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SENSOR LOCATION FOR TRAVEL TIME ESTIMATION BASED ON THE USER EQUILIBRIUM PRINCIPLE: APPLICATION OF LINEAR EQUATIONS

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Travel time is a fundamental measure in any transportation system. With the development of technology, travel time can be automatically collected by a variety of advanced sensors. However, limited by objective conditions, it is difficult for any sensor system to cover the whole transportation network in real time. In order to estimate the travel time of the whole transportation network, this paper gives a system of linear equations which is constructed by the user equilibrium (UE) principle and observed data. The travel time of a link which is not covered by a sensor can be calculated by using the observed data collected by sensors. In a typical transportation network, the minimum number and location of sensors to estimate the travel time of the whole network are given based on the properties of the solution of a systems of linear equations. The results show that, in a typical network, the number and location of sensors follow a certain law. The results of this study can provide reference for the development of transportation and provide a scientific basis for transportation planning.

Keywords: travel time estimation, sensor location, user equilibrium principle, linear equations.

1. Introduction

1.1. Sensor location for travel time estimation. Travel time is a fundamental measure in any transportation system. For the users of the transportation network, travel time is the most easily understood indicator to help them make travel behavior choices and avoid delays. For the managers of the transportation network, the travel time of the whole transportation network provides the most basic measure of the effectiveness of the whole network and can help managers discover congestion areas timely, coordinate traffic management, guide vehicle evacuation, and prevent the spread of congestion. Therefore, it is very important to estimate the travel time of the whole transportation network.

With the development of technology, travel time can be automatically collected by a variety of advanced sensors. Meanwhile, empirical studies have revealed that the vehicle speed collected by the sensors can be adopted to estimate the average speed of a road section, and then the estimated travel time is obtained (Mazaré *et al.*, 2012; Chang *et al.*, 2019). There are two main types of sensors used in the current transportation system: the counting sensors and the automatic vehicle identification (AVI) sensors. Gentili and Mirchandani (2018) compared different sensor location models for travel time estimation with either counting sensors or AVI sensors. In contrast to the counting sensor, the AVI sensor requires vehicles to actively provide their identification. AVI sensors more widely used than counting sensors since they can more directly collect travel time information. License plate readers (Sánchez-Cambronero *et al.*, 2017), Bluetooth sniffers (Haghani *et al.*, 2010; Asudegi and Haghani, 2013) and radio frequency identification (RFID) (Soriguera *et al.*, 2007) are examples of AVI sensors. However, limited by objective conditions, it is difficult

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for any sensor system to cover the whole transportation network in real time.

Decisions on where to locate AVI sensors on the transportation network depend on the optimality criterion and various factors. Sherali *et al*. (2006) proposed a nonlinear location model whose objective function is to be maximized is the total benefit obtained from the location, and the constraints are the total number of sensors that can be installed and the total budget that can be spent. Zhu *et al*. (2014) considered the error variance as an explicit function of the sampling size based on prior information and proposed an optimization model to design a dynamic sensing network in a short-term period. Li and Ouyang (2012) as well as Zhu *et al*. (2017) studied the sensor location problem by considering the impacts of sensor disruptions and failures. Patan and Kowalów (2018) investigated optimization of the percentage of the total number of observations spent at given sensor nodes in such a way as to maximize the accuracy of system parameter estimates. Sun *et al*. (2021) proposed two traffic sensor location models to account for minimizing the error of travel time estimation and maximizing the collected traffic flow.

Chen *et al*. (2004) proposed two AVI sensor location models for travel time estimation. One model aims at minimizing the number of sensors in the network to ensure that sall OD pairs (O is the origination and D is the destinations, one origination and one destination constitute an OD pair; there could have at least one path between an OD pair) are covered by sensors. The objective of the other model is to maximize the total number of covered OD pairs. Actually, the travel times of the paths between the same OD pair are related to one another (except for the OD pairs which have only one path between them). For example, when the traffic state satisfies the user equilibrium (UE) principle, the travel times of each path between the same OD pair are equal.

In the literature, the existing sensor location models for the travel time estimation concentrate on various purposes. These models are summarized in Table 1 in terms of the modeling approach, consideration of various factors, and sensor location rules for a special transportation network.

1.2. UE principle.

Wardorp (1952) proposed two principles of equilibrium for the traffic assignment problem. The first principle leads to a user equilibrium, and is known as the UE principle.

Definition 1. (*UE principle*) The travel times of the paths used between an OD pair are equal, the travel time of any unused path between the same OD pair is larger than or equal to that of the used path.

The UE principle can be simply explained in mathematical form. For a network shown in Fig. 1,

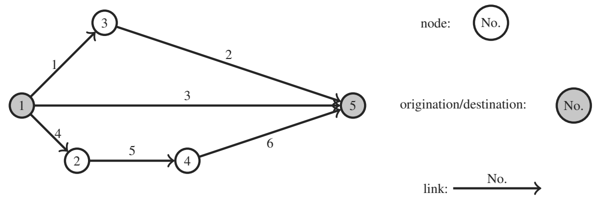


Fig. 1. Physical transportation network.

