Int. J. Appl. Math. Comput. Sci., 2018, Vol. 28, No. 2, 375–386
DOI: 10.2478/amcs-2018-0028

ANALYSIS OF AN N–POLICY GI/M/1 QUEUE IN A MULTI–PHASE SERVICE ENVIRONMENT WITH DISASTERS

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This paper investigates an N-policy GI/M/1 queue in a multi-phase service environment with disasters, where the system tends to suffer from disastrous failures while it is in operative service environments, making all present customers leave the system simultaneously and the server stop working completely. As soon as the number of customers in the queue reaches a threshold value, the server resumes its service and moves to the appropriate operative service environment immediately with some probability. We derive the stationary queue length distribution, which is then used for the computation of the Laplace–Stieltjes transform of the sojourn time of an arbitrary customer and the server's working time in a cycle. In addition, some numerical examples are provided to illustrate the impact of several model parameters on the performance measures.

Keywords: N-policy, GI/M/1 queue, multi-phase service environment, disasters, sojourn time.

1. Introduction

Recently, queueing systems with disasters/catastrophes have often been encountered in practice, and they have proved widely useful in analyzing communication systems and computer networks. In the classical queueing models with disasters, the occurrence of disasters made the server inoperative and forced all customers to abandon the system simultaneously.

During the past two decades, the topic of disasters has been studied extensively by many researchers; the interested readers are referred to the works of Towsley and Tripathi (1991), Artalejo and Gómez-Corral (1998), Krishna Kumar and Arivudainambi (2000), Economou and Fakinos (2003), Gani and Swift (2007), Yechiali (2007), and others. Recently, Dimou and Economou (2013) studied a single server queue with catastrophes and geometric abandonments, in which customers may become impatient and leave the system according to a geometric distribution when the system resides in repair period. Mytalas and Zazanis (2015) considered a queueing system with batch Poisson arrivals subject to disasters and repairs under a multiple adapted vacation policy, and gave an explicit analysis of the queue length distribution and important performance measures. Kim and Lee (2014) discussed an M/G/1 queue with disasters and working breakdown. Jiang and Liu (2017) investigated a GI/M/1 queue in a multi-phase service environment with disasters by a matrix analytic approach.

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Sudhesh *et al.* (2016) analyzed an N-policy M/M/1 queue with disastrous breakdown, and derived the closed-form expressions of system size probabilities in transient state and in steady state. For the discrete-time queueing models, there also exist a large volume of references (see, e.g., Atencia and Moreno, 2004; Park *et al.*, 2009; 2010; Atencia, 2014; 2016). There are also a number of recent publications closely related to queueing systems with disasters. However, in these papers, the authors studied the queueing systems from an economic viewpoint (see, e.g., Boudali and Economou, 2012; 2013; Economou and Manou, 2013).

The N-policy was first considered by Yadin and Naor (1963), who analyzed an N-policy with a removable service station. Since the introduction of the N-policy, there has been considerable attention on this topic. For more literature on N-policy queues, we may refer interested readers to the works of Moreno (2007), Ke (2003), Ke and Wang (2002), Zhang and Tian (2004), Lim *et al.* (2013), Lee and Yang (2013), and others.

In fact, the queueing model in consideration is practical and reasonable, and it can be applied to a power saving scheme in wireless sensor networks (WSNs) under unreliable network connections. Specifically, WSNs are allocated in various areas, especially for environmental sensing and forest fire monitoring, etc. However, they are mainly allocated in some extreme or hostile environments. Facing the not-easily-accessible environments, it may be difficult to replace their batteries. Therefore, a threshold control policy is a relative important power saving scheme in prolonging the usage period of WSNs. Compared with the T-policy, the N-policy may also hold more effective for a power saving scheme because it can reduce the setup power consumption required to switch between a busy mode and an idle mode, and we can find an optimal N-policy to minimize the power consumption. (For more details, see the work of Lee and Yang (2013).)

Hence, motivated by the applications of the queueing model, in this paper, we further develop the excellent work presented by Jiang and Liu (2017), who studied a GI/M/1 queue in a multi-phase service environment with disasters. It is different from Jiang and Liu (2017) in that, in the present paper, the server resumes its service as soon as the number of customers in the queue reaches a threshold value N, rather than experiencing an exponential repair time. Meanwhile, the system may suffer from disasters while the server is idle. Following the idea presented by Lim *et al.* (2013), who used a trial solution approach in dealing with the N-policy queue, we derive the stationary queue length distribution at arrival epochs. Furthermore, we give the stationary queue length distribution at arbitrary epochs by using the method of the semi-Markov process. Moreover, we present an elaborate analysis on the sojourn time of an arbitrary customer and the server's working time in a cycle.

The rest of this paper is organized as follows. In Section 2, we provide the model formulation. In Section 3, by investigating an embedded Markov chain and using a trial solution approach, we give the stationary queue length distribution at arrival epochs. By means of the semi-Markov process, we further derive the stationary queue length distribution at arbitrary epochs in Section 4. Sections 5 and 6 are devoted to deriving the Laplace–Stieltjes transform (LST) of the sojourn time of an arbitrary customers and the server's working time in a cycle. Some numerical examples are presented in Section 7. We conclude the paper and give some future research directions in Section 8.

2. Preliminaries

In this paper, we consider an N-policy GI/M/1 queue in a multi-phase service environment with disasters. The queuing model is described in detail below:

- Interarrival times $\{A_k, k \geq 1\}$ of customers are independent and identically distributed (iid) with a general distribution function, denoted by $A(\nu)$ with mean $1/\lambda$ and an LST, denoted by $A^*(s)$.
- Under operative service environment i, the service times S_i are exponentially distributed with parameter μ_i, and LST $B_i^*(s) = \mu_i/(s+\mu_i), i = 1, 2, \ldots, n$.
- D_i is the duration of times that the system resides in operative service environment i, i.e., the interarrival times of disasters in service environment i also follow an exponential distribution with parameter $\eta_i > 0, i = 1, 2, \ldots, n$.
- Whenever a disaster occurs while the system is in operative service environment (no matter the sever is idle or busy), all customers are forced to be removed from the system simultaneously, and the server is rendered inoperative. As soon as the number of customers in the queue reaches the threshold N, the system resumes its service and jumps to the operative service environment i with probability q_i, where $\sum_{i=1}^{n} q_i = 1$.

