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DELAY–DEPENDENT GENERALIZED H_2 CONTROL FOR DISCRETE T–S FUZZY LARGE–SCALE STOCHASTIC SYSTEMS WITH MIXED DELAYS

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This paper is concerned with the problem of stochastic stability and generalized H_2 control for discrete-time fuzzy large-scale stochastic systems with time-varying and infinite-distributed delays. Large-scale interconnected systems consist of a number of discrete-time interconnected Takagi–Sugeno (T–S) subsystems. First, a novel Delay-Dependent Piecewise Lyapunov–Krasovskii Functional (DDPLKF) is proposed, in which both the upper and the lower bound of delays are considered. Then, two improved delay-dependent stability conditions are established based on this DDPLKF in terms of Linear Matrix Inequalities (LMIs). The merit of the proposed conditions lies in its reduced conservatism, which is achieved by circumventing the utilization of some bounding inequalities for cross products of two vectors and by considering the interactions among the fuzzy subsystems in each subregion. A decentralized generalized H_2 state feedback fuzzy controller is designed for each subsystem. It is shown that the mean-square stability for discrete T–S fuzzy large-scale stochastic systems can be established if a DDPLKF can be constructed and a decentralized controller can be obtained by solving a set of LMIs. Finally, an illustrative example is provided to demonstrate the effectiveness of the proposed method.

Keywords: fuzzy large-scale stochastic system, delay-dependent, generalized H_2 control, infinite-distributed delays, linear matrix inequality.

1. Introduction

Large-scale interconnected systems have received much attention in recent years. It is an effective mathematical modelling way to deal with physical, engineering, and societal systems, which are usually of high dimensions or exhibit interacting dynamic phenomena. Many methods have been developed to investigate the stability analysis and controller design of interconnected systems. In particular, a decentralized control scheme is preferred in control design of large-scale interconnected systems (Zhang, 1985). However, due to the effects of nonlinear interconnection among subsystems, there is still no efficient way to deal with the decentralized control problem of nonlinear interconnected systems.

Fuzzy logic control has been proposed as a simple and effective approach for complex nonlinear systems or even nonanalytic systems. The Takagi–Sugeno (T–S) model is one of the most popular fuzzy systems in model-based fuzzy control. It is well suited to model-based nonlinear control. By using a T–S fuzzy plant model, one can describe a nonlinear system as a weighted sum of some simple linear subsystems. This fuzzy model is described by a family of fuzzy IF-THEN rules that represent local linear input/output relations of the system. The overall fuzzy model of the system is achieved by smoothly blending these local linear models together through membership functions (Takagi, 1985). During the past few years, various techniques have been developed for stability analysis and controller synthesis of T–S fuzzy systems (Chen *et al*., 2009; Wang *et al*., 2007, Gao *et al*., 2009a; 2009b and Guerra *et al*., 2004, Johansson *et al*., 1999, Lam, 2008; Zhang *et al*., 2008b). Recently, stability analysis and stabilization of fuzzy large-scale interconnected systems was discussed by Tseng (2009) and Zhang *et al*. (2006; 2008a; 2009a; 2010). Stability analysis of discrete fuzzy large-scale systems was discussed by Wang *et al*. (2010). Fuzzy adaptive output feedback control for large-scale nonlinear systems with dynamical uncertainties was studied by Tong *et al*. (2010). However, these results are derived based on the Common Lyapunov–Krasovskii Functional (CLKF). A similar can be seen in the works of Zhang *et al*. (2006; 2008a; 2009a; 2010), in which the authors consider the H_∞ controller and filter design for both continuous-time and discrete-time fuzzy large-scale interconnected systems based on Piecewise Lyapunov–Krasovskii Functionals (PLKFs). It is shown that the PLKFs constitute a much richer class of Lyapunov–Krasovskii functional candidates than the CLKF. PLKFs are able to deal with a large class of fuzzy systems and obtained results are less conservative.

As the dual of the robust control problem, the generalized H_2 control for dynamic systems has been extensively investigated. The generalized H_2 control problem is the one in which the conventional H_2 norm is replaced by an operator norm. The closed-loop system is described in terms of a mapping between the space of time-domain input disturbances in l_2 and the space of time-domain controlled outputs in l_∞. Consequently, generalized H_2 performance is useful for handling stochastic aspects such as measurement noise and random disturbances (Wang *et al*., 2004). However, to the best of our knowledge, the problem of generalized H_2 stability analysis and controller design for T–S fuzzy large-scale systems has not been fully investigated based on PLKFs despite its theoretical and practical significance.

It is also known that there are many stochastic perturbations that affect the stability and control performance of practical systems. The study of stochastic systems has been of great interest since stochastic modeling has come to play an important role in many engineering applications. Therefore, analysis and synthesis of stochastic systems have been investigated extensively and many fundamental results for deterministic systems have been extended to stochastic cases. For example, the stochastic stabilization problem for time-delay systems was dealt with by Gao *et al*. (2007a), Gong *et al*. (2009) and Wang *et al*. (2010). Zhang *et al*. (2009b) studied the H_∞ stochastic control problem for uncertain stochastic piecewise-linear systems, where the controller was designed based on a PLKF. Stochastic H_∞ filtering and fuzzy filtering problems for nonlinear networked systems and Itô systems have also been studied by Dong *et al*. (2010), Wu *et al*. (2008) and Halabi *et al*. (2009). So far, in comparison with the extensive literature available for stochastic dynamic systems, H_∞/H_2 stochastic control results for fuzzy large-scale stochastic systems are relatively few.

On another research frontier, time delay exists commonly in many practical systems, such as chemical processes and networked systems, which has been generally regarded as the main source of instability and poor performance. Many authors have studied time-delay systems. To mention a few, Zhang *et al*. (2010) studied delay-independent robust H_∞ filtering design for nonlinear interconnected systems with multiple time delays based on PLKFs. More recently, to reduce the design conservatism, Gao *et al*. (2007a; 2007b; 2009a), Wang *et al*. (2009), Zhang *et al*. (2009), Chen *et al*. (2008; 2009), Qiu *et al*. (2009) and Li *et al*. (2009) studied delay-dependent stabilization control and H_∞ filtering design for time delay systems and fuzzy time delay systems.

Recently, another type of time-delay, namely, distributed time-delays, has drawn much research interest. This is mainly because signal propagation is often distributed during a certain time period with the presence of an amount of parallel pathways with a variety of axon sizes and lengths (Wei *et al*., 2009). In fact, both discrete and distributed delays should be taken into account when modeling realistic complex systems, and it is not surprising that various systems with discrete and distributed delays have drawn increasing research attention. However, almost all available results have been focused on continuous-time systems with distributed delays that are described in the form of a finite or infinite integral. It is well known that discrete-time systems better lend themselves to model digitally transmitted signals in a dynamic way than their continuous-time analogues. Therefore, it becomes desirable to study discrete-time systems with time varying and infinite-distributed delays. Li *et al*. (2010) derived a new passivity result for discrete-time stochastic neural networks with mixed delays. Wang *et al*. (2010) studied the state feedback control problem for a class of discrete-time stochastic systems with mixed delay and nonlinear disturbances. These results rely on the existence of a CLKF for all local models, which tend to be conservative. However, based on the PLKFs, the problem of decentralized generalized H_2 controller design for T–S fuzzy large-scale stochastic systems with mixed delays remains to be investigated. The aim of this paper is to lessen such a gap.

Motivated by the aforementioned discussion, in this paper, we aim to investigate the decentralized generalized H_2 fuzzy-control problem for discrete-time fuzzy large-scale stochastic systems with mixed delays. Large-scale fuzzy systems consist of J interconnected discrete-time T–S fuzzy subsystems. A novel DDPLKF will be introduced, in which both the upper and lower bounds to delays are considered. Two improved delay-dependent conditions for the stochastic stability of the closed-loop discrete-time T–S fuzzy large-scale stochastic delay systems are obtained based on DDPLKFs, while a prescribed generalized H_2 attenuation level is guaranteed. The explicit expression of the desired decentralized fuzzy generalized H_2 controller parameters is also derived. A numerical simulation example is used to demonstrate the effectiveness of the proposed control scheme. The main contributions of this paper, which concern primarily new research problems and a new method, are summarized as follows:

(i) The investigation of the T–S fuzzy large-scale model is carried out for a class of complex systems that account for stochastic perturbations, time-varying delays and infinitely distributed delays, and disturbances within the same framework.

(ii) A delay-dependent approach is developed to solve the problems of stochastic stability analysis and controller synthesis for discrete-time T–S fuzzy large-scale stochastic systems.

(iii) By applying some new slack matrices in each region, a more relaxed stabilization criterion, in which the interactions among the fuzzy subsystems are considered, is derived.

The merits of the proposed conditions consist in the reduced conservatism, which is achieved by circumventing the utilization of some bounding inequalities for cross products between two vectors and by considering the interactions among the fuzzy subsystems in each subregion Ω_j^i.

It is noted that, in recent years, several decentralized control approaches have also been developed by Zhang *et al.* (2006; 2008a; 2009a) for T–S fuzzy large-scale systems based on PLKFs. However, the approaches of Zhang *et al.* (2006; 2008a; 2009a) did not consider time delays and stochastic factors that affect the stability and control performance of fuzzy large-scale systems. Although the approaches of Li *et al.* (2010) and Wang *et al.* (2010) considered mixed delays as in this paper, these results rely on the existence of a CLKF for all local models and delay-dependent criteria only for time-varying delays, which tend to be conservative.

2. Problem formulation and some preliminaries

The following fuzzy dynamic model is described by fuzzy IF-THEN rules and will be employed here to represent a complex stochastic nonlinear infinite-distributed delay large-scale interconnected system S consisting of J interconnected subsystems $S^i, i = 1, 2, \dots, J$. The fuzzy model of subsystem S^i is proposed in the following form:

Rule j: If $\theta_{i1}(t)$ is W_{j1}^i and ... and $\theta_{ip}(t)$ is W_{jp}^i, then

$$\begin{aligned}
&x_i(t+1)\\
&\quad= A_j^i x_i(t) + B_j^i u_i(t) + A_{1dj}^i \sum_{d=1}^{\infty} \mu_d x_i(t-d)\\
&\qquad + A_{2dj}^i x_i(t-d(t)) + D_j^i v_i(t) + \Big[\bar{A}_j^i x_i(t)\\
&\qquad + \bar{A}_{1dj}^i \sum_{d=1}^{\infty} \mu_d x_i(t-d) + \bar{A}_{2dj}^i x_i(t-d(t))\\
&\qquad + \bar{D}_j^i v_i(t)\Big]\omega_i(t) + \sum_{n=1,n\neq i}^{J} C_n^i x_n(t),\\
&z_i(t) = C_j^i x_i(t),\\
&x_i(t) = \phi_i(t), \quad \forall t \in \mathbb{Z}^{-1}, \quad j = 1, 2, \dots, r_i,
\end{aligned} \tag{1}$$

where $j \in N = \{1, 2, \dots, r_i\}$ denotes a fuzzy inference rule; r_i is the number of inference rules; W_{jl}^i, $l = 1, 2, \dots, p$, is a fuzzy set; $\theta_i(t) = [\theta_{i1}(t), \theta_{i2}(t), \dots, \theta_{ip}(t)] \in \mathbb{R}^{s_i}$ is the premise variable vector; $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector; $u_i(t) \in \mathbb{R}^{m_i}$ is the control input vector; $z_i(t) \in \mathbb{R}^{q_i}$ is the controlled output vector; $v_i(t) \in l_2[0, \infty)$ is the disturbance input; $\omega_i(t)$ is the Brownian motion that satisfies $\mathbb{E}[\omega_i(t)] = 0, \mathbb{E}[\omega_i^2(t)] = 1, \mathbb{E}[\omega_i(j)\omega_i(l)] = 0, (j \neq l)$; $\phi_i(t)\,(\forall t \in \mathbb{Z}^{-1})$ represent a given initial condition sequence, independent of the process $\omega_i(t)$; $(A_j^i, B_j^i, A_{1dj}^i, A_{2dj}^i, D_j^i, \bar{A}_j^i, \bar{A}_{1dj}^i, \bar{A}_{2dj}^i, \bar{D}_j^i, C_j^i)$ are all constant matrices with appropriate dimensions, which represent the j-th local model of the i-th fuzzy subsystem, and C_n^i is the interconnection between the n-th and i-th subsystems. The constants $\mu_d \geq 0$ $(d = 1, 2, \dots)$ satisfy the following convergence conditions:

$$\bar{\mu} = \sum_{d=1}^{\infty} \mu_d \leq \sum_{d=1}^{\infty} d\mu_d < \infty, \tag{2}$$

where $d(t)$ is a time-varying delay in the state. A natural assumption on $d(t)$ can be made as follows.

Assumption 1. The time delay $d(t)$ is assumed to be time varying and satisfy $0 < d_m \leq d(t) \leq d_M$, where d_m and d_M are constant positive scalars representing the lower and upper delay bounds, respectively.

Remark 1. The delay term $\sum_{d=1}^{\infty} \mu_d x_i(t-d)$ in the fuzzy system (1) is the so-called infinitely distributed delay in the discrete-time setting. The description of discrete-time distributed delays and the H_∞ control problem for fuzzy systems was proposed by Wei *et al.* (2008). In this paper, based on a DDPLKF, we aim to study stochastic stability and generalized H_2 control for discrete-time fuzzy large-scale stochastic systems with time-varying and infinite-distributed delays.

Remark 2. In this paper, in much the same way as for the convergence restriction on delay kernels to infinite-distributed delays for continuous-time systems, the constants μ_d $(d = 1, 2, \dots)$ are assumed to satisfy the convergence condition (2), which can guarantee the convergence of the terms of infinite delays as well as the Lyapunov function defined later.

