin{bmatrix} G_1(s) & 0 \\ 0 & G_2(s) \end{bmatrix}$$

with

$$G_1(s) = \frac{\omega}{v_1} = \frac{1}{(s - s_1)(s - s_1')},$$
$$G_2(s) = \frac{|\phi|}{v_1} = \frac{1}{(s - s_2)(s - s_2')}. \tag{17}$$

The eigenvalues $\{s_1, s_1'\}$ and $\{s_2, s_2'\}$ are chosen to determine respectively the dynamics of the rotor speed and the rotor flux modulus in order to consider the physical time constants of the system.

Consider the particular sliding surface

$$\sigma(\underline{x}, t) = \sigma_1 \underline{x} + \sigma_2 \underline{\dot{x}} \tag{18}$$

with $\sigma_1 = [\ S_1 \quad S_2\ ]$ and $\sigma_2 = [\ S_3 \quad S_4\ ]$ standing for matrices to be determined. Developing (18), we can write

$$\sigma(\underline{x}, t) = S_1 \underline{x}_1 + S_2 \underline{x}_2 + S_3 \underline{\dot{x}}_1 + S_4 \underline{\dot{x}}_2. \tag{19}$$

Using (6) yields

$$\underline{x}_2 = -(S_2 + S_3)^{-1} \left[S_1 \underline{x}_1 + S_4 \underline{\ddot{x}}_1 \right]. \tag{20}$$

Substituting the nonlinear regular expression of (3) gives

$$\underline{\dot{x}}_1 = -A_{12} \left[(S_2 + S_3)^{-1} (S_1 \underline{x}_1 + S_4 \underline{\ddot{x}}_1) \right]. \tag{21}$$

Finally,

$$\underline{\ddot{x}}_1 = -S_4^{-1} S_1 \underline{x}_1 - S_4^{-1} (S_2 + S_3) \underline{\dot{x}}_1. \tag{22}$$

Consider the particular case where $S_4 = S_2 = I$,

$$S_1 = \begin{bmatrix} s_1 s_1' & 0 \\ 0 & s_2 s_2' \end{bmatrix},$$
$$S_3 = \begin{bmatrix} -(1 + s_1 + s_1') & 0 \\ 0 & -(1 + s_2 + s_2') \end{bmatrix}. \tag{23}$$

Consequently, the sliding surface is

$$\begin{aligned} \sigma(\underline{x}, t) &= \begin{bmatrix} s_1 s_1' & 0 & 1 & 0 \\ 0 & s_2 s_2' & 0 & 1 \end{bmatrix} \underline{x} \\ &+ \begin{bmatrix} -(1 + s_1 + s_1') & 0 & 1 & 0 \\ 0 & -(1 + s_2 + s_2') & 0 & 1 \end{bmatrix} \underline{\dot{x}}. \end{aligned} \tag{24}$$

In order to find the control law $u(t)$ which imposes $\sigma(\underline{x}, t) = 0$, we use the equivalent control method (Utkin, 1992). Using (18) and (11), we write

$$\sigma(\underline{x}, t) = \sigma_1 \underline{x} + \sigma_2 f(\underline{x}, t) + \sigma_2 B(\underline{x}, t) \underline{u}_{eq} = 0. \tag{25}$$

Suppose that the matrix σ_2 is such that the square matrix $[\sigma_2 B(\underline{x}, t)]$ is invertible. Then the equivalent control is defined by the following expression:

$$\underline{u}_{eq}(\underline{x}, t) = -\left[\sigma_2 B(\underline{x}, t) \right]^{-1} \left[\sigma_1 \underline{x} + \sigma_2 f(\underline{x}, t) \right]. \tag{26}$$

Finally, the linearizing control is given by

$$\underline{u} = \underline{u}_{eq} + \left[\sigma_2 B(\underline{x}, t) \right]^{-1} \underline{v}. \tag{27}$$

The equivalent system with the input vector $\underline{v}$ is also defined by (14) and its representation is shown in Fig. 1.