Actually, in the present queueing model, there are no direct jumps among the operative service environments. Once a disaster occurs, the system must undergo a period depending on the total interarrival times of N customers, and then moves to the operative service environment i with probability $q_i, i = 1, 2, \ldots, n$.

3. Stationary queue length distribution at arrival epochs

In this section, we construct an embedded Markov chain to obtain the stationary queue length distribution at arrival epochs. Whenever a disaster occurs while the system is in operative service environment, all customers are forced to leave the system, which means that the system will be empty and the number of customers never goes to infinity. Therefore, the system can be analyzed in steady state.

We suppose that τ_k is the k-th customer arriving instant with $\tau_0 = 0$. Let $L(t)$ denote the number of customers in the system at time t, and $L_k = L(\tau_k - 0)$ denote the number of customers seen by the k-th arrival instant. Define

$$J_k = \begin{cases} 0 & \text{if the } k\text{-th arrival occurs while the server is} \\ & \text{inoperative,} \\ i & \text{if the } k\text{-th arrival occurs during} \\ & \text{operative service environment } i, \\ & i = 1, 2, \dots, n, \end{cases}$$

Then the system can be described by the process $\{(L_k, J_k), k \geq 1\}$, which is an embedded Markov chain with the state space

$$\begin{aligned} \Omega = &\{(m, 0),\, m = 0, 1, \dots, N-1\} \\ &\cup \{(h, i),\, h \geq 0,\, i = 1, 2, \dots, n\}. \end{aligned}$$

Next, in order to form the transition matrix of $\{(L_k, J_k), k \geq 1\}$, we give the form of transition probabilities as follows:

$$\begin{aligned} P_{(h,l),(m,j)} = &P(L_{k+1} = m, J_{k+1} = j | L_k = h, J_k = l), \\ &0 \leq m \leq h+1, \quad l, j = 0, 1, 2, \dots, n. \end{aligned}$$

First, consider transitions when the server is in operative service environment $i, i = 1, 2, \dots, n$.

Case 1: Consider the transition from (h, i) to (m, i). The transition occurs if the next customer arrives earlier than a disaster arrives and $h+1-m$ customers are served during the arrival of the next customer. Then we have

$$\begin{aligned} P_{(h,i),(m,i)} &= \int_0^\infty e^{-\eta_i t} \frac{(\mu_i t)^{h+1-m}}{(h+1-m)!} e^{-\mu_i t} \,\mathrm{d}A(t) \\ &= b_{i,h+1-m}, \quad 1 \leq m \leq h+1. \end{aligned}$$

Case 2: Consider the transition from (h, i) to $(0, i)$. The transition occurs if the next customer arrives earlier than a disaster arrives and all present customers are served before the arrival of the next customer. Then we have

$$\begin{aligned} P_{(h,i),(0,i)} &= \int_0^\infty e^{-\eta_i t} \int_0^t \frac{\mu_i (\mu_i x)^h}{h!} e^{-\mu_i x} \,\mathrm{d}x \,\mathrm{d}A(t) \\ &= \int_0^\infty e^{-\eta_i t} \,\mathrm{d}A(t) - \sum_{m=0}^{h} b_{i,m} \\ &= c_{i,h+1}, \quad h \geq 0. \end{aligned}$$

Case 3: Consider the transition from (h, i) to $(0, 0)$. The transition occurs if the next customer arrives later than a disaster arrives. Then for $h \geq 0$ we obtain

$$P_{(h,i),(0,0)} = \int_0^\infty (1 - e^{-\eta_i t}) \,\mathrm{d}A(t) = d_i.$$

Next, consider transitions when the server is not available.

Case 4: Consider the transition from $(h, 0)$ to $(h+1, 0)$. In this case, only an arrival can occur. Therefore, the transition occurs with probability 1. Then we have

$$P_{(h,0),(h+1,0)} = 1, \quad 0 \leq h \leq N-2.$$

Case 5: Consider the transition from $(N-1, 0)$ to (m, i). Then, we have

$$\begin{aligned} P_{(N-1,0),(m,i)} &= q_i P_{(N-1,i),(m,i)} \\ &= q_i b_{i,N-m}, \quad m \geq 0. \end{aligned}$$

Case 6: Consider the transition from $(N-1, 0)$ to $(0, i)$. Then we derive

$$P_{(N-1,0),(0,i)} = q_i P_{(N-1,i),(0,i)} = q_i c_{i,N}.$$

Case 7: Consider the transition from $(N-1, 0)$ to $(0, 0)$. Then we obtain

$$P_{(N-1,0),(0,0)} = \sum_{i=1}^{n} q_i P_{(N-1,i),(0,0)} = \sum_{i=1}^{n} q_i d_i.$$

Once these transition probabilities are given, we can obtain the stationary queue length distribution at arrival epochs. Let

$$\pi_{m,j} = \lim_{k \to \infty} P\{(L_k, J_k) = (m, j)\},$$

i.e., $\pi_{m,j}$ is the stationary probability that a new arrival finds m customers in the system and the server is in phase j, where $(m, j) \in \Omega$. According to the aforementioned transition probabilities, $\pi_{m,j}$ should satisfy the following balance equations:

$$\pi_{0,0} = \pi_{1,0} = \dots = \pi_{N-1,0}, \tag{1a}$$

$$\begin{aligned} \pi_{m,i} = \pi_{N-1,0} q_i b_{i,N-m} + \sum_{h=0}^{\infty} \pi_{h+m-1,i} b_{i,h}, \\ 1 \leq m \leq N, \quad i = 1, 2, \dots, n, \end{aligned} \tag{1b}$$

$$\pi_{m,i} = \sum_{h=0}^{\infty} \pi_{h+m-1,i} b_{i,h}, \quad m \geq N+1, \tag{1c}$$

$$\pi_{0,i} = \pi_{N-1,0} q_i c_{i,N} + \sum_{h=0}^{\infty} \pi_{h,i} c_{i,h+1}, \tag{1d}$$

$$\pi_{0,0} = \pi_{N-1,0} \sum_{i=1}^{n} q_i d_i + \sum_{i=1}^{n} \sum_{h=0}^{\infty} \pi_{h,i} d_i. \tag{1e}$$

Using a trial solution approach and referring to Lim *et al.* (2013), we proceed to solve these balance equations.

