Int. J. Appl. Math. Comput. Sci., 2020, Vol. 30, No. 1, 61–74
DOI: 10.34768/amcs-2020-0005

FAULT TOLERANT MULTICONTROLLERS FOR NONLINEAR SYSTEMS: A REAL VALIDATION ON A CHEMICAL PROCESS

SONDESS MEJDI[a,*], ANIS MESSAOUD[a], RIDHA BEN ABDENNOUR[a]

[a] Laboratory of Numerical Control of Industrial Processes
University of Gabes
Omar Ibn Khattab St., 6029, Gabes, Tunisia
e-mail: sondessmejdi@gmail.com

An active sensor fault tolerant controller for nonlinear systems represented by a decoupled multimodel is proposed. Active fault tolerant control requires accurate fault estimation. Thus, to estimate both state variables and sensor faults, a discrete unknown input multiobserver, based on an augmented state multimodel, is designed. The multiobserver gains are computed by solving linear matrix inequalities with equality constraints. A multicontrol strategy is proposed for the compensation of the sensor fault and recovering the desired performances. This strategy integrates a bank of controllers, corresponding to a set of partial models, to generate a set of control laws compensating the fault effect. Then, a switching strategy between the generated local control laws is established in order to apply the most suitable control law that tolerates the fault and maintains good closed loop performances. The effectiveness of the proposed strategy is proven through a numerical example and also through a real time application on a chemical reactor. The obtained results confirm satisfactory closed loop performance in terms of trajectory tracking and fault tolerance.

Keywords: multicontroller, experimental validation, transesterification reactor, discrete unknown input multiobserver, fault tolerant control, sensor fault estimation.

1. Introduction

Fault diagnosis (FD) and fault tolerant control (FTC) are integrated in many applications to prevent performances losses, preserve stability and maintain desirable performances. It is crucial to point out that, when a fault occurs in actuators or sensors, the characteristics of the entire system can endure significant changes, degrading its performances and even causing instability (Bonfè *et al.*, 2011; Noura *et al.*, 2009; Du *et al.*, 2014). Therefore, FD and FTC are increasingly recommended because they enhance system safety due to their reliability (Khelassi *et al.*, 2011; Gao *et al.*, 2015; Sojoudi *et al.*, 2011).

Two types of FTC are mentioned in survey papers (Gao *et al.*, 2015; Yu and Jiang, 2015; Jiang and Yu, 2012; Zhang and Jiang, 2008): passive and active approaches. The former aim to design the same controller for normal and faulty cases. They use robust control techniques to ensure that the closed-loop system remains insensitive to certain faults which are presumed and regarded as disturbances and uncertainties of modeling. Nevertheless, the latter react actively even in the occurrence of a fault. They require necessarily fault identification as well as an updated control mechanism.

Many research works have dealt with the design of FTC, and important results are developed for linear and nonlinear systems (Pazera *et al.*, 2017; Noura *et al.*, 2009; Du *et al.*, 2014; Theilliol *et al.*, 2008; Rodrigues *et al.*, 2014; Abdelkrim *et al.*, 2012; Ichalal *et al.*, 2016). Since real industrial processes are nonlinear, it is difficult to perform fault diagnosis and control. To surmount these problems, multimodel and multicontrol approaches are considered in several research works. The multimodel approach has received increasing attention. It represents an interesting technique and an efficient solution to perform the modeling, control and fault diagnosis of nonlinear systems (Orjuela *et al.*, 2013; Tanaka and Wang, 2001; Takagi and Sugeno, 1985; Ben Atia *et al.*, 2014). Indeed, the nonlinear system can be represented by a set of partial systems, which makes the modeling and application of some techniques possible thanks to the simplicity of multimodel representation. Then, it

*Corresponding author.

is possible to design and implement local control laws generated by controllers associated with partial models.

The controller design requires the knowledge of the state variables, which are not measurable and inaccessible in practice (Ben Atia *et al.*, 2014). Regarding state estimation and control problems, many works exploiting observers for linear and nonlinear systems have been conducted. Thus, many tools are used to perform fault tolerant control and several real-time applications are proposed to preserve human safety, maintain system reliability and prevent process breakdowns (Pico and Adam, 2017; Ichalal *et al.*, 2016; Theilliol *et al.*, 2000; Odgaard and Stoustrup, 2012).

In contrast to model-based approaches, Jiang and Yin (2018) as well as Jiang *et al.* (2018) developed data driven approaches, where no model representation can be used to describe the system studied. Thus, the monitoring system has to collect and store process data to perform state estimation, fault diagnosis and fault tolerant control. Data driven based plant monitoring, prognosis and fault diagnosis require failure determination, affecting key performances indicators of the systems (Jiang and Yin, 2018) exploiting various approaches and several algorithms. Using data driven based techniques, the model parameters are not known *a priori* and it is not clear which model matrices are unavailable for observers and controllers design. Thus, model based approaches are crucial to overcome this problem.

Motivated by the decoupled multimodel approach, a multicontrol strategy is proposed in this work. An integrated sensor fault estimation and FTC is discussed, where a switching mechanism is established to select the appropriate control law accommodating faults. The proposed decoupled multimodel representation results from the association of partial models only in the output equation of the multimodel. The retained decoupled multimodel structure is well known for its flexibility in the modeling of complex systems with a variable structure. Indeed, it introduces partial models with different dimensions adapted to each operating zone. Consequently, the number of the identified parameters will be reduced.

With this multimodel structure, the output of each partial model, depending on the operating zone of the system, contributes more or less to the global approximative system behavior. The contribution of each partial model to the global multimodel representation is defined by an activation function. However, for switching systems, a switching law indicates which partial model of a set of partial models is active at each instant. In fact, a logical rule organizes transition between these models and total activation of the partial model is registered. Therefore, the output of each partial model of switching systems, depending on the zone where the system evolves, contributes solely to the overall approximation of the system behavior. Activation functions are used for multimodel representation to ensure transition between local models. These functions point out the degree of contribution of each partial model in the global representation of complex systems.

Activation functions, which are mentioned as membership function for the fuzzy logic case, represent each local model with a membership value between 0 and 1. Moreover, gain scheduling is a technique similar to activation functions, interpolating a set of local models. However, the interpolation between models, in this case, is achieved according to the scheduling variables.

In the present paper, the main contribution is to design active FTC for nonlinear systems subject to sensor faults. Nonlinear systems are represented by a decoupled state multimodel. An integrated online fault estimation is needed to achieve this active control strategy. Then, a discrete unknown input multiobserver is synthesized to provide accurate and rapid state variables and sensor fault estimation. To achieve this task, an augmented decoupled multimodel is constructed where the sensor fault is added as an auxiliary state variable. Based on the Lyapunov approach, a stability analysis is achieved and sufficient conditions for the convergence of the state estimation error are established in terms of linear matrix inequalities (LMIs) with equality constraints.

Thanks to the features of a decoupled multimodel, multiple controllers, associated with each local model, are designed to generate a set of local controls compensating the occurring fault. Thereafter, a switching strategy is established to select the suitable control law that provides satisfactory closed loop performances. Finally, we prove the effectiveness of the proposed multicontrol strategy through a numerical example and a real time application on a transesterification reactor.

This paper is organized as follows. Section 2 describes the design of a partial nominal tracking controller. Section 3 is reserved for fault estimation based on a discrete decoupled state multimodel. A switching strategy for fault tolerant control is proposed in Section 4. Section 5 provides an illustrative example to prove the effectiveness of the proposed control strategy. A practical application on a chemical reactor is presented in Section 6. The last section concludes the paper.

2. Partial feedback controller design for nominal trajectory tracking

An active FTC strategy is founded on the design of a nominal control law, fault estimation and modification of the control law in order to compensate the fault effect.

As a first step, we are interested in the design of a nominal tracking controller. A state feedback integral control is designed to ensure the trajectory tracking.

Consider a nonlinear system described by the following discrete decoupled state multimodel:

$$\begin{cases} x_i(k+1) = A_i x_i(k) + B_i u(k), \\ y_i(k) = C_i x_i(k), \\ y_{\mathrm{MM}}(k) = \sum\limits_{i=1}^{Nm} \mu_i(\nu_{k-1}) y_i(k), \end{cases} \tag{1}$$

where Nm is the number of partial models, $x_i(k) \in \mathbb{R}^{n_i}$ and $y_i(k)$ are the state and the measured output of the i-th partial model, respectively, $u(k)$ represents the control input and $y_{\mathrm{MM}}(k)$ is the multimodel output, $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times 1}$ and $C_i \in \mathbb{R}^{1 \times n_i}$ are known and appropriately dimensioned matrices, $\mu_i(\nu_{k-1})$ are the activation functions that ensure the transition between partial models. They depend on the decision variable (ν_{k-1}) which can be the signal control. These activation functions satisfy the convex sum property

$$\begin{cases} \sum\limits_{i=1}^{Nm} \mu_i(\nu_{k-1}) = 1, \\ 0 \le \mu_i(\nu_{k-1}) \le 1, \quad \forall\, i = 1, \dots, Nm. \end{cases} \tag{2}$$

The activation functions can be chosen as normalized Gaussian functions given as follows:

$$\mu_i(\nu_{k-1}) = \frac{\exp\left(-\frac{(\nu_{k-1}-c_i)^2}{\sigma^2}\right)}{\sum\limits_{i=1}^{Nm} \exp\left(-\frac{(\nu_{k-1}-c_i)^2}{\sigma^2}\right)}, \tag{3}$$

$i = 1, \dots, Nm$, where $c_i\ (i = 1 \dots Nm)$ are the centers and σ is the dispersion.