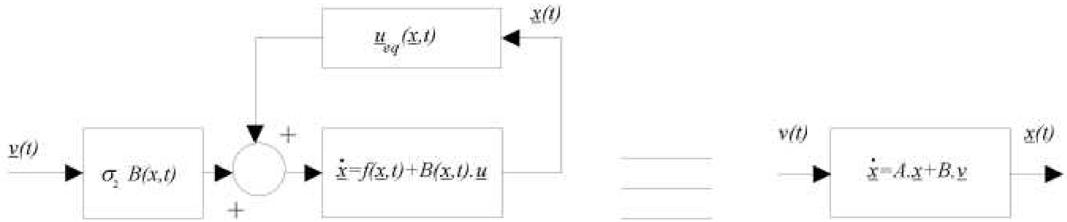

Fig. 1. Sliding mode linearization principle.

Notice that this control is equivalent to a classical exact input-output linearization technique with poles placement (Chiasson, 1993; Isidori, 1989). In practice, it is impossible to use directly this type of linearization. The first difficulty is the loss of decoupling when parametric variations appear. This robustness problem is usually solved using an adaptive solution. Another difficulty concerns linearization stability toward the perturbation signal. This stability degree is conditioned by the choice of the pole placement $\{s_1, s_1'\}$, $\{s_2, s_2'\}$ and the external signals (tracking references, perturbation signals, etc.). In order to improve the limits of linearization, we propose to use the Lyapunov theory.

3.3. Linearization Stability

The Lyapunov approach is used for deriving the condition control $u(\underline{x}, t)$ that will drive the state trajectory to the sliding surface. Consider the quadratic Lyapunov function

$$V(t, \underline{x}, \sigma) = \sigma^T(\underline{x}, t)\sigma(\underline{x}, t), \tag{28}$$

with $\sigma(\underline{x}, t)$ being the sliding surface defined in (25). To prove linearization stability, we must verify that the control law (27) allows us to obtain the Lyapunov condition

$$\dot{V}(t, \underline{x}, \sigma) = 2\sigma^T\dot{\sigma} < 0, \quad \nabla\sigma \neq 0, \tag{29}$$

where we have cancelled the specific $\underline{x}$ and t dependencies. We write

$$\dot{\sigma} = \frac{\delta\sigma}{\delta t} + \frac{\delta\sigma}{\delta x}\dot{\underline{x}}. \tag{30}$$

Equation (1) yields

$$\dot{\sigma} = \frac{\delta\sigma}{\delta t} + \frac{\delta\sigma}{\delta x}f(\underline{x}, t) + \frac{\delta\sigma}{\delta x}B(\underline{x}, t)\underline{u}. \tag{31}$$

The expression (29) can be written down as follows:

$$\begin{aligned}\dot{V} &= 2\sigma^T\frac{\delta\sigma}{\delta t} + 2\sigma^T\frac{\delta\sigma}{\delta x}f(\underline{x}, t) \\ &\quad + 2\sigma^T\frac{\delta\sigma}{\delta x}B(\underline{x}, t)\underline{u} < 0, \quad \nabla\sigma \neq 0.\end{aligned} \tag{32}$$

Equations (27) and (32) yield

$$\dot{V} = 2\sigma^T\underline{v} < 0, \quad \forall\sigma \neq 0. \tag{33}$$

We see that the Lyapunov condition depends on the pole placement and, consequently, on the designed surface. Moreover, the reference signal may exert influence on linearization stability. Hence, the tracking references are considered as perturbation signals.

Notice that the result (33) was obtained using the equivalent control (26), which depends on the nominal system (1). In the case of parametric variations or modelling errors, the Lyapunov condition might not be checked. Consequently, for tracking behaviours, the discussed linearization technique cannot be used alone.

4. Sliding Mode Control

In the presence of parametric variations, the linearized system is modified as follows (Cauet *et al*., 2001a):

$$\begin{aligned}\dot{\underline{x}} &= (A_n + A_R\delta R + A_L\delta L)\underline{x} \\ &\quad + (B_n + B_L\delta L)\underline{v} + d(\underline{x})\end{aligned} \tag{34}$$

with

$$A_n = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -s_1s_1' & s_1 + s_1' & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -s_2s_2' & s_2 + s_2' \end{bmatrix},$$

$$A_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\gamma_n - \alpha_n & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & A_{R4,3} & -\gamma_n - 3\alpha_n \end{bmatrix},$$