Node 1 is the origination and node 5 is the destination. There are three paths between OD pair $(1, 5)$: link 1 $\rightarrow$ link 2, link 3 and link 4 $\rightarrow$ link 5 $\rightarrow$ link 6. The path travel time is equal to the sum of passing link travel times. Let t_a be the travel time of link a. The travel times of the three paths are respectively t_1+t_2, t_3 and $t_4+t_5+t_6$. When the transportation network in Fig. 1 satisfies the UE principle, assume path link 4 $\rightarrow$ link 5 $\rightarrow$ link 6 is an unused path between OD pair $(1, 5)$ and others are used paths. The travel time of the three paths satisfies the constraint

$$\begin{cases} t_1 + t_2 = t_3, \\ t_3 \leq t_4 + t_5 + t_6. \end{cases} \tag{1}$$

Obviously, the UE principle can be expressed in terms of equality and inequality constraints. In particular, when all the paths in the network are used paths, the UE principle could be expressed as a system of linear equations. Take the network in Fig. 1 as an example. When all paths between OD pair $(1, 5)$ are used paths, the travel time of the three paths satisfies the constraints

$$\begin{cases} t_1 + t_2 = t_3, \\ t_3 = t_4 + t_5 + t_6. \end{cases} \tag{2}$$

1.3. Structure of the transportation network.

There are many types of the urban transportation network, such as grid type networks, ring-radial networks, freestyle network, hybrid networks, etc. It is of great practical significance to study sensor location for travel time estimation in grid networks as grid networks are typical structures of the urban transportation network.

The grid network is a kind of a parallel arrangement of urban trunk roads and branch roads along the north-south and east-west directions, which divides the urban land into rectangular areas of appropriate sizes. Its characteristic is dividing each section of the city by a Cartesian grid. This type of transportation network has the advantages of equal accessibility to all parts of the city, good procedural and directional sense, dividing the urban land and layout buildings advantageously, making the traffic flow assignment even and increasing the traffic flow of unit-time.

Table 1. Literature summary of sensor location for travel time estimation.

Literature	Modeling approach	Consideration of factors	Sensor location rule for special transportation network
Sherali *et al.* (2006)	mathematical programming	total benefit obtained by location	
Zhu *et al.* (2014)	mathematical programming	error variance	
Li and Ouyang (2012)	mathematical programming	sensor disruptions and failures	
Zhu *et al.* (2017)	mathematical programming	sensor disruptions and failures	
Chen *et al.* (2004)	mathematical programming	covered OD pair	
This paper	linear equations	UE principle	✓

1.4. Potential contributions and the structure of this paper. Travel time is a direct description of the traffic state. With the development of technology, travel time can be automatically collected by a variety of advanced sensors. However, limited by objective conditions (the quantity and accuracy of sensors, the influence of the environment, etc.), it is difficult for any sensor system to cover the whole transportation network in real time. There have been many studies on sensor location for travel time estimation. Most of these studies solve this problem from the perspective of the total number of sensors to be installed, the number of covered routes in the network, the number of covered OD pairs and the error in travel time estimation.

This paper studies sensor location for travel time estimation based on the UE principle and the main contributions are summarized as follows:

- A travel time estimation framework is establised based on the UE principle and observed data. An optimal sensor location strategy is found based on the properties of linear equations.
- A rule for sensor location for a typical structure of the urban transportation network is found based on the proposed framework.

The remainder of this paper is organized as follows. The next section sets up a system of linear equations to solve the sensor location problem based on the UE principle. Section 3 studies the sensor location problem for travel time estimation in one-way and two-way rectangular grid type transportation networks.

2. Sensor location for travel time estimation based on the UE principle and observed data

2.1. Symbols and assumptions. Table 2 contains the notation employed in this paper.

The proposed model is established on the following assumptions:

A1. The transportation network satisfies UE principle.

A2. All links of the network are passable and all paths are used.

A3. Traffic users always travel along a direction which is far away from the origination and near the destination.

A4. The originations and destinations in the network are at the vertexes of the network edge.

2.2. System of linear equations for travel time estimation. Let δ_{ai} be a link-path association binary variable. When path i goes through link a, $\delta_{ai} = 1$; otherwise, $\delta_{ai} = 0$. Then the travel time of path i is

$$T_i = \sum_a \delta_{ai} t_a. \tag{3}$$

For OD pair (r, s), under the assumption that the transportation network satisfies the UE principle, the travel times of paths between OD pair (r, s) are equal to one another,

$$T_i^{rs} = T_{i+1}^{rs}, \tag{4}$$

or

$$\sum_a \delta_{ai}^{rs} t_a = \sum_a \delta_{a,i+1}^{rs} t_a. \tag{5}$$

In particular, when there is only one path between OD pair (r, s), the travel time of this path is equal to itself. In this case, Eqns. (4) and (5) are $0 = 0$.

The equations of all OD pairs in the transportation network can be combined and converted into a matrix form as follows:

$$\Delta_1 t = \Delta_2 t, \tag{6}$$

or

$$\Delta t = \mathbf{0}, \tag{7}$$

where $\Delta = \Delta_1 - \Delta_2$, Δ_1 and Δ_2 are the link-path association matrices constructed by different link-path association variables, $\Delta_1 \neq \Delta_2$. Here $\mathbf{0}$ is a zero vector.

Table 2. Indices, parameters and variables.