By using a standard fuzzy inference method, that is, using a center-average defuzzifier product fuzzy inference, and a singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model:

$$\begin{aligned} x_i(t+1) &= \sum_{j=1}^{r_i} h_j^i(\theta_i(t))\Big\{A_j^i x_i(t) + B_j^i u_i(t) \\ &\quad + A_{1dj}^i \sum_{d=1}^{\infty} \mu_d x_i(t-d) + A_{2dj}^i x_i(t-d(t)) \\ &\quad + D_j^i v_i(t) + \Big[\bar{A}_j^i x_i(t) \\ &\quad + \bar{A}_{1dj}^i \sum_{d=1}^{\infty} \mu_d x_i(t-d) \\ &\quad + \bar{A}_{2dj}^i x_i(t-d(t)) + \bar{D}_j^i v_i(t)\Big]\omega_i(t)\} \\ &\quad + \sum_{n=1, n\neq i}^{J} C_n^i x_n(t), \\ z_i(t) &= \sum_{j=1}^{r_i} h_j^i(\theta_i(t))\{C_j^i x_i(t)\Big\}, \end{aligned} \tag{3}$$

where

$$h_j^i(\theta_i(t) = \frac{\omega_j^i(\theta_i(t))}{\sum_{i=1}^{r_i} \omega_j^i(\theta_i(t))},$$

$$\omega_j^i(\theta_i(t)) = \prod_{l=1}^{p} W_{jl}^i(\theta_i(t)),$$

with $W_{jl}^i(\theta_i(t))$ being the grade of membership of $\theta_{il}(t)$ in W_{jl}^i. Here $\omega_j^i(\theta_i(t)) \geq 0$ has the following basic property:

$$\omega_j^i(\theta_i(t)) \geq 0, \qquad \sum_{j=1}^{r_i} \omega_j^i(\theta_i(t)) > 0, \tag{4}$$

and therefore

$$h_j^i(\theta_i(t)) \geq 0, \qquad \sum_{j=1}^{r_i} h_j^i(\theta_i(t)) = 1. \tag{5}$$

In order to facilitate the design of a less conservative H_2 controller, we partition the premise variable space $\Omega^i \subseteq \mathbb{R}^{s_i}$ into s polyhedral regions by the boundaries (Chen and Feng, 2009)

$$\begin{aligned} \partial(\Omega_j^i)^v = \Big\{&\theta_i(t) \mid h_j^i(\theta_i(t)) = 1, \\ &0 \leq \underbrace{h_j^i(\theta_i(t+\delta))}_{0\leq\|\delta\|<1} < 1, j \in \mathbb{N}\Big\}, \end{aligned} \tag{6}$$

where v is the set of the face indices of the polyhedral hull satisfying $\partial\Omega_j^i = \cup_v \partial(\Omega_j^i)^v$. Based on the boundaries (6), s independent polyhedral regions $\Omega_j^i, j \in L^i = \{1, 2, \dots, s\}$ can be obtained satisfying

$$\Omega_j^i \cap \Omega_l^i = \partial(\Omega_k^i)^v, \quad j \neq l, \quad j, l \in L^i, \tag{7}$$

where L^i denotes the set of polyhedral region indices. In each region Ω_j^i, we define the set

$$\begin{aligned} M^i(j) &:= \Big\{l \mid h_{jl}^i(\theta_i(t)) > 0, \theta_i(t) \in \Omega_j^i\Big\}, \quad j \in L^i, \\ M_j^i &:= \mathrm{card}(M^i(j)). \end{aligned} \tag{8}$$

That is, the variable M_j^i is used to represent the number of demands in the set $M^i(j)$ for the region Ω_j^i. Note that $M_j^i \geq 1,\ j \in L^i$.

Considering (5) and (8), in each region Ω_j^i, we have

$$\sum_{l=1}^{M_j^i} h_{jl}^i(\theta_i(t)) = 1. \tag{9}$$

Then, we follow the idea of Wang *et al.* (2007) to rewrite the fuzzy infinite-distributed delays system (1) to be an equivalent discrete-time switching fuzzy system in the following form:

Region Rule i:

If $\theta_i(t) \in \Omega_j^i$, then

Local plant Rule l:

If $\theta_{i1}(t)$ is M_{jl1}^i and … and $\theta_{ip}(t)$ is M_{jlp}^i, then

$$\begin{aligned} x_i(t+1) &= A_{jl}^i x_i(t) + B_{jl}^i u_i(t) + A_{1djl}^i \sum_{d=1}^{\infty} \mu_d x_i(t-d) \\ &\quad + A_{2djl}^i x_i(t-d(t)) + D_{jl}^i v_i(t) + \Big[\bar{A}_{jl}^i x_i(t) \\ &\quad + \bar{A}_{1djl}^i \sum_{d=1}^{\infty} \mu_d x_i(t-d) + \bar{A}_{2djl}^i x_i(t-d(t)) \\ &\quad + \bar{D}_{jl}^i v_i(t)\Big]\omega_i(t) + \sum_{n=1, n\neq i}^{J} C_n^i x_n(t), \\ z_i(t) &= C_{jl}^i x_i(t), \\ &\quad i = 1, 2, \dots, J,\ l \in M^i(j), \quad j \in L^i, \end{aligned} \tag{10}$$

where Ω^i_j denotes the j-th subregion.

Given a pair of $[x_i(t), u_i(t)]$, the final output of the switching fuzzy system is inferred as

$$\begin{aligned} x_i(t+1) &= \sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))\{A^i_{jl}x_i(t) + B^i_{jl}u_i(t) \\ &\quad + A^i_{1djl}\sum_{d=1}^{\infty}\mu_d x_i(t-d) + A^i_{2djl}x_i(t-d(t)) \\ &\quad + D^i_{jl}v_i(t) + \Big[\bar{A}^i_{jl}x_i(t) \\ &\quad + \bar{A}^i_{1djl}\sum_{d=1}^{\infty}\mu_d x_i(t-d) \\ &\quad + \bar{A}^i_{2djl}x_i(t-d(t)) + \bar{D}^i_{jl}v_i(t)\Big]\omega_i(t)\} \\ &\quad + \sum_{n=1,n\neq i}^{J} C^i_n x_n(t), \\ z_i(t) &= \sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))\{C^i_{jl}x_i(t)\}, \quad \theta_i(t) \in \Omega^i_j. \end{aligned} \tag{11}$$

In this paper, we consider the generalized H_2 controller problem for the fuzzy system (1) or, equivalently, (11).

Assumption 2. When the state of the system transits from the region Ω^i_j to Ω^i_l at the time t, the dynamics of the system are governed by the dynamics of the region model of Ω^i_j at that time t.

For future use, we define a set Θ_i that represents all possible transitions from one region to itself or other regions, that is,

$$\Theta_i = \Big\{(j,l) \mid \theta_i(t) \in \Omega^i_j, \theta_i(t+1) \in \Omega^i_l, \forall j,l \in L^i\Big\}. \tag{12}$$

Here $j = l$ when $\theta_i(t)$ stays in the same region Ω^i_j, and $j \neq l$ when $\theta_i(t)$ transits from the region Ω^i_j to another one Ω^i_l .

Consider the switching fuzzy system (11) and choose the following decentralized piecewise fuzzy controller:

$$u_i(t) = -\sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))F^i_{jl}x_i(t), \quad \theta_i(t) \in \Omega^i_j. \tag{13}$$

Then the final output of the closed-loop switching fuzzy system with (11) and (13) is

$$\begin{aligned} x_i(t+1) &= \sum_{l=1}^{M^i_j}\sum_{k=1}^{M^i_j} h^i_{jl}(\theta_i(t))h^i_{jk}(\theta_i(t)) \\ &\quad \Big\{(A^i_{jl} - B^i_{jl}F^i_{jk})x_i(t) \\ &\quad + A^i_{1djl}\sum_{d=1}^{\infty}\mu_d x_i(t-d) + A^i_{2djl}x_i(t-d(t)) \\ &\quad + D^i_{jl}v_i(t) + \Big[\bar{A}^i_{jl}x_i(t) + \bar{A}^i_{1djl}\sum_{d=1}^{\infty}\mu_d x_i(t-d) \\ &\quad + \bar{A}^i_{2djl}x_i(t-d(t)) + \bar{D}^i_{jl}v_i(t)\Big]\omega_i(t)\Big\} \\ &\quad + \sum_{n=1,n\neq i}^{J} C^i_n x_n(t), \\ z_i(t) &= \sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))\{C^i_{jl}x_i(t)\}, \quad \theta_i(t) \in \Omega^i_j. \end{aligned} \tag{14}$$

Before formulating the problem to be investigated, we first introduce the following concept for the system (14).

Definition 1. The closed-loop stochastic fuzzy system (14) is said to be *mean-square stable with generalized H_2 performance* if for any $\epsilon > 0$ there is a $\delta(\epsilon) > 0$ such that $\mathbb{E}[\|x_i(t)\|^2] < \epsilon$ for every $t > 0$ when $\mathbb{E}[\|x_i(0)\|^2] < \delta(\epsilon)$. In addition, if $\lim_{t\to\infty}\mathbb{E}[\|x_i(t)\|^2] = 0$ for any initial condition, then the closed-loop stochastic fuzzy system (14) is said to be mean-square asymptotically stable.

Definition 2. Let a constant γ be given. The closed-loop fuzzy system (14) is said to be *stable with generalized H_2 performance* if both of the following conditions are satisfied:

1. The disturbance-free fuzzy system is mean-square globally asymptotically stable.
2. Assuming zero initial conditions, the controlled output satisfies

$$\|z\|_{\mathbb{E}_\infty} < \gamma\|v\|_{\mathbb{E}_2}, \tag{15}$$

$$z(t) = [z^T_1(t), \dots, z^T_J(t)]^T,$$
$$v(t) = [v^T_1(t), \dots, v^T_J(t)]^T$$

for all non-zero $v \in l_2$, the l_2-norm being defined as

$$\|v\|^2_{\mathbb{E}_2} = \mathbb{E}\Big\{\sum_{i=1}^{J}\sum_{t=0}^{\infty}\{v^{\mathrm{T}}_i(t)v_i(t)\Big\},$$

and

$$\|z\|_{\mathbb{E}_\infty} = \mathbb{E}\Big\{\sum_{i=1}^{J}\underbrace{\max}_{1\leq\prec j\leq n}|z_{ij}(t)|\Big\}.$$

Now, we introduce the following lemma that will be used in the development of our main results.

Lemma 1. (Wei *et al.*, 2008) *Let $M \in \mathbb{R}^{n\times n}$ be a positive semi-definite matrix, $x_i(t) \in \mathbb{R}^n$ and constant $a_i > 0$ $(i = 1, 2, \dots)$. If the series concerned is convergent, then we have*

$$\Big(\sum_{i=1}^{\infty} a_i x_i\Big)^{\mathrm{T}} M \Big(\sum_{i=1}^{\infty} a_i x_i\Big) \le \Big(\sum_{i=1}^{\infty} a_i\Big) \sum_{i=1}^{\infty} a_i x_i M x_i. \quad (16)$$

Lemma 2. (Zhang *et al.*, 2008) *Given three matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{m\times n}$ and $M \in R^{n\times n}$ and two positive definite matrices $M \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$ such that*

$$\begin{aligned} A^{\mathrm{T}}PA - P + Q \le 0,\\ B^{\mathrm{T}}PB - P + Q \le 0, \end{aligned} \quad (17)$$

we have

$$A^{\mathrm{T}}PB + B^{\mathrm{T}}PA - 2P + 2Q \le 0. \quad (18)$$

3. Main results

For notational simplicity, we write

$$\begin{aligned} H^i_{jlk} =& \Big[\sum_{l=1}^{M^i_j}\sum_{k=1}^{M^i_j} h^i_{jl} h^i_{jk}(\theta_i(t))\mathbb{A}^i_{jlk},\ \sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))A^i_{1djl},\\ & \sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))A^i_{1dj2},\ \sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))D^i_{jl}\Big],\\ W^i_{jl} =& \Big[\sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))\bar{A}^i_{jlk},\ \sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))\bar{A}^i_{1djl},\\ & \sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))\bar{A}^i_{2djl},\ \sum_{l=1}^{M^i_j} h^i_{jl}(\theta_i(t))\bar{D}^i_{jl}\Big],\\ \xi_i(t) =& \Big[x_i(t),\ \sum_{d=1}^{\infty}\mu_d x_i(t-d),\ x_i(t-d(t)),\ v_i(t)\Big],\\ \mathbb{A}^i_{jlk} =& A^i_{jl} - B^i_{jl}F^i_{jk}. \end{aligned} \quad (19)$$

Theorem 1. *Given a constant $\gamma > 0$, the closed-loop discrete-time fuzzy stochastic large-scale system composed of J fuzzy subsystems as (14) with both the time-varying delay $d(t)$ satisfying $0 < d_m \le d(k) \le d_M$ and infinite-distributed delays is mean-square stable with generalized H_2 performance γ, if there exist a set of symmetric positive definite matrices P^i_j, $P^i_0 \ge P^i_j$, Q^i, R^i, S^i, Z^i, matrices X^i_{1j}, X^i_{2j}, Y^i_{1j}, Y^i_{2j} and positive constants $\alpha_i \le 1$, ε^i_1, ε^i_2, for all $i = 1, 2, \dots, J$ and $j \in L^i$, such that the following LMIs hold:*

$$(C^i_{jl})^{\mathrm{T}} C^i_{jl} - \gamma^2 P^i_j < 0, \quad (20)$$

$$\Pi^i_{jmll} < 0, \quad (21)$$

$$\Pi^i_{jmlk} + \Pi^i_{jmkl} < 0, \quad (22)$$

where $j, m \in L^i$, $l, k \in M^i(j)$, $l \ne k$, $(j, m) \in \Theta_i$, and

$$\Pi^i_{jmlk} = \begin{bmatrix} \Psi^i_{0j} & \Psi^i_{1j} & \Psi^i_{2j} & \Psi^i_{3jml} & \Psi^i_{4jmlk}\\ * & \Psi^i_{5j} & 0 & 0 & 0\\ * & * & \frac{1}{2}\Psi^i_{6j} & 0 & 0\\ * & * & * & \Psi^i_{7j} & 0\\ * & * & * & * & \Psi^i_{8j} \end{bmatrix},$$

with

$$\Psi^i_{0j} = \bar{P}^i_j + \mathrm{sym}\Xi^i_{1j} + \mathrm{sym}\Xi^i_{2j},$$

$$\bar{P}^i_j = \begin{bmatrix} -\mathbb{P}^i_j & 0 & 0 & 0\\ * & -\frac{1}{\mu}Q^i & 0 & 0\\ * & * & -R^i & 0\\ * & * & * & -\alpha_i I \end{bmatrix},$$

$$-\mathbb{P}^i_j = -P^i_j + \sum_{d=1}^{\infty} d\mu_d Z^i + d_M S^i + \tau R^i + \bar{\mu} Q^i,$$