$$A_{R4,3} = 2\alpha_n(-\gamma_n - \alpha_n + \alpha_n\beta_nM_{srn}),$$

$$A_L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ s_1 s_1' & -s_1 s_1' & 0 & 0 \\ 0 & 0 & 2\alpha_n & 0 \\ 0 & 0 & A_{L4,3} & A_{L4,4} \end{bmatrix},$$

$$A_{L4,3} = s_2 s_2' - 2\alpha_n^2 \beta_n M_{srn} - 4\alpha_n^2,$$

$$A_{L4,4} = -2\alpha_n - (s_2 + s_2'),$$

$$B_n = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_L = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix},$$

$$d(\underline{x}) = \begin{bmatrix} 0 \\ d_1(\underline{x}) = \dfrac{-\delta_L p \mu}{M_{srn}} \left(\dfrac{x_{11} x_{22}}{2\alpha_n} + x_{11} x_{21} \right) \\ 0 \\ d_2(\underline{x}) \end{bmatrix},$$

with

$$\begin{aligned} d_2(\underline{x}) &= 2p\alpha_n M_{srn} \delta_L \frac{x_{11} x_{12}}{\mu} \\ &\quad + 2\alpha_n^2 M_{srn}^2 (\delta_R + \delta_L) \\ &\quad \times \left\{ \frac{1}{x_{21}} \left[\left(\frac{x_{22} + 2\alpha_n x_{21}}{2\alpha_n M_{srn}} \right)^2 + \frac{x_{12}^2}{\mu^2} \right] \right\}. \end{aligned}$$

We propose to use the particular robust properties of sliding mode control to minimize the consequences of these parametric variations. Commonly, in order to introduce tracking requirements, a model-based approach is chosen. A similar approach is proposed in (Cauet *et al.*, 2001b) for induction motor control.

Assume that the plant is defined by (14), which produces the following tracking model:

$$\underline{\dot{\omega}} = A_m \underline{\omega} + B_m \underline{r}. \tag{35}$$

Define the error state

$$\underline{e}(t) = \underline{x}(t) - \underline{\omega}(t). \tag{36}$$

The objective of the second step of the design procedure is to choose the control input vector $\underline{v}$ which imposes $\underline{e}(t) \longrightarrow 0$ in a short time. The following sliding surface is proposed:

$$\sigma_e(\underline{e}, t) = S\underline{e}(t) = 0 \tag{37}$$

with

$$S = \begin{bmatrix} 1 & 0 & -\dfrac{1}{s_{e\omega}} & 0 \\ 0 & 1 & 0 & -\dfrac{1}{s_{e\Phi}} \end{bmatrix}. \tag{38}$$

We have $-1/s_{e\omega}$ and $-1/s_{e\Phi}$ as the two poles that impose the dynamic errors.

The equivalent control $\underline{v}_{eq}(t)$ (Utkin, 1992) that satisfies the condition $\sigma_e(\underline{e}, t) = 0$ is

$$\underline{v}_{eq}(t) = -(SB)^{-1} S \big[A_m \underline{e} + (A - A_m)\underline{x} - B_m \underline{r} \big]. \tag{39}$$

To complete the control design, we have to solve the reachability problem (Edwards and Spurgeon, 1998). In the presence of parametric variations, the control $\underline{v}_{eq}(t)$ cannot impose the condition $\sigma_e(\underline{e}, t) = 0$. The solution is to find complementary control $\underline{v}_N(\sigma_e, t)$ which drives the state trajectory error to the equilibrium manifold. Using the Lyapunov theory, we choose the continuous control (DeCarlo *et al.*, 1996):

$$\underline{v}_N = -P\sigma_e(\underline{e}, t), \quad P = P^T > 0. \tag{40}$$

In this case, the derivative of the Lyapunov function can be written as follows:

$$\dot{V} = 2\sigma_e^T(\underline{e}, t)\underline{v}_N = 2\sigma_e^T(\underline{e}, t) P \sigma_e(\underline{e}, t). \tag{41}$$

The Lyapunov condition $\dot{V} < 0$, $\forall \sigma_e(\underline{e}, t) \neq 0$ is satisfied for all $P = P^T > 0$. Finally, we obtain the global control

$$\underline{v}(t) = \underline{v}_{eq}(t) + (SB)^{-1} \underline{v}_N(t). \tag{42}$$

The control scheme is summarized in Fig. 2.