The objective is to design a partial controller to make the system output follow the reference input $y_c(k)$ as close as possible. To track the reference input, a comparator integrator is added satisfying (Theilliol *et al.*, 2008; Noura *et al.*, 2000; Prajapati and Roy, 2016)

$$\begin{aligned} e_{u0_i}(k+1) = e_{u0_i}(k) + T\big(&y_c(k+1) \\ &- C_i x_i(k+1)\big), \end{aligned} \tag{4}$$

$i = 1, \dots, Nm$, where $e_{u0_i}(k)$ is the integral error and T is the sampling time. Taking into account (1) and (4), a new augmented decoupled state multimodel is obtained:

$$\begin{cases} X_{a_i}(k+1) = A_{a_i} X_{a_i}(k) + B_{a_i} u(k) \\ \qquad\qquad\qquad + G_{a_i} y_c(k+1), \\ y_i(k) = C_{a_i} X_{a_i}(k), \end{cases} \tag{5}$$

where $X_{a_i}(k)$ is the new augmented state vector defined by

$$X_{a_i}(k) = \begin{bmatrix} x_i(k) \\ e_{u0_i}(k) \end{bmatrix} \in \mathbb{R}^{n_i+1}.$$

Define the new augmented matrices:

$$A_{a_i} = \begin{bmatrix} A_i & 0_{n_i \times 1} \\ -TC_iA_i & 1 \end{bmatrix} \in \mathbb{R}^{(n_i+1)\times(n_i+1)},$$

$$B_{a_i} = \begin{bmatrix} B_i \\ -TC_iB_i \end{bmatrix} \in \mathbb{R}^{(n_i+1)\times 1},$$

$$G_{a_i} = \begin{bmatrix} 0_{n_i \times 1} \\ T \end{bmatrix} \in \mathbb{R}^{(n_i+1)\times 1}$$

and

$$C_{a_i} = \begin{bmatrix} C_i & 0 \end{bmatrix} \in \mathbb{R}^{1\times(n_i+1)}.$$

Hence, the classical feedback control law, which guarantees both stability and dynamic behavior of the closed-loop system, is modified and computed:

$$u(k) = -\begin{bmatrix} k_{x_i} & k_{e_i} \end{bmatrix} \begin{bmatrix} x_i(k) \\ e_{u0_i}(k) \end{bmatrix} = -K_i X_{a_i}(k), \tag{6}$$

where $K_i \in \mathbb{R}^{1\times(n_i+1)}$ represents the feedback gain calculated based on an augmented partial model.

The most popular techniques, such as eigenstructure assignment or linear quadratic (LQ) optimization, can be used to determine the feedback gain matrix.

3. Fault estimation based on an unknown input multiobserver

In the presence of a sensor fault, the closed-loop system behavior is corrupted. Indeed, the real output does not converge to the desired input reference. Furthermore, fault compensation is carried out by the addition of a new control law. The newly added control law relies on the occurring fault estimation.

Considering a decoupled state multimodel subject to an additive sensor fault, the state representation of the faulty decoupled multimodel is given by

$$\begin{cases} x_i(k+1) = A_i x_i(k) + B_i u(k), \\ y_i(k) = C_i x_i(k) + F_c f_c(k), \\ y_{\mathrm{MM}}(k) = \sum\limits_{i=1}^{Nm} \mu_i(\nu_{k-1}) y_i(k), \end{cases} \tag{7}$$

where $f_c(k)$ and F_c are the sensor fault and its fault distribution matrix, respectively.

3.1. Discrete unknown input multiobserver synthesis. Recall the faulty decoupled multimodel (7) rewritten in compact form (Orjuela *et al.*, 2009):

$$\begin{cases} x_{cf}(k+1) = A_{cf} x_{cf}(k) + B_{cf} u(k), \\ y_{\mathrm{MM}}(k) = C_{cf}(k) x_{cf}(k) + F_c f_c(k), \end{cases} \tag{8}$$

where $x_{cf}(k)$ is the compact state vector defined by

$$\begin{aligned} x_{cf}(k) = \begin{bmatrix} x_1^T(k) & \cdots & x_i^T(k) & \cdots & x_{Nm}^T(k) \end{bmatrix}^T \\ \in \mathbb{R}^n, \quad n = \sum_{i=1}^{Nm} n_i, \end{aligned} \tag{9}$$

A_{cf} and B_{cf} are the augmented matrices

$$A_{cf} = \begin{bmatrix} A_1 & 0 & \cdots & & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & & A_i & & \\ & & & \ddots & 0 \\ 0 & \cdots & & 0 & A_{Nm} \end{bmatrix} \in \mathbb{R}^{n\times n},$$

$$B_{cf} = \begin{bmatrix} {B_1}^T & \cdots & {B_i}^T & \cdots & {B_{Nm}}^T \end{bmatrix}^T \in \mathbb{R}^{n\times m},$$

$C_{cf}(k)$ is a time varying matrix. It can be written, taking into account the properties (2), as a weighted sum of matrices

$$\begin{aligned} C_{cf}(k) &= \sum_{i=1}^{Nm} \mu_i(\nu_{k-1})\widetilde{C}_{cf_i}, \\ \widetilde{C}_{cf_i} &= \begin{bmatrix} 0 & \cdots & C_i & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{1\times n}. \end{aligned} \tag{10}$$

To estimate both the state vector and the sensor fault, we propose to design an unknown input multiobserver. Nevertheless, the decoupled multimodel (8) seems appropriate for the proposed multiobserver. To make this decoupled multimodel suitable for unknown input multiobserver synthesis, assume that a sensor fault has as a dynamic (Theilliol *et al.*, 2000; Sobhani and Poshtan, 2012; Chaves *et al.*, 2019)

$$f_c(k+1) = f_c(k) + T u_c(k), \tag{11}$$

where $u_c(k)$ represents the sensor error input and T is the sampling time.

Then, a new augmented decoupled multimodel is defined including Eqn. (11),

$$\begin{cases} \bar{x}_{cf}(k+1) = \bar{A}_{cf}\bar{x}_{cf}(k) + \bar{B}_{cf}u(k) + \bar{T}u_c(k), \\ y_{\mathrm{MM}}(k) = \bar{C}_{cf}(k)\bar{x}_{cf}(k), \end{cases} \tag{12}$$

denoting by

$$\bar{x}_{cf}(k) = \begin{bmatrix} x_{cf}(k) \\ f_c(k) \end{bmatrix} \in \mathbb{R}^{n+1} \tag{13}$$

the new augmented state vector.

This augmented state vector is defined based on the compact form, where the state vector $x_{cf}(k) \in \mathbb{R}^n$. In the case of a SISO nonlinear system, the real output is subject to only one sensor fault. The new matrices are as follows:

$$\bar{A}_{cf} = \begin{bmatrix} A_{cf} & 0_{n\times 1} \\ 0_{1\times n} & 1 \end{bmatrix} \in \mathbb{R}^{(n+1)\times(n+1)},$$

$$\bar{B}_{cf} = \begin{bmatrix} B_{cf} \\ 0 \end{bmatrix} \in \mathbb{R}^{(n+1)\times 1},$$

$$\bar{C}_{cf}(k) = \begin{bmatrix} C_{cf}(k) & F_c \end{bmatrix} \in \mathbb{R}^{1\times(n+1)}$$

$$\bar{T} = \begin{bmatrix} 0_{n\times 1} \\ T \end{bmatrix} \in \mathbb{R}^{(n+1)\times 1}.$$

The decoupled multimodel (8) affected by a sensor fault is rewritten as a decoupled multimodel affected by one actuator fault.

The unknown input observer has been largely conceived and exploited in several research works. Buciakowski *et al.* (2017) and Witczak *et al.* (2016) designed a robust unknown input observer to estimate states and faults for nonlinear systems. However, the observer design seems difficult in the general case where nonlinear models have several structures since the nonlinear terms are involved indifferent ways.