$$\Xi^i_{1j} = [X^i_j\ \ -X^i_j\ \ 0\ \ 0],\quad \Xi^i_{2j} = [Y^i_j\ \ 0\ \ -Y^i_j\ \ 0],$$

$$X^i_j = [X^i_{1j}\ \ X^i_{2j}\ \ 0\ \ 0]^{\mathrm{T}},\quad Y^i_j = [Y^i_{1j}\ \ 0\ \ Y^i_{2j}\ \ 0]^{\mathrm{T}},$$

$$\Psi^i_{1j} = \Big[\sqrt{\sum_{d=1}^{\infty} d\mu_d}X^i_j\ \ \sqrt{d_M}Y^i_j\Big],$$

$$\Psi^i_{5j} = \mathrm{diag}\{-Z^i\ \ -S^i\},$$

$$\tau = d_M + d_m - 1,$$

$$\Psi^i_{2j} = \begin{bmatrix} (\hat{C}^i_p)^{\mathrm{T}} (\hat{C}^i_{p\varepsilon^i_1})^{\mathrm{T}} (\hat{C}^i_z)^{\mathrm{T}} (\hat{C}^i_{z\varepsilon^i_1}))^{\mathrm{T}} & (\hat{C}^i_s)^{\mathrm{T}} (\hat{C}^i_{s\varepsilon^i_1}))^{\mathrm{T}}\\ 0\quad 0\quad 0\quad 0 & 0\quad 0\\ 0\quad 0\quad 0\quad 0 & 0\quad 0\\ 0\quad 0\quad 0\quad 0 & 0\quad 0 \end{bmatrix},$$

$$\Psi^i_{3jml} = \begin{bmatrix} (\bar{A}^i_{jl})^{\mathrm{T}} P^i_m & (\bar{A}^i_{jl})^{\mathrm{T}} Z^i & (\bar{A}^i_{jl})^{\mathrm{T}} S^i\\ (\bar{A}^i_{1djl})^{\mathrm{T}} P^i_m & (\bar{A}^i_{1djl})^{\mathrm{T}} Z^i & (\bar{A}^i_{1djl})^{\mathrm{T}} S^i\\ (\bar{A}^i_{2djl})^{\mathrm{T}} P^i_m & (\bar{A}^i_{2djl})^{\mathrm{T}} Z^i & (\bar{A}^i_{2djl})^{\mathrm{T}} S^i\\ (\bar{D}^i_{jl})^{\mathrm{T}} P^i_m & (\bar{D}^i_{jl})^{\mathrm{T}} Z^i & (\bar{D}^i_{jl})^{\mathrm{T}} S^i \end{bmatrix},$$

$$\Psi^i_{4jmlk} = \begin{bmatrix} (\mathbb{A}^i_{jlk})^{\mathrm T} P^i_m & (\mathbb{A}^i_{jlk})^{\mathrm T} P^i_m & \rho(\mathbb{A}^i_{jlk}-I)^{\mathrm T} Z^i & \rho(\mathbb{A}^i_{jlk}-I)^{\mathrm T} Z^i & \sqrt{d_M}(\mathbb{A}^i_{jlk}-I)^{\mathrm T} S^i & \sqrt{d_M}(\mathbb{A}^i_{jlk}-I)^{\mathrm T} S^i \\ (A^i_{1djl})^{\mathrm T} P^i_m & (A^i_{1djl})^{\mathrm T} P^i_m & (A^i_{1djl})^{\mathrm T} Z^i & (A^i_{1djl})^{\mathrm T} Z^i & (A^i_{1djl})^{\mathrm T} S^i & (A^i_{1djl})^{\mathrm T} S^i \\ (A^i_{2djl})^{\mathrm T} P^i_m & (A^i_{2djl})^{\mathrm T} P^i_m & (A^i_{2djl})^{\mathrm T} Z^i & (A^i_{2djl})^{\mathrm T} Z^i & (A^i_{2djl})^{\mathrm T} S^i & (A^i_{2djl})^{\mathrm T} S^i \\ (D^i_{jl})^{\mathrm T} P^i_m & (D^i_{jl})^{\mathrm T} P^i_m & (D^i_{jl})^{\mathrm T} Z^i & (D^i_{jl})^{\mathrm T} Z^i & (D^i_{jl})^{\mathrm T} S^i & (D^i_{jl})^{\mathrm T} S^i \end{bmatrix},$$

$$\Psi^i_{6j} = \mathrm{diag}\{-\hat P^i_0 \;\; -\hat P^i_{\varepsilon^i_1} \;\; -\hat Z^i \;\; -\hat Z^i_{\varepsilon^i_2} \;\; -\hat S^i \;\; -\hat S^i_{\varepsilon^i_2}\},$$

$$\Psi^i_{7j} = \mathrm{diag}\{-P^i_m \;\; -Z^i \;\; -S^i\},\quad \rho = \sqrt{\sum_{d=1}^{\infty} d\mu_d},$$

$$\Psi^i_{8j} = \mathrm{diag}\{-P^i_m \;\; -\varepsilon^i_1 P^i_m \;\; -Z^i \;\; -\varepsilon^i_2 Z^i \;\; -S^i \;\; -\varepsilon^i_2 S^i\},$$

$$\hat P^i_0 = \mathrm{diag}\{P^1_0,\dots,P^1_{0,n\neq i},\dots,P^J_0\},$$

$$\hat P^i_{\varepsilon^i_1} = \mathrm{diag}\{\varepsilon^1_1 P^1_0,\dots,\varepsilon^n_1 P^1_{0,n\neq i},\dots,\varepsilon^J_1 P^J_0\},$$

$$\hat Z^i = \mathrm{diag}\{\rho Z^i,\dots,\rho Z^i, n\neq i,\dots,\rho Z^i\},$$

$$\hat Z^i_{\varepsilon^i_2} = \mathrm{diag}\{\varepsilon^1_2\rho Z^i,\dots,\varepsilon^n_2\rho Z^i, n\neq i,\dots,\varepsilon^J_2\rho Z^i\},$$

$$\hat S^i = \mathrm{diag}\{\sqrt{d_M}S^i,\dots,\sqrt{d_M}S^i,\quad n\neq i, \dots,\sqrt{d_M}S^i\},$$

$$\hat S^i_{\varepsilon^i_2} = \mathrm{diag}\{\varepsilon^1_2\sqrt{d_M}S^i,\dots,\varepsilon^n_2\sqrt{d_M}S^i,\quad n\neq i, \dots,\varepsilon^J_2\sqrt{d_M}S^i\},$$

$$\hat C^i_p = \Big[(C^i_1)^{\mathrm T}P^1_0,\dots,(C^i_{n,n\neq i})^{\mathrm T}P^n_{0,n\neq i}, \dots,(C^i_J)^{\mathrm T}P^J_0\Big]^{\mathrm T},$$

$$\hat C^i_{p\varepsilon^i_1} = \Big[\varepsilon^1_1(C^i_1)^{\mathrm T}P^1_0,\dots,\varepsilon^n_1(C^i_{n,n\neq i})^{\mathrm T}P^n_{0,n\neq i}, \dots,\varepsilon^J_1(C^i_J)^{\mathrm T}P^J_0\Big]^{\mathrm T},$$

$$\hat C^i_z = \Big[\rho(C^i_1)^{\mathrm T}Z^i,\dots,\rho(C^i_{n,n\neq i})^{\mathrm T}Z^i, \dots,\rho(C^i_J)^{\mathrm T}Z^i\Big]^{\mathrm T},$$

$$\hat C^i_{z\varepsilon^i_2} = \Big[\varepsilon^1_2\rho(C^i_1)^{\mathrm T}Z^i,\dots,\varepsilon^n_2\rho(C^i_{n,n\neq i})^{\mathrm T}Z^i, \dots,\varepsilon^J_2\rho(C^i_J)^{\mathrm T}Z^i\Big]^{\mathrm T},$$

$$\hat C^i_s = \Big[\sqrt{d_M}(C^i_1)^{\mathrm T}S^i,\dots,\sqrt{d_M}(C^i_{n,n\neq i})^{\mathrm T}S^i, \dots,\sqrt{d_M}(C^i_J)^{\mathrm T}S^i\Big]^{\mathrm T},$$

$$\hat C^i_{s\varepsilon^i_2} = \Big[\varepsilon^1_2\sqrt{d_M}(C^i_1)^{\mathrm T}S^i,\dots,\varepsilon^n_2\sqrt{d_M}(C^i_{n,n\neq i})^{\mathrm T}S^i, \dots,\varepsilon^J_2\sqrt{d_M}(C^i_J)^{\mathrm T}S^i\Big]^{\mathrm T}. \tag{23}$$

Proof. To investigate the stability problem of the system (14), we construct the following DDPLKF candidate:

$$\begin{aligned}
V(t) &:= \sum_{i=1}^{J}\sum_{j=1}^{6} V^i_j(t),\\
V^i_1(t) &= x_i(t)^{\mathrm T}P^i_j x_i(t),\\
V^i_2(t) &= \sum_{d=1}^{\infty}\mu_d \sum_{k=t-d}^{t-1} x_i(k)^{\mathrm T}Q^i x_i(k),\\
V^i_3(t) &= \sum_{d=1}^{\infty}\mu_d \sum_{i=-d}^{-1}\sum_{l=t+i}^{t-1} \eta_i(l)^{\mathrm T}Z^i\eta_i(l),\\
V^i_4(t) &= \sum_{l=t-d(t)}^{t-1} x_i(l)^{\mathrm T}R^i x_i(l),\\
V^i_5(t) &= \sum_{j=-d_M+2}^{-d_m+1}\sum_{l=t+j-1}^{t-1} x_i(l)^{\mathrm T}R^i x_i(l),\\
V^i_6(t) &= \sum_{i=-d_M}^{-1}\sum_{l=t+i}^{t-1} \eta_i(l)^{\mathrm T}S^i\eta_i(l),\\
&\qquad \theta_i(t)\in\Omega^i_j,
\end{aligned}\tag{24}$$

where $P^i_j,\ Q^i,\ Z^i,\ R^i, S^i,\ i=1,2,\dots,J$ and $j\in L^i$, are symmetric positive-definite matrices, and

$$\begin{aligned}
\eta_i(t) &= x_i(t+1)-x_i(t),\\
\eta_i(t) &= \sum_{l=1}^{M^i_j}\sum_{k=1}^{M^i_j} h^i_{jl}(\theta_i(t))h^i_{jk}(\theta_i(t))\\
&\quad\times\{[(A^i_{jl}-B^i_{jl}F^i_{jk})-I]x_i(t)\\
&\quad + A^i_{1djl}\sum_{d=1}^{\infty}\mu_d x_i(t-d)\\
&\quad + A^i_{2djl}x_i(t-d(t)) + D^i_{jl}v_i(t)\\
&\quad + \Big[\bar A^i_{jl}x_i(t) + \bar A^i_{1djl}\sum_{d=1}^{\infty}\mu_d x_i(t-d)\\
&\quad + \bar A^i_{2djl}x_i(t-d(t)) + \bar D^i_{jl}v_i(t)\Big]\omega_i(t)\}\\
&\quad + \sum_{n=1,n\neq i}^{J} C^i_n x_n(t).
\end{aligned}$$

Taking the difference of every term of V^i_j along the system (14) and taking the mathematical expectation, by Lemmas 1 and 2, we have

$$\begin{aligned}
&\mathbb{E}\{\Delta V^i_1(t)\}\\
&\quad = \mathbb{E}\Big\{\xi^{\mathrm T}_i(t)[(H^i_{jlk})^{\mathrm T}P^i_m H^i_{jlk}
\end{aligned}$$