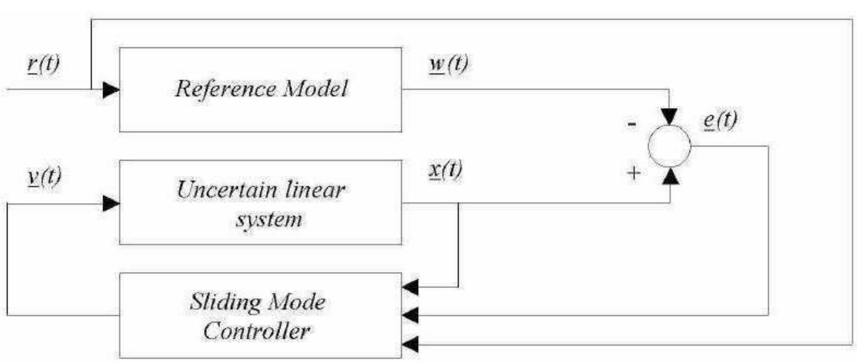

Fig. 2. Model-based sliding mode control.

The reference model imposes the dynamics to be followed by the linearized system. Sliding mode control guarantees the convergence of the error ($\underline{e}(t) \longrightarrow 0$). The reference model is chosen as a copy of the theorical linearized system in the nominal case. The external loop leads to robust linearization.

5. Experimental Results

The control law is tested on an experimental plant composed of a 1.1 kW induction motor. The control command is designed with a digital signal processor TMS320C32 board. The rotor speed and two stator current measurements are used to estimate the error flux component (Verghese and Sanders, 1988). The two poles of the linearized system are $s_1 = -10$, $s_1' = -200$, $s_2 = s_2' = -300$.

Figure 3 shows the benchmark signals used in practice. To show the advantages of the model-based control, we introduce parameter deviations in the control algorithm ($\delta R_r = \pm 50\%$ and $\delta L_s = \pm 20\%$). The results are provided in two cases: Figs. 4–7 provide the responses obtained without the reference model and Figs. 8–11 illustrate the improvements due to the reference model (RM). Figures 12 and 13 show the sliding surfaces.

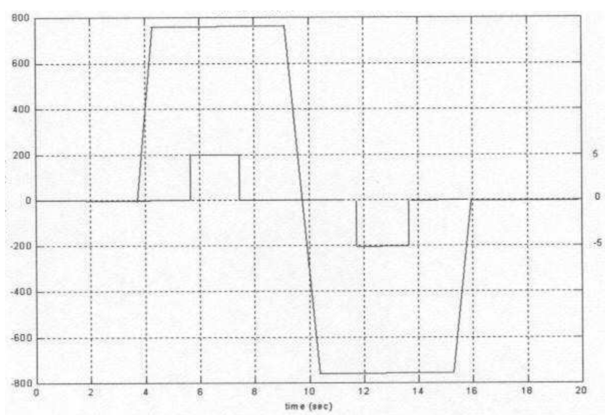

Fig. 3. Benchmark test: rotor speed (turns/min) and load torque (Nm).

Without the reference model, the parametric variations introduced in the control algorithm provide an important coupling action between the rotor flux and the rotor speed. Moreover, the speed is obtained with an important steady-state error. With the use of the reference model, the decoupling is ensured and correct tracking is obtained for the components $|\Phi|^2 = \Psi_{r\alpha}^2 + \Psi_{r\beta}^2$ and ω.

6. Conclusion

In this paper, we propose induction motor control using sliding mode theory in two steps. A particular sliding sur-

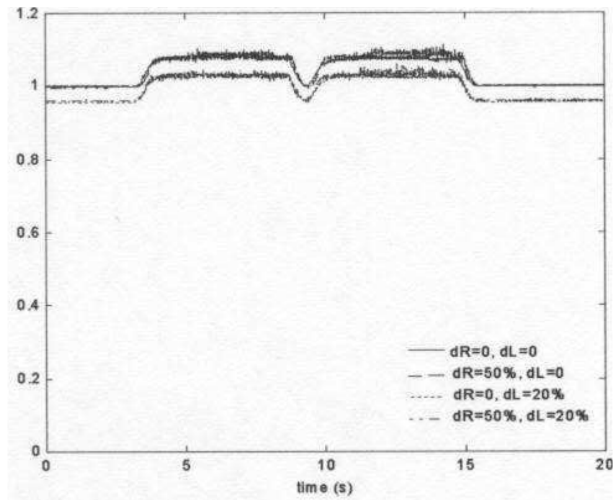

Fig. 4. Rotor flux response without the RM.