Nevertheless, the multimodel approach highlights a set of simple and linear local models offering the possibility of an easier synthesis of observers extended from linear cases irrespective of the system complexity. Based on the multimodel representation that is recognized by its ability to decompose the nonlinear system in a set of partial models, a local observer is associated with each local model where a global observer known as the multiobserver is defined. Compared with the existing discrete unknown input observers, the proposed unknown input multiobserver is able to estimate states of any nonlinear system irrespective of its complexity. Indeed, the existing discrete unknown input observers are dedicated to estimate state variables of a specific class of nonlinear systems. However, based on the multimodel representation, the proposed multiobserver structure is known as a general form of observers that estimate states of a wide range of nonlinear systems. The proposed unknown input multiobserver (UIMO) provides estimates of the nonlinear system states, particularly the decoupled states of the multimodel. Moreover, it has an analytical form resulting from the aggregation of local observers that performs the stability and convergence study of the estimation error. It is synthesized based on mixing the outputs of the partial models. Therefore, it yields an excellent compromise between the generality and the practical usability, independently of the classes of nonlinear systems.

According to the state representation (12), an unknown input multiobserver can be easily built:

$$\begin{cases} \bar{z}_{cf}(k+1) = \bar{N}_{cf}\bar{z}_{cf}(k) + \bar{G}_{cf}u(k) + \bar{L}_{cf}y_{\mathrm{MM}}(k), \\ \hat{\bar{x}}_{cf}(k) = \bar{z}_{cf}(k) - \bar{E}_{cf}y_{\mathrm{MM}}(k), \\ \hat{y}_{\mathrm{MM}}(k) = \bar{C}_{cf}(k)\hat{\bar{x}}_{cf}(k), \end{cases} \tag{14}$$

where $\bar{z}_{cf}(k) \in \mathbb{R}^{n+1}$ is the multiobserver state vector and $\hat{\bar{x}}_{cf}(k) \in \mathbb{R}^{n+1}$ is the estimated state vector,$\bar{N}_{cf} \in \mathbb{R}^{(n+1)\times(n+1)}$, $\bar{L}_{cf} \in \mathbb{R}^{(n+1)\times 1}$, $\bar{G}_{cf} \in \mathbb{R}^{(n+1)\times 1}$ and $\bar{E}_{cf} \in \mathbb{R}^{(n+1)\times 1}$ are the multiobserver gains determined later.

Define the state estimation error

$$\bar{e}_x(k) = \bar{x}_{cf}(k) - \hat{\bar{x}}_{cf}(k). \tag{15}$$

It can be obtained as

$$\bar{e}_x(k) = \left(\mathrm{I}_{n+1} + \bar{E}_{cf}\bar{C}_{cf}(k)\right)\bar{x}_{cf}(k) - \bar{z}_{cf}(k). \quad (16)$$

Setting

$$\begin{aligned}\bar{P}_{cf}(k) &= \mathrm{I}_{n+1} + \bar{E}_{cf}\bar{C}_{cf}(k),\\ \bar{P}_{cf}(k) &\in \mathbb{R}^{(n+1)\times(n+1)},\end{aligned} \quad (17)$$

Equation (16) can be rewritten as

$$\bar{e}_x(k) = \bar{P}_{cf}(k)\overline{x}_{cf}(k) - \bar{z}_{cf}(k). \quad (18)$$

The evolution of the state estimation error can be expressed by

$$\begin{aligned}&\bar{e}_x(k+1)\\ &= \bar{N}_{cf}\bar{e}_x(k) + \left[\bar{P}_{cf}(k+1)\bar{B}_{cf} - \bar{G}_{cf}\right]u(k)\\ &\quad + \left[\bar{P}_{cf}(k+1)\bar{A}_{cf} - \bar{N}_{cf} - \bar{K}_{cf}\bar{C}_{cf}(k)\right]\bar{x}_{cf}(k)\\ &\quad + \bar{P}_{cf}(k+1)\bar{T}u_c(k),\end{aligned} \quad (19)$$

where

$$\bar{K}_{cf} = \bar{L}_{cf} + \bar{N}_{cf}\bar{E}_{cf}, \quad \bar{K}_{cf} \in \mathbb{R}^{(n+1)\times 1}. \quad (20)$$

Based on the decoupling technique, the discrete unknown input multiobserver gains are computed when the following relation skips are satisfied:

$$\begin{cases}\bar{P}_{cf}(k+1)\bar{A}_{cf} - \bar{N}_{cf} - \bar{K}_{cf}\bar{C}_{cf}(k) = 0,\\ \bar{P}_{cf}(k+1)\bar{B}_{cf} - \bar{G}_{cf} = 0,\\ \bar{P}_{cf}(k+1)\bar{T} = 0.\end{cases} \quad (21)$$

Retaining the previous conditions, the state estimation error is reduced to

$$\bar{e}_x(k+1) = \bar{N}_{cf}\bar{e}_x(k). \quad (22)$$

The convergence of the state estimation error relies on matrix $\bar{N}_{cf}$, which must be a Hurwitz matrix.

The stability of the multiobserver is studied with the Lyapunov approach in terms of linear matrix inequalities. Exponential convergence of the estimation error towards zero is achieved if a symmetric positive definite matrix $\bar{X} = \bar{X}^T > 0$ exists satisfying the following theorem.

Theorem 1. *The augmented state estimation error converges exponentially to zero, under the condition of the existence of matrices $\bar{S}_{cf}$ and $\bar{W}_{cf}$ and a symmetric positive definite matrix $\bar{X} = \bar{X}^T > 0$ of appropriate dimensions, if the following LMIs hold for $i, j = 1, \ldots, N_m$:*

$$\begin{bmatrix}(1-2\alpha)\bar{X} & Z_{ij}^T\\ Z_{ij} & \bar{X}\end{bmatrix} > 0 \quad (23)$$

with $Z_ij^T = \bar{X}\bar{A}_{cf} + \bar{S}_{cf}\tilde{\bar{C}}_{cf_i}\bar{A}_{cf} - \bar{W}_{cf}\tilde{\bar{C}}_{cf_j}$ while the following equality constraints are satisfied:

$$\bar{X}\bar{T} + \bar{S}_{cf}\tilde{\bar{C}}_{cf_i}\bar{T} = 0, \quad i = 1, \ldots, N_m, \quad (24)$$

where α is the decay rate, serving to quantify the convergence speed of the estimation error.

For a chosen decay rate $0 < \alpha < 0.5$, the LMIs (23) are solved. Thereafter, the multiobserver gains are calculated as follows:

$$\begin{cases}\bar{K}_{cf} = \bar{X}^{-1}\bar{W}_{cf},\\ \bar{E}_{cf} = \bar{X}^{-1}\bar{S}_{cf},\\ \bar{G}_{cf}(k+1) = \bar{P}_{cf}(k+1)\bar{B}_{cf},\\ \bar{N}_{cf}(k+1) = \bar{P}_{cf}(k+1)\bar{A}_{cf} - \bar{K}_{cf}\bar{C}_{cf}(k),\\ \bar{L}_{cf}(k+1) = \bar{K}_{cf} - \bar{N}_{cf}(k+1)\bar{E}_{cf}.\end{cases} \quad (25)$$

Once the augmented state vector is estimated with a good accuracy, the estimated sensor fault is the last component of the estimated augmented state vector.

The fault estimation shows a powerful ability to provide exact information about the fault online.

3.2. Decoupling conditions.

The unknown input multiobserver provides supplementary conditions compared with classical multiobservers. It decouples unknown inputs and removes their effect from the error estimate.

To prove the effectiveness of the supplementary technique, it is crucial to satisfy the condition

$$\bar{P}_{cf}(k+1)\bar{T} = 0. \quad (26)$$

Taking into account (17), the previous equation is transformed to

$$\bar{T} + \bar{E}_{cf}\bar{C}_{cf}(k+1)\bar{T} = 0 \quad (27)$$

Since

$$\bar{C}_{cf}(k+1) = \sum_{i=1}^{Nm}\mu_i(\nu_k)\tilde{\bar{C}}_{cf_i}, \quad (28)$$

taking into account (28), Eqn. (27) can be rewritten as

$$\sum_{i=1}^{Nm}\mu_i(\nu_k)\left[\bar{T} + \bar{E}_{cf}\tilde{\bar{C}}_{cf_i}\bar{T}\right] = 0, \quad (29)$$

A possible solution among several others, owing to the convex sum properties, can be proposed:

$$\bar{T} + \bar{E}_{cf}\tilde{\bar{C}}_{cf_i}\bar{T} = 0, \quad i = 1, \ldots, Nm. \quad (30)$$

A single solution of (30) exists if the following Nm conditions are satisfied:

$$\mathrm{rank}(\tilde{\bar{C}}_{cf_i}\bar{T}) = \mathrm{rank}(\bar{T}), \quad i = 1, \ldots, Nm. \quad (31)$$