Indices	Definition
a	index of a link
i	index of a path
(r, s)	index of an OD pair
Parameters	Definition
δ_{ai}	link-path association binary variable; when path i goes through link a, $\delta_{ai} = 1$, otherwise, $\delta_{ai} = 0$
Δ_1, Δ_2	link-path association matrices which are constructed by link-path association binary variable, $\Delta_1 \neq \Delta_2$;
Δ	$\Delta = \Delta_1 - \Delta_2$
t_a^*	observed travel time of link a
n_l	number of links in the transportation network
r_Δ	rank of matrix Δ
f	row number of the grid type transportation network
g	column number of the grid type transportation network
n_{ol}	number of observed links in the transportation network
p_{ol}^1	proportion of the observed links in all links on the one-way rectangular grid type network
p_{ol}^2	proportion of the observed links in all links on the two-way rectangular grid type network
Variables	Definition
t_a	travel time of link a
T_i	travel time of path i
k_a	binary variable associated with each link a; when link a is the observed link, $k_a = 1$, otherwise $k_a = 0$

The link covered by a sensor is called the observed link. Then the sensor location problem is to choose which link is the observed link:

$$k_a(t_a - t_a^*) = 0, \tag{8}$$

where k_a is a binary variable and t_a^* is the observed travel time of link a. When link a is the observed link, $k_a = 1$ and Eqn. (8) is $t_a = t_a^*$ (the travel time of link a is equal to the observed travel time); otherwise, $k_a = 0$ and Eqn. (8) is $0 = 0$.

Equations (7) (constructed by the UE principle) and (8) (constructed by the travel time of observed links) can be combined into a system of linear equations

$$\begin{cases} \Delta t = \mathbf{0}, \\ k_a(t_a - t_a^*) = 0, \quad a = 1, 2, \ldots, n_l, \\ k_a = 0 \text{ or } 1. \end{cases} \tag{9}$$

2.3. Solution of linear equations. Once the observed links are selected, the value of k_a can be determined, and the solution/solutions of Eqn. (9) is/are the estimated value of the link travel time in the network. In the best case, Eqn. (9) have a unique solution after the observed links have been selected. Then the problem of sensor location becomes the choice of observed links which can yield a unique solution to the equations (9).

When the transportation network structure and OD pairs are determined, the equations $\Delta t = \mathbf{0}$ constructed by the UE principle in Eqn. (9) and r_Δ (the rank of Δ) are determined to be invariant. The unknown quantity in Eqn. (9) is the travel time of each link in the network, and the number of links in the network is n_l. In order to have a unique solution to (9), the observed links are going to be the free unknowns in the equations $\Delta t = \mathbf{0}$, and the number of observed links could satisfy

$$\sum_a k_a = n_l - r_\Delta. \tag{10}$$

3. Sensor location for travel time estimation for a rectangular grid type transportation network

It can always be seen from equation (10) that the number of observed links is related to the rank of the equations constructed by the UE principle (Eqn. (7)). In view of the universality and typification of the grid network, the rank of the systems of equations constructed by the UE principle on rectangular grid network is studied as follows.

3.1. One-way rectangular grid type network. In order to study the rank of equations constructed by the UE principle in the one-way rectangular grid type network, the 2×2 one-way rectangular grid type network in Fig. 2 is taken as an example. In this network, there are 9 nodes and 12 links. Each row and each column have two small grid networks.

According to the assumptions and the directions of the links in the network, let Node 1 be the origination and Node 9 be the destination. Table 3 lists the links passed

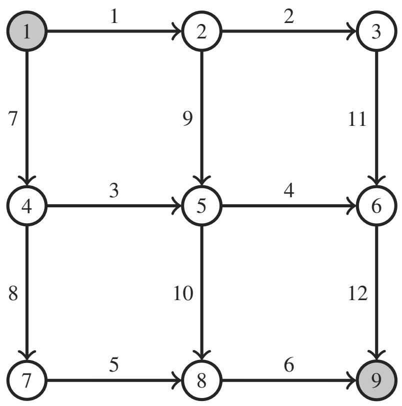


Fig. 2. One-way rectangular grid type network (2×2).

Table 3. Links passed by each path between OD pair $(1, 9)$.

Path no.	Link sequence
1	$1 \to 2 \to 11 \to 12$
2	$1 \to 9 \to 4 \to 12$
3	$1 \to 9 \to 10 \to 6$
4	$7 \to 3 \to 4 \to 12$
5	$7 \to 3 \to 10 \to 6$
6	$7 \to 8 \to 5 \to 6$

by each path between OD pair $(1, 9)$.

For OD pair $(1, 9)$, the equations constructed by the UE principle are

$$\begin{cases} t_1 + t_2 + t_{11} + t_{12} = t_1 + t_9 + t_4 + t_{12}, \\ t_1 + t_9 + t_4 + t_{12} = t_1 + t_9 + t_{10} + t_6, \\ t_1 + t_9 + t_{10} + t_6 = t_7 + t_3 + t_4 + t_{12}, \\ t_7 + t_3 + t_4 + t_{12} = t_7 + t_3 + t_{10} + t_6, \\ t_7 + t_3 + t_{10} + t_6 = t_7 + t_8 + t_5 + t_6. \end{cases} \tag{11}$$

The rank of the systems of equations (11) is 4 (or 2×2), and (11) is equivalent to

$$\begin{cases} t_1 + t_9 = t_7 + t_3, \\ t_2 + t_{11} = t_9 + t_4, \\ t_3 + t_{10} = t_8 + t_5, \\ t_4 + t_{12} = t_{10} + t_6. \end{cases} \tag{12}$$

It is easy to see that each equation in (12) represents each small grid in the 2×2 one-way rectangular grid type network satisfying the UE principle. For example, the first equation in (12) represents that the OD pair $(1, 9)$ satisfies the UE principle. The same conclusion can be obtained for OD pairs $(2, 6)$, $(4, 8)$ and $(5, 9)$. By further transforming (11), the OD pairs $(1, 6)$, $(1, 8)$, $(4, 9)$ and $(2, 9)$ also satisfy the constraints of the UE principle.