$$
\begin{aligned}
&+ (W_{jl}^i)^{\mathrm{T}} P_m^i W_{jl}^i]\xi_i(t) - x_i(t)^{\mathrm{T}} P_j^i x_i(t) \\
&+ 2\xi_i^{\mathrm{T}}(t)(H_{jlk}^i)^{\mathrm{T}} P_m^i \sum_{n=1,n\neq i}^{J} C_n^i x_n(t) \\
&+ \Big(\sum_{n=1,n\neq i}^{J} C_n^i x_n(t)\Big)^{\mathrm{T}} P_m^i \sum_{n=1,n\neq i}^{J} C_n^i x_n(t)\Big\} \\
\le{}& \mathbb{E}\Big\{\xi_i^{\mathrm{T}}(t)\Big[(1+(\varepsilon_1^i)^{-1})(H_{jlk}^i)^{\mathrm{T}} P_m^i H_{jlk}^i \\
&+ (W_{jl}^i)^{\mathrm{T}} P_m^i W_{jl}^i\Big]\xi_i(t) \\
&+ (1+\varepsilon_1^i)\Big(\sum_{n=1,n\neq i}^{J} C_n^i x_n(t)\Big)^{\mathrm{T}} P_m^i \sum_{n=1,n\neq i}^{J} C_n^i x_n(t) \\
&- x_i(t)^{\mathrm{T}} P_j^i x_i(t)\Big\} \\
\le{}& \mathbb{E}\Big\{\xi_i^{\mathrm{T}}(t)\Big[(1+(\varepsilon_1^i)^{-1})(H_{jlk}^i)^{\mathrm{T}} P_m^i H_{jlk}^i \\
&+ (W_{jl}^i)^{\mathrm{T}} P_m^i W_{jl}^i\Big]\xi_i(t) \\
&+ x_i(t)\Big[2\sum_{n=1,n\neq i}^{J}(1+\varepsilon_1^n)(C_n^i)^{\mathrm{T}} P_0^n C_n^i - P_j^i\Big] x_i(t),
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_2^i(t)\} \\
&= \mathbb{E}\Big\{\sum_{d=1}^{\infty}\mu_d \sum_{k=t+1-d}^{t} x_i(k)^{\mathrm{T}} Q^i x_i(k) \\
&\quad - \sum_{d=1}^{\infty}\mu_d \sum_{k=t-d}^{t-1} x_i(k)^{\mathrm{T}} Q^i x_i(k)\Big\} \\
&= \mathbb{E}\{\bar{\mu} x_i(t)^{\mathrm{T}} Q^i x_i(t) \\
&\quad - \sum_{d=1}^{\infty}\mu_d x_i(t-d)^{\mathrm{T}} Q^i x_i(t-d)\} \\
&\le \mathbb{E}\Big\{\bar{\mu} x_i(t)^{\mathrm{T}} Q^i x_i(t) \\
&\quad - \frac{1}{\bar{\mu}}\Big(\sum_{d=1}^{\infty}\mu_d x_i(t-d)\Big)^{\mathrm{T}} Q^i \sum_{d=1}^{\infty}\mu_d x_i(t-d)\Big\},
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_3^i(t)\} \\
&= \mathbb{E}\Big\{\sum_{d=1}^{\infty}\mu_d \sum_{i=-d}^{-1}\sum_{l=t+i+1}^{t} \eta_i(l)^{\mathrm{T}} Z^i \eta_i(l) \\
&\quad - \sum_{d=1}^{\infty}\mu_d \sum_{i=-d}^{-1}\sum_{l=t+i}^{t-1} \eta_i(l)^{\mathrm{T}} Z^i \eta_i(l)\Big\} \\
&= \mathbb{E}\Big\{\sum_{d=1}^{\infty}\mu_d \sum_{i=-d}^{-1} \eta_i(t)^{\mathrm{T}} Z^i \eta_i(t) \\
&\quad - \sum_{d=1}^{\infty}\mu_d \sum_{i=-d}^{-1} \eta_i(t+i)^{\mathrm{T}} Z^i \eta_i(t+i)\Big\} \\
&= \mathbb{E}\Big\{\sum_{d=1}^{\infty} d\mu_d \eta_i(t)^{\mathrm{T}} Z^i \eta_i(t) \\
&\quad - \sum_{d=1}^{\infty}\mu_d \sum_{l=t-d}^{t-1} \eta_i(l)^{\mathrm{T}} Z^i \eta_i(l)\Big\},
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_4^i(t)\} \\
&= \mathbb{E}\Big\{\sum_{l=t+1-d(t)}^{t} x_i(l)^{\mathrm{T}} R^i x_i(l) \\
&\quad - \sum_{l=t-d(t)}^{t-1} x_i(l)^{\mathrm{T}} R^i x_i(l)\Big\} \\
&= \mathbb{E}\Big\{x_i(t)^{\mathrm{T}} R^i x_i(t) - x_i(t-d(t))^{\mathrm{T}} R^i \\
&\quad \times x_i(t-d(t)) + \sum_{l=t+1-d(t+1)}^{t-1} x_i(l)^{\mathrm{T}} R^i x_i(l) \\
&\quad - \sum_{l=t+1-d(t)}^{t-1} x_i(l)^{\mathrm{T}} R^i x_i(l)\Big\} \\
&\le \mathbb{E}\Big\{x_i(t)^{\mathrm{T}} R^i x_i(t) \\
&\quad - x_i(t-d(t))^{\mathrm{T}} R^i x_i(t-d(t)) \\
&\quad + \sum_{l=t-d_M+1}^{t-d_m} x_i(l)^{\mathrm{T}} R^i x_i(l)\Big\},
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_5^i(t)\} \\
&= \mathbb{E}\Big\{\sum_{j=-d_M+2}^{-d_m+1} x_i(t)^{\mathrm{T}} R^i x_i(t) \\
&\quad - \sum_{j=-d_M+2}^{-d_m+1} x_i(t+j-1)^{\mathrm{T}} R^i x_i(t+j-1)\Big\} \\
&= \mathbb{E}\Big\{(d_M - d_m) x_i(t)^{\mathrm{T}} R^i x_i(t) \\
&\quad - \sum_{l=t-d_M+1}^{t-d_m} x_i(l)^{\mathrm{T}} R^i x_i(l)\Big\},
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_6^i(t)\} \\
&= \mathbb{E}\Big\{\sum_{i=-d_M}^{-1}\sum_{l=t+i+1}^{t} \eta_i(l)^{\mathrm{T}} S^i \eta_i(l) \\
&\quad - \sum_{i=-d_M}^{-1}\sum_{l=t+i}^{t-1} \eta_i(l)^{\mathrm{T}} S^i \eta_i(l)\Big\} \\
&= \mathbb{E}\Big\{\sum_{i=-d_M}^{-1} \eta_i(t)^{\mathrm{T}} S^i \eta_i(t) \\
&\quad - \sum_{i=-d_M}^{-1} \eta_i(t+i)^{\mathrm{T}} S^i \eta_i(t+i)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&\leq \mathbb{E}\Big\{ d_M \eta_i(t)^{\mathrm{T}} S^i \eta_i(t) \\
&- \sum_{l=t-d(t)}^{t-1} \eta_i(l)^{\mathrm{T}} S^i \eta_i(l) \Big\}. \qquad (30)
\end{aligned}
$$

Based on $\eta_i(t)$, for any matrices X_j^i and Y_j^i, we have

$$
\begin{aligned}
\Lambda_1^i &= 2\Big[x_i^{\mathrm{T}}(t) \ \sum_{d=1}^{\infty} \mu_d x_i^{\mathrm{T}}(t-d) \ 0 \ 0\Big] X_j^i \Big[\bar{\mu} x_i(t) \\
&\quad - \sum_{d=1}^{\infty} \mu_d x_i(t-d) - \sum_{d=1}^{\infty} \mu_d \sum_{l=t-d}^{t-1} \eta_i(l)\Big] = 0, \quad (31)\\
\Lambda_2^i &= 2\Big[x_i^{\mathrm{T}}(t) \ 0 \ x_i^{\mathrm{T}}(t-d(t)) \ 0\Big] Y_j^i \Big[x_i(t) \\
&\quad - x_i(t-d(t)) - \sum_{l=t-d(t)}^{t-1} \eta_i(l)\Big] = 0. \quad (32)
\end{aligned}
$$

Therefore, by considering (25)–(30) and adding the left hand side of (31)–(32) to $\mathbb{E}\{\Delta V(t)\}$, we eventually obtain

$$
\begin{aligned}
&\mathbb{E}\{\Delta V(t)\} \\
&\leq \mathbb{E}\Big\{ \sum_{i=1}^{J} \sum_{j=1}^{6} \Delta V_j^i(t) + \Lambda_1^i + \Lambda_2^i \Big\} \\
&\leq \mathbb{E}\Big\{ \sum_{i=1}^{J} \sum_{j=1}^{6} (\xi_i^{\mathrm{T}}(t)[(1+(\varepsilon_1^i)^{-1})(H_{jlk}^i)^{\mathrm{T}} P_m^i H_{jlk}^i \\
&\quad + (W_{jl}^i)^{\mathrm{T}} P_m^i W_{jl}^i]\xi_i(t) \\
&\quad + x_i(t)\Big[2 \sum_{n=1,n\neq i}^{J} (1+\varepsilon_1^n)(C_n^i)^{\mathrm{T}} P_0^n C_n^i \\
&\quad - P_j^i\Big] x_i(t) + \bar{\mu} x_i(t)^{\mathrm{T}} Q^i x_i(t) \\
&\quad - \frac{1}{\bar{\mu}} (\sum_{d=1}^{\infty} \mu_d x_i(t-d))^{\mathrm{T}} Q^i \sum_{d=1}^{\infty} \mu_d x_i(t-d) \\
&\quad + \sum_{d=1}^{\infty} d\mu_d \eta_i(t)^{\mathrm{T}} Z^i \eta_i(t) \\
&\quad - x_i(t-d(t))^{\mathrm{T}} R^i x_i(t-d(t)) \tau x_i(t)^{\mathrm{T}} R^i x_i(t) \\
&\quad + d_M \eta_i(t)^{\mathrm{T}} S^i \eta_i(t) - \sum_{d=1}^{\infty} \mu_d \sum_{l=t-d}^{t-1} \eta_i(l)^{\mathrm{T}} Z^i \eta_i(l) \\
&\quad - \sum_{l=t-d(t)}^{t-1} \eta_i(l)^{\mathrm{T}} S^i \eta_i(l) + \Lambda_1^i + \Lambda_2^i) \Big\} \\
&= \mathbb{E}\Big\{ \xi_i^{\mathrm{T}}(t) \Big[\sum_{l=1}^{M_j^i} (h_{jl}^i \theta_i(t))^2 \Upsilon_{jmll}^i \\
&\quad + \sum_{l<k}^{M_j^i} h_{jl}^i \theta_i(t) h_{jk}^i \theta_i(t) (\Upsilon_{jmlk}^i + \Upsilon_{jmkl}^i) \Big] \xi_i(t) \\
&\quad + \alpha_i v_i^{\mathrm{T}}(t) v_i(t) \Big\}, \qquad (33)
\end{aligned}
$$

where $\Xi_{1j}^i, \Xi_{2j}^i, \Psi_{1j}^i, \Psi_{5j}^i$ are defined in (23) and

$$
\begin{aligned}
\Upsilon_{jmlk}^i &= \Gamma_{jmlk}^i + \bar{P}_j^i + \mathrm{sym}\Xi_{1j}^i + \mathrm{sym}\Xi_{2j}^i \\
&\quad + \Psi_{1j}^i (\Psi_{5j}^i)^{-1} \Psi_{1j}^i, \\
\Gamma_{jmlk}^i &= (1+(\varepsilon_1^i)^{-1})(H_{jlk}^i)^{\mathrm{T}} P_m^i H_{jlk}^i \\
&\quad + (W_{jl}^i)^{\mathrm{T}} P_m^i W_{jl}^i + (1+(\varepsilon_2^i)^{-1})(\bar{H}_{jlk}^i)^{\mathrm{T}} \\
&\quad \times \Big(\sum_{d=1}^{\infty} d\mu_d Z^i + d_M S^i\Big) \bar{H}_{jlk}^i \\
&\quad + (W_{jl}^i)^{\mathrm{T}} \Big(\sum_{d=1}^{\infty} d\mu_d Z^i + d_M S^i\Big) W_{jl}^i + 2\mathbb{C}^i, \\
\mathbb{C}^i &= \Big[\sum_{n=1,n\neq i}^{J} (1+\varepsilon_1^n)(C_n^i)^{\mathrm{T}} P_0^n C_n^i \\
&\quad + (1+\varepsilon_2^n)(C_n^i)^{\mathrm{T}} \Big(\sum_{d=1}^{\infty} d\mu_d Z^i + d_M S^i\Big) C_n^i \\
&\quad 0 \ 0 \ 0\Big], \\
\bar{H}_{jlk}^i &= \Big[\sum_{l=1}^{M_j^i} \sum_{k=1}^{M_j^i} h_{jl}^i(\theta_i(t)) h_{jk}^i(\theta_i(t)) (\mathbb{A}_{jlk}^i - I) \\
&\quad \sum_{l=1}^{M_j^i} h_{jl}^i(\theta_i(t)) A_{1djl}^i \ \ \sum_{l=1}^{M_j^i} h_{jl}^i(\theta_i(t)) A_{2djl}^i \\
&\quad \sum_{l=1}^{M_j^i} h_{jl}^i(\theta_i(t)) D_{jl}^i \Big]. \qquad (34)
\end{aligned}
$$

On the other hand, noticing that $0 < d_m \leq d(t) \leq d_M$, by the Schur complement, from the LMIs (21) and (22), it is not difficult to get

$$
\Upsilon_{jmll}^i < 0, \qquad (35)
$$

$$
\Upsilon_{jmlk}^i + \Upsilon_{jmkl}^i < 0, \qquad (36)
$$

for

$$
j, m \in L^i, \quad l, k \in M^i(j), \quad l \neq k, \quad (j, m) \in \Theta_i.
$$

We have

$$
\begin{aligned}
&\mathbb{E}\Big\{ \sum_{i=1}^{J} \sum_{j=1}^{6} \Delta V_j^i(t) \Big\} \\
&\quad < \mathbb{E}\Big\{ \sum_{i=1}^{J} \alpha_i v_i^{\mathrm{T}}(t) v_i(t) \Big\} \leq \mathbb{E}\Big\{ \sum_{i=1}^{J} v_i^{\mathrm{T}}(t) v_i(t) \Big\}, \\
&\qquad x_i(t) \neq 0, \quad v_i(t) \neq 0. \qquad (37)
\end{aligned}
$$

It is noted that if $v_i(t) = 0$, from (37) we have

$$\mathbb{E}\{\Delta V(t)\} < 0. \quad (38)$$

By Definition 1, this means that the closed-loop fuzzy large-scale system composed of J fuzzy subsystems as (14) is mean-square asymptotical stable.

The aforementioned conditions (20)–(22) are LMIs in the variables $P_j^i,\ P_0^i,\ Q^i,\ R^i,\ S^i,\ Z^i$ and matrices $X_{1j}^i,\ X_{2j}^i,\ Y_{1j}^i,\ Y_{2j}^i$. A solution to those inequalities ensures $V(t)$ defined in (24) to be a DDPLKF for fuzzy large-scale stochastic systems. When $l = k$, the LMIs in (21) guarantee that the function decreased along all subsystems' trajectories within each region. When $l \neq k$, the LMIs in (22) guarantee that the function decreases when the states of the subsystem transit from one region to another region.