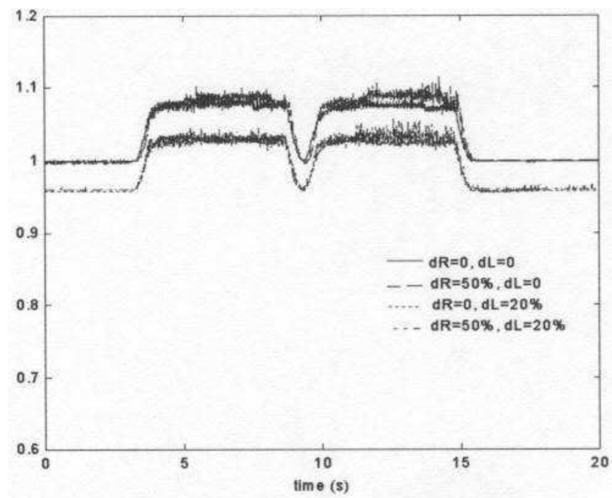

Fig. 5. Zoom of the rotor flux response without the RM.

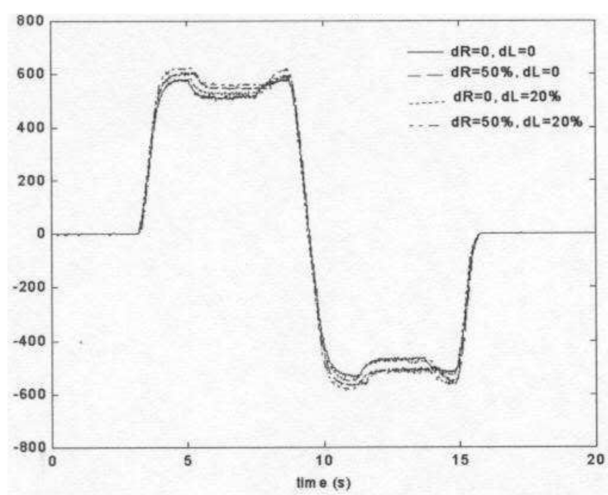

Fig. 6. Rotor speed response without the RM.

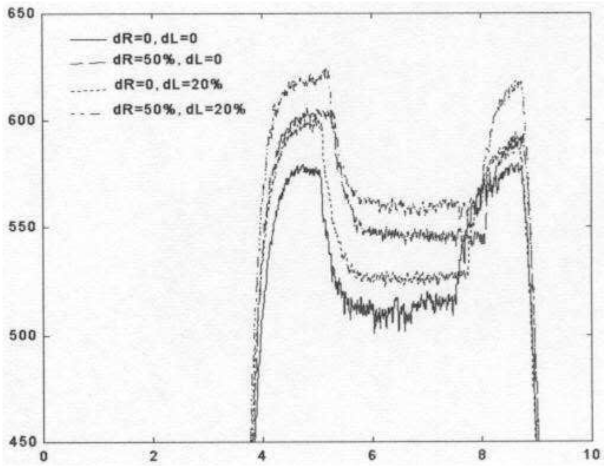

Fig. 7. Zoom of the rotor speed response without the RM.

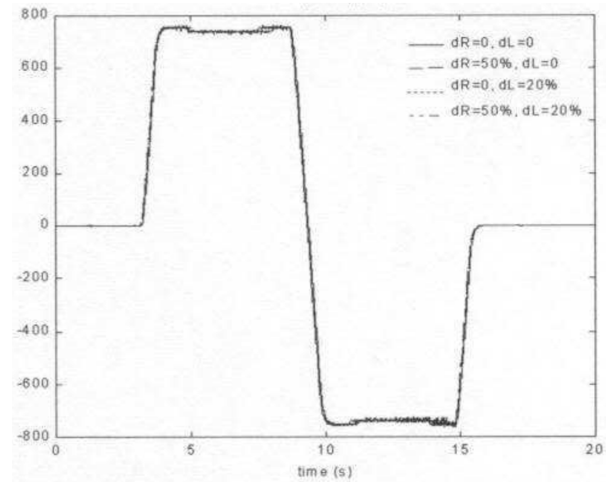

Fig. 10. Rotor speed response with the RM.