By studying the rank of the systems of equations constructed by the UE principle on the 2×2 one-way rectangular grid type network, we conjecture that the rank of the equations constructed by the UE principle on the $f \times g$ one-way rectangular grid type network (the network has g unit grids per row and f unit grids per column) is $f \times g$, and when the OD pair on the vertex at the edge of the network satisfies the UE principle, the OD pair of any gird in the network satisfies the UE principle.

Let $f = 1$ for any g. The network is shown as Fig. 3.

No matter which path has been chosen between the origination $(1, 1)$ and the destination $(2, g + 1)$, the path must go through $g + 1$ links, and must go through only one of $v_{(1,j)}$, $(j = 1, 2, \dots, g + 1)$. Therefore, the number of the paths between the origination $(1, 1)$ and the destination $(2, g + 1)$ is $g + 1$. Each path passes through the links as follows:

Path 1: $v_{11} \to h_{21} \to \cdots \to h_{2j} \to \cdots \to h_{2,g}$.
Path 2: $h_{11} \to v_{12} \to h_{22} \to \cdots \to h_{2j} \to \cdots \to h_{2,g}$.
Path 3: $h_{11} \to h_{12} \to v_{13} \to h_{23} \to \cdots \to h_{2j} \to \cdots \to h_{2,g}$.
$\vdots$
Path j: $h_{11} \to \cdots \to h_{1,(j-1)} \to v_{1,j} \to h_{2,j} \to \cdots \to h_{2,g}$.
Path $j + 1$: $h_{11} \to \cdots \to h_{1j} \to v_{1,(j+1)} \to h_{2,(j+1)} \to \cdots \to h_{2,g}$.
$\vdots$
Path $g - 1$: $h_{11} \to \cdots \to h_{1j} \to \cdots \to h_{1,(g-2)} \to v_{1,(g-1)} \to h_{2,(g-1)} \to h_{2,g}$.
Path g: $h_{11} \to \cdots \to h_{1j} \to \cdots \to h_{1,(g-1)} \to v_{1,g} \to h_{2,g}$.
Path $g + 1$: $h_{11} \to \cdots \to h_{1j} \to \cdots \to h_{1,g} \to v_{1,(g+1)}$.

As the network satisfies the UE principle, these travel times of the paths should be equal to one another. Let $h_{1,j}$ and $h_{2,j}$ be the travel times of horizontal links, and $v_{1,j}$ be the travel time of the vertical links. Then

$$\begin{cases} h_{11} + \cdots + h_{1j} + \cdots + \boldsymbol{h_{1,g}} + \boldsymbol{v_{1,(g+1)}} \\ \quad = h_{11} + \cdots + h_{1j} + \cdots + h_{1,(g-1)} + \boldsymbol{v_{1,g}} + \boldsymbol{h_{2,g}}, \\ h_{11} + \cdots + h_{1,(g-2)} + \boldsymbol{h_{1,(g-1)}} + \boldsymbol{v_{1,g}} + h_{2,g} \\ \quad = h_{11} + \cdots + h_{1,(g-2)} + \boldsymbol{v_{1,(g-1)}} + \boldsymbol{h_{2,(g-1)}} + h_{2,g}, \\ \vdots \\ h_{11} + \cdots + \boldsymbol{h_{1j}} + \boldsymbol{v_{1,(j+1)}} + h_{2,(j+1)} + \cdots + h_{2,g} \\ \quad = h_{11} + \cdots + h_{1,(j-1)} + \boldsymbol{v_{1,j}} + \boldsymbol{h_{2,j}} + \cdots + h_{2,g}, \\ \vdots \\ h_{11} + \boldsymbol{h_{12}} + \boldsymbol{v_{13}} + h_{23} + \cdots + h_{2j} + \cdots + h_{2,g} \\ \quad = h_{11} + \boldsymbol{v_{12}} + \boldsymbol{h_{22}} + h_{23} + \cdots + h_{2j} + \cdots + h_{2,g}, \\ \boldsymbol{h_{11}} + \boldsymbol{v_{12}} + h_{22} + \cdots + h_{2j} + \cdots + h_{2,g} \\ \quad = \boldsymbol{v_{11}} + \boldsymbol{h_{21}} + h_{22} + \cdots + h_{2j} + \cdots + h_{2,g}. \end{cases} \tag{13}$$

Eliminate the same terms on both the sides of these

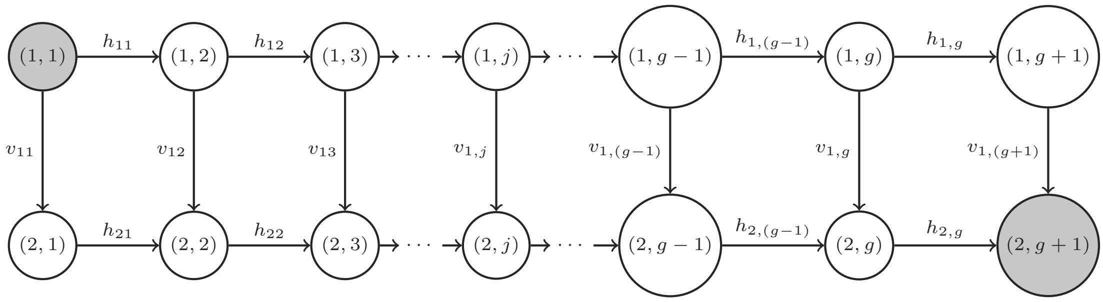


Fig. 3. One-way rectangular grid type network $(1 \times g)$.

