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FURTHER RESULTS ON ROBUST FUZZY DYNAMIC SYSTEMS WITH LMI $\mathcal{D}$-STABILITY CONSTRAINTS

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This paper examines the problem of designing a robust $\mathcal{H}_\infty$ fuzzy controller with $\mathcal{D}$-stability constraints for a class of nonlinear dynamic systems which is described by a Takagi–Sugeno (TS) fuzzy model. Fuzzy modelling is a multi-model approach in which simple sub-models are combined to determine the global behavior of the system. Based on a linear matrix inequality (LMI) approach, we develop a robust $\mathcal{H}_\infty$ fuzzy controller that guarantees (i) the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value, and (ii) the closed-loop poles of each local system to be within a specified stability region. Sufficient conditions for the controller are given in terms of LMIs. Finally, to show the effectiveness of the designed approach, an example is provided to illustrate the use of the proposed methodology.

Keywords: fuzzy controller, robust $\mathcal{H}_\infty$ control, LMI approach, $\mathcal{D}$-stability, Takagi–Sugeno fuzzy model.

1. Introduction

In the last few decades, nonlinear $\mathcal{H}_\infty$ theories have been extensively developed and well applied by many researchers (see Fu *et al.*, 1992; Isidori and Astolfi, 1992; van der Schaft, 1992; Ball *et al.*, 1993; 1994; Mansouri *et al.*, 2009; Rezac and Hurak, 2013). $\mathcal{H}_\infty$ control problems basically involve MIMO systems as well as disturbance and model error problems. The nonlinear $\mathcal{H}_\infty$ control problem can be stated as follows: Given a dynamic system with exogenous input noise and a measured output, find a controller such that the $\mathcal{L}_2$ gain of the mapping from the exogenous input noise to the regulated output is less than or equal to a prescribed value.

Currently, there are two commonly practical methods for solving solutions to nonlinear $\mathcal{H}_\infty$ control problems. The first one is based on the nonlinear version of the classical bounded real lemma (see Isidori and Astolfi, 1992; van der Schaft, 1992; Ball *et al.*, 1994). The other is based on dissipativity theory and the theory of differential games (see Hill and Moylan, 1980; Willems, 1972; Wonham, 1970; Basar and Olsder, 1982). Both methods show that the solution of the nonlinear $\mathcal{H}_\infty$ control problem is in fact related to the solvability of Hamilton–Jacobi inequalities (HJIs). To the best of our knowledge, there has been no easy computation technique for solving those inequalities yet.

Recently, many problems in $\mathcal{H}_\infty$ control theories have been extensively investigated (see Chen *et al.*, 2000; Chilali and Gahinet, 1996; Chilali *et al.*, 1999; Vesely *et al.*, 2011), with the desired controllers designed in terms of the solution to linear matrix inequalities (LMIs). So far, it has been proven that the LMI technique is one of the best alternatives for the basic analytical method and can be supported by efficient interior-point optimization (see Yakubovich, 1976a, 1976b; Boyd *et al.*, 1994; Gahinet *et al.*, 1995; Scherer *et al.*, 1997). A prominent advantage of the LMI approach is the feasibility to combine various design multi-objectives in a numerically tractable manner. However, most of the existing results are restricted to linear dynamic systems.

So far, there have been numerous research advances devoted to the design of an $\mathcal{H}_\infty$ fuzzy controller for a class of nonlinear systems which can be represented by a Takagi–Sugeno (TS) fuzzy model (see Yakubovich, 1967a; Han and Feng, 1998; Han *et al.*, 2000; Tanaka *et al.*, 1996; Assawinchaichote and Nguang, 2004a; 2004b; 2006; Assawinchaichote, 2012; Yeh *et al.*, 2012). Fuzzy system theory utilizes qualitative, linguistic information for a complex nonlinear system to construct a mathematical model for it. Recent studies (Zadeh, 1965; Tanaka and Sugeno, 1992; Tanaka and Sugeno, 1995;

Teixeira and Zak, 1999; Wang *et al*., 1996; Yoneyama *et al*., 2000; Zhang *et al*., 2001; Joh *et al*., 1998; Ma *et al*., 1998; Park *et al*., 2001; Bouarar *et al*., 2013) show that fuzzy submodels can be used to approximate global behaviors of a uncertain nonlinear system.

Since fuzzy sub-models in different state space regions are represented by local linear systems, the global model of the system is obtained by combining these linear models through nonlinear fuzzy membership functions. It is a fact that fuzzy modelling is a multi-model approach in which simple submodels are combined to determine the global system behavior while conventional modelling uses a single model to describe the global system behavior. Recent contributions (Chayaopas and Assawinchaichote, 2013; Assawinchaichote and Chayaopas, 2013) have considered an $\mathcal{H}_\infty$ fuzzy controller based on an LMI approach and a robust $\mathcal{H}_\infty$ fuzzy control design. However, these works did not address satisfactorily the system dynamic characteristics which might change on the transient response.