Thus, it is necessary to determine the multiobserver gain $\bar{E}_{cf}$. Since the conditions (31) are satisfied, a unique $\bar{E}_{cf}$ is determined exploiting the pseudo-inverse formula,

$$\begin{aligned}\bar{E}_{cf} &= -\bar{T}\left(\tilde{\bar{C}}_{cf_i}\bar{T}\right)^+\\ &\quad - \Phi_i\left[\mathrm{I} - \left(\tilde{\bar{C}}_{cf_i}\bar{T}\right)\left(\tilde{\bar{C}}_{cf_i}\bar{T}\right)^+\right],\end{aligned} \quad (32)$$

where $\left(\tilde{\bar{C}}_{cf_i}\bar{T}\right)^+$ is the generalized inverse of $\tilde{\bar{C}}_{cf_i}\bar{T}$ defined by (33) and Φ_i is a real arbitrarily chosen matrix of appropriate dimensions,

$$\left(\tilde{\bar{C}}_{cf_i}\bar{T}\right)^+ = \left(\left(\tilde{\bar{C}}_{cf_i}\bar{T}\right)^T\left(\tilde{\bar{C}}_{cf_i}\bar{T}\right)\right)^{-1}\left(\tilde{\bar{C}}_{cf_i}\bar{T}\right)^T. \quad (33)$$

Proof. (of Theorem 1) Based on the second method of Lyapunov, the stability of the multiobserver is studied. Indeed, the exponential convergence of the state estimation error is guaranteed if there exists a Lyapunov function $V(k) > 0$ and $\alpha > 0$ such that

$$\exists \bar{X} = \bar{X}^T > 0, \quad \Delta V(k) + 2\alpha V(k) < 0, \quad (34)$$

where

$$\Delta V(k) = V(k+1) - V(k), \quad (35)$$

with $V(k)$ being a candidate Lyapunov function which guarantees the convergence of the estimation error near zero, defined by

$$V(k) = \bar{e}_x(k)^T \bar{X} \bar{e}_x(k), \quad \bar{X} = \bar{X}^T > 0. \quad (36)$$

In this case, the difference $\Delta V(k)$, given in Eqn. (35), is equivalent to

$$\Delta V(k) = \bar{e}_x(k+1)^T \bar{X}\,\bar{e}_x(k+1) - \bar{e}_x(k)^T \bar{X}\,\bar{e}_x(k) \quad (37)$$

By reference to (22), (37) is given as follows:

$$\Delta V(k) = \bar{e}_x(k)^T (\bar{N}_{cf}^T \bar{X} \bar{N}_{cf} - \bar{X}) \bar{e}_x(k). \quad (38)$$

Taking account of the Lyapunov function and its difference given by the previous relation, Eqn. (34) can be rewritten as

$$\begin{aligned}&\Delta V(k) + 2\alpha V(k)\\ &= \bar{e}_x(k)^T \left(\bar{N}_{cf}^T \bar{X} \bar{N}_{cf} + (2\alpha - 1)\bar{X}\right) \bar{e}_x(k) < 0. \end{aligned}\quad (39)$$

Replacing $\bar{N}_{cf}$ by its defining expression (see Eqn. (25)), (39) can be written as follows:

$$\begin{aligned}\bar{e}_x(k)^T \{ &\left[\bar{P}_{cf}(k+1)\bar{A}_{cf} - \bar{K}_{cf}\bar{C}_{cf}(k)\right]^T \bar{X}\\ &\left[\bar{P}_{cf}(k+1)\bar{A}_{cf} - \bar{K}_{cf}\bar{C}_{cf}(k)\right]\\ &+ (2\alpha - 1)\,\bar{X}\}\bar{e}_x(k) < 0,\end{aligned} \quad (40)$$

which can be reduced to

$$\begin{aligned}&\left[\bar{P}_{cf}(k+1)\bar{A}_{cf} - \bar{K}_{cf}\bar{C}_{cf}(k)\right]^T \bar{X}\\ &\left[\bar{P}_{cf}(k+1)\bar{A}_{cf} - \bar{K}_{cf}\bar{C}_{cf}(k)\right] + (2\alpha - 1)\,\bar{X} < 0.\end{aligned} \quad (41)$$

Replacing $\bar{P}_{cf}(k+1)$ and $\bar{C}_{cf}(k)$ by their expressions in (17) and (10) and taking into account of (2), the previous inequality can be rewritten as

$$\begin{aligned}&\left[\bar{A}_{cf} + \bar{E}_{cf}\tilde{\bar{C}}_{cf_i}\bar{A}_{cf} - \bar{K}_{cf}\tilde{\bar{C}}_{cf_j}\right]^T \bar{X}\\ &\left[\bar{A}_{cf} + \bar{E}_{cf}\tilde{\bar{C}}_{cf_i}\bar{A}_{cf} - \bar{K}_{cf}\tilde{\bar{C}}_{cf_j}\right] + (2\alpha - 1)\,\bar{X} < 0.\end{aligned} \quad (42)$$

Setting

$$\bar{W}_{cf} = \bar{X}\bar{K}_{cf}, \qquad \bar{S}_{cf} = \bar{X}\bar{E}_{cf} \quad (43)$$

and taking into account the property of the matrix $\bar{X}$, we get

$$\begin{aligned}&(1 - 2\alpha)\,\bar{X} - \left[\bar{X}\bar{A}_{cf} + \bar{S}_{cf}\tilde{\bar{C}}_{cf_i}\bar{A}_{cf} - \bar{W}_{cf}\tilde{\bar{C}}_{cf_j}\right]^T\\ &\bar{X}^{-1}\left[\bar{X}\bar{A}_{cf} + \bar{S}_{cf}\tilde{\bar{C}}_{cf_i}\bar{A}_{cf} - \bar{W}_{cf}\tilde{\bar{C}}_{cf_j}\right] > 0.\end{aligned} \quad (44)$$

To obtain the LMIs (23), the Schur complement is applied to the latest inequality. Taking into account the relation (43), the equality constraints (30) can be rewritten as follows:

$$\bar{X}\bar{T} + \bar{S}_{cf}\tilde{\bar{C}}_{cf_i}\bar{T} = 0, \quad i = 1, \dots, N_m. \quad (45)$$

■

Afterwards, the proposed UIMO is synthesized and estimates the state vector and the sensor fault simultaneously. The estimated augmented state vector is needed to reconfigure the control law and maintain the tracking performances.

4. Switching strategy for an effective fault tolerant control

In most practical systems, controllers are synthesized neglecting that faults can occur (a fault-free case). The nominal control law is updated and modified according to the occurrence of a sensor fault. Furthermore, it is necessary to take account of the fact that direct sensor fault accommodation should be considered to prevent performances losses, maintain the system stability and ensure trajectory tracking.

A single controller can hardly tolerate faults affecting the nonlinear system represented by a decoupled state multimodel. Indeed, a multimodel control strategy is studied.

According to the interesting feature of the proposed multimodel approach, a bank of controllers is built. Each designed controller generates a local control law.

Therefore, an additive control law is computed and added to the nominal one when a sensor fault occurs. The new partial control law is computed as follows:

$$u_i(k) = -k_{x_i}\hat{x}_i(k) - k_{e_i}e_{u0_i}(k) + u_{\text{add}_i}(k). \quad (46)$$

The partial measured output and the integral error $e_{u0_i}(k)$ are changed since the occurrence of a sensor fault and their expressions are given as follows:

$$\begin{cases} y_i(k) = C_i x_i(k) + F_c f_c(k), \\ e_{u_i}(k) = e_{u0_i}(k) + \tilde{e}_{f_i}(k), \end{cases} \tag{47}$$

where

$$\begin{cases} e_{u_i}(k+1) = e_{u_i}(k) + T(y_c(k+1) \\ \qquad\qquad - C_i\hat{x}_i(k+1) - F_c\hat{f}_c(k+1)), \\ \tilde{e}_{f_i}(k+1) = e_{u_i}(k) - e_{u0_i}(k) - TF_c\hat{f}_c(k+1). \end{cases} \tag{48}$$

These conditions lead the additional control law to compensate the sensor fault. It is computed as follows:

$$u_{\mathrm{add}_i}(k) = -k_{x_i}C_i^+F_c\hat{f}_c(k) - k_{e_i}\tilde{e}_{f_i}(k). \tag{49}$$

Moreover, the faulty nonlinear system is maintained as the nominal one when the partial control laws are modified, avoiding huge performances losses and improving trajectory tracking.

Then, it is crucial to calculate a global control law which makes the nonlinear system output track the reference input and compensates the sensor fault effect. A set of control laws are generated. Hence, the sensor fault is accommodated by one of a set of selected controllers under a suitable switching strategy.