Now, to establish the generalized H_2 performance for the closed-loop system (14), summing from $t = 0$ to $t = T$ with a zero-initial condition $x_i(0) = 0$ and $v_i(t) \neq 0$, (37) leads to

$$\mathbb{E}\{\ V(T+1)\} < \mathbb{E}\Big\{\sum_{i=1}^{J}\sum_{t=0}^{T} v_i^{\mathrm{T}}(t)v_i(t)\Big\}. \quad (39)$$

From (20), we have

$$\begin{aligned}
&\mathbb{E}\{\ z^{\mathrm{T}}(t)z(t)\}\\
&= \sum_{i=1}^{J}\mathbb{E}\Big\{\sum_{l=1}^{M_j^i}\sum_{k=1}^{M_j^i} h_{jl}^i(\theta_i(t))h_{jk}^i(\theta_i(t))\\
&\quad \times x_i^{\mathrm{T}}(C_{jl}^i)^{\mathrm{T}}C_{jk}^i x_i(t)\Big\}\\
&\le \sum_{i=1}^{J}\mathbb{E}\Big\{\sum_{l=1}^{M_j^i}(h_{jl}^i)^2(\theta_i(t))x_i^{\mathrm{T}}(C_{jl}^i)^{\mathrm{T}}C_{jl}^i x_i(t)\Big\}\\
&= \sum_{i=1}^{J}\mathbb{E}\Bigg\{\sum_{l=1}^{M_j^i}(h_{jl}^i)^2(\theta_i(t))\lambda_i(t)\\
&\quad \times \begin{bmatrix} (C_{jl}^i)^{\mathrm{T}}C_{jl}^i & 0 & 0 & 0 & 0 & 0\\ * & 0 & 0 & 0 & 0 & 0\\ * & * & 0 & 0 & 0 & 0\\ * & * & * & 0 & 0 & 0\\ * & * & * & * & 0 & 0\\ * & * & * & * & * & 0\end{bmatrix}\lambda_i(t)\Bigg\}\\
&< \gamma^2\sum_{i=1}^{J}\mathbb{E}\Bigg\{\lambda_i^{\mathrm{T}}(t)\begin{bmatrix} P_j^i & 0 & 0 & 0 & 0 & 0\\ * & Q^i & 0 & 0 & 0 & 0\\ * & * & Z^i & 0 & 0 & 0\\ * & * & * & R^i & 0 & 0\\ * & * & * & * & R^i & 0\\ * & * & * & * & * & S^i\end{bmatrix}\lambda_i(t)\Bigg\}\\
&= \gamma^2\mathbb{E}\{V(t)\},
\end{aligned} \quad (40)$$

with

$$\begin{aligned}
\lambda_i(t) = \Big[&x_i^{\mathrm{T}}(t)\ \ \sum_{d=1}^{\infty}\mu_d x_i^{\mathrm{T}}(t-d)\\
&\sum_{d=1}^{\infty}\mu_d\sum_{i=-d}^{-1}\sum_{l=t-d}^{t-1}\eta_i^{\mathrm{T}}(l)\\
&\sum_{l=t-d(t)}^{t-1}x_i^{\mathrm{T}}(l)\ \ \sum_{j=-d_M+2}^{-d_m+1}\sum_{l=t+j-1}^{t-1}x_i^{\mathrm{T}}(l)\\
&\sum_{i=-d_M}^{-1}\sum_{l=t+i}^{t-1}\eta_i^{\mathrm{T}}(l)\Big].
\end{aligned}$$

From (39) and (40), we have

$$\|z\|_{\mathbb{E}_\infty}^2 < \gamma^2\|v\|_{\mathbb{E}_2}^2. \quad (41)$$

Therefore, it can be concluded that the closed-loop fuzzy stochastic large-scale system composed of J fuzzy subsystems is mean-square stable with generalized H_2 performance γ and thus the proof is completed. ■

According to Theorem 1, the following theorem presents an LMI-based delay-dependent condition for the existence of the decentralized piecewise fuzzy controller (13) for the system (11).

Theorem 2. *Given a constant $\gamma > 0$, consider the discrete-time fuzzy stochastic large-scale system composed of J fuzzy subsystems as (11) with both the time-varying delay $d(t)$ satisfying $0 < d_m \le d(k) \le d_M$ and infinite-distributed delays. A stabilizing controller in the form of (13) exists, such that the closed-loop fuzzy system in (14) is mean-square stable with generalized H_2 performance γ, if there exist a set of symmetric positive definite matrices $\tilde{P}_j^i,\ \tilde{P}_0^i \ge \tilde{P}_j^i,\ \tilde{Q}^i,\ \tilde{R}^i,\ \tilde{S}^i,\ \tilde{Z}^i$, matrices $X_{1j}^i,\ X_{2j}^i,\ Y_{1j}^i,\ Y_{2j}^i,\ G^i,\ \bar{F}_{jl}^i$, and positive constants $\alpha_i \le 1,\ \varepsilon_1^i,\ \varepsilon_2^i$, for all $i = 1, 2, \ldots, J$ and $j \in L^i$, such that the following LMIs hold:*

$$(C_{jl}^i)^{\mathrm{T}}C_{jl}^i - \gamma^2 P_j^i < 0, \quad (42)$$

$$\Phi_{jmll}^i < 0, \quad (43)$$

$$\Phi_{jmlk}^i + \Phi_{jmkl}^i < 0, \quad (44)$$

for

$$j, m \in L^i, \quad l, k \in M^i(j), \quad l \neq k, \quad (j, m) \in \Theta_i,$$

where

$$\Phi_{jmlk}^i = \begin{bmatrix} \tilde{\Psi}_{0j}^i & \varepsilon\tilde{\Psi}_{1j}^i & \varepsilon\tilde{\Psi}_{2j}^i & \varepsilon\tilde{\Psi}_{3jml}^i & \tilde{\Psi}_{4jmlk}^i\\ * & \tilde{\Psi}_{5j}^i & 0 & 0 & 0\\ * & * & \frac{1}{2}\tilde{\Psi}_{6j}^i & 0 & 0\\ * & * & * & \tilde{\Psi}_{7j}^i & 0\\ * & * & * & * & \tilde{\Psi}_{8j}^i\end{bmatrix},$$

with

$$\tilde{\Psi}^i_{0j} = \hat{P}^i_j + \mathrm{sym}\tilde{\Xi}^i_{1j} + \mathrm{sym}\tilde{\Xi}^i_{2j},$$

$$\hat{P}^i_j = \begin{bmatrix} -\tilde{\mathbb{P}}^i_j & 0 & 0 & 0 \\ * & -\frac{1}{\mu}\tilde{Q}^i & 0 & 0 \\ * & * & -\tilde{R}^i & 0 \\ * & * & * & -\tilde{I}^i \end{bmatrix},$$

$$-\tilde{\mathbb{P}}^i_j = -\tilde{P}^i_j + \varepsilon^{-2}\Big(\sum_{d=1}^{\infty} d\mu_d \tilde{Z}^i + d_M\tilde{S}^i + \tau\tilde{R}^i + \bar{\mu}\tilde{Q}^i\Big),$$

$$\tilde{\Xi}^i_{1j} = [\tilde{X}^i_j \ \ -\varepsilon\tilde{X}^i_j \ \ 0 \ \ 0], \quad \tilde{\Xi}^i_{2j} = [\tilde{Y}^i_j \ \ 0 \ \ -\varepsilon\tilde{Y}^i_j \ \ 0],$$

$$\tilde{X}^i_j = [\tilde{X}^i_{1j} \ \ \tilde{X}^i_{2j} \ \ 0 \ \ 0]^{\mathrm{T}}, \quad \tilde{Y}^i_j = [\tilde{Y}^i_{1j} \ \ 0 \ \ \tilde{Y}^i_{2j} \ \ 0]^{\mathrm{T}},$$

$$\tilde{\Psi}^i_{1j} = \left[\sqrt{\sum_{d=1}^{\infty} d\mu_d}\tilde{X}^i_j \ \ \sqrt{d_M}\tilde{Y}^i_j\right],$$

$$\tilde{\Psi}^i_{5j} = \mathrm{diag}\{-\tilde{Z}^i \ \ -\tilde{S}^i\}, \quad \tau = d_M + d_m - 1,$$

$$\tilde{\Psi}^i_{2j} = \begin{bmatrix} (G^i)^{\mathrm{T}}(\tilde{C}^i_p)^{\mathrm{T}} & (G^i)^{\mathrm{T}}(\tilde{C}^i_{p\varepsilon^i_1})^{\mathrm{T}} & (G^i)^{\mathrm{T}}(\tilde{C}^i_z)^{\mathrm{T}} & (G^i)^{\mathrm{T}}(\tilde{C}^i_{z\varepsilon^i_1}))^{\mathrm{T}} & (G^i)^{\mathrm{T}}(\tilde{C}^i_s)^{\mathrm{T}} & (G^i)^{\mathrm{T}}(\tilde{C}^i_{s\varepsilon^i_1}))^{\mathrm{T}} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} \tilde{\Psi}^i_{6j} = \ & \mathrm{diag}\{\check{P}^i_0 - \mathrm{sym}(G^i) \ \ \check{P}^i_{\varepsilon^i_1} - \mathrm{sym}(\varepsilon G^i) \\ & \check{Z}^i - \mathrm{sym}(\varepsilon G^i) \ \ \check{Z}^i_{\varepsilon^i_2} - \mathrm{sym}(\varepsilon G^i) \\ & \check{S}^i - \mathrm{sym}(\varepsilon G^i) \ \ \check{S}^i_{\varepsilon^i_2} - \mathrm{sym}(\varepsilon G^i)\}, \end{aligned}$$

$$\tilde{\Psi}^i_{7j} = \mathrm{diag}\Big\{\tilde{P}^i_m - \mathrm{sym}(G^i) \ \tilde{Z}^i - \mathrm{sym}(\varepsilon G^i) \ \ \tilde{S}^i - \mathrm{sym}(\varepsilon G^i)\Big\},$$

$$\tilde{\Psi}^i_{3jml} = \begin{bmatrix} (G^i)^{\mathrm{T}}(\bar{A}^i_{jl})^{\mathrm{T}} & (G^i)^{\mathrm{T}}(\bar{A}^i_{jl})^{\mathrm{T}} & (G^i)^{\mathrm{T}}(\bar{A}^i_{jl})^{\mathrm{T}} \\ (K^i)^{\mathrm{T}}(\bar{A}^i_{1djl})^{\mathrm{T}} & (K^i)^{\mathrm{T}}(\bar{A}^i_{1djl})^{\mathrm{T}} & (K^i)^{\mathrm{T}}(\bar{A}^i_{1djl})^{\mathrm{T}} \\ (K^i)^{\mathrm{T}}(\bar{A}^i_{2djl})^{\mathrm{T}} & (K^i)^{\mathrm{T}}(\bar{A}^i_{2djl})^{\mathrm{T}} & (K^i)^{\mathrm{T}}(\bar{A}^i_{2djl})^{\mathrm{T}} \\ (K^i)^{\mathrm{T}}(\bar{D}^i_{jl})^{\mathrm{T}} & (K^i)^{\mathrm{T}}(\bar{D}^i_{jl})^{\mathrm{T}} & (K^i)^{\mathrm{T}}(\bar{D}^i_{jl})^{\mathrm{T}} \end{bmatrix},$$

$$\tilde{\Psi}^i_{4jmlk} = \begin{bmatrix} \Gamma^i_{jlk} & \Gamma^i_{jlk} & \rho\Sigma^i_{jlk} & \rho\Sigma^i_{jlk} & \sqrt{d_M}\Sigma^i_{jlk} & \sqrt{d_M}\Sigma^i_{jlk} \\ (K^i)^{\mathrm{T}}(A^i_{1djl})^{\mathrm{T}} & (K^i)^{\mathrm{T}}(A^i_{1djl})^{\mathrm{T}} & \rho(K^i)^{\mathrm{T}}(A^i_{1djl})^{\mathrm{T}} & \rho(K^i)^{\mathrm{T}}(A^i_{1djl})^{\mathrm{T}} & \sqrt{d_M}(K^i)^{\mathrm{T}}(A^i_{1djl})^{\mathrm{T}} & \sqrt{d_M}(K^i)^{\mathrm{T}}(A^i_{1djl})^{\mathrm{T}} \\ (K^i)^{\mathrm{T}}(A^i_{2djl})^{\mathrm{T}} & (K^i)^{\mathrm{T}}(A^i_{2djl})^{\mathrm{T}} & \rho(K^i)^{\mathrm{T}}(A^i_{2djl})^{\mathrm{T}} & \rho(K^i)^{\mathrm{T}}(A^i_{2djl})^{\mathrm{T}} & \sqrt{d_M}(K^i)^{\mathrm{T}}(A^i_{2djl})^{\mathrm{T}} & \sqrt{d_M}(K^i)^{\mathrm{T}}(A^i_{2djl})^{\mathrm{T}} \\ (K^i)^{\mathrm{T}}(D^i_{jl})^{\mathrm{T}} & (K^i)^{\mathrm{T}}(D^i_{jl})^{\mathrm{T}} & \rho(K^i)^{\mathrm{T}}(D^i_{jl})^{\mathrm{T}} & \rho(K^i)^{\mathrm{T}}(D^i_{jl})^{\mathrm{T}} & \sqrt{d_M}(K^i)^{\mathrm{T}}(D^i_{jl})^{\mathrm{T}} & \sqrt{d_M}(K^i)^{\mathrm{T}}(D^i_{jl})^{\mathrm{T}} \end{bmatrix},$$

$$\Sigma^i_{jlk} = (G^i)^{\mathrm{T}}(A^i_{jl} - I)^{\mathrm{T}} + (\bar{F}^i_{jk})^{\mathrm{T}}(B^i_{jl})^{\mathrm{T}},$$

$$\Gamma^i_{jlk} = (G^i)^{\mathrm{T}}(A^i_{jl})^{\mathrm{T}} + (\bar{F}^i_{jk})^{\mathrm{T}}(B^i_{jl})^{\mathrm{T}},$$

$$\check{P}^i_0 = \mathrm{diag}\{\tilde{P}^1_0, \dots, \tilde{P}^1_{0,n\neq i}, \dots, \tilde{P}^J_0\},$$

$$\check{P}^i_{\varepsilon^i_1} = \mathrm{diag}\{\varepsilon^1_1\tilde{P}^1_0, \dots, \varepsilon^n_1\tilde{P}^1_{0,n\neq i}, \dots, \varepsilon^J_1\tilde{P}^J_0\},$$

$$\check{Z}^i = \mathrm{diag}\{\rho\tilde{Z}^i, \dots, \rho\tilde{Z}^i, n\neq i, \dots, \rho\tilde{Z}^i\},$$

$$\check{Z}^i_{\varepsilon^i_2} = \mathrm{diag}\{\varepsilon^1_2\rho\tilde{Z}^i, \dots, \varepsilon^n_{2,n\neq i}\rho\tilde{Z}^i, \dots, \varepsilon^J_2\rho\tilde{Z}^i\},$$

$$\check{S}^i = \mathrm{diag}\{\sqrt{d_M}\tilde{S}^i, \dots, \sqrt{d_M}\tilde{S}^i, \quad n\neq i, \dots, \sqrt{d_M}\tilde{S}^i\},$$

$$\check{S}^i_{\varepsilon^i_2} = \mathrm{diag}\{\varepsilon^1_2\sqrt{d_M}\tilde{S}^i, \dots, \varepsilon^n_{2,n\neq i}\sqrt{d_M}\tilde{S}^i, \dots, \varepsilon^J_2\sqrt{d_M}\tilde{S}^i\},$$

$$\tilde{C}^i_p = [(C^i_1)^{\mathrm{T}}, \dots, (C^i_{n,n\neq i})^{\mathrm{T}}, \dots, (C^i_J)^{\mathrm{T}}]^{\mathrm{T}},$$