Although the robustness and/or the stability of the closed-loop system are basically the first issue needed to be considered, the system dynamic characteristic sometimes does not meet the desired objectives such as the rise time, the settling time, and transient oscillations in many applications or real physical systems due to poor transient responses. A satisfactory transient response can be obtained by enforcing the closed-loop pole to lie within a suitable region. The problem of assigning all poles of a system in a specified region is the so-called $\mathcal{D}$-stable pole placement problem. Recently, Han *et al*. (2000) have studied $\mathcal{H}_\infty$ controller design of fuzzy dynamic systems with pole placement constraints. However, their methods require the system to be in a state subspace for a period of time and also require switching controllers. Therefore, with this motivation, we examine the problem of designing a robust $\mathcal{H}_\infty$ fuzzy controller for a class of fuzzy dynamic systems. First, we approximate this class of uncertain nonlinear systems by a Takagi–Sugeno fuzzy model. Then, based on an LMI approach, we develop a technique for designing robust $\mathcal{H}_\infty$ fuzzy controllers such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value and the closed-loop system is $\mathcal{D}$-stable, i.e., we enforce eigenvalue clustering in a specified region. It is necessary to note that the requirement of the system to be in a state subspace for a period of time is not mandatory, and also our proposed robust $\mathcal{H}_\infty$ fuzzy controller is not a switching controller.

This paper is organized as follows. In Section 2, preliminaries and definitions are presented. In Section 3, based on an LMI approach, we develop a technique for designing robust $\mathcal{H}_\infty$ fuzzy controllers such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value and the closed-loop poles of each local system are stable within a pre-specified region for the system described in Section 2. The validity of this approach is demonstrated by an example from the literature in Section 4. Finally, conclusions are given in Section 5.

2. Preliminaries and definitions

In this paper, we first examine the following standard TS fuzzy system with parametric uncertainties:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\nu(t)) \Big[[A_i + \Delta A_i] x(t) \\ &\qquad + B_w w(t) + [B_i + \Delta B_i] u(t) \Big], \\ z(t) &= \sum_{i=1}^{r} \mu_i(\nu(t)) \Big[[C_i + \Delta C_i] x(t) \\ &\qquad + [D_i + \Delta D_i] u(t) \Big], \end{aligned} \tag{1}$$

where $x(0) = 0$, $\nu(t) = [\nu_1(t) \ \cdots \ \nu_\vartheta(t)]$ is the premise variable vector that may depend on states in many cases, $\mu_i(\nu(t))$ denotes the normalized time-varying fuzzy weighting functions for each rule (i.e., $\mu_i(\nu(t)) \geq 0$ and $\sum_{i=1}^{r} \mu_i(\nu(t)) = 1$), ϑ is the number of fuzzy sets, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^p$ is the disturbance which belongs to $\mathcal{L}_2[0, \infty)$, $z(t) \in \mathbb{R}^s$ is the controlled output, the matrices A_i, B_w, B_i, C_i, and D_i are of appropriate dimensions, and r is the number of IF-THEN rules. The matrices ΔA_i, ΔB_i, ΔC_i, and ΔD_i represent the system uncertainties and satisfy the following assumption.

Assumption 1.

$$\begin{aligned} \Delta A_i &= E_{1_i} F(x(t), t) H_{1_i}, \\ \Delta B_i &= E_{2_i} F(x(t), t) H_{2_i}, \\ \Delta C_i &= E_{3_i} F(x(t), t) H_{3_i}, \\ \Delta D_i &= E_{4_i} F(x(t), t) H_{4_i}, \end{aligned}$$

where E_{j_i} and H_{j_i}, $j = 1, \ldots, 4$ are known matrix functions which characterize the structure of the uncertainties. Furthermore,

$$\|F(x(t), t)\| \leq \rho \tag{2}$$

for some known positive constant ρ.

Throughout this paper, we assume that the fuzzy model satisfies the following assumption.

Assumption 2. The pairs (A_i, B_i) are locally controllable for every $i \in \{1, 2, \ldots, r\}$.

Next, let us recall the following definition.

Definition 1. Let γ be a given positive number. The system (1) is said to have an $\mathcal{L}_2$-*gain less than or equal to* γ if

$$\int_0^{T_f} z^T(t)z(t)\,\mathrm{d}t \le \gamma^2 \int_0^{T_f} w^T(t)w(t)\,\mathrm{d}t \tag{3}$$

for all $T_f \ge 0$, $x(0) = 0$, and $w(t) \in \mathcal{L}_2\,[0, T_f]$.

Note that, for the symmetric block matrices, we use the asterisk $(*)$ as a placeholder for a term that is induced by symmetry.

3. Main results

In this section, we first consider the problem of designing a robust $\mathcal{H}_\infty$ fuzzy controller based on an LMI approach so that the inequality (3) holds. Then, LMI-based sufficient conditions for each local system (1) to have all its closed-loop poles within a prescribed LMI region are presented. Finally, the problem of designing an $\mathcal{H}_\infty$ fuzzy controller with $\mathcal{D}$-stability constraints is examined.

3.1. Robust $\mathcal{H}_\infty$ fuzzy control design.

A robust $\mathcal{H}_\infty$ fuzzy state-feedback controller is readily established in the form

$$u(t) = \textstyle\sum_{j=1}^{r} \mu_j K_j x(t), \tag{4}$$

where K_j is the controller gain such that (3) holds. The state space form of the fuzzy system model (1) with the controller (4) is given by

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j \Big[&[(A_i + B_iK_j) \\ &+ (\Delta A_i + \Delta B_i K_j)]x(t) + B_w w(t)\Big].\end{aligned} \tag{5}$$

The following theorem provides sufficient conditions for the existence of a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller. These sufficient conditions can be derived by the Lyapunov approach.