The switching strategy is based on the evaluation of the quadratic criterion that selects the most appropriate generated fault tolerant control law guaranteeing trajectory tracking and system stability. The most appropriate controller is the one that yields the smallest value of the criterion (ben Atia *et al.*, 2015; 2018; Allaoui *et al.*, 2017; Messaoud *et al.*, 2009).

The criterion is given by

$$J_i(k) = \varsigma e_{p_i}^2(k) + \beta\sum_{h=1}^{k} e^{-\delta(k-h)}e_{p_i}^2(h), \qquad i = 1,\dots,Nm, \tag{50}$$

where $e_{p_i}(k) = y(k) - \hat{y}_{c_i}(k)$ means the error between the real output and the i-th predicted reference input, ς, β and δ are positive tuning parameters. They determine the switching speed between controllers, ς, β represent the weighting factors of the instantaneous and the long-term measures accuracy. Moreover, δ denotes the forgetting factor that ensures the boundness of the partial criterion $J_i(k)$ for the bounded error $e_{p_i}(k)$.

Despite the presence of a fault, a stable and certain commutation to the correct and suitable controller is established. The evaluation of the criterion (50) leads to correct identification and selection of the appropriate controller that is convenient to satisfy the closed-loop performances in terms of trajectory tracking and fault compensation (Narendra and Balakrishnan, 1997).

Algorithm 1. Final design procedure of the proposed approach.

1: An offline multimodel identification is adopted to represent the nonlinear system.
2: An unknown input multiobserver, based on an augmented decoupled state multimodel, is designed to estimate accurately both sensor faults and state variables (Eqn. (14)).
3: A set of controllers are designed where nominal control laws are reconfigured by computation of additive control laws when the sensor fault appears (Eqn. (46)).
4: A switching strategy is investigated to select the most suitable control in the sense of the desired performance criterion (Eqn. (50)).

The local predicted reference input $\hat{y}_{c_i}(k)$, deduced from (46), (48) and (49), can be written as follows:

$$\begin{aligned}\hat{y}_{ci}(k) = (k_{e_i}T)^{-1}(&-u_i(k) - k_{x_i}\hat{x}_i(k) \\ &- k_{e_i}e_{u0_i}(k-1) + k_{e_i}TC_i\hat{x}_i(k) \\ &+ u_{\mathrm{add}_i}(k))\end{aligned} \tag{51}$$

The final design procedure of the proposed approach is detailed in Algorithm 1.

5. Numerical example

We consider nonlinear SISO system described by a discrete decoupled multimodel including two heterogenous partial models (Orjuela *et al.*, 2009):

Partial model 1:

$$A_1 = \begin{bmatrix} -0.5 & -0.7 \\ 0.4 & 0.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ -0.8 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.7 & 0.4 \end{bmatrix}.$$

Partial model 2:

$$A_2 = \begin{bmatrix} -0.7 & 0.2 & 0.5 \\ 0.3 & -0.4 & -0.1 \\ -0.2 & -0.3 & 0.6 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.4 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.3 & 0 & 0.5 \end{bmatrix}.$$

The input $u(k) \in [0,1]$ is retained as a decision variable. The centers and dispersion, in this case, are as follows: $c_1 = 0.25$, $c_2 = 0.75$ and $\sigma = 0.4$.

Real processes operate over a wide range of conditions and it seems difficult to use approximate models linearized around a single operating point. To deal with this problem, a multimodel identification procedure can be applied. An off-line identification procedure is established to obtain a decoupled multimodel, with a set

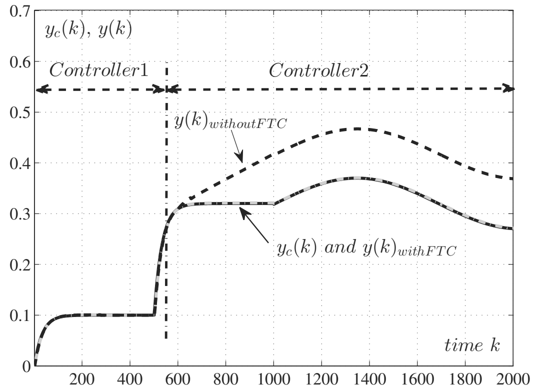


Fig. 1. Evolution of the output and the reference trajectory (with and without FTC).

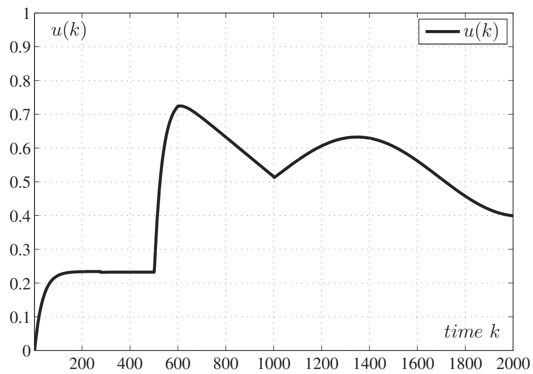


Fig. 2. Evolution of effective control.

of partial models representing the nonlinear system. The parameters of the resulting partial models are constant and, consequently, the decoupling conditions, in this case, are satisfied and a relatively perfect decoupling can be achieved in numerical and experimental studies.

The evolution of the output and the reference input, when an additive sensor fault appears at $k = 600$, is depicted in Fig. 1. The proposed strategy shows its capacity in improving the closed-loop performances in terms of tracking and sensor fault compensation.

The smallest value of the criterion allows choosing an appropriate controller according to a switching mechanism. A control law is obtained from switching between local control signals,

$$u(k) = u_i(k), \quad i = 1, 2. \tag{52}$$

The evolution of the effective control applied to the nonlinear system is shown in Fig. 2.

An active FTC requires accurate fault estimation to accomplish fault compensation. Moreover, the control performance relies on the sensor fault estimation quality, which must be rapid and accurate. In the present paper, the synthesized unknown input multiobserver is an efficient one that estimates rapidly and accurately both sensor faults and states. Therefore, the accurate estimation provides an efficient control law that allows reference trajectory tracking and fault tolerance.

Safety problems are due to a different kind of failures affecting actuators, sensors and process that may cause process performance degradation and even system breakdowns. Sensor faults, which can be evoked by many different kinds of problems, are generally related to wrong readings due to a failure in the sensor components causing the loss of effectiveness. They are considered additive signals on the measurements.

Thus, the estimation of the occurring fault is accomplished by the proposed UIMO as illustrated in Fig. 3.

The unknown input multiobserver performs accurate fault estimation that allows the control law to be the most suitable one to maintain the trajectory tracking and fault compensation goals.

The proposed discrete multiobserver shows its capacity in estimating state variables with a good accuracy (Figs. 4–8).

Compared with a proportional integral multiobserver (PIMO) in the case of a time varying fault, the proposed unknown input multiobserver performs rapid and accurate sensor fault estimation. Hence, the proposed strategy provides satisfactory trajectory tracking and fault tolerance. The outputs and the reference trajectory are compared in Fig. 9.

The proposed UIMO shows its capacity in terms of accuracy in estimation despite the fact that the fault is a time varying one. Thus, satisfactory trajectory tracking and fault tolerance are fulfilled.

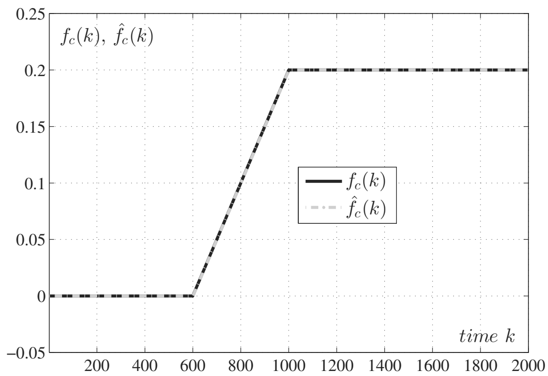


Fig. 3. Evolution of the sensor fault and its estimate.

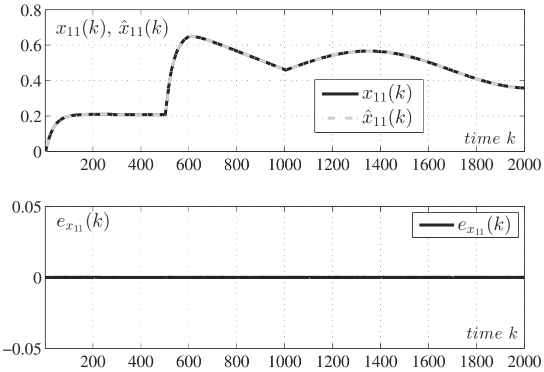


Fig. 4. Evolution of the first state variable of the first partial model, its estimate and the estimation error.