First, we assume the trial solution is of the form

$$\pi_{m,i} = \pi_{0,0} K_{N,i} \alpha_i^m, \quad m \geq N, \quad 0 < \alpha_i < 1, \tag{2}$$

where $K_{N,i}$ is a constant which can be determined later. Substituting (2) into (1c), we have

$$\begin{aligned}\alpha_i &= \sum_{h=0}^{\infty} \alpha_i^h b_{i,h} \\ &= A^*(\mu_i + \eta_i - \mu_i \alpha_i), \quad i = 1, 2, \dots, n.\end{aligned}$$

Since $\eta_i > 0$, α_i is the unique root of the equation

$$z = A^*(\mu_i + \eta_i - \mu_i z), \quad i = 1, 2, \dots, n$$

in $|z| < 1$. The proof is similar to that in Jiang *et al.* (2015).

To solve $\pi_{h,i}, h = 0, 1, 2, \dots, N-1$, we must deal with N equations of (1b) and (1d), and derive the results by these recursive relations. For example, in (1b), consider $m = N$; applying (2) and (1a), we have

$$\pi_{N-1,i} b_{i,0} = \pi_{0,0} K_{N,i} {\alpha_i}^{N-1} b_{i,0} - \pi_{0,0} q_i b_{i,0},$$

i.e.,

$$\begin{aligned}\pi_{N-1,i} &= \pi_{0,0}(K_{N,i}\alpha_i^{N-1} - q_i) \\ &= \pi_{0,0}(K_{N,i}\alpha_i^{N-1} - K_{0,i}).\end{aligned}$$

Then, consider $m = N - 1$. We have

$$\begin{aligned}\pi_{N-2,i} b_{i,0} = {} & \pi_{N-1,i}(1 - b_{i,1}) - \pi_{0,0} q_i b_{i,1} \\ & - \sum_{k=2}^{\infty} \pi_{k+N-2,i} b_{i,k}.\end{aligned} \tag{3}$$

Substituting $\pi_{N-1,i}$ and $\pi_{m,i}, m \geq N$ into (3), we obtain

$$\begin{aligned}\pi_{N-2,i} &= \pi_{0,0}\Big(K_{N,i}\alpha_i^{N-2} - \frac{q_i}{b_{i,0}}\Big) \\ &= \pi_{0,0}\Big(K_{N,i}\alpha_i^{N-2} - K_{1,i}\Big).\end{aligned}$$

Similarly, the remaining $\pi_{m,i}, 0 \leq m \leq N-3$ can be obtained by the same method.

Then, the results are summarized as

$$\begin{aligned}&\pi_{m,i} = \pi_{0,0}(K_{N,i}\alpha_i^m - K_{N-m-1,i}), \\ &\qquad 0 \leq m \leq N-1, \quad i = 1, 2, \dots, n,\end{aligned}$$

where

$$K_{0,i} = q_i, \quad b_{i,0} K_{1,i} = K_{0,i},$$

and

$$b_{i,0} K_{m,i} = K_{m-1,i} - \sum_{h=1}^{m-1} b_{i,m-h} K_{h,i}, \qquad 2 \leq m \leq N-1.$$

Finally, substituting $\pi_{m,i}, m \geq 0$ into (1d) and using the normalization condition

$$\pi_{0,0} + \dots + \pi_{N-1,0} + \sum_{i=1}^{n} \sum_{m=0}^{\infty} \pi_{m,i} = 1,$$

we obtain

$$K_{N,i} = \frac{K_{N-1,i} - \sum\limits_{h=0}^{N-2} K_{N-h-1,i} c_{i,h+1}}{1 - \sum\limits_{h=0}^{\infty} \alpha_i^h c_{i,h+1}},$$

$$\pi_{0,0} = \left(N + \sum_{i=1}^{n} \frac{K_{N,i} - (1-\alpha_i)\sum\limits_{m=0}^{N-1} K_{N-m-1,i}}{1-\alpha_i}\right)^{-1}.$$

For all aforementioned results, the stationary queue length distribution at arrival epochs can be summarized in the following theorem.

Theorem 1. *The stationary queue length distribution at arrival epochs is given as follows:*

$$\pi_{m,0} = \left(N + \sum_{i=1}^{n} \frac{K_{N,i} - (1-\alpha_i)\sum\limits_{k=0}^{N-1} K_{N-k-1,i}}{1-\alpha_i}\right)^{-1}, \qquad 0 \leq m \leq N-1, \tag{4a}$$

$$\pi_{m,i} = \pi_{0,0}(K_{N,i}\alpha_i^m - K_{N-m-1,i}), \qquad 0 \leq m \leq N-1, \tag{4b}$$

$$\pi_{m,i} = \pi_{0,0} K_{N,i} \alpha_i^m, \quad m \geq N, \tag{4c}$$

where α_i *is the unique root of the equation*

$$z = A^*(\mu_i + \eta_i - \mu_i z), \quad i = 1, 2, \dots, n$$

in the range $0 < z < 1$*, and*

$$K_{0,i} = q_i, \tag{5a}$$

$$b_{i,0} K_{1,i} = K_{0,i}, \tag{5b}$$

$$b_{i,0} K_{m,i} = K_{m-1,i} - \sum_{h=1}^{m-1} b_{i,m-h} K_{h,i}, \qquad N-1 \geq m \geq 2, \tag{5c}$$

$$K_{N,i} = \frac{K_{N-1,i} - \sum\limits_{h=0}^{N-2} K_{N-h-1,i} c_{i,h+1}}{1 - \sum\limits_{h=0}^{\infty} \alpha_i^h c_{i,h+1}}. \tag{5d}$$

4. Stationary queue length distribution at arbitrary epochs

In this section, by using the method of the semi-Markov process, we will derive the limit distribution of $L(t)$.