$$\tilde{C}^i_{p\varepsilon^i_1} = [\varepsilon^1_1(C^i_1)^{\mathrm{T}}, \dots, \varepsilon^n_1(C^i_{n,n\neq i})^{\mathrm{T}}, \dots, \varepsilon^J_1(C^i_J)^{\mathrm{T}}]^{\mathrm{T}},$$

$$\tilde{C}^i_z = [\rho(C^i_1)^{\mathrm{T}}, \dots, \rho(C^i_{n,n\neq i})^{\mathrm{T}}, \dots, \rho(C^i_J)^{\mathrm{T}}]^{\mathrm{T}},$$

$$\tilde{C}^i_{z\varepsilon^i_2} = [\varepsilon^1_2\rho(C^i_1)^{\mathrm{T}}, \dots, \varepsilon^n_2\rho(C^i_{n,n\neq i})^{\mathrm{T}}, \dots, \varepsilon^J_2\rho(C^i_J)^{\mathrm{T}}]^{\mathrm{T}},$$

$$\tilde{C}^i_s = [\sqrt{d_M}(C^i_1)^{\mathrm{T}}, \dots, \sqrt{d_M}(C^i_{n,n\neq i})^{\mathrm{T}}, \dots, \sqrt{d_M}(C^i_J)^{\mathrm{T}}]^{\mathrm{T}},$$

$$\tilde{C}^i_{s\varepsilon^i_2} = [\varepsilon^1_2\sqrt{d_M}(C^i_1)^{\mathrm{T}}, \dots, \varepsilon^n_2\sqrt{d_M}(C^i_{n,n\neq i})^{\mathrm{T}}, \dots, \varepsilon^J_2\sqrt{d_M}(C^i_J)^{\mathrm{T}}]^{\mathrm{T}},$$

$$\begin{aligned} \tilde{\Psi}^i_{8j} = \mathrm{diag}\{&\tilde{P}^i_m - \mathrm{sym}(G^i) \ \varepsilon^i_1\tilde{P}^i_m - \mathrm{sym}(\varepsilon G^i) \\ &\tilde{Z}^i - \mathrm{sym}(\varepsilon G^i) \ \varepsilon^i_2\tilde{Z}^i - \mathrm{sym}(\varepsilon G^i) \\ &\tilde{S}^i - \mathrm{sym}(\varepsilon G^i) \ \ \varepsilon^i_2\tilde{S}^i - \mathrm{sym}(\varepsilon G^i)\}. \end{aligned} \tag{45}$$

Furthermore, if the aforementioned conditions are satisfied, the matrix gains F^i_{jl} of the controller are given by

$$F^i_{jl} = \bar{F}^i_{jl}G^i. \tag{46}$$

Proof. Suppose that there exist positive definite matrices $\tilde{P}^i_j$, $\tilde{Q}^i$, $\tilde{Z}^i$, $\tilde{R}^i$, $\tilde{S}^i$, and matrices $\tilde{X}^i_j$, $\tilde{Y}^i_j$, G^i satisfying (43) and (44). Since $\tilde{P}^i_j > 0$, $\tilde{Z}^i > 0$, $\tilde{S}^i > 0$, we have

$$\begin{aligned} [\tilde{P}^i_j - G^i](\tilde{P}^i_j)^{-1}[\tilde{P}^i_j - G^i]^{\mathrm{T}} &\geq 0, \\ [\tilde{Z}^i - \varepsilon G^i](\tilde{Z}^i)^{-1}[\tilde{Z}^i - \varepsilon G^i]^{\mathrm{T}} &\geq 0, \\ [\tilde{S}^i - \varepsilon G^i](\tilde{S}^i)^{-1}[\tilde{S}^i - \varepsilon G^i]^{\mathrm{T}} &\geq 0, \end{aligned} \tag{47}$$

which imply that

$$\begin{aligned} -G^i(\tilde{P}^i_j)^{-1}(G^i)^{\mathrm{T}} &\leq \tilde{P}^i_j - G^i - (G^i)^{\mathrm{T}}, \\ -\varepsilon^2 G^i(\tilde{Z}^i)^{-1}(G^i)^{\mathrm{T}} &\leq \tilde{Z}^i - \varepsilon G^i - \varepsilon(G^i)^{\mathrm{T}}, \end{aligned}$$

$$-\varepsilon^2 G^i(\tilde{S}^i)^{-1}(G^i)^{\mathrm{T}} \le \tilde{S}^i - \varepsilon G^i - \varepsilon (G^i)^{\mathrm{T}}. \quad (48)$$

Define

$$K^i = \varepsilon G^i. \quad (49)$$

From (43) and (44) we have that

$$\tilde{P}_j^i - G^i - (G^i)^{\mathrm{T}} < 0.$$

Since $\tilde{P}_j^i > 0$, we have

$$G^i + (G^i)^{\mathrm{T}} > 0,$$

which ensures that $(G^i)^{-1}$ exist. Define matrix

$$\Delta_{11}^i = \mathrm{diag}\{(G^i)^{-1}\ (K^i)^{-1}\ (K^i)^{-1}\ (K^i)^{-1}\},$$

and

$$\Delta_1^i = \mathrm{diag}\{\Delta_{11}^i\ I\ I\ I\ I\ I\}.$$

By pre- and postmultiplying (43) and (44) by $(\Delta_1^i)^{\mathrm{T}}$ and Δ_1^i, respectively, and by considering (48) and (49) as well as defining

$$\begin{aligned}
P_j^i &= (G^i)^{-\mathrm{T}}\tilde{P}_j^i(G^i)^{-1}, & Q^i &= (K^i)^{-\mathrm{T}}\tilde{Q}^i(K^i)^{-1},\\
Z^i &= (K^i)^{-\mathrm{T}}\tilde{Z}^i(K^i)^{-1}, & S^i &= (K^i)^{-\mathrm{T}}\tilde{S}^i(K^i)^{-1},\\
X_{j1}^i &= (K^i)^{-\mathrm{T}}\tilde{X}_{j1}^i(K^i)^{-1}, & X_{j2}^i &= (G^i)^{-\mathrm{T}}\tilde{X}_{j2}^i(K^i)^{-1},\\
Y_{j1}^i &= (K^i)^{-\mathrm{T}}\tilde{Y}_{j1}^i(K^i)^{-1}, & Y_{j2}^i &= (G^i)^{-\mathrm{T}}\tilde{Y}_{j2}^i(K^i)^{-1},\\
\alpha_i I &= (K^i)^{-\mathrm{T}}\tilde{I}^i(K^i)^{-1}, & P_0^i &= (G^i)^{-\mathrm{T}}\tilde{P}_0^i(G^i)^{-1},\\
R^i &= (K^i)^{-\mathrm{T}}\tilde{R}^i(K^i)^{-1}, & i &= 1,2,\dots,J,
\end{aligned}$$

we have

$$(\Delta_1^i)^{\mathrm{T}}\Phi_{jmll}^i\Delta_1^i < 0, \quad (50)$$

$$(\Delta_1^i)^{\mathrm{T}}\Phi_{jmlk}^i\Delta_1^i + (\Delta_1^i)^{\mathrm{T}}\Phi_{jmkl}^i\Delta_1^i < 0, \quad (51)$$

$$(\Delta_1^i)^{\mathrm{T}}\Phi_{jmlk}^i\Delta_1^i = \begin{bmatrix} \Psi_{j0}^i & \Psi_{1j}^i & \check{\Psi}_{2j}^i & \check{\Psi}_{3jml}^i & \check{\Psi}_{4jmlk}^i \\ * & \Psi_{5j}^i & 0 & 0 & 0 \\ * & * & \frac{1}{2}\check{\Psi}_{6j}^i & 0 & 0 \\ * & * & * & \check{\Psi}_{7j}^i & 0 \\ * & * & * & * & \check{\Psi}_{8j}^i \end{bmatrix},$$

where

$$\check{\Psi}_{2j}^i = \begin{bmatrix} (\tilde{C}_p^i)^{\mathrm{T}} & (\tilde{C}_{p\varepsilon_1^i}^i)^{\mathrm{T}} & (\tilde{C}_z^i)^{\mathrm{T}} & (\tilde{C}_{z\varepsilon_1^i}^i)^{\mathrm{T}} & (\tilde{C}_s^i)^{\mathrm{T}} & (\tilde{C}_{s\varepsilon_1^i}^i)^{\mathrm{T}} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\check{\Psi}_{3jml}^i = \begin{bmatrix} (\bar{A}_{jl}^i)^{\mathrm{T}} & (\bar{A}_{jl}^i)^{\mathrm{T}} & (\bar{A}_{jl}^i)^{\mathrm{T}} \\ (\bar{A}_{1djl}^i)^{\mathrm{T}} & (\bar{A}_{1djl}^i)^{\mathrm{T}} & (\bar{A}_{1djl}^i)^{\mathrm{T}} \\ (\bar{A}_{2djl}^i)^{\mathrm{T}} & (\bar{A}_{2djl}^i)^{\mathrm{T}} & (\bar{A}_{2djl}^i)^{\mathrm{T}} \\ (\bar{D}_{jl}^i)^{\mathrm{T}} & (\bar{D}_{jl}^i)^{\mathrm{T}} & (\bar{D}_{jl}^i)^{\mathrm{T}} \end{bmatrix},$$

$$\check{\Psi}_{4jmlk}^i = \begin{bmatrix} \Gamma_{jlk}^i & \Gamma_{jlk}^i & \rho\bar{\Sigma}_{jlk}^i & \rho\bar{\Sigma}_{jlk}^i & \sqrt{d_M}\bar{\Sigma}_{jlk}^i & \sqrt{d_M}\bar{\Sigma}_{jlk}^i \\ (A_{1djl}^i)^{\mathrm{T}} & (A_{1djl}^i)^{\mathrm{T}} & \rho(A_{1djl}^i)^{\mathrm{T}} & \rho(A_{1djl}^i)^{\mathrm{T}} & \sqrt{d_M}(A_{1djl}^i)^{\mathrm{T}} & \sqrt{d_M}(A_{1djl}^i)^{\mathrm{T}} \\ (A_{2djl}^i)^{\mathrm{T}} & (A_{2djl}^i)^{\mathrm{T}} & \rho(A_{2djl}^i)^{\mathrm{T}} & \rho(A_{2djl}^i)^{\mathrm{T}} & \sqrt{d_M}(A_{2djl}^i)^{\mathrm{T}} & \sqrt{d_M}(A_{2djl}^i)^{\mathrm{T}} \\ (D_{jl}^i)^{\mathrm{T}} & (D_{jl}^i)^{\mathrm{T}} & \rho(D_{jl}^i)^{\mathrm{T}} & \rho(D_{jl}^i)^{\mathrm{T}} & \sqrt{d_M}(D_{jl}^i)^{\mathrm{T}} & \sqrt{d_M}(D_{jl}^i)^{\mathrm{T}} \end{bmatrix},$$

$$\begin{aligned}
\check{\Psi}_{6j}^i &= \mathrm{diag}\{-(\hat{P}_0^i)^{-1}\ -(\hat{P}_{\varepsilon_1^i}^i)^{-1}\ -(\hat{Z}^i)^{-1}\\
&\quad -(\hat{Z}_{\varepsilon_2^i}^i)^{-1}\ -(\hat{S}^i)^{-1}\ -(\hat{S}_{\varepsilon_2^i}^i)^{-1}\},\\
\check{\Psi}_{7j}^i &= \mathrm{diag}\{-(P_m^i)^{-1}\ -(Z^i)^{-1}\ -(S^i)^{-1}\},\\
\bar{\Sigma}_{jlk}^i &= (A_{jl}^i - I)^{\mathrm{T}} + (F_{jk}^i)^{\mathrm{T}}(B_{jl}^i)^{\mathrm{T}},\\
\check{\Psi}_{8j}^i &= \mathrm{diag}\{-(P_m^i)^{-1}\ -\varepsilon_1^i(P_m^i)^{-1}\ -(Z^i)^{-1}\\
&\quad -\varepsilon_2^i(Z^i)^{-1}\ -(S^i)^{-1}\ -\varepsilon_2^i(S^i)^{-1}\},
\end{aligned}$$

and Ξ_{1j}^i, Ξ_{2j}^i, Ψ_{1j}^i, Ψ_{5j}^i are defined in (23). Define matrices

$$\Delta_{13}^i = \mathrm{diag}\{\hat{P}_0^i\ \hat{P}_{\varepsilon_1^i}^i\ \hat{Z}^i\ \hat{Z}_{\varepsilon_2^i}^i\ \hat{S}^i\ \hat{S}_{\varepsilon_2^i}^i\},$$

$$\Delta_{14}^i = \mathrm{diag}\{P_m^i\ Z^i\ S^i\},$$

$$\Delta_{15}^i = \mathrm{diag}\{P_m^i\ P_m^i\ Z^i\ Z^i\ S^i\ S^i,$$

$$\Delta_2^i = \mathrm{diag}\{I\ I\ \Delta_{13}^i\ \Delta_{14}^i\ \Delta_{15}^i\},$$

by pre- and postmultiplying (50) and (51) by $(\Delta_2^i)^{\mathrm{T}}$ and Δ_2^i. We have

$$(\Delta_1^i\Delta_2^i)^{\mathrm{T}}\Phi_{jmll}^i\Delta_1^i\Delta_2^i < 0, \quad (52)$$

$$(\Delta_1^i\Delta_2^i)^{\mathrm{T}}\Phi_{jmlk}^i\Delta_1^i\Delta_2^i + (\Delta_1^i\Delta_2^i)^{\mathrm{T}}\Phi_{jmkl}^i\Delta_1^i\Delta_2^i < 0, \quad (53)$$

where

$$(\Delta_1^i\Delta_2^i)^{\mathrm{T}}\Phi_{jmlk}^i\Delta_1^i\Delta_2^i = \begin{bmatrix} \Psi_{0j}^i & \Psi_{1j}^i & \Psi_{2j}^i & \Psi_{3jml}^i & \Psi_{4jmlk}^i \\ * & \Psi_{5j}^i & 0 & 0 & 0 \\ * & * & \frac{1}{2}\Psi_{6j}^i & 0 & 0 \\ * & * & * & \Psi_{7j}^i & 0 \\ * & * & * & * & \Psi_{8j}^i \end{bmatrix}.$$

We can obtain that (52) and (53) yield (43) and (44), which means that there exist matrices $P_m^i > 0$, $Q^i >$

$0,\ Z^i > 0,\ R^i > 0,\ S^i > 0$, and $X^i_{1j},\ X^i_{2j},\ Y^i_{1j},\ Y^i_{2j}$ satisfying (43) and (44), and the controller gains defined in (46) render the closed-loop system in (14) mean-square asymptotically stable. ■

Remark 3. If the global state space replaces the transitions Θ_i and all P^i_js in Theorem 2 become a common P^i, Theorem 2 is regressed to Corollary 1, shown in the following. However, in Theorem 2, the state transition is considered and there are different P^i_js instead of a common P^i to satisfy the inequalities. Therefore, Theorem 2 can be less conservative than Corollary 1, but the number of inequalities of Theorem 2 is generally larger than that of Corollary 1.