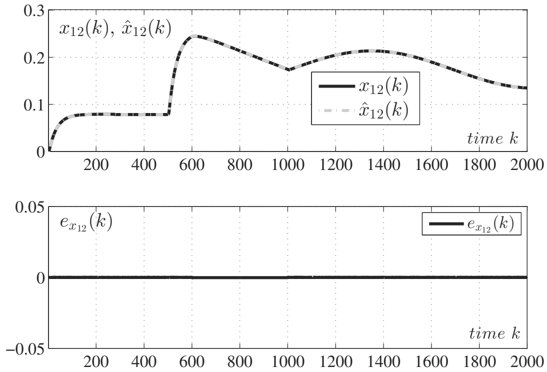


Fig. 6. Evolution of the first state variable of the second partial model, its estimate and the estimation error.

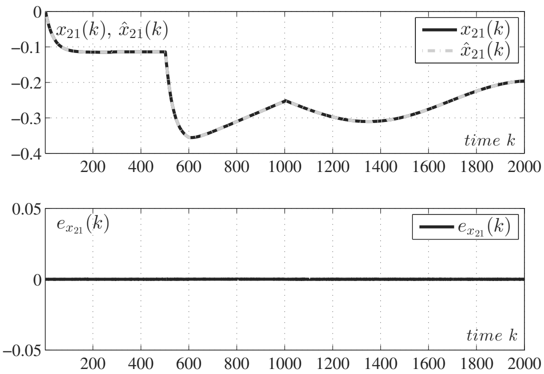


Fig. 5. Evolution of the second state variable of the first partial model, its estimate and the estimation error.

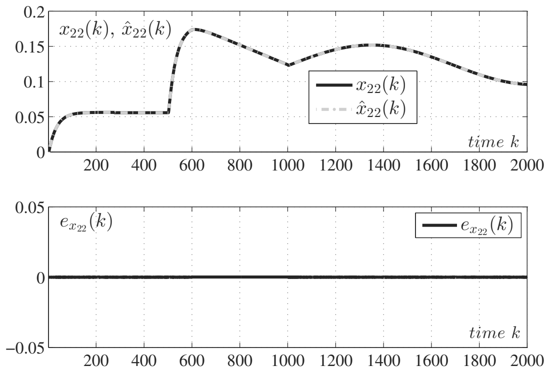


Fig. 7. Evolution of the second state variable of the second partial model, its estimate and the estimation error.

6. Practical application to a transesterification reactor

6.1. Process description. Chemical reactors are the most often encountered industrial process in chemical and pharmaceutical applications (Messaoud *et al.*, 2009). Their flexibility and their polyvalent character made them attractive for several applications in research. However, these batch reactors have nonlinear behavior, which makes diagnosis and control more difficult. Batch reactors can be used in the extraction of oils. Plant oils are generally extracted and undergo reactions in order to produce fuels. Indeed, oils need small modification such as a transesterification reaction to produce cleaner biofuels. To accelerate the conversion, a catalyst such as a strong acid or base is needed.

The chemical process (Fig. 10) consists of a stirred tank equipped with a jacket where heat exchange between a cooling fluid and the reaction mixture is provided. The fluid flow rate of the heating cooling reactor is constant. The fluid temperature within the jacket is regulated with an external servo system including a plate heat exchange with electric resistors. The heating of the fluid is insured by electric resistors, whereas the cooling fluid which is also named tap water is provided by a plate exchanger. To measure the chemical process' temperatures and the inlet and the outlet jacket temperature, many temperature sensors are implemented.

The chemical reactor is exploited in batch mode with which biodiesel can be used. The transesterification reaction takes place in these steps. Tallow (animal fat) or vegetable oils (virgin or used) which are considered fatty materials (FM) are mixed with the alcohol to produce ester and glycerol. The transesterification is called also alcoholysis. The mentioned reaction is described as follows (Meher *et al.*, 2006; Ma and Hanna, 1999):

$$\text{FM} + \text{Alcohol} \rightleftharpoons \text{Esters} + \text{Glycerol}.$$

Triglycerides and a minor amount of mono and diglycerides are associated to form FM. Indeed, a molecule of triglycerides is composed from a three carbon glycerol head group chemically altered to three fatty acid chains,

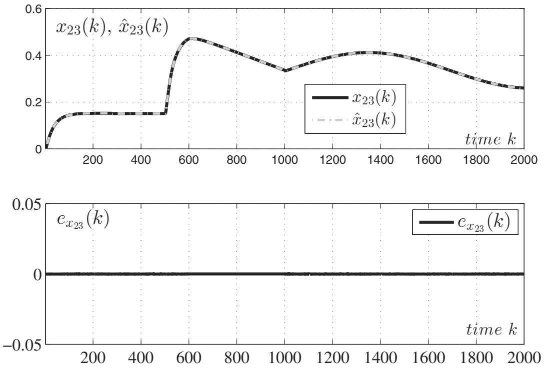


Fig. 8. Evolution of the third state variable of the second partial model, its estimate and the estimation error.

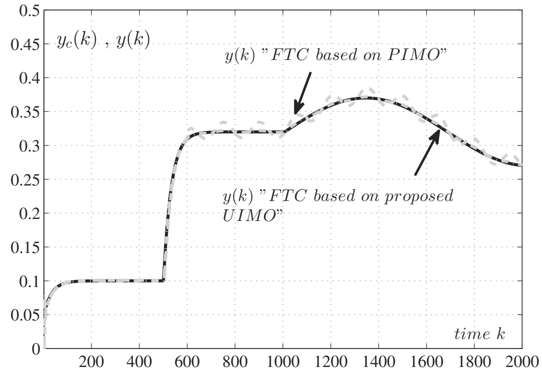
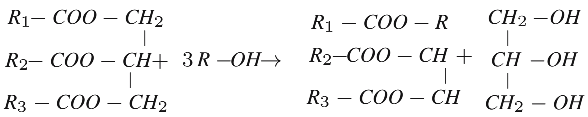


Fig. 9. Evolutions of reference trajectory and outputs.



Biodiesels are considered an interesting alternative to bio-based fuel, which make up for fossil fuel resources that will be depleted in the near future. Because they are eco-friendly, biodiesels have attracted increasing attention and have been developed to be used as a renewable energy source. In fact, biofuels help to reduce global warming by decreasing the emissions of particles, sulfur and carbon dioxide.

The system can be considered as a single-input single-output one. The heating power Q [W] is an input and the output is the chemical reactor temperature T_R°C. An experimental study identified the sampling period to be 120 s.

6.2. Experimental results. Based on a set of data, a multimodel identification procedure (Messaoud and Ben Abdennour, 2018) is established and yields a decoupled multimodel with three heterogenous partial models:

Partial model 1:

$$A_1 = \begin{bmatrix} 0.4581 & 0.4871 \\ 0.8951 & 0.0680 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0008 \\ 0.0210 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Partial model 2:

$$A_2 = \begin{bmatrix} 0.4699 & 0.4881 \\ 0.8950 & 0.0670 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0019 \\ 0.0185 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Partial model 3:

$$A_3 = 0.9906, \quad B_3 = 0.0033, \quad C_3 = 1.$$

The desired reference trajectory T_C is defined in three phases: heating, reaction and cooling.

When an additive fault occurs, the monitored chemical reactor temperature changes. The experimental results of the proposed control strategy are presented and show, again, the efficiency of the strategy in terms of trajectory tracking.

Figure 11 illustrates the evolution of the reactor output.

When the sensor fault is estimated with a good accuracy, the right control law is considered after computing an additive control law. Therefore, the reactor's temperature is maintained and the trajectory tracking is carried out. In this condition, the evolution of the heating power Q [W], which denotes the control input,

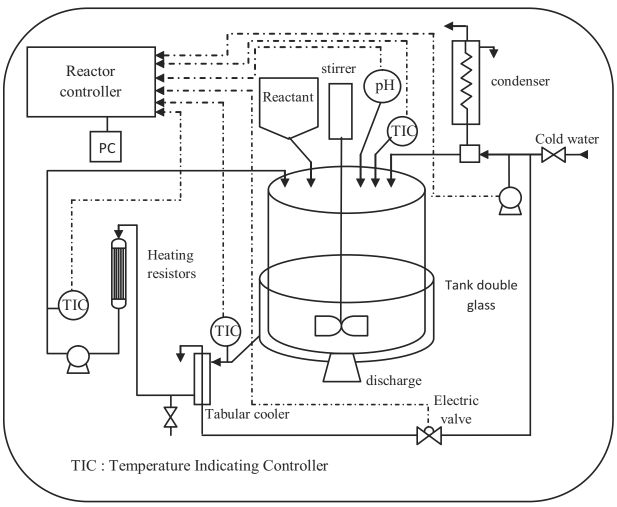


Fig. 10. Synoptic scheme of the transesterification reactor considered.