Let L denote the number of customers in the system at an arbitrary epoch. Define

$$P_m = P\{L = m\} = \lim_{t\to\infty} P\{L(t) = m\}.$$

Next, we will construct a semi-Markov process to find P_m. We define a new process $\{(Z(t), K(t)), t \geq 0\}$, where $Z(t) = L_k, \tau_k \leq t < \tau_k + 1$, and $K(t) = J_k, \tau_k \leq t < \tau_k + 1$. Obviously, $\{(Z(t), K(t)), t \geq 0\}$ should be a semi-Markov process having $\{(L_k, J_k), k \geq 1\}$ for its embedded Markov chain. Let $w_{m,i}$ be the expected time that the semi-Markov process is in state (m, i). Then, we have $w_{m,i} = 1/\lambda$ for all $(m, i) \in \Omega$. Let

$$f_{m,i} = \lim_{t\to\infty} P\{(Z(t), K(t)) = (m, i)\},$$

that is, $f_{m,i}$ denotes the limiting probability that the semi-Markov process is in state (m, i). From the theory of semi-Markov processes (see Gross *et al.*, 2008, p. 298), we have

$$f_{m,i} = \frac{\pi_{m,i} w_{m,i}}{\sum\limits_{j=1}^{n}\sum\limits_{h=0}^{\infty} \pi_{h,j} w_{h,j} + \sum\limits_{h=0}^{N-1} \pi_{h,0} w_{h,0}}.$$

Substituting $w_{m,i} = 1/\lambda$ into the above equation, we have

$$f_{m,i} = \frac{\pi_{m,i}}{\sum\limits_{j=1}^{n}\sum\limits_{h=0}^{\infty} \pi_{h,j} + \sum\limits_{h=0}^{N-1} \pi_{h,0}} = \pi_{m,i}.$$

Define A_E as the elapsed interarrival time of customers at an arbitrary epoch in steady state. Then, the density function of A_E is $\lambda(1 - A(t))$. Let $\delta_{i,h}$ denote the probability that h customers are served and no disasters occur during A_E.

According to Gross *et al.* (2008, p. 292), the limiting distribution of $L(t)$ has the following expressions:

$$\begin{aligned}
P_k &= \sum_{(j,i)\in\Omega} f_{j,i} \sum_{m=0}^{n} \int_0^{\infty} P(\text{required changes in } t \\
&\qquad \text{to bring state from } (j,i) \text{ to } (k,m)\,)\lambda(1 - A(t))\,\mathrm{d}t
\end{aligned}$$

for $k > 0$. For $m \geq N$, using the relationship between $\{L(t), t \geq 0\}$ and $\{(Z(t), K(t)), t \geq 0\}$, we have

$$\begin{aligned}
P_{m+1} &= \sum_{i=1}^{n}\sum_{h=m}^{\infty} f_{h,i}\delta_{h-m} \\
&= \sum_{i=1}^{n}\sum_{h=m}^{\infty} \pi_{0,0} K_{N,i} \alpha_i^h \\
&\quad \int_0^{\infty} \frac{(\mu_i t)^{h-m}}{(h-m)!} e^{-\mu_i t} e^{-\eta_i t} \lambda P(A > t)\,\mathrm{d}t \\
&= \sum_{i=1}^{n} \pi_{0,0} K_{N,i} \alpha_i^m \\
&\quad \int_0^{\infty} e^{-(\mu_i + \eta_i - \alpha_i\mu_i)t} \lambda(1 - P(A < t))\,\mathrm{d}t \\
&= \sum_{i=1}^{n} \pi_{0,0} K_{N,i} \alpha_i^m \lambda \frac{1 - A^*(\mu_i + \eta_i - \alpha_i\mu_i)}{\mu_i + \eta_i - \alpha_i\mu_i} \\
&= \sum_{i=1}^{n} \pi_{0,0} K_{N,i} \alpha_i^m \lambda \frac{1 - \alpha_i}{\mu_i + \eta_i - \alpha_i\mu_i}.
\end{aligned} \tag{6}$$

For $1 \leq m \leq N$, let

$$\varepsilon_{m,N} = \begin{cases} 0, & m = N, \\ 1, & 1 \leq m < N. \end{cases}$$

We obtain

$$\begin{aligned}
P_m &= f_{m-1,0}\varepsilon_{m,N} + \sum_{i=1}^{n} f_{N-1,0} q_i \delta_{i,N-m} \\
&\quad + \sum_{i=1}^{n}\sum_{h=m-1}^{\infty} f_{h,i}\delta_{i,h+1-m} \\
&= \pi_{m-1,0}\varepsilon_{m,N} + \sum_{i=1}^{n} \pi_{N-1,0} q_i \delta_{i,N-m} \\
&\quad + \sum_{i=1}^{n} \Bigg(\sum_{h=m-1}^{\infty} \pi_{0,0} K_{N,i} \alpha_i^h \delta_{i,h+1-m} \\
&\quad - \sum_{h=m-1}^{N-1} \pi_{0,0} K_{N-h-1,i} \delta_{i,h+1-m} \Bigg) \\
&= \pi_{m-1,0}\varepsilon_{m,N} \\
&\quad + \sum_{i=1}^{n} \Bigg(\sum_{h=m-1}^{\infty} \pi_{0,0} K_{N,i} \alpha_i^h \delta_{i,h+1-m} \\
&\quad - \sum_{h=m-1}^{N-2} \pi_{0,0} K_{N-h-1,i} \delta_{i,h+1-m} \Bigg) \\
&= \pi_{0,0}\varepsilon_{m,N} - \sum_{i=1}^{n}\sum_{h=m-1}^{N-2} \pi_{0,0} K_{N-h-1,i} \delta_{i,h+1-m} \\
&\quad + \sum_{i=1}^{n} \pi_{0,0} K_{N,i} \alpha_i^{m-1} \lambda \frac{1 - A^*(\mu_i + \eta_i - \alpha_i\mu_i)}{\mu_i + \eta_i - \alpha_i\mu_i}.
\end{aligned} \tag{7}$$

In order to simplify (7), we first give the following lemma.

Lemma 1. *The relationships between $b_{i,h}$ and $\delta_{i,h}$ are given by*

$$b_{i,0} = 1 - \frac{\mu_i + \eta_i}{\lambda}\delta_{i,0},$$
$$b_{i,h} = \frac{\mu_i}{\lambda}\delta_{i,h-1} - \frac{\mu_i + \eta_i}{\lambda}\delta_{i,h}, \quad h \geq 1.$$

Proof. We use integration by parts to yield the following relationships between $b_{i,h}$ and $\delta_{i,h}$:

$$\begin{aligned} b_{i,0} &= \int_0^\infty e^{-(\eta_i+\mu_i)t}\,\mathrm{d}A(t) \\ &= 1 - \frac{\mu_i + \eta_i}{\lambda}\delta_{i,0}, \\ b_{i,h} &= \int_0^\infty e^{-\eta_i t}\frac{(\mu_i t)^h}{h!}e^{-\mu_i t}\,\mathrm{d}A(t) \\ &= \frac{\mu_i}{\lambda}\delta_{i,h-1} - \frac{\mu_i + \eta_i}{\lambda}\delta_{i,h}, \quad h \geq 1. \end{aligned}$$