Theorem 1. *Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, if there exist a matrix $P = P^T$ and matrices Y_j, $j = 1, 2, \dots, r$, satisfying the following linear matrix inequalities:*

$$P > 0, \tag{6}$$

$$\Xi_{ii} < 0, \quad i = 1, 2, \dots, r, \tag{7}$$

$$\Xi_{ij} + \Xi_{ji} < 0, \quad i < j \le r, \tag{8}$$

where

$$\Xi_{ij} = \begin{pmatrix} \Psi_{1_{ij}} & (*)^T & (*)^T & (*)^T \\ \Psi_{2_{ij}} & -\Gamma + \tilde{E}_i^T\tilde{E}_i & (*)^T & (*)^T \\ \Psi_{3_{ij}} & 0 & -I & (*)^T \\ \Psi_{4_{ij}} & 0 & 0 & -I \end{pmatrix}, \tag{9}$$

$$\begin{aligned}\Psi_{1_{ij}} &= A_iP + PA_i^T + B_iY_j + Y_j^TB_i^T, \\ \Psi_{2_{ij}} &= \tilde{B}_{w_i}^T + \tilde{E}_i^TC_iP + \tilde{E}_i^TD_iY_j, \\ \Psi_{3_{ij}} &= \tilde{C}_iP + \tilde{D}_iY_j, \\ \Psi_{4_{ij}} &= C_iP + D_iY_j,\end{aligned}$$

with

$$\begin{aligned}\tilde{B}_{w_i} &= \begin{bmatrix} E_{1_i} & E_{2_i} & B_w & 0 & 0 \end{bmatrix}, \\ \tilde{C}_i &= \begin{bmatrix} \rho H_{1_i}^T & \rho H_{3_i}^T & 0 & 0 \end{bmatrix}^T, \\ \tilde{D}_i &= \begin{bmatrix} 0 & 0 & \rho H_{2_i}^T & \rho H_{4_i}^T \end{bmatrix}^T, \\ \tilde{E}_i &= \begin{bmatrix} 0 & 0 & 0 & E_{3_i} & E_{4_i} \end{bmatrix}, \\ \Gamma &= \mathrm{diag}\{I, I, \gamma^2 I, I, I\},\end{aligned}$$

then the inequality (3) holds. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{r} \mu_j K_j x(t), \tag{10}$$

where

$$K_j = Y_jP^{-1}. \tag{11}$$

Proof. According to Assumption 1, the closed-loop fuzzy system (5) can be expressed as follows:

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j \Big(&[A_i + B_iK_j]x(t) \\ &+ \tilde{B}_{w_i}\tilde{w}(t)\Big),\end{aligned} \tag{12}$$

where

$$\tilde{B}_{w_i} = \begin{bmatrix} E_{1_i} & E_{2_i} & B_w & 0 & 0 \end{bmatrix},$$

and the disturbance $\tilde{w}(t)$ is

$$\tilde{w}(t) = \begin{bmatrix} F(x(t),t)H_{1_i}x(t) \\ F(x(t),t)H_{2_i}K_jx(t) \\ w(t) \\ F(x(t),t)H_{3_i}x(t) \\ F(x(t),t)H_{4_i}K_jx(t) \end{bmatrix}. \tag{13}$$

Consider the Lyapunov function

$$V(x(t)) = x^T(t)Qx(t),$$

where $Q = P^{-1}$. Differentiating $V(x(t))$ along the trajectories of the closed-loop system (12) yields

$$\begin{aligned}\dot{V}(x(t)) &= \dot{x}^T(t)Qx(t) + x^T(t)Q\dot{x}(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j \Big(x^T(t)(A_i + B_iK_j)^TQx(t) \\ &\quad + x^T(t)Q(A_i + B_iK_j)x(t) \\ &\quad + \tilde{w}^T(t)\tilde{B}_{w_i}^TQx(t) + x^T(t)Q\tilde{B}_{w_i}\tilde{w}(t)\Big).\end{aligned} \tag{14}$$

Adding and subtracting

$$-z^T(t)z(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\Gamma\tilde{w}(t)]$$

to and from (14), combined with the fact that $\|F(x(t),t)\|\le\rho$, we get

$$\begin{aligned}&\dot{V}(x(t))\\&=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\times\Bigg(\begin{bmatrix} x^T(t) & \tilde{w}^T(t)\end{bmatrix}\\&\times\begin{pmatrix}\begin{pmatrix}(A_i+B_iK_j)^TQ\\+Q(A_i+B_iK_j)\\+(\tilde{C}_i+\tilde{D}_iK_j)^T\\\times(\tilde{C}_m+\tilde{D}_mK_n)\\+(C_i+D_iK_j)^T\times\\(C_m+D_mK_n)\end{pmatrix} & (*)^T\\ \begin{pmatrix}\tilde{B}_{w_i}^TQ+\\\tilde{E}_i^T(C_i+D_iK_j)\end{pmatrix} & -\Gamma+\tilde{E}_i^T\tilde{E}_i\end{pmatrix}\\&\times\begin{bmatrix}x(t)\\\tilde{w}(t)\end{bmatrix}\Bigg)-z^T(t)z(t)+\gamma^2w^T(t)w(t),\end{aligned}\tag{15}$$

where

$$\begin{aligned}\tilde{C}_i&=\begin{bmatrix}\rho H_{1_i}^T & \rho H_{3_i}^T & 0 & 0\end{bmatrix}^T,\\\tilde{D}_i&=\begin{bmatrix}0 & 0 & \rho H_{2_i}^T & \rho H_{4_i}^T\end{bmatrix}^T,\\\tilde{E}_i&=\begin{bmatrix}0 & 0 & 0 & E_{3_i} & E_{4_i}\end{bmatrix},\\\Gamma&=\mathrm{diag}\{I,I,\gamma^2I,I,I\}.\end{aligned}$$