equation. Then Eqn.(13) are

$$\begin{cases} h_{1,g} + v_{1,(g+1)} = v_{1,g} + h_{2,g}, \\ h_{1,(g-1)} + v_{1,g} = v_{1,(g-1)} + h_{2,(g-1)}, \\ \vdots \\ h_{1j} + v_{1,(j+1)} = v_{1,j} + h_{2,j}, \\ \vdots \\ h_{12} + v_{13} = v_{12} + h_{22}, \\ h_{11} + v_{12} = v_{11} + h_{21}. \end{cases} \tag{14}$$

It is also easy to see that each equation in (14) represents a small grid in the $1 \times g$ one-way rectangular grid type network also satisfying the UE principle. By further transforming (14), all possible OD pairs in the network also satisfy the constraints of the UE principle. Similarly, the same holds for any f and g. The rank of the systems of equations (7) on the $f \times g$ one-way rectangular grid type network is $f \times g$.

For a $f \times g$ one-way rectangular grid type network, the number of links is

$$n_l^1 = [f \times (g+1) + (f+1) \times g] = 2fg + f + g, \tag{15}$$

and the rank of the coefficient matrix for the equations constructed by the UE principle is

$$r_\Delta = fg. \tag{16}$$

Let the free unknown variables in the equations constructed by the UE principle be observed links. Then the number of observed links is

$$n_{\text{ol}}^1 = n_l^1 - r_\Delta = 2fg + f + g - fg = fg + f + g, \tag{17}$$

and the proportion of the observed links in all links on the transportation network is

$$p_{\text{ol}}^1(f,g) = \frac{n_{\text{ol}}}{n_l} = \frac{fg + f + g}{2fg + f + g}. \tag{18}$$

For $p_{\text{ol}}^1(f,g)$, we have

$$p_{\text{ol}}^1(f+1,g) < p_{\text{ol}}^1(f,g), \tag{19}$$

$$p_{\text{ol}}^1(f,g+1) < p_{\text{ol}}^1(f,g), \tag{20}$$

$$p_{\text{ol}}^1(f+1,g+1) < p_{\text{ol}}^1(f,g), \tag{21}$$

$$\lim_{\substack{f\to+\infty \\ g\to+\infty}} p_{\text{ol}}^1(f,g) = \lim_{\substack{f\to+\infty \\ g\to+\infty}} \frac{fg + f + g}{2fg + f + g} = \frac{1}{2}. \tag{22}$$

That is to say, if the network satisfies the assumptions of the UE principle, as the size of one-way rectangular grid type transportation network increases, the proportion of links that need to be observed in the network decreases, but the proportion is no less than half of all the links in the network.

3.2. Two-way rectangular grid type transportation network.

Let $f = 2$ and $g = 2$. On the basis of the above study for one-way grid network, a 2×2 two-way rectangular grid type network in Fig. 4 will be an example to study two-way traffic networks.

As the traffic users always travel along the direction which is far away from the origination and near the destination, the 2×2 two-way rectangular grid type network above can be split into four one-way rectangular grid type networks.

The following can be found from the four networks in Fig. 5:

1. The above four networks have the same characteristics. Each of them has $(f+1)\times(g+1) = 9$ nodes and $2fg + f + g = 12$ links.

2. All vertical links in Fig. 5(b) are exactly the same as those in Fig. 5(a).

3. There is no identical link in Figs. 5(c) and 5(b), but all the horizontal links in Fig. 5(c) are exactly the same as those in Fig. 5(a).

4. All vertical links in Fig. 5(d) are exactly the same as those in Fig. 5(c) and all the horizontal links in Fig. 5(d) are exactly the same as those in Fig. 5(b), but there is no identical link in Figs. 5(d) and 5(a).

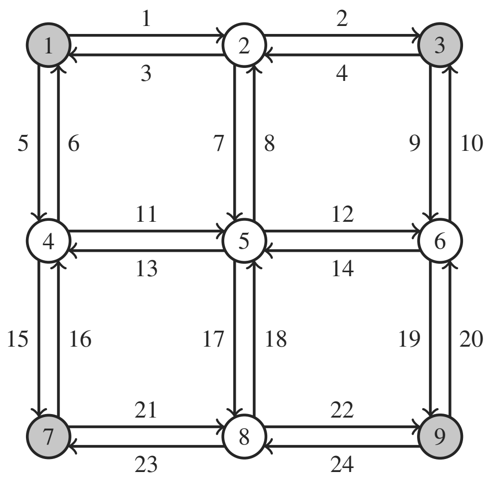
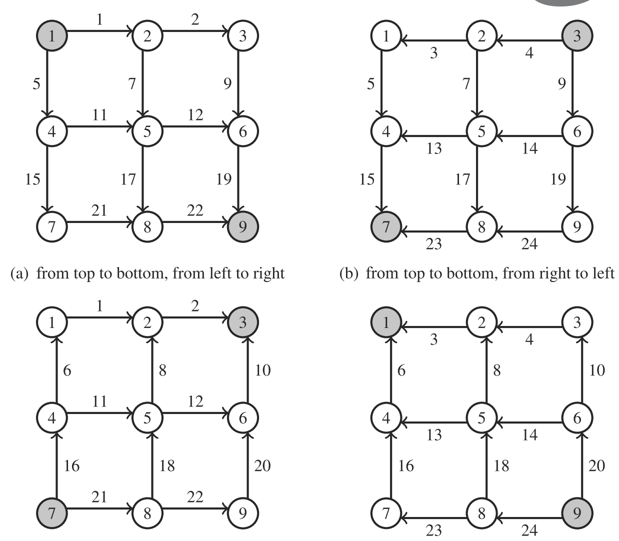


Fig. 4. Two-way rectangular grid type network (2×2).