■

Let

$$E_i = \frac{\mu_i}{\lambda}, \quad F_i = \frac{\mu_i + \eta_i}{\lambda}.$$

By Lemma 1, Eqns. (5b) and (5c) can be rewritten as follows:

$$\begin{aligned} &K_{N-m-1,i} \\ &= b_{i,0}K_{N-m,i} + \sum_{h=1}^{N-m-1} b_{i,N-m-h}K_{h,i} \\ &= b_{i,0}K_{N-m,i} + b_{i,1}K_{N-m-1,i} + \cdots \\ &\quad + b_{i,N-m-1}K_{1,i} \\ &= K_{N-m,i} - F_i(\delta_{i,0}K_{N-m,i} + \cdots \\ &\quad + \delta_{i,N-m-1}K_{1,i}) \\ &\quad + E_i(\delta_{i,0}K_{N-m-1,i} + \cdots \\ &\quad + \delta_{i,N-m-2}K_{1,i}), \end{aligned} \tag{8}$$

where $1 \leq m \leq N-1$.

Further, using a recursion method similar to that by Lim *et al.* (2013), Eqn. (8) can be recursively calculated starting from $m = N-1$, which yields

$$\begin{aligned} &\sum_{h=m-1}^{N-2} K_{N-h-1,i}\delta_{i,h+1-m} \\ &= \sum_{k=0}^{N-m-1} \frac{E_i^k}{F_i^{k+1}}(K_{N-m-k,i} - K_{N-m-k-1,i}). \end{aligned} \tag{9}$$

Substituting (9) into (7), we can obtain the expression of P_m for $1 \leq m \leq N$,

$$\begin{aligned} &P_m \\ &= \pi_{0,0}\varepsilon_{m,N} + \sum_{i=1}^{n} \pi_{0,0}K_{N,i}\alpha_i^{m-1}\lambda\frac{1-\alpha_i}{\mu_i + \eta_i - \alpha_i\mu_i} \\ &\quad - \sum_{i=1}^{n}\sum_{k=0}^{N-m-1} \frac{E_i^k}{F_i^{k+1}}(K_{N-m-k,i} - K_{N-m-k-1,i}). \end{aligned} \tag{10}$$

Finally,

$$P_0 = 1 - \sum_{m=1}^{\infty} P_m.$$

Next, we summarize the stationary queue length distribution at arbitrary epochs in the following theorem.

Theorem 2. *The stationary queue length distribution at arbitrary epochs is given as follows:*

$$\begin{aligned} P_{m+1} &= \sum_{i=1}^{n} \pi_{0,0}K_{N,i}\alpha_i^m\lambda\frac{1-\alpha_i}{\mu_i + \eta_i - \alpha_i\mu_i}, \quad m \geq N, \\ P_m &= \sum_{i=1}^{n} \pi_{0,0}K_{N,i}\alpha_i^{m-1}\lambda\frac{1-\alpha_i}{\mu_i + \eta_i - \alpha_i\mu_i} \\ &\quad - \sum_{i=1}^{n}\sum_{k=0}^{N-m-1} \frac{E_i^k(K_{N-m-k,i} - K_{N-m-k-1,i})}{F_i^{k+1}} \\ &\quad + \pi_{0,0}\varepsilon_{m,N}, \quad 1 \leq m \leq N, \\ P_0 &= 1 - \sum_{m=1}^{\infty} P_m. \end{aligned}$$

where α_i is the unique root of the equation

$$z = A^*(\mu_i + \eta_i - \mu_i z), \quad i = 1, 2, \ldots, n$$

in the range $0 < z < 1$,

$$E_i = \frac{\mu_i}{\lambda}, \quad F_i = \frac{\mu_i + \eta_i}{\lambda}.$$

5. Stationary sojourn time distribution

By introducing a tagged customer, we will obtain the LST of the stationary sojourn time of an arbitrary customer under the first-come-first-served (FCFS) discipline, where the sojourn time is defined as the period from the time it enters into the system until departure, either by occurrence of a disaster or by its service completion. Let W and $W^*(s)$ denote the stationary sojourn time of an arbitrary customer and its LST, D_i denote the random variable of the duration of time during which the system resides in service phase $i, i = 1, 2, \ldots, n$, $B_{i,h}$ denote the total service times of h customers in service environment i. In order to obtain the LST of the sojourn time of an arbitrary customer $W^*(s)$, we consider two cases:

Case 1: The tagged customer arrives in state $(h, 0), 0 \leq h \leq N-1$. Then the sojourn time of this tagged customer consists of the time taken for $N-h-1$ customers to arrive and either the inter-arrival of a disaster or the total service time of $h+1$ customers in operative phase $i, i = 1,2,\dots,n$, i.e., $W_{1,i} = (N-h-1)A + \min(B_{i,h+1}, D_i)$.

Case 2: The tagged customer arrives in state $(h,i), h \geq 0, i = 1,2,\dots,n$. Then the sojourn time of this tagged customer is the time taken as either the inter-arrival of a disaster or the total service time of $h+1$ customers in operative phase $i, i = 1,2,\dots,n$, i.e., $W_2 = \min(B_{i,h+1}, D_i)$.

Combining the two cases, we have

$$\begin{aligned}
W^*(s) &= \sum_{h=0}^{N-1}\sum_{i=1}^{n} \pi_{h,0} q_i W^*_{1,i}(s) \\
&\quad + \sum_{i=1}^{n}\sum_{h=0}^{\infty} \pi_{h,i} W^*_2(s) \\
&= \sum_{h=0}^{N-1} \pi_{h,0} [A^*(s)]^{N-h-1} \\
&\quad \times \sum_{i=1}^{n} q_i E\Big[e^{-s\min\{B_{i,h+1},D_i\}}\Big] \\
&\quad + \sum_{i=1}^{n}\sum_{h=0}^{\infty} \pi_{h,i} E\Big[e^{-s\min\{B_{i,h+1},D_i\}}\Big],
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
&\sum_{h=0}^{N-1} \pi_{h,0}[A^*(s)]^{N-h-1} \sum_{i=1}^{n} q_i E\Big[e^{-s\min\{B_{i,h+1},D_i\}}\Big] \\
&= \pi_{0,0} \sum_{i=1}^{n}\sum_{h=0}^{N-1} [A^*(s)]^{N-h-1} q_i \\
&\quad \times \frac{\eta_i + s[B^*_i(s+\eta_i)]^{h+1}}{s+\eta_i} \\
&= \pi_{0,0}\Big(\sum_{i=1}^{n} \frac{q_i \eta_i}{s+\eta_i} \frac{1-[A^*(s)]^N}{1-A^*(s)} \\
&\quad + \sum_{i=1}^{n} \frac{q_i s}{s+\eta_i} \\
&\quad \times \frac{B^*_i(s+\eta_i)[[A^*(s)]^N - [B^*_i(s+\eta_i)]^N]}{[A^*(s) - B^*_i(s+\eta_i)]}\Big),
\end{aligned}$$