Corollary 1. *Given a constant $\gamma > 0$, consider the discrete-time fuzzy stochastic large-scale system composed of J fuzzy subsystems as given by (11) with both the time-varying delay $d(t)$ satisfying $0 < d_m \le d(k) \le d_M$ and infinite-distributed delays. A stabilizing controller in the form of (13) exists, such that the closed-loop fuzzy system in (14) is mean-square stable with generalized H_2 performance γ, if there exist a set of symmetric positive definite matrices $\tilde{P}^i,\ \tilde{P}^i_0 \ge \tilde{P}^i,\ \tilde{Q}^i,\ \tilde{R}^i,\ \tilde{S}^i,\ \tilde{Z}^i$, matrices $X^i_{1j},\ X^i_{2j},\ Y^i_{1j},\ Y^i_{2j},\ G^i,\ \bar{F}^i_{jl}$, and positive constants $\alpha_i \le 1,\ \varepsilon^i_1,\ \varepsilon^i_2$, for all $i = 1, 2, \dots, J$ and $j \in L^i$, such that the following LMIs hold:*

$$(C^i_{jl})^{\mathrm{T}} C^i_{jl} - \gamma^2 P^i < 0, \tag{54}$$

$$\bar{\Phi}^i_{jll} < 0, \tag{55}$$

$$\bar{\Phi}^i_{jlk} + \bar{\Phi}^i_{jkl} < 0, \tag{56}$$

for

$$j \in L^i, \quad l, k \in M^i(j), \quad l \neq k,$$

where

$$\tilde{\Phi}^i_{jlk} = \begin{bmatrix} \hat{\Psi}^i_{0j} & \varepsilon\tilde{\Psi}^i_{1j} & \varepsilon\tilde{\Psi}^i_{2j} & \varepsilon\tilde{\Psi}^i_{3jl} & \tilde{\Psi}^i_{4jlk} \\ * & \tilde{\Psi}^i_{5j} & 0 & 0 & 0 \\ * & * & \frac{1}{2}\tilde{\Psi}^i_{6j} & 0 & 0 \\ * & * & * & \tilde{\Psi}^i_{7j} & 0 \\ * & * & * & * & \tilde{\Psi}^i_{8j} \end{bmatrix},$$

$$\hat{\Psi}^i_{0j} = \hat{P}^i + \mathrm{sym}\tilde{\Xi}^i_{1j} + \mathrm{sym}\tilde{\Xi}^i_{2j},$$

$$\hat{P}^i = \begin{bmatrix} -\tilde{\mathbb{P}}^i & 0 & 0 & 0 \\ * & -\frac{1}{\mu}\tilde{Q}^i & 0 & 0 \\ * & * & -\tilde{R}^i & 0 \\ * & * & * & -\tilde{I}^i \end{bmatrix},$$

$$-\tilde{\mathbb{P}}^i = -\tilde{P}^i + \varepsilon^{-2}\Big(\sum_{d=1}^{\infty} d\mu_d \tilde{Z}^i + d_M \tilde{S}^i + \tau\tilde{R}^i + \bar{\mu}\tilde{Q}^i\Big),$$

with

$$\begin{aligned} \tilde{\Psi}^i_{7j} &= \mathrm{diag}\{\tilde{P}^i - \mathrm{sym}(G^i)\ \ \tilde{Z}^i - \mathrm{sym}(\varepsilon G^i) \\ &\qquad \tilde{S}^i - \mathrm{sym}(\varepsilon G^i)\}, \\ \tilde{\Psi}^i_{8j} &= \mathrm{diag}\{\tilde{P}^i - \mathrm{sym}(G^i)\ \ \varepsilon^i_1\tilde{P}^i - \mathrm{sym}(\varepsilon G^i) \\ &\qquad \tilde{Z}^i - \mathrm{sym}(\varepsilon G^i)\ \ \varepsilon^i_2\tilde{Z}^i - \mathrm{sym}(\varepsilon G^i) \\ &\qquad \tilde{S}^i - \mathrm{sym}(\varepsilon G^i)\ \ \varepsilon^i_2\tilde{S}^i - \mathrm{sym}(\varepsilon G^i)\}, \\ \tilde{\Psi}^i_{3jl} &= \tilde{\Psi}^i_{3jml}, \quad \tilde{\Psi}^i_{4jlk} = \tilde{\Psi}^i_{4jmlk}, \end{aligned} \tag{57}$$

and $\tilde{\Psi}^i_{1j}, \tilde{\Psi}^i_{2j}, \tilde{\Psi}^i_{5j}, \tilde{\Psi}^i_{6j}$ defined in (45).

For further reduction of the conservatism of the stabilization criterion, we apply some new slack matrices Q^i_{jmlk}. A more relaxed stabilization criterion, in which the interactions among the fuzzy subsystems are considered, is stated in Theorem 3.

Theorem 3. *Given a constant $\gamma > 0$, consider the discrete-time fuzzy stochastic large-scale system composed of J fuzzy subsystems as given by (11) with both the time-varying delay $d(t)$ satisfying $0 < d_m \le d(k) \le d_M$ and infinite-distributed delays. A decentralized piecewise fuzzy controller in the form of (13) exists, such that the closed-loop fuzzy system in (14) is mean-square stable with generalized H_2 performance γ, if there exist symmetric matrices Q^i_{jmlk}, and a set of symmetric positive definite matrices $\tilde{P}^i_j,\ \tilde{P}^i_0 \ge \tilde{P}^i_j,\ \tilde{Q}^i,\ \tilde{R}^i,\ \tilde{S}^i,\ \tilde{Z}^i$, matrices $X^i_{1j},\ X^i_{2j},\ Y^i_{1j},\ Y^i_{2j},\ G^i,\ \bar{F}^i_{jl}$, and positive constants $\alpha_i \le 1,\ \varepsilon^i_1,\ \varepsilon^i_2$, for all $i = 1, 2, \dots, J$ and $j \in L^i$, such that the following LMIs hold:*

$$(C^i_{jl})^{\mathrm{T}} C^i_{jl} - \gamma^2 P^i_j < 0, \tag{58}$$

$$\Phi^i_{jmll} + Q^i_{jmll} < 0, \tag{59}$$

$$\Phi^i_{jmlk} + \Phi^i_{jmkl} + Q^i_{jmlk} + Q^i_{jmkl} < 0, \tag{60}$$

$$N^i_j = \begin{bmatrix} N^i_{jm11} & N^i_{jm12} & \cdots & N^i_{jm1M^i_j} \\ N^i_{jm12} & N^i_{jm22} & \cdots & N^i_{jm2M^i_j} \\ \vdots & \vdots & \ddots & \vdots \\ N^i_{jm1M^i_j} & N^i_{jm2M^i_j} & \cdots & N^i_{jmM^i_jM^i_j} \end{bmatrix} > 0,$$

$$U^i_j = \begin{bmatrix} U^i_{jm11} & U^i_{jm12} & \cdots & U^i_{jm1M^i_j} \\ U^i_{jm12} & U^i_{jm22} & \cdots & U^i_{jm2M^i_j} \\ \vdots & \vdots & \ddots & \vdots \\ U^i_{jm1M^i_j} & U^i_{jm2M^i_j} & \cdots & U^i_{jmM^i_jM^i_j} \end{bmatrix} > 0,$$

$$V^i_j = \begin{bmatrix} V^i_{jm11} & V^i_{jm12} & \cdots & V^i_{jm1M^i_j} \\ V^i_{jm12} & V^i_{jm22} & \cdots & V^i_{jm2M^i_j} \\ \vdots & \vdots & \ddots & \vdots \\ V^i_{jm1M^i_j} & V^i_{jm2M^i_j} & \cdots & V^i_{jmM^i_jM^i_j} \end{bmatrix} > 0,$$

$$W_j^i = \begin{bmatrix} W_{jm11}^i & W_{jm12}^i & \cdots & W_{jm1M_j^i}^i \\ W_{jm12}^i & W_{jm22}^i & \cdots & W_{jm2M_j^i}^i \\ \vdots & \vdots & \ddots & \vdots \\ W_{jm1M_j^i}^i & W_{jm2M_j^i}^i & \cdots & W_{jmM_j^iM_j^i}^i \end{bmatrix} > 0, \quad (61)$$

for

$$j, m \in L^i, l, k \in M^i(j), l \neq k, \ \ (l, m) \in \Theta_i,$$

where

$$Q_{jmlk}^i = \begin{bmatrix} N_{jmlk}^i & 0 & 0 & 0 \\ * & U_{jmlk}^i & 0 & 0 \\ * & * & V_{jmlk}^i & 0 \\ * & * & * & W_{jmlk}^i \end{bmatrix}.$$

Hence, the controller gains are obtained from $F_{jl}^i = \bar{F}_{jl}^i G^i$.

Proof. According to Theorem 2, the following inequalities are satisfied:

$$\Phi_{jmlk}^i = \sum_l^{M_j^i} (h_{jl}^i)^2 \Phi_{jmll}^i + \sum_l^{M_j^i} \sum_{l<k}^{M_j^i} h_{jl}^i h_{jk}^i \Phi_{jmlk}^i < 0. \quad (62)$$

From (59) and (60), we have

$$\sum_l^{M_j^i} (h_{jl}^i)^2 \Phi_{jmll}^i + \sum_l^{M_j^i} \sum_{l<k}^{M_j^i} h_{jl}^i h_{jk}^i \Phi_{jmlk}^i < -\Big(\sum_l^{M_j^i} (h_{jl}^i)^2 Q_{jmll}^i + \sum_l^{M_j^i} \sum_{l<k}^{M_j^i} h_{jl}^i h_{jk}^i Q_{jmlk}^i\Big). \quad (63)$$

Equation (61) guarantees

$$-\Big(\sum_l^{M_j^i} (h_{jl}^i)^2 Q_{jmll}^i + \sum_l^{M_j^i} \sum_{l<k}^{M_j^i} h_{jl}^i h_{jk}^i Q_{jmlk}^i\Big) < 0 \quad (64)$$

Thus, $\mathbb{E}\{\Delta V(t)\} < 0$ and the closed-loop fuzzy system (14) is mean-square stable with generalized H_2 performance if (61) hold. The proof is completed. ■

Remark 4. Since there are matrices Q_{jmlk}^i to represent the interactions among the fuzzy subsystems in the region Ω_j^i, Theorem 3 is more relaxed than Theorem 2 and Corollary 1. However, the number of inequalities of Theorem 3 is generally larger than that of Theorem 2 and Corollary 1.

4. Simulation

In this section, a numerical example is presented to show the effectiveness and advantages of the proposed decentralized control scheme.

Example 1. Consider fuzzy large-scale stochastic systems S composed of two fuzzy subsystems S^i, $i = 1, 2$ as (1). The membership functions h_j^i are

$$h_1^i(\theta_i(t)) = \begin{cases} 1 & \text{if} \quad \theta_i(t) \in [-3, -1], \\ -0.5\theta_i(t) + 0.5 & \text{if} \quad \theta_i(t) \in [-1, 1], \end{cases}$$

$$h_2^i(\theta_i(t)) = \begin{cases} 0.5\theta_i(t) + 0.5 & \text{if} \quad \theta_i(t) \in [-1, 1], \\ 1 & \text{if} \quad \theta_i(t) \in [1, 3]. \end{cases} \quad (65)$$

The system matrices are

$$A_1^1 = \begin{bmatrix} -0.098 & 0.078 \\ 0.076 & 0.087 \end{bmatrix},$$

$$A_2^1 = \begin{bmatrix} 0.075 & -0.060 \\ 0.060 & 0.050 \end{bmatrix},$$

$$A_1^2 = \begin{bmatrix} -0.095 & 0.080 \\ 0.077 & 0.086 \end{bmatrix},$$

$$A_2^2 = \begin{bmatrix} 0.080 & -0.060 \\ 0.060 & 0.050 \end{bmatrix},$$

$$A_{1d1}^1 = \begin{bmatrix} 0.012 & 0.09 \\ 0.011 & 0.07 \end{bmatrix},$$

$$A_{1d2}^1 = \begin{bmatrix} -0.01 & -0.05 \\ 0 & 0 \end{bmatrix},$$

$$A_{1d1}^2 = \begin{bmatrix} 0.010 & 0.08 \\ 0.012 & 0.06 \end{bmatrix}, \quad A_{1d2}^2 = \begin{bmatrix} -0.01 & -0.05 \\ 0 & 0 \end{bmatrix},$$

$$A_{2d1}^1 = A_{2d1}^2 \begin{bmatrix} 0.012 & 0.0014 \\ 0 & 0.0015 \end{bmatrix},$$

$$A_{2d2}^1 = A_{2d2}^2 = \begin{bmatrix} 0.001 & 0 \\ 0.001 & 0.0015 \end{bmatrix},$$

$$D_1^1 = D_2^1 = D_1^2 = D_2^2 = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix},$$

$$B_1^1 = B_1^2 = \begin{bmatrix} 0.0050 \\ 0.0045 \end{bmatrix}, \quad B_2^1 = \begin{bmatrix} 0.0080 \\ 0.0082 \end{bmatrix},$$

$$B_2^2 = \begin{bmatrix} 0.0090 \\ 0.0085 \end{bmatrix}, \quad C_1^1 = C_1^2 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.1 \end{bmatrix},$$

$$C_2^1 = C_2^2 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.3 \end{bmatrix},$$

$$C_{12} = C_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0.001 \end{bmatrix},$$

$$\bar{A}_1^1 = \bar{A}_1^2 = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix},$$

$$\bar{A}_2^1 = \bar{A}_2^2 = \begin{bmatrix} 0.0015 & 0 \\ 0 & 0.0015 \end{bmatrix},$$

$$\bar{A}_{1d2}^1 = \bar{A}_{1d2}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0.001 \end{bmatrix},$$

$$\bar{A}_{2d1}^1 = \bar{A}_{2d1}^2 = \begin{bmatrix} 0.001 & 0.002 \\ 0 & 0.003 \end{bmatrix},$$

$$\bar{A}_{1d1}^1 = \bar{A}_{1d1}^2 = \begin{bmatrix} 0.005 & 0 \\ 0 & 0.001 \end{bmatrix},$$

$$\bar{A}_{2d2}^1 = \bar{A}_{2d2}^2 = \begin{bmatrix} 0.0015 & 0 \\ 0 & 0.0025 \end{bmatrix},$$

$$\bar{D}_1^1 = \bar{D}_2^1 = \bar{D}_1^2 = \bar{D}_2^2 = \begin{bmatrix} 0.0015 \\ 0 \end{bmatrix}.$$

In this example, choosing the constant $\mu_d = 2^{-3-d}$, we can easily find that

$$\bar{\mu} = \sum_{d=1}^{\infty} \mu_d = 2^{-3} \leq \sum_{d=1}^{\infty} d\mu_d = 2 < \infty,$$

which satisfies the convergence condition (2). The normalized membership functions and cell partitions are shown in Figs. 1 and 2, respectively.