(a) from top to bottom, from left to right
(b) from top to bottom, from right to left
(c) from bottom to top, from left to right
(d) from bottom to top, from right to left

Fig. 5. Splitting the 2×2 two-way rectangular grid type network into four one-way rectangular grid type networks.

The observed links of the four 2×2 one-way rectangular grid type networks are selected as follows.

For Fig. 5(a), a 2×2 one-way rectangular grid type network for which the travel directions are from top to bottom and from left to right, it can be seen from Eqns. (16) and (17) that the rank of the systems of equations constructed by the UE principle is 4 and the number of observed links is 8.

The observed links are selected using the full row method or the full column method. The former is to select all horizontal links as observed links, and then randomly select 2 vertical links (because $f = 2$) that are not in the same row as observed links; the latter is to select all vertical links as observed links, and then randomly select 2 horizontal links (because $g = 2$) that are not in the same column as observed links.

The observed travel time of the observed links is substituted into the linear equations (9) to determine the travel times of the other (unobserved) links.

For example, let horizontal links 1, 2, 11, 12, 21, 22 and vertical links 5, 15 be observed links. Then we substitute the observed travel times of links 1, 2, 11, 12, 21, 22, 5 and 15 into the linear equations (9) to find the travel times of links 7, 17, 9 and 19.

For Fig. 5(b), a 2×2 one-way rectangular grid type network for which the travel directions are from top to bottom and from right to left, it can be seen from (16) and (17) that the rank of the systems of equations constructed by the UE principle is 4.

Since the travel times of all links in Fig. 5(a) are already known (observed or solved) and all the vertical links in Fig. 5(b) are exactly the same as those in Fig. 5(a), the observed links can be selected by the full column method. Meanwhile, since the travel time of all vertical links is known, two horizontal links (because $g = 2$) that are not in the same column are only needed to be selected as observed links.

We now substitute the known and observed travel time of the links into the linear equations (9) to determine the travel times of other links.

For example, given the travel times of all links in Fig. 5(a), let horizontal links 3 and 4 be observed links. We substitute the known travel times of links 5, 15, 7, 17, 9, 19 and observed travel time of links 3, 4 into the linear equations (9) to determine the travel time of links 13, 14, 23 and 24.

For Fig. 5(c), a 2×2 one-way rectangular grid type network for which the travel directions are from bottom to top and from left to right, it can be seen from (16) and (17) that the rank of the systems of equations constructed by the UE principle is 4.

Since the travel times of all links in Fig. 5(a) are known (observed or solved) and all the horizontal links in Fig. 5(c) are exactly the same as those in Fig. 5(a), the observed links can be selected by the full row method. Meanwhile, since the travel times of all horizontal links are known, two vertical links (because $f = 2$) that are not in the same row only need to be selected as observed links.

The known and observed travel times of the links are substituted into the linear equations (9) to find the travel times of the other links.

For example, once the travel times of all links in Fig. 5(a) are known, if horizontal links 6 and 16 are observed links, we substitute the known travel times of links 1, 2, 11, 12, 21, 22 and the observed travel times of links 6, 16 into the linear equations (9) to find the travel times of links 8, 18, 10 and 20.

For Fig. 5(d), a 2×2 one-way rectangular grid type network for which the travel directions are from bottom to top and right to left, since all vertical links in Fig. 5(d) are exactly the same as those in Fig. 5(c) and all the horizontal links in Fig. 5(d) are exactly the same as those in Fig. 5(b), once the travel times of all links in Figs. 5(b) and 5(c) are known (observed or solved), the travel time of all links in Fig. 5(d) are already known.

Then two special selections of observed links in the 2×2 two-way rectangular grid type network are shown in Fig. 6, where the gray links are the observed links. Figure 6(a) is a special selection of observed links when the first one-way rectangular grid type network (from top to bottom, from left to right) uses the full row method to select the observed links, and Fig. 6(b) is a special selection of observed links when the first one-way rectangular grid type network (from top to bottom, from left to right) uses the full column method to select the observed links.

Similarly, the $f \times g$ two-way rectangular grid type network (the network has g unit grids per row and f unit grids per column) can be split into four one-way rectangular grid type networks:

Network 1: a $f \times g$ one-way rectangular grid type network for which travel directions are from top to bottom and from left to right;

Network 2: a $f \times g$ one-way rectangular grid type network for which travel directions are from top to bottom and from right to left;

Network 3: a $f \times g$ one-way rectangular grid type network for which travel directions are from bottom to top and from left to right;

Network 4: a $f \times g$ one-way rectangular grid type network for which travel directions are from bottom to top and from right to left.

Obviously, the four one-way networks share some repeating links: all vertical links in Network 2 are exactly the same as those in Network 1; all horizontal links in network 3 are exactly the same as those in Network 1; all vertical links in Network 4 are exactly the same as those in Network 3, and all horizontal links in Network 4 are exactly the same as those in Network 2. Because of this repeating, the selection of observed links and the computation of unknown links in the four one-way networks are performed in the order shown in Fig. 7. Firstly, select the observed links in Network 1, and then solve the unknown links in Network 1. Secondly, on the basis the travel times of all the links in Network 1 are known (observed or solved), select the observed links to solve the unknown links in networks 2 and 3, respectively. Finally, once the travel times of all the links in Networks 2 and 3 is known, the travel time of all the links in Network 4 known.