$$\begin{aligned}
&\sum_{i=1}^{n}\sum_{h=0}^{\infty} \pi_{h,i} E\Big[e^{-s\min\{B_{i,h+1},D_i\}}\Big] \\
&= \sum_{i=1}^{n}\sum_{h=0}^{\infty} \frac{\eta_i}{s+\eta_i} \pi_{h,i} \\
&\quad + \sum_{i=1}^{n} \frac{s B^*_i(s+\eta_i)}{s+\eta_i} \pi_i(B^*_i(s+\eta_i)),
\end{aligned}$$

with

$$B^*_i(s+\eta_i) = \frac{\mu_i}{s+\eta_i+\mu_i},$$

$$\pi_i(B^*_i(s+\eta_i)) = \sum_{k=0}^{\infty} \pi_{k,i}[B^*_i(s+\eta_i)]^k.$$

The expected sojourn time of an arbitrary customer is

$$\begin{aligned}
E[W] &= -\frac{\mathrm{d}W^*(s)}{\mathrm{d}s}\Big|_{s=0} \\
&= \pi_{0,0} \sum_{i=1}^{n} \left[N\frac{q_i}{\eta_i} + \frac{q_i N(N-1)}{2\lambda} \right] \\
&\quad - \pi_{0,0} \sum_{i=1}^{n} \frac{q_i}{\eta_i} \frac{B^*_i(\eta_i)[1-[B^*_i(\eta_i)]^N]}{1-B^*_i(\eta_i)} \\
&\quad + \pi_{0,0} \sum_{i=1}^{n} \left(\frac{K_{N,i}}{1-\alpha_i} - \sum_{h=0}^{N-1} K_{N-h-1,i} \right) \frac{1}{\eta_i} \\
&\quad - \sum_{i=1}^{n} \frac{B^*_i(\eta_i)}{\eta_i} \pi_{0,0} \Bigg[K_{N,i} \frac{1}{1-\alpha_i B^*_i(\eta_i)} \\
&\quad - \sum_{k=0}^{N-1} K_{N-k-1,i} [B^*_i(\eta_i)]^k \Bigg].
\end{aligned} \tag{12}$$

6. Length of working time in a cycle

In this section, we mainly focus on the LST of the length of the server's working time in a cycle. To avoid terminological confusion, we define a cycle as the time between two consecutive instants at which the inoperative environment commences, and the working time is the time interval during which the server is busy in an operative service environment (it does not contain the time that the server is idle). Let C denote the length of a cycle, U denote the working time in a cycle, U_i denote the working time of the server in operative service environment i, $H_{i,h}, h \geq 1, i = 1,2,\dots,n$ denote the length of busy period caused by h customers in operative service environment i. Since the interarrival times of customers follow a general distribution, the memoryless property is no longer satisfied. We further define A_R as the residual lifetime of an interarrival time.

From the work of Haviv (2013, p. 24) or Cohen (1982), we know that A_E and A_R have the same limiting distribution, so the density function of A_R is also equal to $\lambda(1-A(t))$. Similarly to the analysis by Jiang and Liu (2017), according to our assumptions and the ordering of various times (time to a disaster, residual lifetime of next interarrival time, overall time of busy period), U_i can be obtained as follows:

$$U_i = \begin{cases} V_1, & H_{i,N} > D_i, \\ V_2 + U_{i,0}, & H_{i,N} < D_i, \end{cases}$$

where $V_1 = (D_i | H_{i,N} > D_i)$ and $V_2 = (H_{i,N} | H_{i,N} < D_i)$,

$$U_{i,0} = \begin{cases} 0, & A_R > D_i, \\ T, & A_R < D_i, \end{cases}$$

$$T = \begin{cases} T_1, & H_{i,1} > D_i, \\ T_2 + U_{i,0}, & H_{i,1} < D_i, \end{cases}$$

$$T_1 = (D_i | H_{i,1} > D_i), \quad T_2 = (H_{i,1} | H_{i,1} < D_i).$$

First, the LST of T_1 and T_2 can be obtained by

$$\begin{aligned} T_1^*(s) &= E[e^{-sD_i} | H_{i,1} > D_i] \\ &= \frac{\eta_i}{s+\eta_i} \frac{1 - H_{i,1}^*(s+\eta_i)}{P(H_{i,1} > D_i)}, \end{aligned}$$

$$\begin{aligned} T_2^*(s) &= E[e^{-sH_{i,1}} | H_{i,1} < D_i] \\ &= \frac{H_{i,1}^*(s+\eta_i)}{P(H_{i,1} < D_i)}. \end{aligned}$$

Then, substituting $T_1^*(s)$ and $T_2^*(s)$ into

$$\begin{aligned} T^*(s) &= P(H_{i,1} > D_i) T_1^*(s) \\ &\quad + P(H_{i,1} < D_i) T_2^*(s) U_{i,0}^*(s), \end{aligned}$$

we have

$$\begin{aligned} T^*(s) &= \frac{\eta_i}{s+\eta_i}(1 - H_{i,1}^*(s+\eta_i)) \\ &\quad + H_{i,1}^*(s+\eta_i) U_{i,0}^*(s). \end{aligned}$$