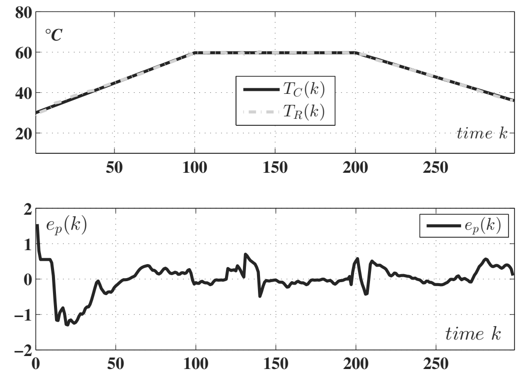


Fig. 11. Evolution of the real output, the reference trajectory and the tracking error.

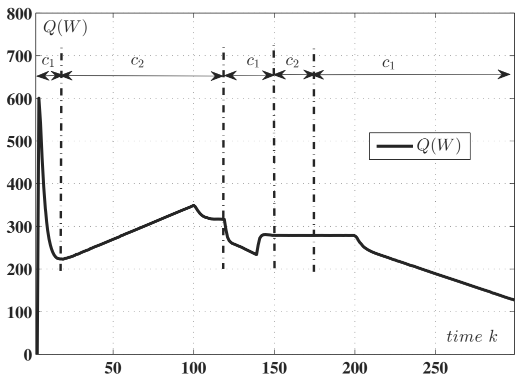


Fig. 12. Evolution of control input.

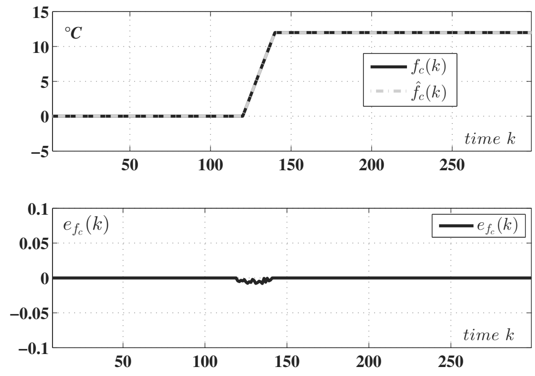


Fig. 13. Evolution of the sensor fault, its estimate and the estimation error.

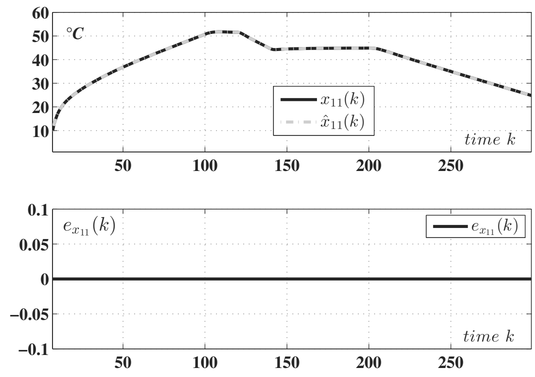


Fig. 14. Evolution of the first state variable of the first partial model, its estimate and the estimation error.

is shown in Fig. 12. The suitable control law is selected by the proposed switching strategy between controllers ($c_i, i = 1, 2, 3$).

Figure 13 illustrates the evolution of a sensor fault, its estimate and the estimation error.

Since fault estimation is achieved accurately, the most suitable effective control law is chosen. Consequently, the FTC strategy is actively concerned and the reactor's temperature is maintained. The accuracy of the model relies on the best states estimation established by the proposed unknown input multiobserver.

The evolution of state variables, their estimates and the estimation errors are illustrated in Figs. 14–18 to prove the efficiency of the proposed unknown input multiobserver in terms of accuracy.

7. Conclusion

An active fault tolerant control for a nonlinear system subject to a sensor fault was developed. A decoupled state multimodel was retained to represent the nonlinear system. An unknown input multiobserver was designed to estimate both state variables and sensor fault. Once accurate estimation was achieved, an additive control law was computed to tolerate the fault effect. Thanks to the features of the multimodel approach, a bank of controllers corresponding to local models was designed. Then, the most suitable controller was selected, based on a switching mechanism, in order to generate the appropriate control law that compensates the fault effect and improves the closed loop performances. The simulation results demonstrate the efficiency of the proposed control scheme. A real time application on a chemical reactor was realized to further evaluate, again, the efficiency of the adopted FTC strategy.

In future works, a multimodel structure considering noise will be adopted, a multimodel identification procedure will be developed and a robust multiobserver for nonlinear systems subject to external (habitual and harmonic) disturbances will be synthesized.

Acknowledgment

This work was supported by the Ministry of Higher Education and Scientific Research of Tunisia.

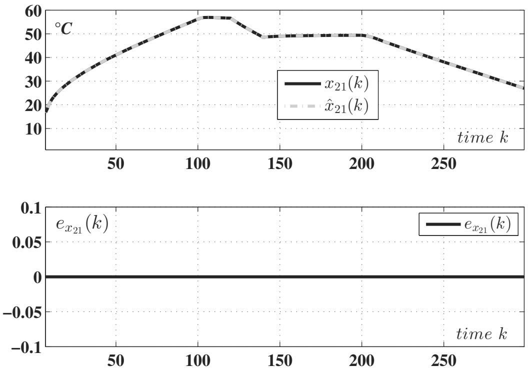


Fig. 15. Evolution of the second state variable of the first partial model, its estimate and the estimation error.

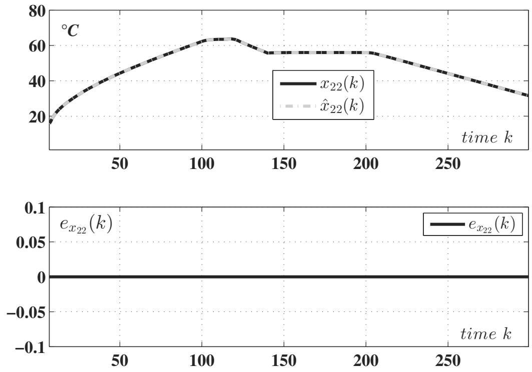


Fig. 17. Evolution of the second state variable of the second partial model, its estimate and the estimation error.

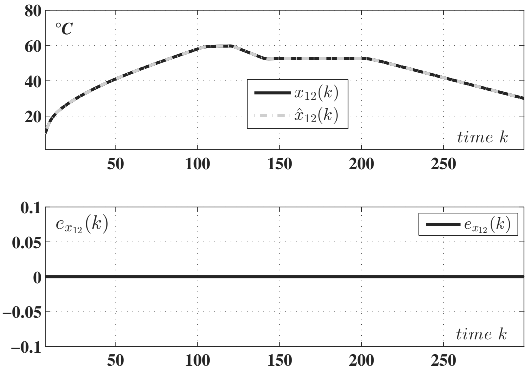


Fig. 16. Evolution of the first state variable of the second partial model, its estimate and the estimation error.

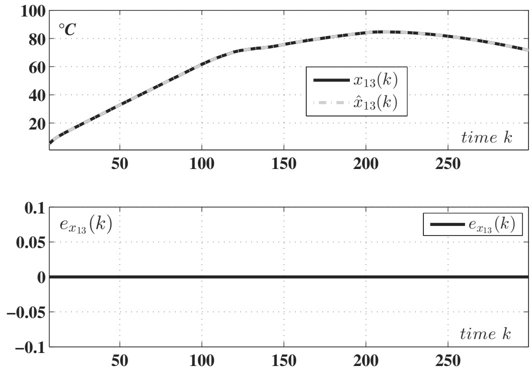


Fig. 18. Evolution of the state of the third partial model, its estimate and the estimation error.

References

Abdelkrim, N., Tellili, A. and Abdelkrim, M.N. (2012). Additive fault tolerant control applied to delayed singularly perturbed system, *Journal of Software Engineering and Applications* **5**(04): 217.

Allaoui, M., Messaoud, A., Dehri, K. and Ben Abdennour, R. (2017). Multimodel repetitive-predictive control of nonlinear systems: Rejection of unknown non-stationary sinusoidal disturbances, *International Journal of Control* **90**(7): 1478–1494.

Ben Atia, S., Messaoud, A. and Ben Abdennour, R. (2015). Decoupled multimodel predictive control based on multi-observer for discrete-time uncertain nonlinear systems, *12th International Multi-Conference on Systems, Signals and Devices (SSD), Mahdia, Tunisia*, pp. 1–8.

Ben Atia, S., Messaoud, A. and Ben Abdennour, R. (2018). An online identification algorithm of unknown time-varying delay and internal multimodel control for discrete non-linear systems, *Mathematical and Computer Modelling of Dynamical Systems* **24**(1): 26–43.

Ben Atia, S., Messaoud, A., Ltaief, M. and Ben Abdennour, R. (2014). Synthesis of multi-observers for discrete-time nonlinear systems with delayed output, *International Journal of Sciences and Techniques of Automatic Control and Computer Engineering* **8**(1): 1966–1981.