For each $f \times g$ one-way rectangular grid type network above, the observed links are selected according to the full row method or the full column method. Once the travel times of some links are known, the travel times of the other links can be calculated by substituting these known link travel time into the linear equations (9).

For Network 1, we use the full row method to select the observed links: we select all $g(f+1)$ horizontal links and f vertical links that are not in the same row as the observed links. Then we substitute the observed travel times of the observed links into the linear equations (9) to find the travel times of the other fg vertical links in Network 1.

For Network 2, since the travel time of all links in Network 1 are already known (observed or solved) and all the vertical links in Network 2 are exactly the same as those in Network 1, the observed links can be selected by the full column method. Meanwhile, since the travel times of all vertical links in Network 2 are known, g horizontal links that are not in the same column are only needed to be selected as observed links. Then we substitute the known and observed travel times of the links into the linear equations (9) to find the travel times of the other fg horizontal links in Network 2.

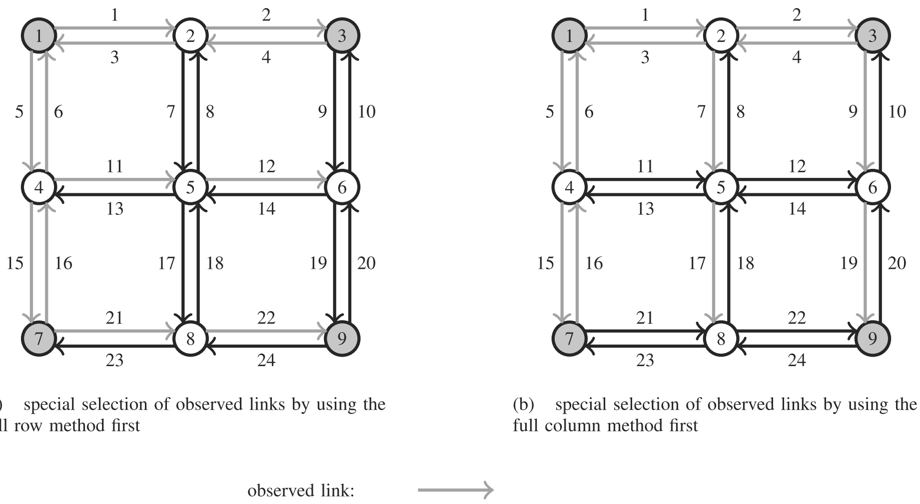
For Network 3, since the travel times of all links in Network 1 are known (observed or solved) and all the horizontal links in Network 3 are exactly the same as those in Network 1, the observed links can be selected by the full row method. Meanwhile, since the travel times of all horizontal links in Network 3 are known, f vertical links that are not in the same row are only needed to be selected as observed links. Then we substitute the known and observed travel times of the links into the linear Eqn. (9) to determine the travel times of the other fg vertical links in Network 3.

For Network 4, since all the horizontal links in Network 4 are exactly the same as those in Network 2 and all the vertical links in Network 4 are exactly the same as those in Network 3, once the travel times of all links in Network 2 and Network 3 are already known (observed or solved), the travel time of all links in Network 4 are known.

Overall, the four $f \times g$ one-way rectangular grid type networks which are divided by the $f \times g$ two-way rectangular grid type network need $g(f+1)+f$, g, f and 0 observed link/links, respectively. The number of observed links for the $f \times g$ two-way rectangular grid type network to determine all the link travel times is

$$n_{\text{ol}}^2 = [g(f+1)+f] + g + f + 0 = fg + 2f + 2g. \quad (23)$$

For a $f \times g$ two-way rectangular grid type network, the



(a) special selection of observed links by using the full row method first

(b) special selection of observed links by using the full column method first

Fig. 6. Two special selections of observed links in the 2×2 two-way rectangular grid type network.

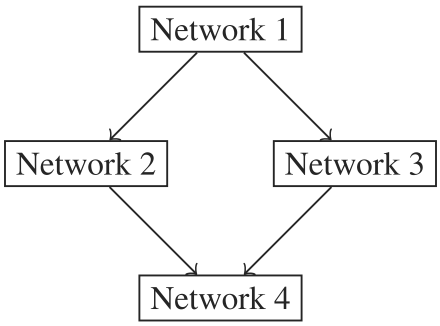


Fig. 7. Order for the selection of observed links and the solution of unknown links in four one-way networks.

number of links is

$$n_l^2 = 2[f(g+1) + (f+1)g] = 4fg + 2f + 2g, \quad (24)$$

and the proportion of the observed links in all links of the transportation network is

$$p_{\mathrm{ol}}^2(f,g) = \frac{n_{\mathrm{ol}}^2}{n_l^2} = \frac{fg + 2f + 2g}{4fg + 2f + 2g}. \quad (25)$$

For $p_{\mathrm{ol}}^2(f,g)$, we have

$$p_{\mathrm{ol}}^2(f+1,g) < p_{\mathrm{ol}}^2(f,g), \quad (26)$$

$$p_{\mathrm{ol}}^2(f,g+1) < p_{\mathrm{ol}}^2(f,g), \quad (27)$$

$$p_{\mathrm{ol}}^2(f+1,g+1) < p_{\mathrm{ol}}^2(f,g), \quad (28)$$

$$\lim_{\substack{f\to+\infty\\ g\to+\infty}} p_{\mathrm{ol}}^2(f,g) = \lim_{\substack{f\to+\infty\\ g\to+\infty}} \frac{fg + 2f + 2g}{4fg + 2f + 2g} = \frac{1}{4}. \quad (29)$$