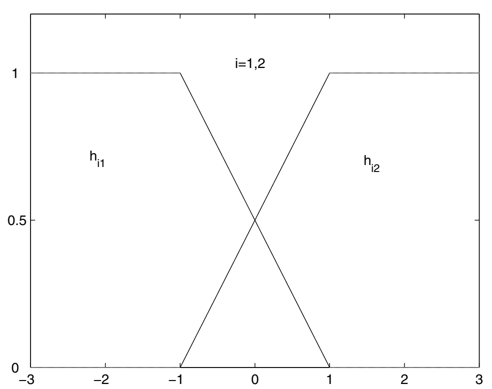

Fig. 1. Normalized membership functions.

Remark 5. As illustrated by Wang *et al.* (2007), according to the membership functions h_j^i shown in Fig. 1, we can divide the state space into three subregions shown in Fig. 2. In the operating regions Ω_{i1}, Ω_{i3}, there is only one rule in these regions (i.e, $M^i(1) = M^i(3) = 1$). In the interpolation region Ω_{i2}, there are two rules in this region (i.e., $M^i(2) = 2$). The possible region transitions Θ_i contain nine subregion transitions: (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2) and (3,3).

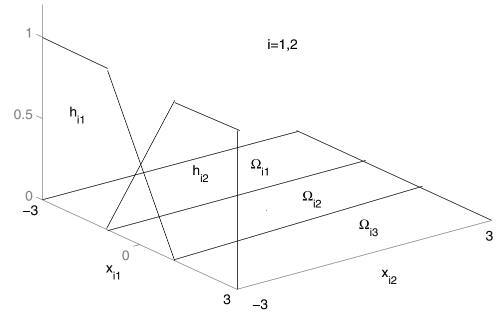

Fig. 2. Cell partitions of the state space.

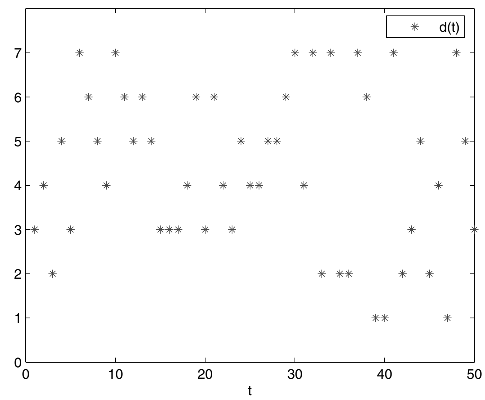

Fig. 3. Time-varying delays.

The system matrices are

$$A_{11}^i = A_{21}^i = A_1^i,$$
$$A_{22}^i = A_{31}^i = A_2^i,$$
$$A_{1d11}^i = A_{1d21}^i = A_{1d1}^i,$$
$$A_{1d22}^i = A_{1d31}^i = A_{1d2}^i,$$
$$A_{2d11}^i = A_{2d21}^i = A_{2d1}^i,$$
$$A_{2d22}^i = A_{2d31}^i = A_{2d2}^i,$$
$$B_{11}^i = B_{21}^i = B_1^i,$$
$$B_{22}^i = B_{31}^i = B_2^i,$$
$$D_{11}^i = D_{21}^i = D_1^i,$$
$$D_{22}^i = D_{31}^i = D_2^i,$$
$$\bar{A}_{11}^i = \bar{A}_{21}^i = \bar{A}_1^i,$$
$$\bar{A}_{22}^i = \bar{A}_{31}^i = \bar{A}_2^i,$$
$$\bar{A}_{1d11}^i = \bar{A}_{1d21}^i = \bar{A}_{1d1}^i,$$
$$\bar{A}_{1d22}^i = \bar{A}_{1d31}^i = \bar{A}_{1d2}^i,$$
$$\bar{A}_{2d11}^i = \bar{A}_{2d21}^i = \bar{A}_{2d1}^i,$$
$$\bar{A}_{2d22}^i = \bar{A}_{2d31}^i = \bar{A}_{2d2}^i,$$

$$\bar{D}^i_{11} = \bar{D}^i_{21} = \bar{D}^i_1,$$
$$\bar{D}^i_{22} = \bar{D}^i_{31} = \bar{D}^i_2,$$
$$C^i_{11} = C^i_{21} = C^i_1,$$
$$C^i_{22} = C^i_{31} = C^i_2.$$

The membership functions are $h^i_{11} \cup h^i_{21} = h^i_1$, $h^i_{22} \cup h^i_{31} = h^i_2$. With the choice of $\gamma = 0.0015$, $\varepsilon = 10$, $\varepsilon_{i1} = \varepsilon_{i1} = 20$ and $v_i(t) = 0.1\cos(t)e^{-0.05t}$, it is found that the system is generalized H_2 stable for all $d_M = 7$. When $d_M = 7$, $d(t)$ presenting a time-varying state delay, let the delay $d(t)$ change randomly between $d_m = 1$ and $d_M = 7$ (see Fig. 3).

Table 1. Comparison of the values of d_M by different methods.

Methods	Corollary 1	Theorem 2	Theorem 3
d_M	3	4	7

Remark 6. Here, $d(k)$ represents a time-varying state delay. Now assume that the lower delay bound of d_k is $d_m = 1$, and we are interested in the upper delay bound d_M below which Example 1 is mean-square stable with generalized H_2 performance $\gamma = 0.0015$ for all $d_m \leq d(k) \leq d_M$. By using Corollary 1, it is found that the upper delay bound $d_M = 3$. By applying Theorem 2, we obtain the upper delay bound $d_M = 4$. However, applying Theorem 3 yields $d_M = 7$. It is clearly shown in Table 1 that the upper delay bound obtained by Theorem 2 based on the PLKF is larger than that obtained by Corollary 1 based on the CLKF. The result clearly demonstrates much better performance of PLKF-based approaches over CLKF-based approaches. It can also be easily seen that the upper delay bound obtained by Theorem 3 is much larger than those obtained by Corollary 1 and Theorem 2, which indicates that Theorem 3 is much less conservative than Corollary 1 and Theorem 2.

By implementing Theorem 3 and using the MATLAB LMI toolbox, we can find

$$P^1_1 = \begin{bmatrix} 0.1184 & 0.0243 \\ 0.0243 & 0.0361 \end{bmatrix},$$
$$P^1_2 = \begin{bmatrix} 0.1329 & 0.0258 \\ 0.0258 & 0.0383 \end{bmatrix},$$
$$P^1_3 = \begin{bmatrix} 0.1282 & 0.0218 \\ 0.0218 & 0.0347 \end{bmatrix},$$
$$Q^1 = \begin{bmatrix} 0.1547 & 0.0338 \\ 0.0338 & 0.1701 \end{bmatrix},$$
$$R^1 = \begin{bmatrix} 0.0585 & 0.0352 \\ 0.0352 & 0.0311 \end{bmatrix},$$
$$Z^1 = \begin{bmatrix} 0.0296 & 0.0224 \\ 0.0224 & 0.0202 \end{bmatrix},$$
$$S^1 = \begin{bmatrix} 0.0145 & 0.0118 \\ 0.0118 & 0.0108 \end{bmatrix},$$
$$P^2_1 = \begin{bmatrix} 0.1421 & 0.0354 \\ 0.0354 & 0.0347 \end{bmatrix},$$
$$P^2_2 = \begin{bmatrix} 0.1594 & 0.0384 \\ 0.0384 & 0.0374 \end{bmatrix},$$
$$P^2_3 = \begin{bmatrix} 0.1535 & 0.0335 \\ 0.0335 & 0.0333 \end{bmatrix},$$
$$Q^2 = \begin{bmatrix} 0.1735 & 0.0588 \\ 0.0588 & 0.1692 \end{bmatrix},$$
$$R^2 = \begin{bmatrix} 0.0659 & 0.0376 \\ 0.0376 & 0.0296 \end{bmatrix},$$
$$Z^2 = \begin{bmatrix} 0.0309 & 0.0234 \\ 0.0234 & 0.0198 \end{bmatrix},$$
$$S^2 = \begin{bmatrix} 0.0150 & 0.0120 \\ 0.0120 & 0.0103 \end{bmatrix},$$

and the fuzzy controller gains

$$F^1_{11} = \begin{bmatrix} -1.2986 & -0.4902 \end{bmatrix},$$
$$F^1_{21} = \begin{bmatrix} -1.0960 & -0.4160 \end{bmatrix},$$
$$F^1_{22} = \begin{bmatrix} -0.5120 & -0.1977 \end{bmatrix},$$
$$F^1_{31} = \begin{bmatrix} -0.6021 & -0.2342 \end{bmatrix},$$
$$F^2_{11} = \begin{bmatrix} -1.8846 & -0.6779 \end{bmatrix},$$
$$F^2_{21} = \begin{bmatrix} -1.6270 & -0.5841 \end{bmatrix},$$
$$F^2_{22} = \begin{bmatrix} -0.6041 & -0.2241 \end{bmatrix},$$
$$F^2_{31} = \begin{bmatrix} -0.7598 & -0.2840 \end{bmatrix},$$

such that (58)–(61) hold. By Theorem 3, the closed-loop T–S fuzzy large-scale stochastic system is mean-square stable with generalized H_2 performance. Simulation results about states, inputs and outputs of the system (11) and the PDF controller (13) with $d_m = 1$ and $d_M = 7$ are shown in Figs. 4–6. From the simulation results, the proposed piecewise decentralized H_2 state-feedback fuzzy control scheme can solve the state-feedback control problem for T–S fuzzy large-scale stochastic systems effectively and systematically with the LMI-based method. This example clearly demonstrates the effectiveness of the results proposed in this paper.

Table 2. Number of inequalities by different methods.

Methods	Corollary 1	Theorem 2	Theorem 3
Number	36	62	70

Remark 7. In Example 1, the numbers of inequalities by different methods are shown in Table 2. From Tables 1 and 2 it is clear that, although the number of inequalities of Theorem 3 is largest, it has the most relaxed results.

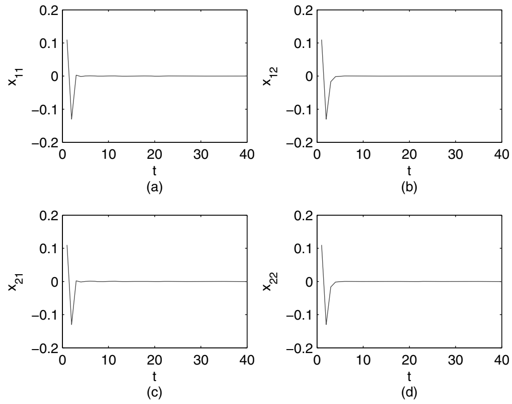

Fig. 4. Simulation of states.

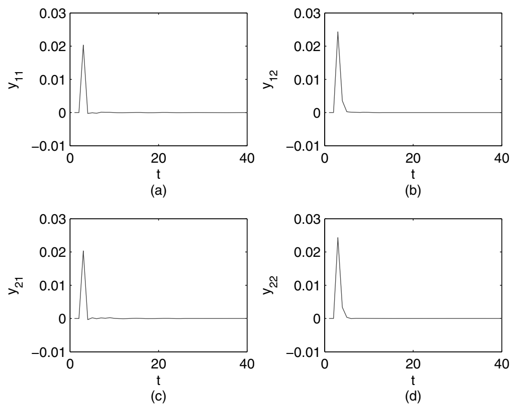

Fig. 5. Simulation of outputs.

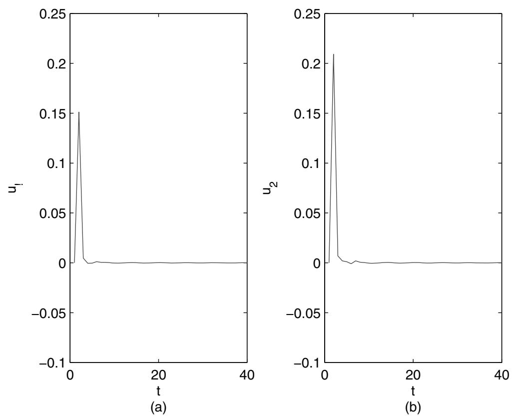

Fig. 6. Simulation of inputs.

5. Conclusion

In this paper, the problem of stochastic stability analysis and stabilization was investigated for discrete-time fuzzy large-scale stochastic systems with time-varying and infinite-distributed delays. By defining a novel delay-dependent piecewise Lyapunov–Krasovskii functional and by making use of novel techniques, two improved delay-dependent stability conditions were established in terms of linear matrix inequalities in which both the upper and lower bounds to delays are considered. The merit of the proposed conditions lies in their reduced conservatism, which is achieved by circumventing the utilization of some bounding inequalities for cross products between two vectors and by considering the interactions among the fuzzy subsystems in each subregion Ω_j^i. A decentralized generalized H_2 piecewise fuzzy controller was developed based on this DDPLKF for each subsystem. It is shown that the stability in the mean square for discrete-time fuzzy large-scale stochastic systems can be established if a DDPLKF can be constructed and a decentralized controller can be obtained by solving a set of LMIs. The effectiveness of the proposed approach is illustrated by a simulation example and some comparisons.

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