Further, substituting $T^*(s)$ into

$$U_{i,0}^*(s) = P(A_R > D_i) E[e^{-s0}] + P(A_R < D_i) T^*(s),$$

we have

$$\begin{aligned} U_{i,0}^*(s) &= P(A_R > D_i) E[e^{-s0}] \\ &\quad + P(A_R < D_i) T^*(s) \\ &= P(A_R < D_i) \Big[\frac{\eta_i}{s+\eta_i}(1 - H_{i,1}^*(s+\eta_i)) \\ &\quad + H_{i,1}^*(s+\eta_i) U_{i,0}^*(s) \Big] + P(A_R > D_i) \end{aligned}$$

i.e.,

$$\begin{aligned} U_{i,0}^*(s) &= \frac{(s+\eta_i) P(A_R > D_i)}{(s+\eta_i)[1 - P(A_R < D_i) H_{i,1}^*(s+\eta_i)]} \\ &\quad + \frac{+\eta_i P(A_R < D_i)[1 - H_{i,1}^*(s+\eta_i)]}{(s+\eta_i)[1 - P(A_R < D_i) H_{i,1}^*(s+\eta_i)]}, \end{aligned}$$

where

$$\begin{aligned} P(A_R < D_i) &= \frac{\lambda}{\eta_i}[1 - A^*(\eta_i)], \\ P(A_R > D_i) &= \frac{\eta_i - \lambda[1 - A^*(\eta_i)]}{\eta_i}. \end{aligned}$$

After some calculations, we can derive the result for $U_{i,0}^*(s)$.

Finally, with the same method

$$\begin{aligned} V_1^*(s) &= E[e^{-sD_i} | H_{i,N} > D_i] \\ &= \frac{\eta_i}{s+\eta_i} \frac{1 - H_{i,N}^*(s+\eta_i)}{P(H_{i,N} > D_i)}, \\ V_2^*(s) &= E[e^{-sH_{i,N}} | H_{i,N} < D_i] \\ &= \frac{H_{i,N}^*(s+\eta_i)}{P(H_{i,N} < D_i)}. \end{aligned}$$

Substituting $V_1^*(s)$ and $V_2^*(s)$ into $U_i^*(s) = P(H_{i,N} > D_i) V_1^*(s) + P(H_{i,N} < D_i) V_2^*(s) U_{i,0}^*(s)$, we have

$$\begin{aligned} U_i^*(s) &= \frac{\eta_i}{s+\eta_i}(1 - H_{i,N}^*(s+\eta_i)) \\ &\quad + H_{i,N}^*(s+\eta_i) U_{i,0}^*(s). \end{aligned}$$

In fact, $H_{i,1}^*(s)$ and $H_{i,N}^*(s)$ have very complex expressions, and the results can be seen in Cohen (1982) (see page 227).

Once $U_i^*(s)$ is derived, the LST of U can be obtained by

$$U^*(s) = \sum_{i=1}^{n} q_i U_i^*(s).$$

7. Numerical examples

In this section, we show some numerical examples for the present queueing model. We first assume that the interarrival times of customers follow an exponential distribution with parameter λ. Then, the system translates into an M/M/1 queue in a multi-phase service environment with disasters and an N-policy. Accordingly, α_i is the unique root of $\mu_i(1-z)z + (\lambda+\eta_i)z - \lambda = 0$ in $(0,1)$, and an immediate result is

$$\alpha_i = \frac{(\lambda+\mu_i+\eta_i) - \sqrt{(\lambda+\mu_i+\eta_i)^2 - 4\lambda\mu_i}}{2\mu_i}, \qquad i = 1, 2, \ldots, n.$$

Using (5a)–(5d) and the transition probabilities, we can derive the constant values of $K_{m,i}$, for $m = 0, 1, \ldots, N$ and $i = 1, 2, \ldots, n$. In succession, the stationary queue length distribution at arbitrary epochs and the sojourn time distribution of an arbitrary customer can be respectively obtained by Theorem 2 and Eqn. (12). Similarly, when the interarrival time distribution is 2-Erlangian or deterministic, α_i is the unique root of

$$z = \frac{\lambda^2}{[\lambda + \eta_i + \mu_i(1-z)]^2}$$

or

$$z = e^{-\frac{\eta_i + \mu_i(1-z)}{\lambda}}$$

in $(0,1)$. The stationary queue length distribution at arbitrary epochs and the sojourn time distribution can be obtained by the same method.

From a practical perspective, a very useful measure is the sojourn time of an arbitrary customer. Hence, in the following content, we will present some figures to show the impact of parameters on the expected sojourn time $E[W]$ for an N-policy M/M/1 queue in a multi-phase service environment with disasters. With no loss of generality, we assume $n = 2$, $N = 3$, and the system parameters $\mu_1 = 1.5$, $\eta_1 = 0.6$. Figure 1 shows the impact of arrival rate λ on the expected sojourn time $E[W]$ for different values of η_2. From Fig. 1, we can easily find that the expected sojourn time of an arbitrary customer $E[W]$ decreases with an increase in λ for any given values of η_2. It is obvious that, if λ is fixed, the bigger η_2 is, the larger $E[W]$ becomes.

Next, in Fig. 2, we pay attention to the curves of the expected sojourn time $E[W]$ with a change in the probability q_1 for different values of η_2. From Fig. 2, we find that the expected sojourn time of an arbitrary customer $E[W]$ increases with an increase in q_1 for any given value of η_2. Obviously, we also see that as q_1 approaches 1, $E[W]$ tends to a fixed value irrespective of the value of η_2. It is reasonable that when q_1 reaches 1, the queue reduces to a classic N-policy M/M/1 queue in a 2-phase service environment, and η_2 has no impact on $E[W]$.

In Fig. 3, we plot the trend of the change in $E[W]$ as μ_2 increases from 1.2 to 2.5. From Fig. 3, we find that the expected sojourn time of an arbitrary customer $E[W]$ decreases with an increase in μ_2 for any given value of η_2, which is identical to the intuitive expectations. Although we only concentrate on showing the impact of the parameters on the mean sojourn time, we believe that similar results may exist for other performance measures.

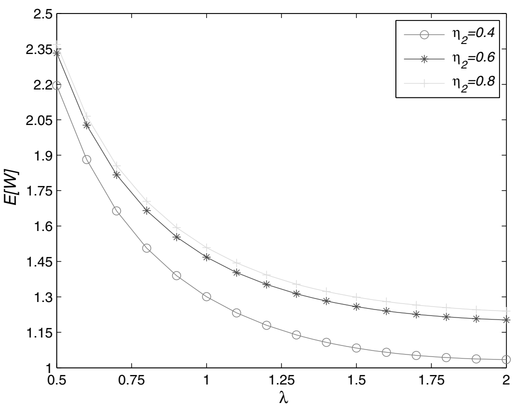

Fig. 1. Expected sojourn time of an arbitrary customer $E[W]$ vs. λ ($q_1 = 0.6, \mu_2 = 2$).