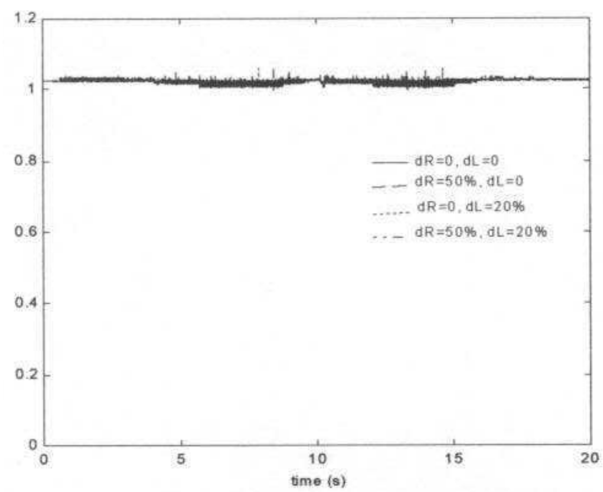

Fig. 8. Rotor flux response with the RM.

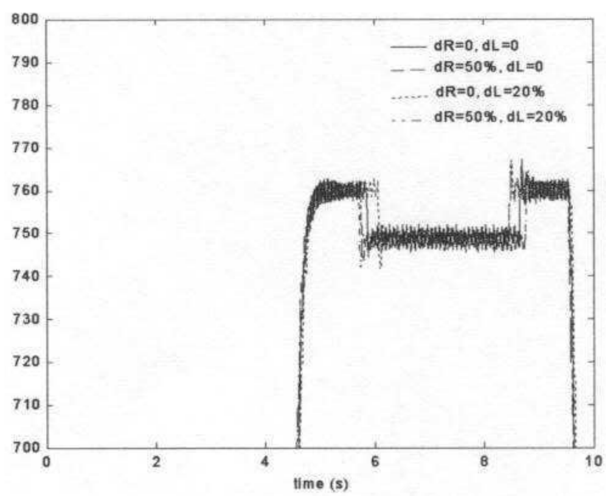

Fig. 11. Zoom of the rotor speed response with the RM.

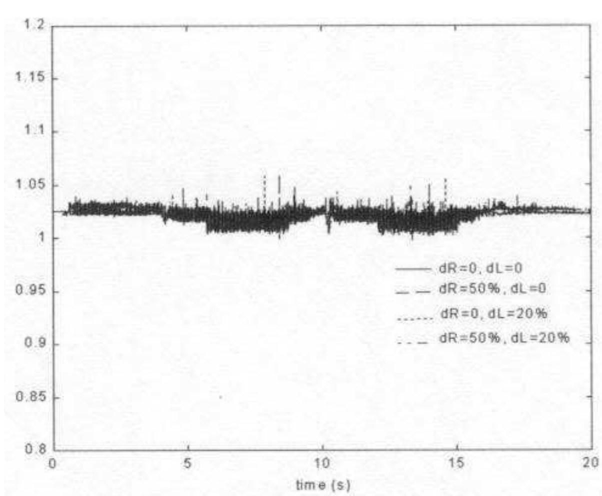

Fig. 9. Zoom of the rotor flux response with the RM.