Bonfè, M., Castaldi, P., Mimmo, N. and Simani, S. (2011). Active fault tolerant control of nonlinear systems: The cart-pole example, *International Journal of Applied Mathematics and Computer Science* **21**(3): 441–445, DOI: 10.2478/v10006-011-0033-y.

Buciakowski, M., Witczak, M., Puig, V., Rotondo, D., Nejjari, F. and Korbicz, J. (2017). A bounded-error approach to simultaneous state and actuator fault estimation for a class of nonlinear systems, *Journal of Process Control* **52**: 14–25.

Chaves, E.R., André, F.A. and Maitelli, A.L. (2019). Robust observer-based actuator and sensor fault estimation for discrete-time systems, *Journal of Control, Automation and Electrical Systems* **30**(2): 160–169.

Du, D., Jiang, B. and Shi, P. (2014). Sensor fault estimation and accommodation for discrete-time switched linear systems, *IET Control Theory and Applications* **8**(11): 960–967.

Gao, Z., Cecati, C. and Ding, S.X. (2015). A survey of fault diagnosis and fault-tolerant techniques I: Fault diagnosis with model-based and signal-based approaches, *IEEE Transactions on Industrial Electronics* **62**(6): 3757–3767.

Ichalal, D., Marx, B., Ragot, J., Mammar, S. and Maquin, D. (2016). Sensor fault tolerant control of nonlinear Takagi–Sugeno systems. Application to vehicle lateral dynamics, *International Journal of Robust and Nonlinear Control* **26**(7): 1376–1394.

Jiang, J. and Yu, X. (2012). Fault-tolerant control systems: A comparative study between active and passive approaches, *Annual Reviews in Control* **36**(1): 60–72.

Jiang, Y. and Yin, S. (2018). Recent advances in key-performance-indicator oriented prognosis and diagnosis with a Matlab toolbox: DB-KIT, *IEEE Transactions on Industrial Informatics* **15**(5): 2849–2858, DOI: 10.1109/TII.2018.2875067.

Jiang, Y., Yin, S. and Kaynak, O. (2018). Data-driven monitoring and safety control of industrial cyber-physical systems: Basics and beyond, *IEEE Access* **6**: 47374–47384.

Khelassi, A., Theilliol, D., Weber, P. and Ponsart, J.C. (2011). Fault-tolerant control design with respect to actuator health degradation: An LMI approach, *IEEE International Conference on Control Applications, Denver, CO, USA*, pp. 983–988.

Ma, F. and Hanna, M.A. (1999). Biodiesel production: A review, *Bioresource Technology* **70**(1): 1–15.

Meher, L., Vidya S., D. and Naik, S. (2006). Technical aspects of biodiesel production by transesterification—A review, *Renewable and Sustainable Energy Reviews* **10**(3): 248–268.

Messaoud, A. and Ben Abdennour, R. (2018). An experimental validation of a new method for multimodel identification, *Journal of Dynamic Systems, Measurement, and Control* **140**(2): 024502.

Messaoud, A., Ltaief, M. and Ben Abdennour, R. (2009). Supervision based on partial predictors for a multimodel generalised predictive control: Experimental validation on a semi-batch reactor, *International Journal of Modelling, Identification and Control* **6**(4): 333–340.

Narendra, K.S. and Balakrishnan, J. (1997). Adaptive control using multiple models, *IEEE Transactions on Automatic Control* **42**(2): 171–187.

Noura, H., Theilliol, D., Ponsart, J.C. and Chamseddine, A. (2009). *Fault-Tolerant Control Systems: Design and Practical applications*, Springer Science & Business Media, Dordrecht.

Noura, H., Theilliol, D. and Sauter, D. (2000). Actuator fault-tolerant control design: Demonstration on a three-tank-system, *International Journal of Systems Science* **31**(9): 1143–1155.

Odgaard, P.F. and Stoustrup, J. (2012). Fault tolerant control of wind turbines using unknown input observers, 8*th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, Mexico City, Mexico*, pp. 313–318.

Orjuela, R., Marx, B., Ragot, J. and Maquin, D. (2009). On the simultaneous state and unknown input estimation of complex systems via a multiple model strategy, *IET Control Theory & Applications* **3**(7): 877–890.

Orjuela, R., Marx, B., Ragot, J. and Maquin, D. (2013). Nonlinear system identification using heterogeneous multiple models, *International Journal of Applied Mathematics and Computer Science* **23**(1): 103–115, DOI: 10.2478/amcs-2013-0009.

Pazera, M., Buciakowski, M. and Witczak, M. (2017). Robust multiple sensor fault-tolerant control for dynamic non-linear systems: Application to the aerodynamical twin-rotor system, *International Journal of Applied Mathematics and Computer Science* **28**(2): 297–308, DOI: 10.2478/amcs-2018-0021.

Pico, M.F. and Adam, E.J. (2017). Fault diagnosis and tolerant control using observer banks applied to continuous stirred tank reactor, *Advances in Science, Technology and Engineering Systems Journal* **1**(3): 171–181.

Prajapati, A.K. and Roy, B. (2016). Multi-fault diagnosis in three coupled tank system using unknown input observer, *IFAC-PapersOnLine* **49**(1): 47–52.

Rodrigues, M., Hamdi, H., Benhadj Braiek, N. and Theilliol, D. (2014). Observer-based fault tolerant control design for a class of LPV descriptor systems, *Journal of the Franklin Institute* **351**(6): 3104–3125.

Sobhani, M.H. and Poshtan, J. (2012). Fault detection and isolation using unknown input observers with structured residual generation, *International Journal of Instrumentation and Control Systems* **2**(2): 1–12.

Sojoudi, S., Lavaei, J. and Murray, R.M. (2011). Fault-tolerant controller design with applications in power systems and synthetic biology, *American Control Conference (ACC), San Francisco, CA, USA*, pp. 4135–4142.

Takagi, T. and Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control, *IEEE Transactions on Systems, Man, and Cybernetics* **15**(1): 116–132.

Tanaka, K. and Wang, H.D. (2001). *Fuzzy Control System Design and Analysis: A Linear Matrix Inequality Approach*, Wiley, New York, NY.

Theilliol, D., Join, C. and Zhang, Y. (2008). Actuator fault tolerant control design based on a reconfigurable reference input, *International Journal of Applied Mathematics and Computer Science* **18**(4): 553–560, DOI: 10.2478/v10006-008-0048-1.

Theilliol, D., Ponsart, J.C. and Noura, H. (2000). Sensor fault diagnosis and accommodation based on analytical redundancy: Application to a three-tank system, *IFAC Proceedings Volumes* **33**(11): 535–540.

Witczak, M., Buciakowski, M., Puig, V., Rotondo, D. and Nejjari, F. (2016). An LMI approach to robust fault estimation for a class of nonlinear systems, *International Journal of Robust and Nonlinear Control* **26**(7): 1530–1548.

Yu, X. and Jiang, J. (2015). A survey of fault-tolerant controllers based on safety-related issues, *Annual Reviews in Control* **39**: 46–57.

Zhang, Y. and Jiang, J. (2008). Bibliographical review on reconfigurable fault-tolerant control systems, *Annual Reviews in Control* **32**(2): 229–252.

Sondess Mejdi received her engineering diploma in electrical and automatic engineering in 2016 from the National School of Engineers of Gabes, Tunisia. Currently, she is a PhD candidate at the Research Laboratory: CONPRI (Numerical Control of Industrial Processes). Her areas of interest deal with fault diagnosis, fault tolerance, multimodel, unknown inputs multiobservers and representation of complex systems.

Anis Messaoud received the engineering and master degrees in electrical engineering and automatic control from the National School of Engineers of Gabes, Tunisia, in 2004 and 2006, respectively. In 2010, he obtained his PhD degree in electrical-automatic engineering from the National School of Engineers of Gabes. His specific research interests are in the areas of predictive control of complex systems, multimodel and multicontrol approaches. He is currently an associate professor in the Electric Engineering Department at the National School of Engineers of Gabes.

Ridha Ben Abdennour received his PhD in automatic control from the Higher School of Technical Education (Tunis) in 1987, and his state doctoral degree in electrical engineering from the National School of Engineers of Tunis in 1996. He is a professor of automatic control at the National School of Engineers of Gabes, Tunisia. He was the founder and a former head of the Electrical Engineering Department and the director of the High Institute of Technological Studies of Gabes. Professor Zen Abdennaur is the head of the Research Laboratory of Numerical Control of Industrial Processes, and the founder and an honorary president of the Tunisian Association of Automatic Control and Digitalization. His research is on identification, multimodel and multicontrol approaches, neural networks, observers, sliding mode, numerical control and fuzzy supervision. He is a co-author of the book on *Identification and Numerical Control of Industrial Processes* (TECHNIP Editions) and more than 350 other publications.

Received: 20 December 2018
Revised: 22 May 2019
Accepted: 22 August 2019