Note that

$$\begin{aligned}&z^T(t)z(t)\\&=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big(x^T(t)\,[C_i+E_{3_i}F(x(t),t)H_{3_i}\\&\quad+D_iK_j+E_{4_i}F(x(t),t)H_{4_i}K_j]^T\\&\quad\times[C_i+E_{3_i}F(x(t),t)H_{3_i}+D_iK_j\\&\quad+E_{4_i}F(x(t),t)H_{4_i}K_j]\,x(t)\Big)\\&=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\begin{bmatrix}x(t)\\\tilde{w}(t)\end{bmatrix}^T\\&\times\begin{bmatrix}\begin{pmatrix}(C_i+D_iK_j)^T\times\\(C_i+D_iK_j)\end{pmatrix} & (*)^T\\\tilde{E}_i^T(C_i+D_iK_j) & \tilde{E}_i^T\tilde{E}_i\end{bmatrix}\begin{bmatrix}x(t)\\\tilde{w}(t)\end{bmatrix}\end{aligned}$$

and

$$\begin{aligned}&\tilde{w}^T(t)\Gamma\tilde{w}(t)\\&=\begin{bmatrix}F(x(t),t)H_{1_i}x(t)\\F(x(t),t)H_{2_i}K_jx(t)\\w(t)\\F(x(t),t)H_{3_i}x(t)\\F(x(t),t)H_{4_i}K_jx(t)\end{bmatrix}^T\Gamma\begin{bmatrix}F(x(t),t)H_{1_i}x(t)\\F(x(t),t)H_{2_i}K_jx(t)\\w(t)\\F(x(t),t)H_{3_i}x(t)\\F(x(t),t)H_{4_i}K_jx(t)\end{bmatrix}\\&\le\gamma^2w^T(t)w(t)+\rho^2x^T\{H_{1_i}^TH_{1_i}+K_j^TH_{2_i}^TH_{2_i}K_j\\&\quad+H_{3_i}^TH_{3_i}+K_j^TH_{4_i}^TH_{4_i}K_j\}x(t).\end{aligned}$$

Note that (9) can be rewritten as follows:

$$\begin{pmatrix}\begin{matrix}(A_iP+B_iY_j)^T\\+(A_iP+B_iY_j)\end{matrix} & (*)^T & (*)^T & (*)^T\\\begin{pmatrix}\tilde{B}_{w_i}^T\\+\tilde{E}_i^TC_iP\\+\tilde{E}_i^TD_iY_j\end{pmatrix} & -\Gamma+\tilde{E}_i^T\tilde{E}_i & (*)^T & (*)^T\\\tilde{C}_iP+\tilde{D}_iY_j & 0 & -I & (*)^T\\C_iP+D_iY_j & 0 & 0 & -I\end{pmatrix}<0.$$

Thus, pre- and post-multiplying (7) and (8) by

$$\begin{pmatrix}Q & 0 & 0 & 0\\0 & I & 0 & 0\\0 & 0 & I & 0\\0 & 0 & 0 & I\end{pmatrix}$$

yields

$$\begin{pmatrix}\begin{matrix}(A_i+B_iK_j)^TQ\\+Q(A_i+B_iK_j)\end{matrix} & (*)^T & (*)^T & (*)^T\\\begin{pmatrix}\tilde{B}_{w_i}^TQ\\+\tilde{E}_i^TC_i\\+\tilde{E}_i^TD_iK_j\end{pmatrix} & -\Gamma+\tilde{E}_i^T\tilde{E}_i & (*)^T & (*)^T\\\tilde{C}_i+\tilde{D}_iK_j & 0 & -I & (*)^T\\C_i+D_iK_j & 0 & 0 & -I\end{pmatrix}<0,\tag{16}$$

$i,j=1,2,\dots,r$. Applying the Schur complement to (16) and rearranging them, we have

$$\begin{pmatrix}\begin{pmatrix}(A_i+B_iK_j)^TQ\\+Q(A_i+B_iK_j)\\+(\tilde{C}_i+\tilde{D}_iK_j)^T\times\\(\tilde{C}_m+\tilde{D}_mK_n)\\+(C_i+D_iK_j)^T\times\\(C_m+D_mK_n)\end{pmatrix} & (*)^T\\\begin{pmatrix}\tilde{B}_{w_i}^TQ+\\\tilde{E}_i^T(C_i+D_iK_j)\end{pmatrix} & -\Gamma+\tilde{E}_i^T\tilde{E}_i\end{pmatrix}<0,$$