In order to find the optimal value of N, we finally investigate the following cost structure by considering holding cost for each customer per unit time c_h as well as a setup cost per cycle c_s. Then the objective function for minimizing the total cost per unit time can be obtained as follows:

$$\min_{N\geq 1} TC(N) = \min_{N\geq 1}\left(c_h E[L] + c_s \frac{1}{E[C]}\right),$$

where $E[L]$ denotes the expected number of customers in the system, $E[C]$ denotes the expected duration of a cycle. According to Lim *et al.* (2013), $1/E[C]$ can be expressed by $\lambda\pi_{0,0}$. Then the expression of the total cost per unit time can be rewritten as

$$\min_{N\geq 1} TC(N) = \min_{N\geq 1}(c_h E[L] + c_s \lambda \pi_{0,0}).$$

Next, we examine the impact of N on the total cost per unit time. With no loss of generality, we assume that

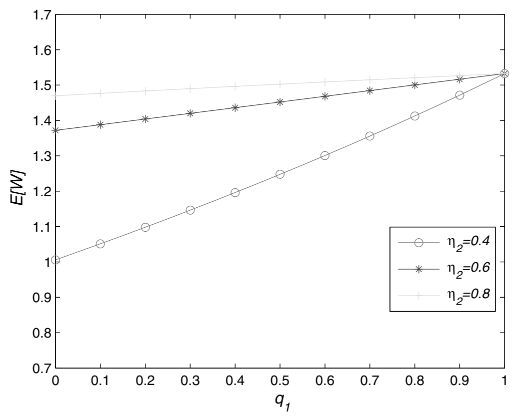

Fig. 2. Expected sojourn time of an arbitrary customer $E[W]$ vs. q_1 ($\lambda = 1, \mu_2 = 2$).

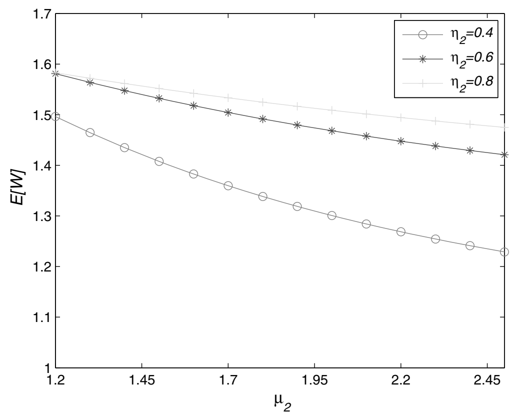

Fig. 3. Expected sojourn time of an arbitrary customer $E[W]$ vs. μ_2 ($\lambda = 1, q_1 = 0.6$).

$q_1 = 0.6, \lambda = 1, \mu_1 = 1.5, \mu_2 = 2, \eta_1 = 0.6, c_s = 3, c_h = 0.1$ and pay attention to the curves of the total cost per unit time with the change in N from 3 to 12 for different values of η_2. From Fig. 4, we find that the total cost per unit time is a convex function of N, which is expected.

A smaller N can be effective to reduce the cost since $E[L]$ is an increasing function of N and bigger N can be effective to save the cost since $1/E[C]$ is a decreasing function of N. Then there exists a trade-off which leads to finding an optimal value of N to minimize the total cost, i.e., as N is smaller, the part $c_s\lambda\pi_{0,0}$ may play a dominant role in increasing the cost, and as N becomes bigger, the part $c_hE[L]$ may play a dominant role in increasing the cost. Furthermore, we can obtain an optimal value of N to minimize the total cost, i.e., for $\eta_2 = 0.4$, as $N = 5$, the total cost has minimum value 0.6674 and for $\eta_2 = 0.6, 0.8$, as $N = 6$, the total cost has minimum values 0.6949 and 0.7050.

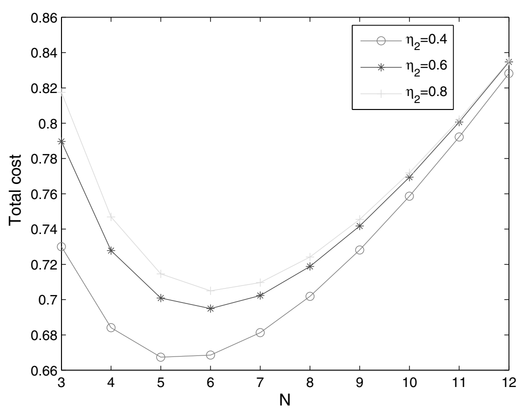

Fig. 4. Total cost in per unit time over N for different values of η_2.

8. Conclusion and future research directions

In this paper, a single server N-policy GI/M/1 queue in a multi-phase service environment with disasters has been considered in terms of the embedded Markov chain method and a trial solution approach following the idea presented by Lim *et al*. (2013). We have focused our analysis on the stationary queue length distribution at arrival epochs, which is an important aspect in dealing with the GI/M/1 models. Furthermore, by using the preceding results and considering the semi-Markov process, we have obtained the stationary queue length distribution at arbitrary epochs. In addition, the LST of the stationary sojourn time of an arbitrary customer and the working time in a cycle have also been obtained. Finally, we have given some numerical examples for the considered queueing model. We expect that the results and the method can be applied to more queueing systems.

Apart from the results obtained by the present queueing model, many interesting and important topics on this queueing model have not yet been fully explored. It would be interesting to consider further extensions of the present queueing model. One direction is to consider the case that service times or time till disaster occurrence (or both) have a phase-type distribution. Despite the structure of the transition matrix will be retained, however, the scalars will become matrices, so we need to find some algorithmic solution for computing the various distributions of interest.

Another interesting direction for future research is to take a non-zero repair time into consideration. At the end of the repair time, the server starts working if there are at least N jobs in the system, or, if less than N customers arrived during the repair time, the server stays dormant until the N-th arrival occurs. Only then the server will start working again. A detailed analysis of this case requires a separate discussion and is left for future research.

Acknowledgment

The authors would like to thank the editor and the referees for their helpful suggestions and comments to improve the quality of an earlier version of this paper. This work is supported by the Scientific Research Foundation of the Shandong University of Science and Technology for Recruited Talents as well as the National Natural Science Foundation of China (grant no. 61773014).

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Received: 19 February 2017
Revised: 24 September 2017
Re-revised: 21 December 2017
Accepted: 13 January 2018