Inequalities (26)–(28) and Eqn. (29) indicate that as the size of two-way rectangular grid type transportation network increases, if the network satisfies the assumptions of the UE principle, the proportion of links that need to be observed in the network decreases, but the proportion is no less than a quarter of all the links in the network. Meanwhile, by comparing the proportion expressions (expressions (18) and (25)) of the observed links in all links of the $f \times g$ one-way and two-way rectangular grid type networks it can be found that

$$\frac{fg + f + g}{2fg + f + g} > \frac{fg + 2f + 2g}{4fg + 2f + 2g}, \quad (30)$$

or

$$p_{\mathrm{ol}}^1(f,g) > p_{\mathrm{ol}}^2(f,g). \quad (31)$$

This indicates that the proportion of the observed links in a $f \times g$ one-way rectangular grid type network is always greater than that in a $f \times g$ two-way rectangular grid type network. In terms of the benefits of the sensor location, the two-way rectangular grid type network is better than the one-way rectangular grid type network.

3.3. Selection of observed links in the 100× 100 rectangular grid type network. Let $f = g = 100$, the number of links on the 100×100 two-way rectangular grid type network is $n_l^2 = 40400$, the number of observed links in the 100×100 two-way rectangular grid type network be $n_{\mathrm{ol}}^2 = 10400$, and the proportion of the observed links in all links in the 100×100 two-way rectangular grid type network be $p_{\mathrm{ol}}^2(100, 100) = 25.743\%$. Then a special selection of observed links in the 100×100 two-way rectangular grid type network is shown in Fig. 8, where the gray lines are the observed links.

4. Conclusions

In contrast to previous studies, this paper studied sensor location (selection of observed links) for travel time

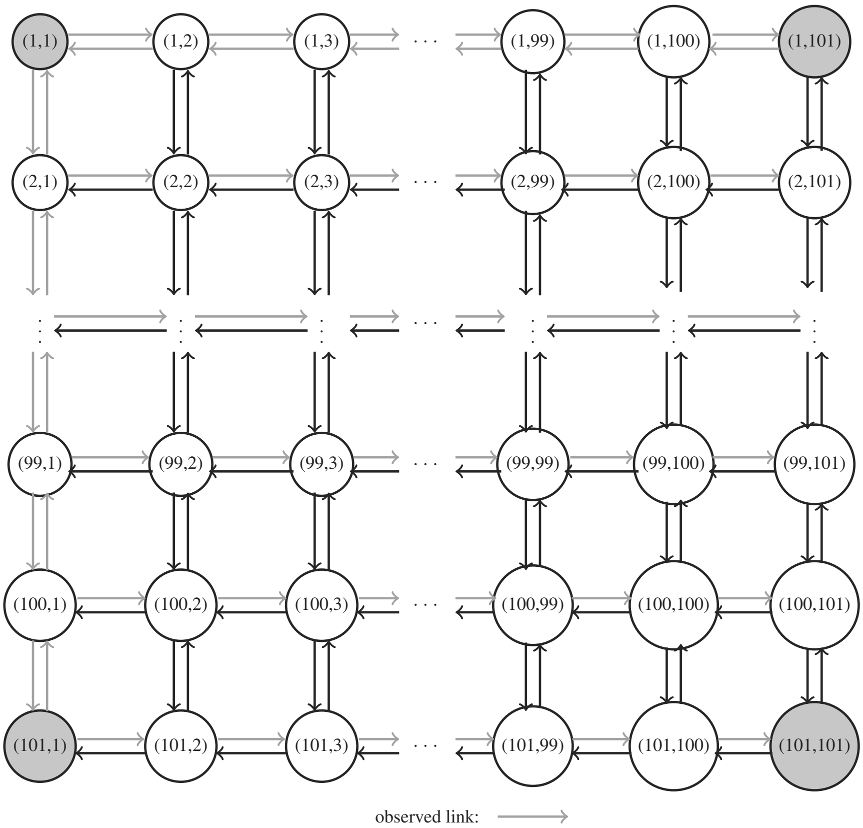


Fig. 8. Special selection of observed links in the 100×100 two-way rectangular grid type network.

estimation based on the relationship (UE principle) between paths (the paths are between the same OD pair). A system of linear equations was established by using the UE principle and travel times of the observed links. When the transportation network structure is determined, the equations constructed by the UE principle are uniquely determined. However, if the selection of observed links is different, the equations constructed by the travel times of observed links are also different. The best case is to obtain a system of linear equations with a unique solution to uniquely estimate the travel time of the unobserved links. By the theory of linear equations and traffic, when the observed links (travel times) are the free unknowns of the equations constructed by UE principle, the established system of linear equations has a unique solution.

After establishing the system of linear equations, this paper also discusses the selection of observed links for travel time estimation in the rectangular grid transportation networks. The results show that the selection of observed sections in the rectangular grid network has certain rules, and the number of observed sections is related to the number of grids in the network. As the number of grids increases, the proportion of links that need to be observed in the network decreases, but there is a lower limit to this proportion.

Overall, the proposed method in this paper does not need complex models and algorithms. It can select the observed links for travel time estimation after formulating the system of linear equations. In some special types of transportation networks, the selection of the observed links can be performed regularly and more quickly. The proposed method has an asset of strong practicality since it uses simple and understandable theory to study the sensor location problem for travel time estimation.

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