$i,j,m,n=1,2,\dots,r$. Using (17) and the fact that

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_nM_{ij}^TN_{mn}\\&\quad\le\frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j[M_{ij}^TM_{ij}+N_{ij}N_{ij}^T],\end{aligned}\tag{17}$$

$\mu_i\ge0$ and $\sum_{i=1}^{r}\mu_i=1$, (15) becomes

$$\dot{V}(x(t))\le-z^T(t)z(t)+\gamma^2w^T(t)w(t).\tag{18}$$

Integrating both the sides of (18) yields

$$\int_0^{T_f} \dot{V}(x(t))\,\mathrm{d}t \leq \int_0^{T_f} \Big[- z^T(t)z(t) + \gamma^2 w^T(t)w(t)\Big]\,\mathrm{d}t,$$

which is

$$V(x(T_f)) - V(x(0)) \leq \int_0^{T_f} \Big[- z^T(t)z(t) + \gamma^2 w^T(t)w(t)\Big]\,\mathrm{d}t.$$

Using the fact that $x(0) = 0$ and $V(x(T_f)) \geq 0$ for all $T_f \neq 0$, we get

$$\int_0^{T_f} z^T(t)z(t)\,\mathrm{d}t \leq \gamma^2 \int_0^{T_f} w^T(t)w(t)\,\mathrm{d}t$$

Hence, the inequality (3) holds. ∎

3.2. $\mathcal{D}$-stability constraints. To begin this subsection, we recall the following definition.

Definition 2. (*Chilali and Gahinet, 1996*) A subset $\mathcal{D}$ of the complex plane is called an *LMI region* if there exist a symmetric matrix $L = [L_{kl}] = [L_{lk}] \in \mathbb{R}^{g\times g}$ and a matrix $M = [M_{kl}] \in \mathbb{R}^{g\times g}$ such that

$$\mathcal{D} = \{z = x + jy \in \mathcal{C} : f_{\mathcal{D}}(z) < 0\}, \tag{19}$$

with the characteristic function

$$\begin{aligned} f_{\mathcal{D}}(z) &= L + Mz + M^T\bar{z} \\ &= [L_{kl} + M_{kl}z + M_{lk}\bar{z}]_{1\leq k,l\leq g}. \end{aligned} \tag{20}$$

The following lemma will be needed to derive the main results in this subsection.

Lemma 1. (Chilali and Gahinet, 1996) *Given a dynamic system $\dot{x}(t) = Ax(t)$, for an LMI region, a matrix $A \in \mathbb{R}^{n\times n}$ is $\mathcal{D}$-stable in an LMI region, i.e., $\Lambda(I, A) \subset \mathcal{D}$ if there exists a matrix $P \in \mathbb{R}^{n\times n}$ such that*

$$\begin{aligned} L\otimes P + M\otimes(AP) + M^T\otimes(AP)^T &= \big[L_{kl}P + M_{kl}AP + M_{lk}PA^T\big]_{1\leq k,l\leq n} < 0, \\ P &> 0, \end{aligned}$$

where $\Lambda(I, A)$ is the set of generalized eigenvalues of the (I, A) pair, i.e., $\det(sI - A) = 0$, and $\otimes$ denotes the Kronecker product of the matrices.

Using Lemma 1, we have the following result.

Theorem 2. *Given any LMI region, if there exist a matrix P_D and matrices Y_j for $j = 1, 2, \ldots, r$, satisfying the following linear matrix constraints:*

$$\Phi_{ii} < 0, \qquad i = 1, 2, \ldots, r, \tag{21}$$

$$\Phi_{ij} + \Phi_{ji} < 0, \qquad i < j \leq r, \tag{22}$$

where

$$\begin{aligned} \Phi_{ij} = {} & L\otimes P_D + M\otimes A_iP_D + M\otimes B_iY_j \\ & + M^T\otimes P_DA_i^T + M^T\otimes {Y_j}^T{B_i}^T, \end{aligned} \tag{23}$$

then the closed-loop poles of each local system of (5) are $\mathcal{D}$-stable in the given LMI region. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{r} \mu_j K_j x(t), \tag{24}$$

where $K_j = Y_jP_D^{-1}$.

Proof. Using Assumptions 1 and 2, the closed-loop fuzzy system (5) can be expressed as follows:

$$\begin{aligned} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j \Big(& [A_i + B_iK_j]x(t) \\ & + \tilde{B}_w\tilde{w}(t)\Big) \end{aligned} \tag{25}$$

where

$$\tilde{B}_{w_i} = \begin{bmatrix} E_{1_i} & E_{2_i} & B_w & 0 & 0 \end{bmatrix},$$

and the disturbance is

$$\tilde{w}(t) = \begin{bmatrix} F(x(t),t)H_{1_i}x(t) \\ F(x(t),t)H_{2_i}K_jx(t) \\ w(t) \\ F(x(t),t)H_{3_i}x(t) \\ F(x(t),t)H_{4_i}K_jx(t) \end{bmatrix}. \tag{26}$$

According to Lemma 1, the system (25) is $\mathcal{D}$-stable if there exists a Q_D such that

$$\begin{aligned} F_{\mathcal{D}} \triangleq {} & M_{kl}\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j(A_i + B_iK_j)\right]Q_D \\ & + M_{lk}Q_D\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j(A_i + B_iK_j)^T\right] \\ & + L_{kl}Q_D < 0. \end{aligned} \tag{27}$$