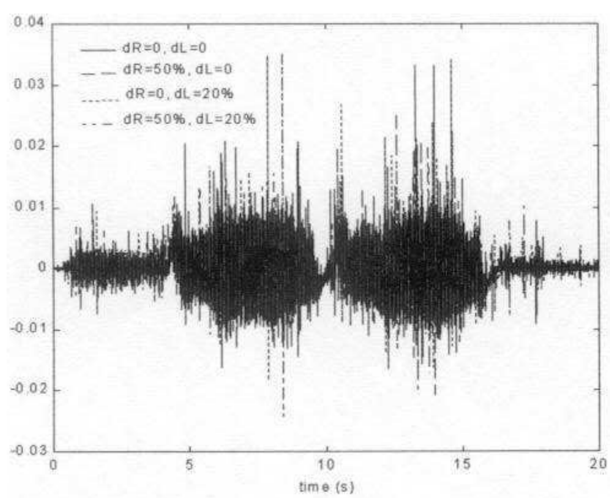

Fig. 12. Sliding surface on the rotor flux error.

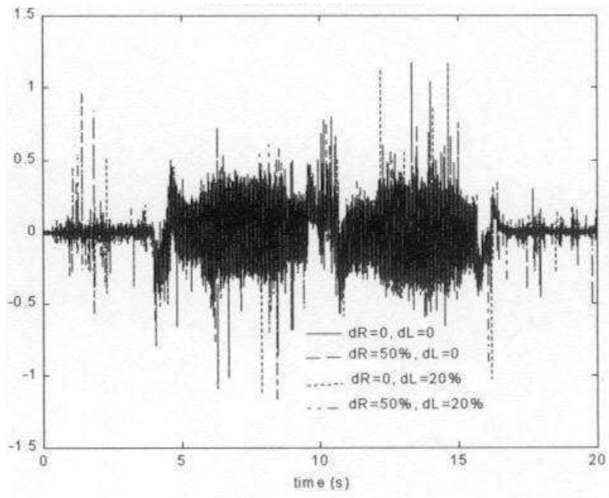

Fig. 13. Sliding surface on the rotor speed error.

face allows us to obtain a linearization effect similar to classical linearization techniques. Second feedback based on a reference model is used to obtain robust properties in terms of parameter variations. Experimental results show good performances obtained with this method.

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Appendix

The equations of the motor in the fixed stator reference frame (α, β) are given by

$$\dot{\underline{z}} = h(\underline{z}) + g u_{s\alpha\beta}, \tag{A1}$$

with $\underline{z}^T = [\ \omega \quad \Psi_{r\alpha} \quad \Psi_{r\beta} \quad i_{s\alpha} \quad i_{s\beta}\ ]$, where $\Psi_{r\alpha}$, $\Psi_{r\beta}$ are the rotor flux dynamics and $i_{s\alpha}$, $i_{s\beta}$ are the stator currents. The control vector is defined by $\underline{u}^T_{s\alpha\beta} = [\ u_{s\alpha} \quad u_{s\beta}\ ]$. With this notation the state space representation is

$$h(\underline{z}) = \begin{bmatrix} \mu(\Psi_{r\alpha} i_{s\beta} - \Psi_{r\beta} i_{s\alpha}) - \frac{T_l}{J} \\ -\alpha\Psi_{r\alpha} - p\omega\Psi_{r\beta} + \alpha M_{sr} i_{s\alpha} \\ p\omega\Psi_{r\alpha} - \alpha\Psi_{r\beta} + \alpha M_{sr} i_{s\beta} \\ \alpha\beta\Psi_{r\alpha} + p\beta\omega\Psi_{r\beta} - \gamma i_{s\alpha} \\ -p\beta\omega\Psi_{r\alpha} + \alpha\beta\Psi_{r\beta} - \gamma i_{s\beta} \end{bmatrix}, \tag{A2}$$

$$g = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{1}{\sigma L_s} & 0 \\ 0 & \dfrac{1}{\sigma L_s} \end{bmatrix}, \tag{A3}$$

and T_l is the load torque. For simplicity, we define the following variables:

$$\alpha = \frac{R_r}{L_r}, \quad \beta = \frac{M_{sr}}{\sigma L_s L_r}, \quad \mu = \frac{pM_{sr}}{JL_r},$$
$$\gamma = \frac{M_{sr}^2 R_r}{\sigma L_s L_r^2} + \frac{R_s}{\sigma L_s}, \tag{A4}$$

where L_s is the stator inductance, M_{sr} is the mutual inductance, L_r is the rotor inductance, R_s is the stator resistance, R_r is the rotor resistance and J denotes the rotor inertia.

Received: 21 May 2001
Revised: 4 July 2002