Now, we have to show that there exists a P_D such that $F_{\mathcal{D}} < 0$. Letting $Q_D = P_D^{-1}$ and substituting it into (27), we get

$$\begin{aligned} F_{\mathcal{D}} = {} & L_{kl}P_D^{-1} \\ & + M_{kl}\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j(A_i + B_iK_j)\right]P_D^{-1} \\ & + M_{lk}P_D^{-1}\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j(A_i + B_iK_j)^T\right]. \end{aligned} \tag{28}$$

Pre-and post-multiplying both the sides of (28) by P_D, we have

$$\begin{aligned}&P_D F_{\mathcal{D}} P_D\\&= L_{kl} P_D\\&\quad + M_{kl}\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j P_D(A_i + B_iK_j)\right]\\&\quad + M_{lk}\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j (A_i + B_iK_j)^T P_D\right].\end{aligned}$$

Using (21),(22) and the fact that $\mu_i \geq 0$ and $\sum_{i=1}^{r}\mu_i = 1$, we deduce that there exists $F_{\mathcal{D}} < 0$. Hence, we show that the closed-loop poles of each local system of (5) are $\mathcal{D}$-stable. ∎

3.3. $\mathcal{H}_\infty$ fuzzy controller with $\mathcal{D}$-stability constraints. In this section, we consider a multi-objective robust $\mathcal{H}_\infty$ fuzzy controller such that the closed-loop poles of each local system of (5) are $\mathcal{D}$-stable in an LMI region and the inequality (3) is satisfied. In order to obtain solutions, we seek a common P, i.e., by enforcing $P = P_D$. The last result in this paper is given by the following theorem.

Theorem 3. *Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, if there exist a matrix $P = P^T$, matrices Y_j, $j = 1, 2, \ldots, r$, a symmetric matrix L and M satisfying the following linear matrix inequalities:*

$$\begin{aligned}P &> 0,\\ \Phi_{ii} &< 0, \quad i = 1,2,\ldots,r,\\ \Phi_{ij} + \Phi_{ji} &< 0, \quad i < j \leq r,\\ \Xi_{ii} &< 0, \quad i = 1,2,\ldots,r,\\ \Xi_{ij} + \Xi_{ji} &< 0, \quad i < j \leq r,\end{aligned}$$

where

$$\begin{aligned}\Phi_{ij} &= L \otimes P + M \otimes A_iP + M \otimes B_iY_j\\ &\quad + M^T \otimes PA_i^T + M^T \otimes Y_j{}^T B_i{}^T,\end{aligned}$$

$$\Xi_{ij} = \begin{pmatrix}\Psi_{1_{ij}} & (*)^T & (*)^T & (*)^T\\ \Psi_{2_{ij}} & -\Gamma + \tilde{E}_i^T\tilde{E}_i & (*)^T & (*)^T\\ \Psi_{3_{ij}} & 0 & -I & (*)^T\\ \Psi_{4_{ij}} & 0 & 0 & -I\end{pmatrix},$$

$$\begin{aligned}\Psi_{1_{ij}} &= A_iP + PA_i^T + B_iY_j + Y_j^TB_i^T,\\ \Psi_{2_{ij}} &= \tilde{B}_{w_i}^T + \tilde{E}_i^TC_iP + \tilde{E}_i^TD_iY_j,\\ \Psi_{3_{ij}} &= \tilde{C}_iP + \tilde{D}_iY_j,\\ \Psi_{4_{ij}} &= C_iP + D_iY_j,\end{aligned}$$

with

$$\begin{aligned}\tilde{B}_{w_i} &= \begin{bmatrix} E_{1_i} & E_{2_i} & B_w & 0 & 0\end{bmatrix},\\ \tilde{C}_i &= \begin{bmatrix}\rho H_{1_i}^T & \rho H_{3_i}^T & 0 & 0\end{bmatrix}^T,\\ \tilde{D}_i &= \begin{bmatrix}0 & 0 & \rho H_{2_i}^T & \rho H_{4_i}^T\end{bmatrix}^T,\\ \tilde{E}_i &= \begin{bmatrix}0 & 0 & 0 & E_{3_i} & E_{4_i}\end{bmatrix},\\ \Gamma &= \mathrm{diag}\{I, I, \gamma^2 I, I, I\},\end{aligned}$$

the inequality (3) holds and the closed-loop poles of each local system of (5) are $\mathcal{D}$-stable in the given LMI region. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{r}\mu_jK_jx(t),$$

where

$$K_j = Y_jP^{-1}.$$

Proof. The desired result can be obtained by using Theorems 1 and 2, together with enforcing $P = P_D$. ∎

4. Illustrative example

Consider a tunnel diode circuit shown in Fig. 1, where the tunnel diode is characterized by (Assawinchaichote and Nguang, 2006)

$$i_D(t) = -0.2v_D(t) - 0.01v_D^3(t).$$

Let $x_1(t) = v_C(t)$ be the capacitor voltage and $x_2(t) =$ $i_L(t)$ be the inductor current. Then, the circuit shown in Figure 1 can be modelled by the following state equations:

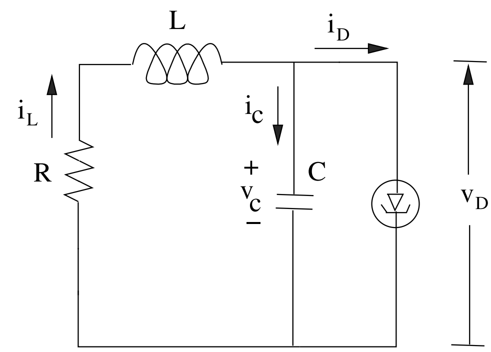

Fig. 1. Tunnel diode circuit (Assawinchaichote and Nguang, 2006).

$$\begin{aligned}C\dot{x}_1(t) &= 0.2x_1(t) + 0.01x_1^3(t) + x_2(t)\\ &\quad + 0.01w_1(t),\\ L\dot{x}_2(t) &= -x_1(t) - Rx_2(t) + u(t)\\ &\quad + 0.1w_2(t),\\ z(t) &= \begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix},\end{aligned} \tag{29}$$

where $u(t)$ is the control input, $w_1(t)$ and $w_2(t)$ are the process disturbances which may represent unmodelled dynamics, $z(t)$ is the controlled output, $x(t) = [x_1^T(t)\ \ x_2^T(t)]^T$ and $w(t) = [w_1^T(t)\ \ w_2^T(t)]^T$. Note that the variables $x_1(t)$ and $x_2(t)$ are treated as the deviation variables (variables deviate from its desired trajectories). The parameters in the circuit are given by $C = 100$ mF, $L = 1000$ mH and $R = 1 \pm 0.3\%\ \Omega$. With these, (29) can be rewritten as

$$\begin{aligned}
\dot{x}_1(t) &= 2x_1(t) + (0.1x_1^2(t)) \cdot x_1(t) + 10x_2(t) \\
&\quad + 0.1w_1(t), \\
\dot{x}_2(t) &= -x_1(t) - (1 \pm \Delta R)x_2(t) + u(t) \\
&\quad + 0.1w_2(t), \\
z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},
\end{aligned} \tag{30}$$

For simplicity, we will use as few rules as possible. Assuming that $|x_1(t)| \le 3$, the nonlinear network system (30) can be approximated by the following TS fuzzy model:

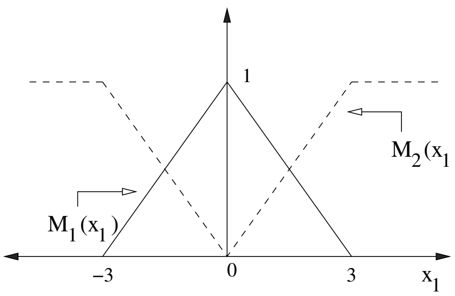

Fig. 2. Membership functions for the two fuzzy sets considered (Assawinchaichote and Nguang, 2006).

Plant Rule 1: IF $x_1(t)$ is $M_1(x_1(t))$ THEN

$$\begin{aligned}
\dot{x}(t) &= [A_1 + \Delta A_1]x(t) + B_w w(t) + B_1 u(t), \\
z(t) &= C_1 x(t).
\end{aligned}$$

Plant Rule 2: IF $x_1(t)$ is $M_2(x_1(t))$ THEN

$$\begin{aligned}
\dot{x}(t) &= [A_2 + \Delta A_2]x(t) + B_w w(t) + B_2 u(t), \\
z(t) &= C_2 x(t),
\end{aligned}$$

where $x(0) = 0$, $x(t) = [x_1^T(t)\ \ x_2^T(t)]^T$, $w(t) = [w_1^T(t)\ \ w_2^T(t)]^T$,

$$A_1 = \begin{bmatrix} 2 & 10 \\ -1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2.9 & 10 \\ -1 & -1 \end{bmatrix},$$

$$B_w = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_1 = E_{1_1} F(x(t), t) H_{1_1},$$

$$\Delta A_2 = E_{1_2} F(x(t), t) H_{1_2}.$$

Now, by assuming that, in (2), $\|F(x(t), t)\| \le \rho = 1$ and since the values of R are uncertain but bounded within 30% of their nominal values given in (29), we have

$$E_{1_1} = E_{1_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$H_{1_1} = H_{1_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

Robust $\mathcal{H}_\infty$ fuzzy controller design with $\mathcal{D}$-stability constraints. Let us place the closed-loop poles of each local system within an LMI disk region with center $q = -20$ and radius $r = 19$.

Note that the LMI disk region has the following characteristic function:

$$f_{\mathcal{D}}(z) = \begin{pmatrix} -r & q + z \\ q + \bar{z} & -r \end{pmatrix},$$

and

$$L = \begin{bmatrix} -r & q \\ q & -r \end{bmatrix}, \qquad M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Using Theorem 3 with $\gamma = 1$, we obtain

$$P = \begin{bmatrix} 0.5602 & -0.4132 \\ -0.4132 & 0.6602 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -9.2411 & -8.0988 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} -8.6991 & -8.0365 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -47.4436 & -41.9590 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -45.5172 & -40.6590 \end{bmatrix}.$$

The resulting fuzzy controller is

$$u(t) = \sum_{j=1}^{2} \mu_j K_j x(t), \tag{31}$$

where

$$\mu_1 = M_1(x_1(t)) \ \text{ and } \ \mu_2 = M_2(x_1(t)).$$

The proposed approach yields a robust $\mathcal{H}_\infty$ fuzzy controller which guarantees that (i) the inequality (3) holds and (ii) the closed-loop poles of each local system are within the given LMI stability region. The responses of the state variables $x_1(t)$ and $x_2(t)$ are shown in Fig. 3 while the disturbance input signal, $w(t)$, which was used during simulation is given in Fig. 4. It is necessary to note that the disturbance cannot always be modelled as white noise, while measurement noise can be quite well described by a random process. The ratio of the regulated output energy to the disturbance input noise energy obtained by using the $\mathcal{H}_\infty$ fuzzy controller (31) is depicted in Fig. 5. After 2 seconds, the ratio of the regulated output energy to the disturbance input noise energy tends to a constant value, which is about 0.145. Accordingly, $\gamma = \sqrt{0.145} = 0.381$, which is less than the prescribed values 1.

Finally, Table 1 shows a comparison of the location of closed-loop poles of each local system of the proposed method and the previous works. It is shown that the closed-loop poles of the proposed method are only located within the pre-specified region, but this is not valid for the other approaches. However, note that the proposed algorithm turns out to be efficient to apply for low-order problems; the computational time might not be suitable for high-order problems since the convergence time depends on the 'size' of the feasible solution set. In addition, due to the increasing size of LMI results produced using the proposed algorithm, the feasibility issue might jeopardize the existence of a solution.

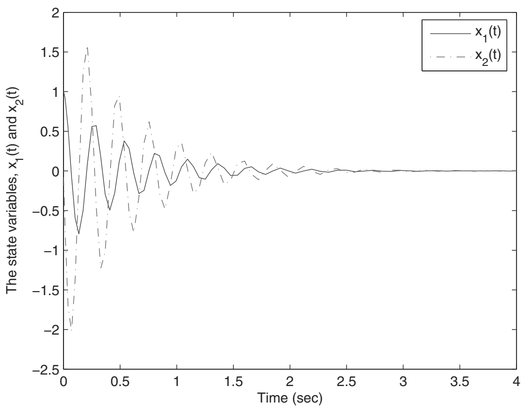

Fig. 3. State variables, $x_1(t)$ and $x_2(t)$.

Table 1. Closed-loop poles of each local system.

Method	Plant Rule 1	Plant Rule 2
Proposed theorem	-15.9088 -25.0502	-13.7934 -24.9656
Chayaopas *et al.* (2013)	-0.1201 -13.6601	-0.8964 -18.1151
Assawinchaichote *et al.* (2013)	-20.9088 -39.1702	-18.7934 -34.3356
Disk region with center $q = -20$ and radius $r = 19$		

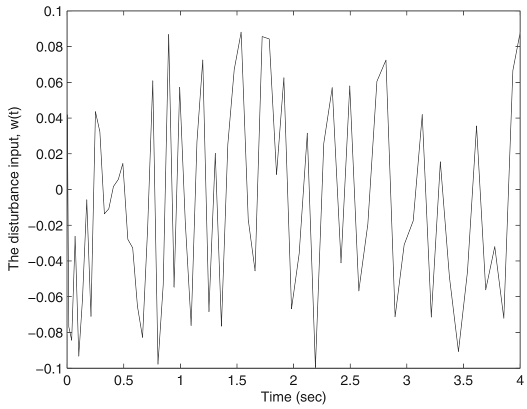

Fig. 4. Disturbance input noise, $w(t)$, used during simulation.

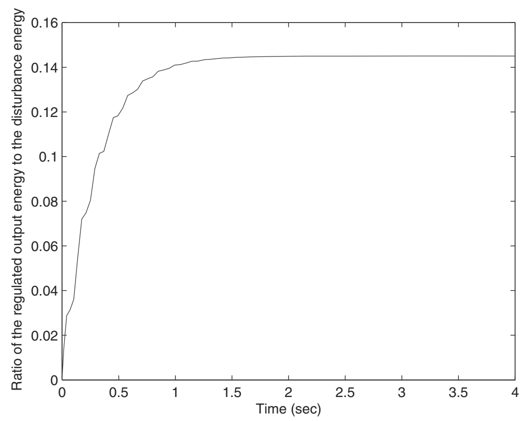

Fig. 5. Ratio of the regulated output energy to the disturbance noise energy, $\left(\int_0^{T_f} z^T(t)z(t)\,\mathrm{d}t / \int_0^{T_f} w^T(t)w(t)\,\mathrm{d}t\right)$.

5. Conclusion

This paper has presented a robust $\mathcal{H}_\infty$ fuzzy controller design procedure for a class of fuzzy dynamic systems with $\mathcal{D}$-stability constraints described by a TS fuzzy model. Based on an LMI approach, we developed a technique for designing a robust $\mathcal{H}_\infty$ fuzzy controller which guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value and the poles of each local system to be within a pre-specified region such that a satisfactory transient response can be obtained by enforcing the closed-loop pole to lie within a suitable region. Finally, a numerical example was given to show the effectiveness of the synthesis procedure developed in this paper. However, since in the designed approach the convergence time depends on the 'size' of the feasible solution set, the proposed method might not be suitable for large-order control problems. Therefore, the designing of a high performance multi-objectives controller can be considered in our possible future research work.

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